ZDM – Mathematics Education
https://doi.org/10.1007/s11858-024-01560-4

ORIGINAL PAPER

Describing the digital competencies of mathematics teachers: theoretical and empirical considerations on the importance of experience and reflection

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Accepted: 16 February 2024

Abstract
This article presents an extension of the well-known TPACK model to describe the professional digital competencies of mathematics teachers. The extension leads to what we want to call MPC-model (Media–Pedagogy–Content) in the following. It additionally includes (1) the consideration of competencies instead of knowledge for a holistic description, (2) the integration of professional digital competencies in a broader context of professional media competencies (including explicitly analog and digital teaching media), (3) the description of concrete individual experiences with digital technology in context-bound subjective domains of experience, and (4) the cross-linking of concrete individual experiences about (digital) technology in specific (subjective) domains of experience. In this article, we first present a motivating literature overview leading to the research question: How can the TPACK model be extended to enable a qualitative description of professional digital competencies of mathematics teachers against the background of situated experiences? This extended framework is developed and presented in a detailed theoretical background. In the empirical part of the article, an exemplary application of the MPC-model is carried out in an explicative case study dealing with the reflections of a mathematics teacher on a planned lesson using virtual reality technology in a guided interview. The qualitative data is interpreted according to the systematic-extensional analysis method. The case study illustrates the importance of taking into account concrete situated experiences opening up a new reflective level analyzing the development of professional mathematics-specific digital competencies.

Keywords Digital competencies · Digital technology · In-service mathematics teacher education · Professional development · TPACK

1 Introduction

In the process of digital transformation of education, mathematics teachers have access to a continuously growing variety of digital-learning technologies. Furthermore, educational policy guidelines strongly demand the integration of digital technologies in mathematics teaching (in Germany, for example, KMK, 2016). This poses new challenges for mathematics teachers in terms of technology integration, as the use of digital technologies has become a fundamental part of mathematics teaching. To face these challenges, professional digital media competencies[1] should be developed to enable mathematics teachers to professionally integrate digital technologies into mathematics teaching–learning processes based on their personal subjective experiences (Drijvers, 2019). In this context, professional digital media competencies are those that enable a mathematics teacher to evaluate and select digital technologies and use them in the classroom. Although different models for describing professional digital competencies already exist (e.g., DigCompEdu by Punie & Redecker, 2017), from our point of view there is still a need of a framework that captures

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[1] This term, which is uncommon in the English language, was chosen deliberately and will be described in more detail in a later section.

Published online: 20 March 2024

domain-specificities of mathematics. What we present as the MPC-model (= **M**edia, **P**edagogy, and **C**ontent) in this paper provides a framework that allows describing and qualitatively classifying the professional (digital) media competencies of teachers in the face of this consideration. It serves as a mathematics-specific extension of the already established TPACK model (Koehler & Mishra, 2009). In our view, and this is what we seek to explain in this paper, an extension of the TPACK model in this way seems to be appropriate and helpful for further descriptive and prescriptive purposes. To illustrate this, we will discuss professional development for mathematics teachers in terms of media competencies with reference to the MPC-model. Finally, the MPC-model is applied as an example to a specific case of a teacher conducting a lesson on analytic geometry using virtual reality technology (VR). The goal of this study is to explicate the basic assumptions of the model and to specify following research concerns.

