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Computation of the stress intensity factor KI for external longitudinal semi-elliptic cracks in the pipelines by FEM and XFEM methods

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Abstract
Evaluation of structural integrity of a cracked structure has become an important matter in the industrial field since couples of decades. However, damage process occurred in a structural component is not yet fixed. The objective of this research was to compute the stress intensity factor KI, in mode I, using in the linear elastic domain, by the finite element method and the extended finite element method. The defect studied in this survey has a form of a longitudinal semi-elliptic crack, located on the outer surface of the tube. A summary of the paper contains a numerical convergence for each method in terms of accuracy and limitations. The proposed methodology and outcomes released from this study act as novel design tool for the industrial engineers when is required to generate a robust solution for product development working in critical conditions.

Keywords Stress intensity factor (SIF) · Finite element method (FEM) · Extended finite element method (XFEM) · Semi-elliptic crack · J integral

1 Introduction

In the field of oil industry, the pipelines are the most used means of transporting petroleum materials such as gas, oil and hydrogen. The pipes installed above ground are of great issue. During their operation, they can generate several unexpected damages that cause significant material and human damage as well as environmental damage, especially if they are in the oceanic environment. To reduce these accidents the costs of maintenance can be very high, triggering a major concern for companies operating in this field. The research carried out by Lam [1] on the statistics of the accidents of the pipes, shows that the majority of the breaks are due to: pits of corrosion, the impact of a bullet lost if one is in a hunting zone (for piping install above ground), or at the impact of a bucket excavating machinery. These defects are in the form of a crack or a scratch; they affect the pipeline resistance and cause a sudden break that can be catastrophic. For that reason, researchers are interested in evaluating the structural integrity of pressurized pipelines.

These defects are generally treated by the fracture mechanics approach, which provide accurate details of the distribution of stresses and deformations near the defect zone, but also can indicate an estimative lifetime of the resistance of these structures according to the critical size of defects.

The Stress Intensity Factor KI is a key parameter used in the fracture mechanics field. Researchers as Moustabchir [2], Berer [3], Zareei [4] use this parameter to predict the initiation and propagation of cracks in the pipes. KI were calculated using several methods such as analytical, semi-analytical, and numerical methods.

Modern FEM simulations permits to reduce considerable the cost of product design process and to estimate in real time potential harmful situation when a crack nucleate and propagate. Besides, applying numerical simulation allows investigate the structural response of a components and predicting its live during working condition [5]. It permits to highlights improvement of design by eliminating the physical/real constrain (incapacity to test large/long products (chain of pipe), visualize their behavior in harsh environ-

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ments, etc.),however they are much easy solved when are transferred to virtual constraint guidance (VCG) [6].

The literature state many researchers who deal with the problem of crack in the pipes using the FEM methods, in contrast a very rare works was noted for the X-FEM method. Moustabchir [2] calculated the Stress Intensity Factor KI, mode I, in the pipes structures containing an axial semi-elliptic crack using the Finite Element Method (FEM). Berer [3] studied the effect of the opening of a crack, in mode I, in cracked cylinders when compressive loading generates the KI factor.

Zareei [4] calculated the KI factor for an internal circumferential semi-elliptic crack in a pipe subjected to any arbitrary load. Study that was based on the finite element analysis in three dimensions. Sahu [7] calculated the KI factor for a semi-elliptic crack located at the inner surface of the tube for different ratios, ratios that depends on the defect geometry (a/t) and (a/c). The finite element method (FEM) embedded was generated on the ANSYS software.

The literature indicates great potential to use the X-FEM method to evaluate the rupture of the pipes. Martin [8] used the X-FEM method to evaluate crack propagation in SUB-SEA equipment.

This research investigates the mechanism of a defect that occurs in a pipeline and propagate at external surface in a longitudinal manner as a semi-elliptic cracks. Shim [9] calculated the Stress Intensity Factor KI in mode I using the X-FEM for various types of plate and pipe cracks. Sharma et al. [10] and Sharma et al. [11] evaluate the stress intensity factors (SIF) of an axial/circumferential semi-elliptical crack in the pipe and elbow using the X-FEM method.

In this work, were computed the Stress Intensity Factor KI numerically by the classical finite element (FEM) and extended (X-FEM) method for an axial semi-elliptic crack located in the outer surface of the pipe. The key objective is to highlight the power of each method using a robust convergence, strategy that provides critical values of the KI parameters in the different positions of the crack.

2 Stress intensity factor KI

The Stress Intensity Factor KI represent the most important parameter in the linear elastic fracture mechanics. It allows to predict whether the crack is stable or not in respect to the toughness of the KIC material. Determination of the KIC is usually performed using the Charpy-V impact test Berer [3]. The cracked engineering components are examined in terms of KI by various analytical or semi-elliptical methods. These methods are based on displacement extrapolation and/or energetic approach such as integral J/integral interaction. Methods detailed in the reference Qian [12]. Zhu [13] showed that the energy approach provides good result in com-
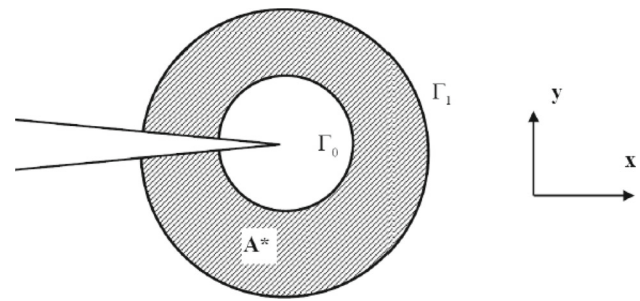

Fig. 1 Evaluation of J by the domain integral

parison with analytical results, reason why the computation of factor KI by the energetic approach is independent of the mesh near the crack.

The strategy applied in this work to calculate the Stress Intensity Factor are presented as follows:

1. Computation of the KI factor by the integral contour method:

In the Abaqus/Standard code calculation the evaluation of the J integral is done in an automatic way by applying the integral method over the energy domain. It is based on the formulation described by Eq. (1) developed by Shih et al. [14, 15]

$$J = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_j}{\partial x} - w\delta_{1i} \right] \frac{\partial q}{\partial x_i} dA \tag{1}$$

where A^* is the area of the surface between the contours Γ_0 and Γ_1 (as presented in Fig. 1). The parameter (q) is a smoothed function that can be choice as $q = 1$ on Γ_0 and $q = 0$ on Γ_1. The contour Γ_0 is reduced in practice at the crack tip. The outline Γ_1 coincides with the edges of the elements.

Further discretization is presented on the Eq. (2) given by formula:

$$J = \sum_{A^*} \sum_{p=1}^{np} \left\{ \left[\left(\sigma_{ij} \frac{\partial u_i}{\partial x} - w\delta_{1i} \right) \frac{\partial q}{\partial x_i} \right] \det \left(\frac{\partial x_j}{\partial \xi_k} \right) \right\} w_p \tag{2}$$

with: np: number of Gauss points, w_p: Integration weight, ξ_k: Coordinates of elements in local landmarks.

There, the integral J was used to calculate the factor KI for the mechanical case with linear elastic rupture. The integral J is equivalent to the rate of the energy restitution G under a single mode for the linear elastic problems. The factor KI were calculated from the integral J according to the following equation Rice [16]:
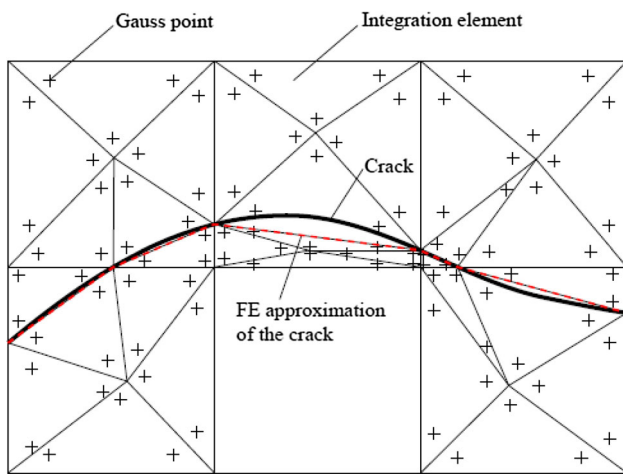
$$K = \sqrt{JE'} \tag{3}$$

Fig. 2 Integration procedure for the presence of a crack in X-FEM method

with $E' = E$ plane stress, $E' = \frac{E}{1-v^2}$ plane strain.

Equation (3) was introduced in the code "ABAQUS" and "ANSYS" to compute the KI Stress Intensity Factor numerically from the integral J.

2. Integration technique (Gauss) in the X-FEM method:

To extract the numerical values of displacements were obtained through numerical integration Eq. (4). Some difficulty appears in classical FEM when is encountered a discontinuous domain, however it was solved much easy, for a cracking problem, applying the method (X-FEM). Digital integration presents two major difficulties: the discontinuity along the crack and the singularity at the bottom of the crack. Therefore, these difficulties require special treatment because of the presence of a discontinuity in integration [17].

$$u(\xi) = \sum_i^n N_i(\xi)u_i + \sum_j^{n_{cf}} N_j(\xi)\big[H(\xi) - H(\xi_j)\big]a_j$$
$$+ \sum_k^{n_{ct}} N_k(\xi) \sum_{\alpha=1}^4 [\psi_\alpha(\xi) - \psi_\alpha(\xi_k)]b_{\alpha k} \quad (4)$$

The integration of these enriched elements were obtained by dividing them into several tetrahedra located above and below the surface, as is highlighted in Fig. 2. For the element comprising the crack tip, it is first necessary to divide it into two types of elements by a vertical line passing through the crack tip, then the numerical integration will be completed in each type of element, as illustrated in figures Fig. 3b–d. For the cracked element, the numerical integration will be completed in two subdomains, as shown in Fig. 3a. The integration of enriched elements generally requires a higher order Gauss quadrature.

3. Calculation of the KI factor by the Raju and Newman method Raju [18]:

$$K_I = \frac{PR_i^2}{R_e^2 - R_i^2} \sqrt{\frac{\pi a}{Q}} \left[2G_0 + 2G_1\left(\frac{a}{R_e}\right) + 3G_2\left(\frac{a}{R_e}\right)^2 + 4G_3\left(\frac{a}{R_e}\right)^3 \right] \quad (5)$$

With $Q = 1.0 + 1.464\left(\frac{a}{c}\right)^{1.65}$ For $a/c \le 1$

G0, G1, G2, G3, are functions dependent on geometry Raju [18].

Ri and Re are the inner radius and the outer radius.

P: Pressure applied to the tube.

3 The principle of the XFEM method

The Extended Finite Element Method (X-FEM) has emerged as a powerful numerical procedure for analyzing crack propagation problems. The approach of XFEM was introduced on 1974 by Benzley [19] who proposed the idea of enrichment near the crack front using asymptotic solutions for static failure problems. Atluri et al. [20] and Nash Gifford et al. [21] subsequently developed this method and obtained highly accurate results for stationary cracks.