2 Theoretical background

2.1 Professional digital competencies—a brief literature survey

Professional development of teachers is a long-standing focus of international educational research. In this context, the description of the knowledge of teachers has long been the main aspect of interest. A well-known and passed on framework is the PCK model according to Shulman (1987), which distinguishes between the two dimensions of content and pedagogy in relation to professional teacher knowledge. However, in recent years there has been a trend towards looking at professional *competencies* instead of professional *knowledge*. This is also due to the limitations of the term knowledge, which Neubrand (2018) sees in the "gap between knowing and acting" and the lack of an "affective component". The concept of competencies is indeed difficult to grasp in its entirety. According to Blömeke et al. (2015), there are traditionally two approaches to defining the concept. In the analytical approach, competency is divided into various cognitive and affective-motivational traits. Competency is then measured by systematically assessing the individual traits. This approach is linked to the assumption that there is a positive correlation between competency and performance. In contrast, the holistic approach defines competency directly in terms of behavior in real-world contexts. Competency is then measured by observing performance in sample real-world tasks. Blömeke et al. (2015) argue that these two approaches should not be seen as contradictory, but rather as parts of a continuum: "The measurement of competence, then, may be viewed along a continuum from traits (cognitive, affective, motivational) that underlie the perception, interpretation, and decision-making that give rise to observed behavior in a particular real-world situation." (p. 11). Well-known and current definitions of the term competency describe both the components of a competency structure and its relationship to performance. For example, Weinert (2001) describes competencies as the cognitive abilities and skills available and learnable by individuals to solve specific problems, as well as the associated motivational, volitional, and social readiness and skills to use in the solution of problems (p. 27).

Models for describing the professional competencies of teachers usually focus on an analytic approach and distinguish between a cognitive and an affective-motivational component. For example, the competency model of the TEDS-M study (Blömeke et al., 2010), based on Shulman (1987) considers subject knowledge, pedagogical knowledge and subject-specific pedagogical knowledge as cognitive components. The affective-motivational component refers to beliefs, motivation and personality attributes. In the COACTIV study (Baumert & Kunter, 2011) beliefs/values/goals, professional knowledge, self-regulation, and motivational orientation are distinguished. These elements also strongly relate to the cognitive and affective-motivational components of competencies.

With regard to the integration of digital technologies in mathematics teaching, knowledge-based approaches continue to be strongly (over-)represented. This is especially true for the TPACK model (Koehler & Mishra, 2009) which, according for example to the meta-study by Hew et al. (2019), can be regarded as one of the most prominent frameworks in educational technology research in the last years. The TPACK model applies the idea of Shulman (1987) to distinguish the knowledge dimensions of content and pedagogy adding a dimension of technology. Of particular interest in this approach is the intersection of the individual dimensions, namely technological pedagogical knowledge, technological content knowledge, and pedagogical content knowledge, as well as the intersection of all three dimensions (technological pedagogical content knowledge).

Since the beginning of the discussion about the use of digital resources and tools in mathematics education in the 1980s, one or maybe the core question was, "How could digital technologies provide an added value for mathematics learning and teaching." To answer this question in specific contexts, the TPACK model has been successfully applied in various studies (e.g., Drijvers et al., 2013) and adapted to the specifics of mathematics (e.g., Niess et al., 2009; Stapf & Martin, 2019). However, since the 1980s, the context of the use of digital technologies has changed enormously. Nowadays, digital technologies are used widely in the professional and private life of everyone, there is a wide range of digital technologies for teaching mathematics, and the use of these technologies is in the interest of educational politics and society. To evaluate and select digital technologies appropriately and use them in the mathematics classroom effectively, teachers need to have professional digital competencies. In an article from 2021, Michal Tabach refers explicitly to a "shift from knowledge to competency" (Tabach, 2021, p. 33) in relation to research on the use of digital technologies in mathematics education—whereby she emphasizes that the concept of professional digital competency has not yet been adequately described. This term is generally considered a "loose concept" in research (Ilomäki et al., 2016, p. 656), as it is not well-defined and authors sometimes associate very different concepts with it. Nevertheless, there are already some models that attempt to define the term and determine areas of competency. DigCompEdu is probably the best-known model for teachers' professional digital competencies (Punie & Redecker, 2017). The model distinguishes between the six areas of professional engagement, digital resources, teaching and learning, assessment, empowering learners and facilitating learners' digital competencies. The strength of the DigCompEdu model lies in its general applicability to teachers of all subjects and school types and its ease of use in the training and further education of teachers. However, it has to be adapted and transferred when it is applied in subject-specific research. Especially, the DigCompEdu model cannot be used to describe mathematics-specific aspects of professional competencies (cf. Osterman et al., 2022).