A few years later, Melenek and Babuska [22] developed the fundamental unit partition method for the finite element method (PUFEM). The first real upgrade "development" effort of X-FEM was made by Belytschkol and Black [23]. Sukumar et al. [24] was the first to extend the XFEM method to model three-dimensional cracks. Stolarska and all [25] coupled the level set method and the X-FEM method to predict crack propagation. Finally, Belytschko et al. [26] developed a new X-FEM formulation for the arbitrary propagation of cracks in hulls.

Yet, several researchers Moes [27], Chahine [28], Budyn [29], Gupta [30], Wang [31], Hou [32] used the X-FEM method to simulate the behavior of fracture mechanics in the case of stationary or dynamic cracks.
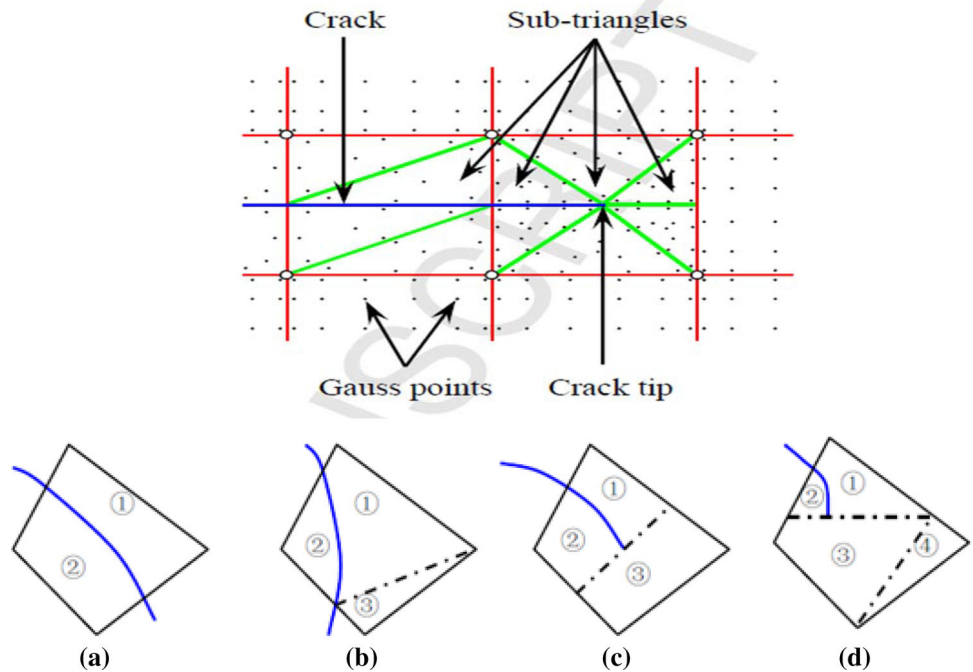
According to Belytschko and Moes [33], the displacement field may be described by the finite element approximation using the following equation:

$$u(x)^h = u^{FE} + u^{enr} \quad (6)$$

$$u(x)^h = \sum_{i \in N_{tot}}^n N_i(x)u_i + \sum_{i \in N_{saut}} N_i(x)H(x)a_i$$
$$+ \sum_{i \in N_{sing}} N_i(x)\left(\sum_{j=1}^4 F_j(x)b_i^j\right) \quad (7)$$

where: Ni (x): Interpolation functions of standard finite elements. H(x): The Heaviside enrichment function presents the 'Jump' of the displacement of the lips. ai: Additional

Fig. 3 Division of elements for digital integration

degrees of freedom associated with the Heaviside function. F_j (x): Enrichment functions with singularity near the crack front. b_i^j: The additional degrees of freedom associated with singular functions.

The function H (x) is presented by the following equation:

$$H(x) = \begin{cases} -1 \text{ si } x > 0 \\ +1 \text{ si } x < 0 \end{cases} \qquad (8)$$

The function H (x) thus takes the values of $+1$ or -1 according to the side of the crack on which one is placed.

The enrichment function, presented as sketch in Fig. 4, shows the singularity of the vicinity of the crack front Fj (x). It can be presented by the following equation:

$$F_j(x) = \sqrt{r}\left\{\cos\left(\frac{\theta}{2}\right),\ \sin\left(\frac{\theta}{2}\right),\ \sin\left(\frac{\theta}{2}\right)\sin(\theta),\ \cos\left(\frac{\theta}{2}\right)\sin(\theta)\right\}$$

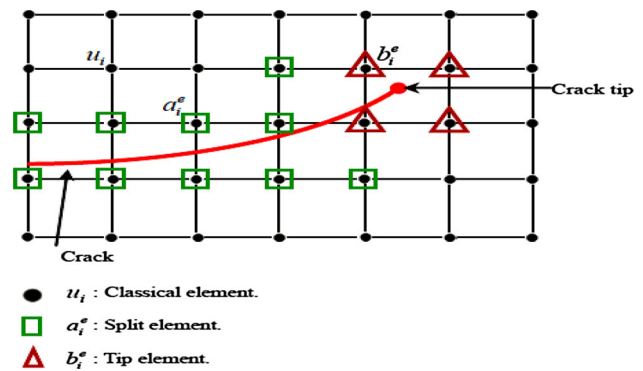where (r, θ) is the polar coordinate system with its origin at the crack tip

Fig. 4 Strategy of enrichment of a crack by the method XFEM Abdelaziz [34]

Table 1 Geometric characteristics of the tube

Inner radius	Ri = 100 mm
Outer radius	Re = 110 mm
Thickness	t = 10 mm
Length	L = 200 mm

4 Numerical simulation with ABAQUS software

4.1 The geometry of the problem

The geometric characteristics of the tube used in this study is present in the following table Moustabchir [35] (Table 1):

The shape of the defect investigated is a longitudinal semi-elliptic crack located on the outer surface of the inner pressure tube P, as shown in Fig. 5.

The material studied is a generally P265GH steel used in boilers and pressure vessels. The values of the mechanical characteristics of the studied material, obtained by the tensile tests, are given in Table 2 Moustabchir [35]:

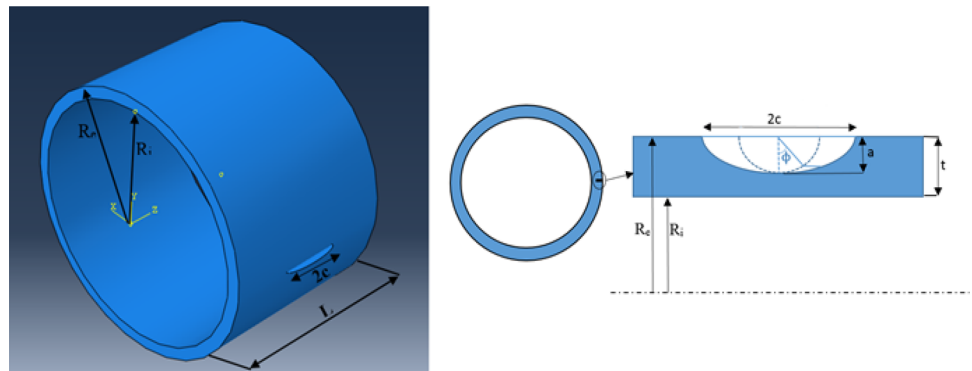**Fig. 5** 3D tube with semi axial elliptic crack, geometric defect characteristic

Table 2 Mechanical properties of P265GH material Moustabchir [35]

Elasticity module	E = 207,000 MPa
Poisson coefficient	$v = 0.3$
The limit of elasticity	Re = 340 MPa
Limit of rupture	$R_M = 440$ MPa
Elongation	A = 35%
Critical stress intensity factor	$K_{IC} = 94.99$ MPa.m$^{0.5}$

4.2 Numerical modeling

In this present study, were extracted the Stress Intensity Factor values of KI in mode I, simulating the behavior in the linear elastic domain. The ABAQUS 6.14 software may offers two different ways to evaluate the full contour. The first is based on the classical finite element method (FEM), which usually requires the user to define explicitly the crack front. In the ABAQUS 6.14 software, the special command (* CONTOUR INTEGRAL) were accessed that is dedicated to calculate the integral J in the crack front. This command uses a predefined model based on the discretized formula of J (Eq. 2).

In the second method of XFEM, the data required for the contour integral is determined automatically by level set for a specified distance of the functions related to the nodes connected to an element Zhu [36].

This study treats a longitudinal crack/cracks in pipes because they are more critical than circumferential cracks. A longitudinal crack/cracks have been studied carefully in this work Khoramishad [37].

The crack configuration is described by some non-dimensional parameters, namely the relative wall thickness (t/R), the relative crack depth (a/t) and the aspect ratio of the crack (a/c). The geometry of the pipe and the position of the longitudinal crack on the pipes and their effects on the so-called Stress Intensity Factor were studied at different positions along the crack front, a/t: 0.2, 0.5, 0.8 and a/c: 0.2, 0.4, 1 with t/R = 0.1
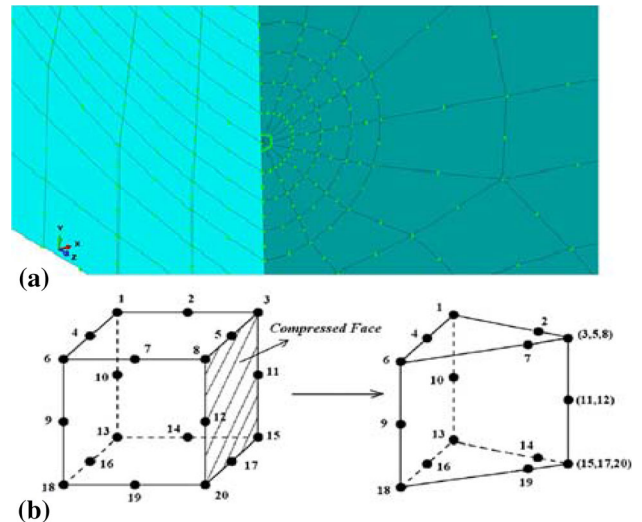

Fig. 6 A modeling sketch of singularity near crack point

4.3 Meshing, loadings and boundary conditions

The mesh step of the geometry studied is a very important phase that determines the precision of the results and the computation time. A satisfactory mesh is generated for a mesh when it makes possible to have precise results in an optimal computation time.

The classical finite element (FEM) method require the mesh to be consistent with the geometry of the crack. Therefore, the choice of the mesh influences the results in terms of displacements and stresses. The stress field near the crack point has an $r^{-0.2}$ singularity. To simulate the singularity in ABAQUS 6.14 software, were considered the method of Barsoum [38] which was developed on a method-based on the (FEM) to determine the KI factor by shifting the median nodes of a quadratic isoparametric element (C3D20R) at a quarter of the point measured from the crack front (Fig. 6a). A 20-node hexahedron (brick) element was used at the crack front, latter converted to an element corner (Fig. 6b).

Were considered a symmetry problem, and we have meshed only a half of a tube as shown in Fig. 7a to model the
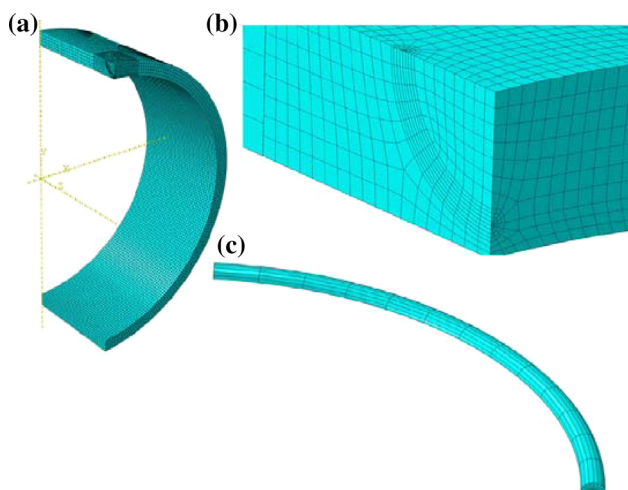
Fig. 7 The mesh of a half-structure using FEM method with refinement near the crack zone, the number of elements is 39,866 and the number of nodes is 185,354
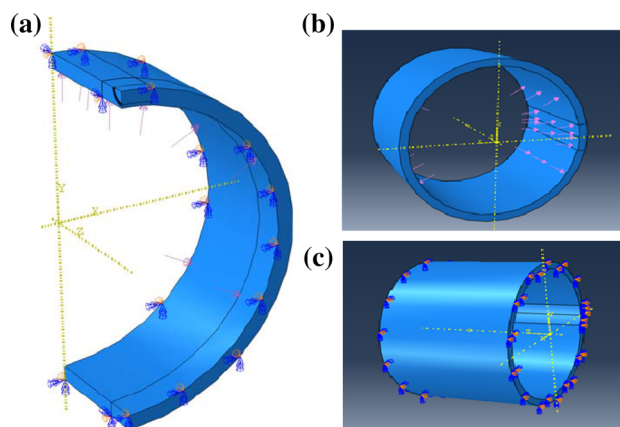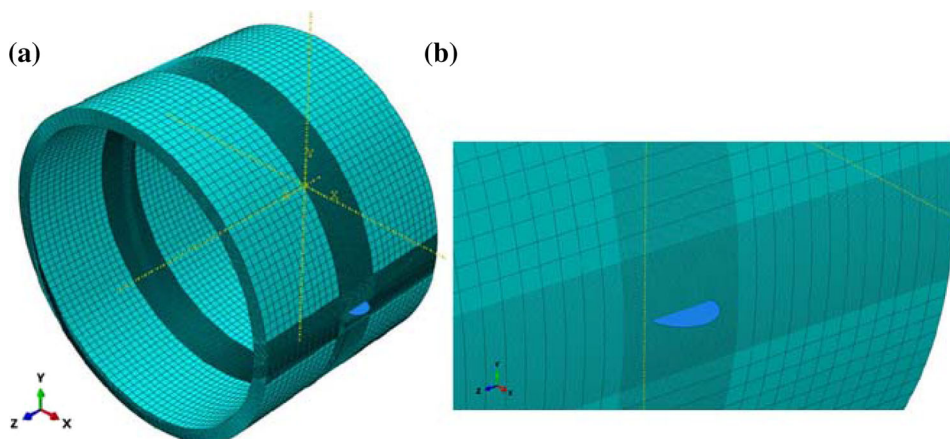

Fig. 9 The boundary conditions and loading, **a** pressure applied on half tube and symmetry following the X and Z axis, **b** pressure applied inside the tube, **c** symmetry following the Z axis (U3 = UR1 = UR2 = 0)

singularity and getting accurate results. There, was refined the mesh around the crack front as shown in Fig. 7b, c. The choice and generation of the mesh size have a great influence on the results, thus, it must take carefully consideration during the simulations. In this study was used a mesh size of 1 mm for both methods (FEM and XFEM). Being a problem of crack propagation, remeshing has become a necessary tool, making the finished element method very difficult to apply.

The new XFEM method permit to eliminate the limitations of the classical finite element method (FEM). This method makes possible to study the problems of crack propagation without remeshing. In addition, the second advantage of this method is that the mesh of the structure is independent of the crack geometry, this is feasible with new enrichment functions that allow dealing with the problem of the singularity at the point of the crack, and the discontinuity of the displacement.

Subsequently, we used this method, which was already integrated on the ABAQUS 6.14 software to calculate the KI factor. A 3D tube was considered with an external axial crack (Fig. 8a). The mesh is independent of the crack, however to obtain accurate stress results the mesh was refined near the crack zone as shown in Fig. 8b. The element used in this study is a C3D8R (An 8-node linear brick, reduced integration, hourglass control).

The internal pressure was set to $P = 2.5$ MPa Moustabchir [35] pressure distributed on the inner side and the boundary conditions are shown in Fig. 9.

5 Results and discussions

The numerical results obtained using the ABAQUS software were compared in terms of KI factor between the two FEM and XFEM methods. In the numerical simulation was considered the energetic approach derived from Eq. (3).
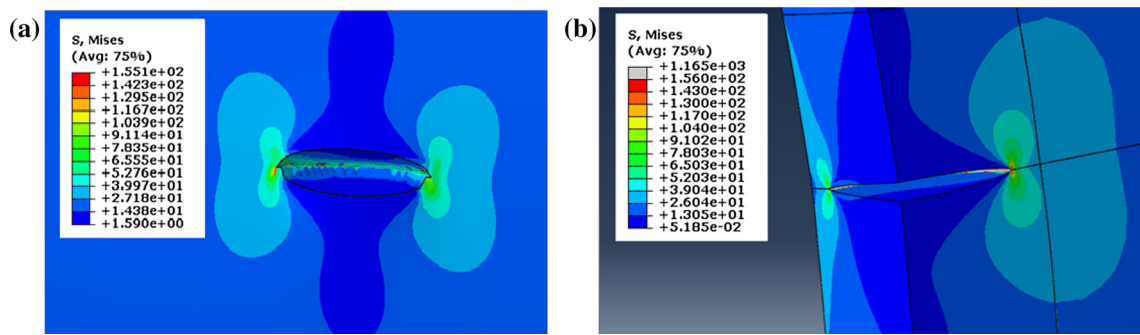
Fig. 8 The mesh of a 3D tube in the XFEM method, the number of elements is 324,240, and the number of nodes is 695,814

Fig. 10 The stress distribution Von mises near the point of the crack for a ratio (a/c = 1, a/c = 0.8), **a** XFEM case, maximum stress is 155.1 MPa, **b** FEM case maximum stress is 155.1 MPa
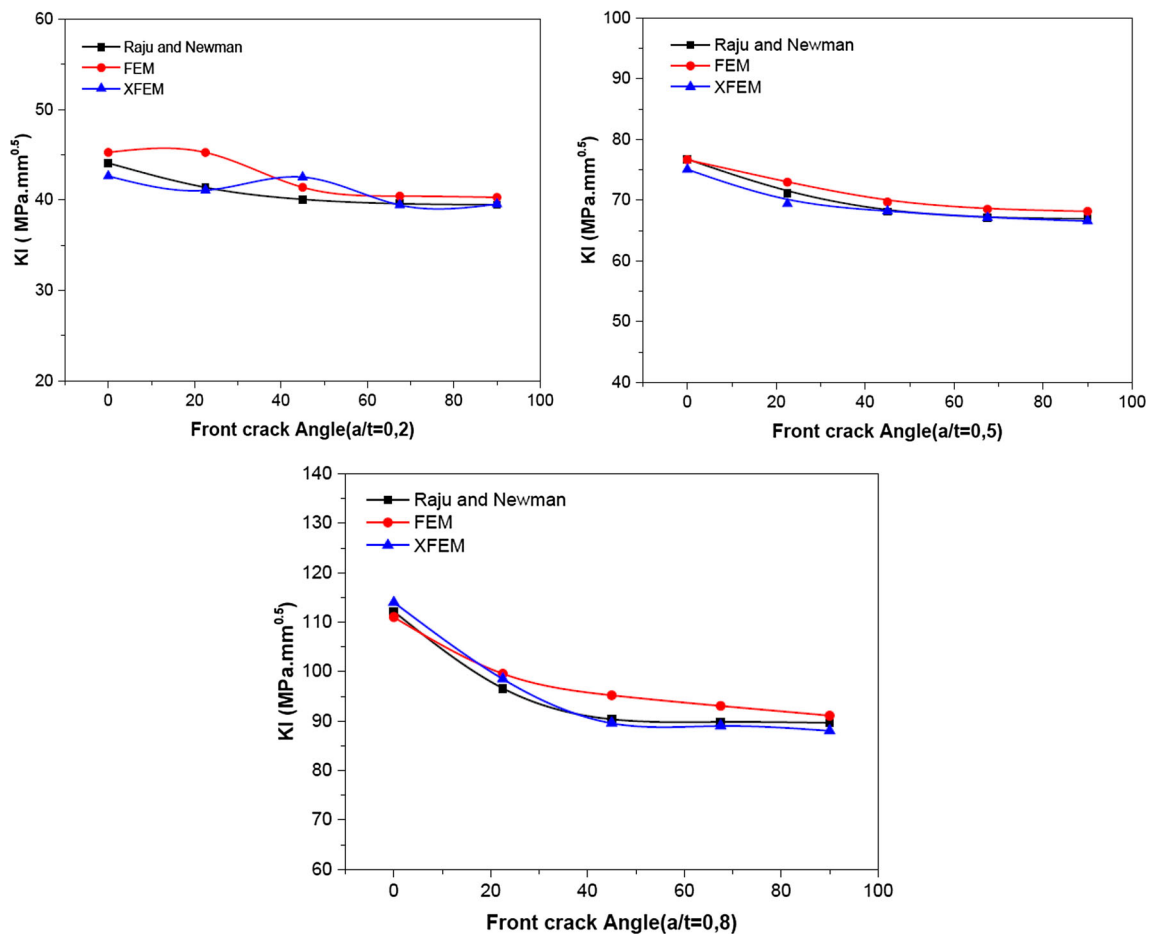

Fig. 11 The variation of the KI factor along the front f the crack for a ratio (a/c = 1)

Figure 10 shows a plot of Von Mises distribution stresses for the ratio (a/c = 1, a/t = 0.8), where the maximum stress reaches the value 151.1 MPa in both methods. There is a small disturbance of the stresses at along the crack front for the FEM method, that may be is due to the presence of singularity at the point of crack.

On the other hand, in the XFEM method, the constraints are more stable and well presented. Here we can note here, the advantage of the XFEM method which permits to overcome the problem of the singularity at the point of the crack tip.

Figure 11 presents the comparison between the XFEM method, the FEM method and the Raju and Newman method Raju [18] along the crack front for the case (a/c = 1). The variation of KI factor was obtained as a function generated by the crack angle where the maximum value lies in the angle $\varphi = \pi/2$, and represent the deepest point of the crack.
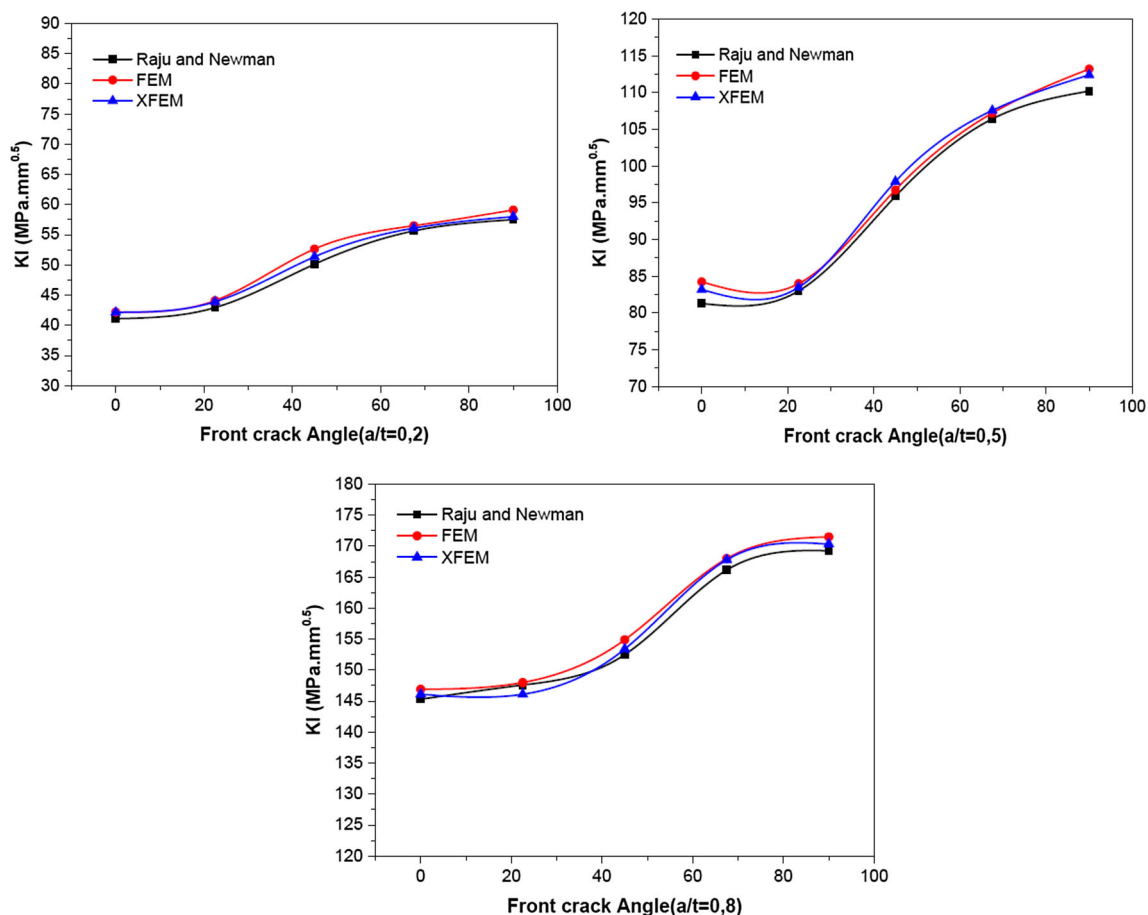
Fig. 12 The variation of the KI factor along the front of the crack for a ratio (a/c = 0.4)

The approximation highlights good agreement between the numerical results calculated by the two methods (FEM and XFEM) and the analytical results found by Raju [18].

Figures 12 and 13 shows the variation of the KI factor around the crack front for the case (a/c = 0.4 and a/c = 0.2). The results acquired show good agreement between the numerical and analytical results, and the maximum value of the KI factor was obtained for an angle $\varphi = \pi/2$.

In Table 3, were provided a comparison between the two numerical methods (FEM and XFEM) and the analytical method of Raju and Newman [18], in terms of errors and KI results values. They proves that the modelling strategy implemented using the XFEM method does not exceed an error of 1.81%, however, were detected a larger error when is used the FEM method, error that is in turn of 6.81%. This difference is due to the power of the XFEM method, once modelling cracked structures. The robustness is derived from the enrichment functions that they are added to the formulation of the classical FEM method.

In Fig. 14 presents the variation of the KI factor as a function of the depth of the crack in the tube. It is noted that when the crack depth increases the value of the KI factor also

increases until it reaches the critical value 280 MPa.mm$^{0.5}$ for the case (a/c = 0.2).

Besides, when the value of the ratio a/c decreases the values of the KI also increases until it reaches critical values.

The outcomes of this study can be summarized as:

- The problem of stress singularity on the crack tip is treated better by the XFEM method, compared to the classical method (FEM), FEM approach that require to use a very fine and very regular mesh around the point of the crack that may have great influence on the results of the stresses; however, this is not the case for the XFEM method where the mesh is independent of the geometry of the crack. The treatment of the singularity problem is evaluated using the enrichment functions.
- The value of the Stress Intensity Factor KI is maximum at the 90° angle of the crack front. This represents the deep point of the crack near the same time of the inner surface of the tube where the maximum value of the pressure is applied.
- The numerical and analytical results present a good agreement, having only a difference of 1.81% by using XFEM
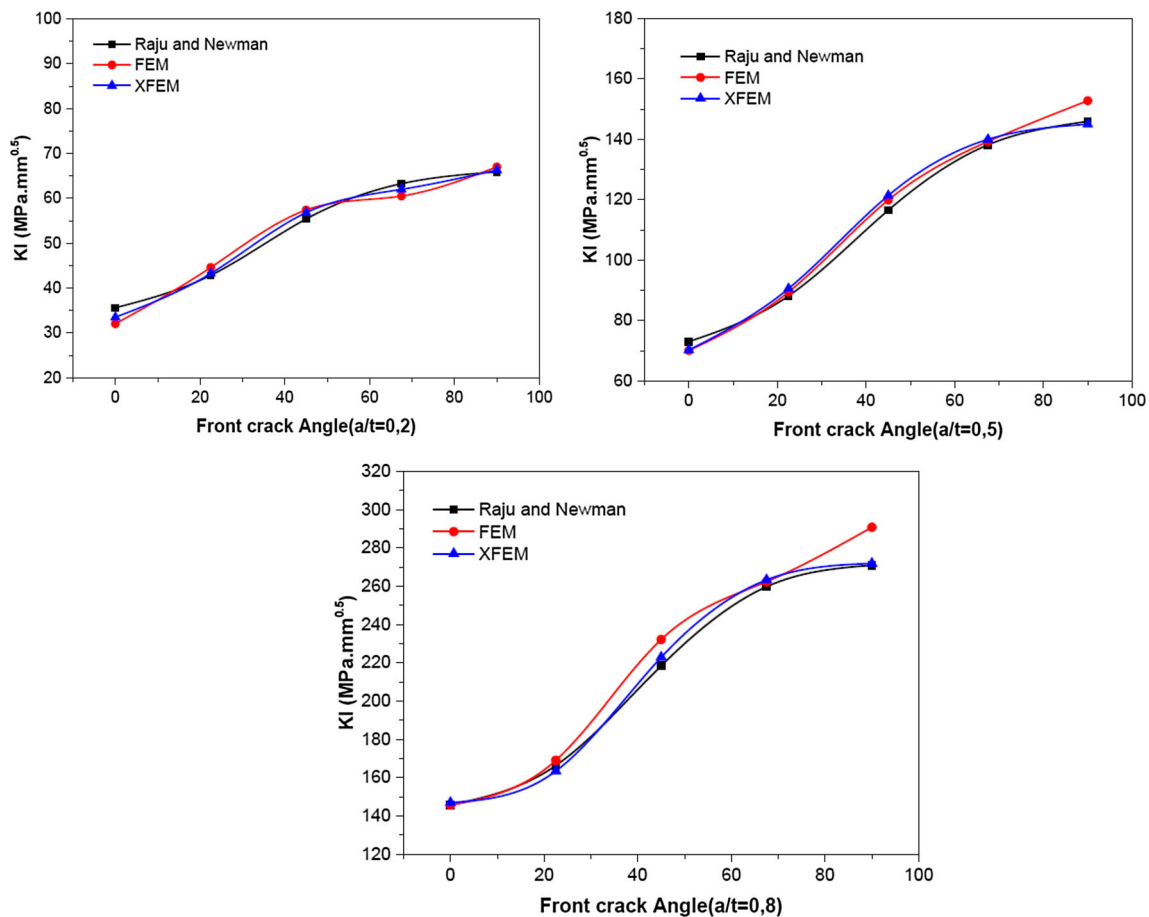
Fig. 13 The variation of the KI factor along the front of the crack for a ratio (a/c = 0.2)

Table 3 The error of the KI values for the critical default

Default size	KI (MPa mm$^{0.5}$) (angle $\varphi = \pi/2$)		
	a/c = 0.2, a/t = 0.8	a/c = 0.4, a/t = 0.8	a/c = 1, a/t = 0.8
Analytical	270.99	169.25	89.65
FEM method	290.8	171.5	92.1
X-FEM method	271.9	170.33	88.02
FEM method error (%)	*6.81*	*1.31*	*2.66*
XFEM method error (%)	*0.33*	*0.63*	*1.81*

Bolditalic values indicate the importance of accuracy

method and excending 6.81% in FEM method. In addition, the results by the XFEM method are closer to the analytical results of Raju [16]. The global agreement observed gives confidence for the use of the XFEM method for the determination of KI values.

6 Conclusion

The two numerical methods (FEM and XFEM) used for calculation of the Stress Intensity Factor KI, in mode I, in the elastic linear domain prove a robust tool for assessment of structural components. It was validated well against the analytical results existing in the literature. The results also show that the position of the longitudinal crack on the pipe has a significant influence on the stress intensity factor KI; and the XFEM method permits to overcome the stress singularity problem at the point of the crack tip. This strategy gives confidence in the use of the XFEM method to deal with cracking problems in complex structures.

The success of this methods can be extended, such as this work using the XFEM method permits to create robust plat-
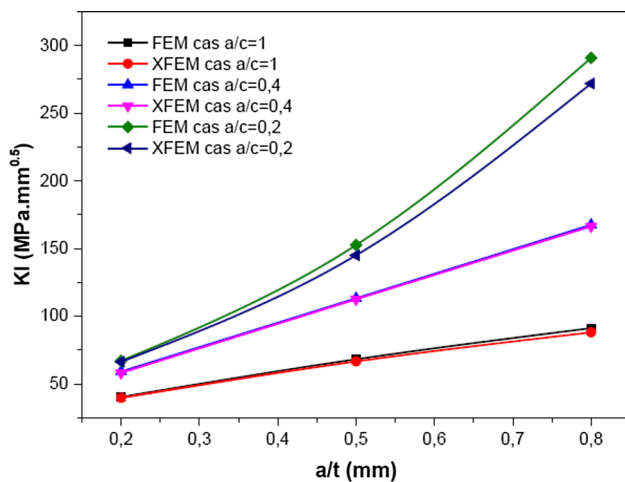
Fig. 14 The variation of the KI factor based on the ratio a/t and a/c

forms to study the problem of fatigue in cracked pipes, that can be easy coupled to the Paris law. And having the final objective to obtain KI, in dynamic case, while applying the XFEM method.

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