Geraniou and Jankvist (2020) provide initial approaches for a mathematics-specific model of teachers' professional digital competency. They address the concept from a networking theories perspective and combine elements of the TPACK model, instrumental orchestration (Drijvers et al., 2014), the Danish KOM framework (Niss & Hojgaard, 2011) and the mathematical knowledge for teaching concept (Loewenberg Ball & Bass, 2009). The authors' theoretical explanations suggest that a combination of different existing theories on the use of digital technologies can be a promising approach to developing a competency model for teachers.

The literature survey reveals a desideratum for a model of teachers' professional mathematics-specific digital competencies. This article aims to take first steps towards developing such a model by extending the TPACK model outlined above. The main reason for its selection is that it is the central model in the context of the professional development of teachers (Clark-Wilson et al., 2020) and is therefore compatible with previous research in this area. Drijvers et al. (2014) also describe the simplicity and accessibility of the model as advantages, but also mention the ambiguities and limited clarity as problems. Ruthven (2014) therefore emphasizes that it should be supplemented by other theories in order to achieve an adequate depth of analysis. Moreover, the TPACK model has to be seen in the frame of the situation it is used or applied for (see Sect. 2.4). This leads to the research question of the present article:

How can the TPACK model (Koehler & Mishra, 2009) be extended to enable a qualitative description of professional digital competencies of mathematics teachers against the background of situated experiences?

To address the current conditions regarding the use of digital technology, four extensions to the TPACK model were conducted, which are described and justified in the following sections and lead to the MPC-model (cf. Dilling et al., 2022a): (1) consideration of competencies instead of knowledge for a holistic description (Sect. 2.2), (2) integration of professional digital competencies in a broader context of professional media competencies (covering analog <u>and</u> digital teaching media) (Sect. 2.3), (3) description of concrete individual experiences with digital technology in context-bound subjective domains of experience (Bauersfeld et al., 1983) (Sect. 2.4), and (4) characterization of the reflective level as a superordinate subjective domain of experience that enables the cross-linking of concrete individual experiences with (digital) technology (Sect. 2.5).

2.2 Extension 1: competencies instead of knowledge

The TPACK model describes *knowledge* dimensions related to teaching with digital teaching media. By contrast, the goal of the MPC-model developed here is to describe *competency* dimensions in relation to teaching media. Professional digital media competencies are understood in this paper as competencies that enable a teacher to evaluate and select digital technologies and use them in the mathematics classroom effectively. Knowledge, as described in the TPACK model, is only one part of competencies. Only its combination with motivational, volitional, and social aspects makes the successful application of knowledge for teaching possible. The shift from the consideration of knowledge to the consideration of competencies has multiple good reasons, as it could be seen in the previous section. In the MPC-model, in addition to the knowledge component, beliefs—as affective-cognitive components (Pehkonen & Pietilä, 2004)—receive special attention (see also Sect. 2.4). Ertmer and Ottenbreit-Leftwich (2010) demonstrated the evident correlations between beliefs and the use of digital technologies in the classroom. In a quantitative questionnaire study, Thurm and Barzel (2022) were also able to establish links between beliefs about teaching with technology, self-efficacy beliefs and epistemological beliefs with different modes of technology use in the classroom.

2.3 Extension 2: digital technologies in the context of teaching media

For a long time, research on digital technologies was carried out based on the perspective of possibilities for the use of particular technologies in mathematics classrooms. The development of the TPACK model was also based on this perspective. Koehler and Mishra (2009) posed the question, "How can teachers integrate technology into their teaching?" (p. 62).

In the last few years there has been a change shifting this perspective away from specific digital technologies and appropriate mathematical usage scenarios towards "technology and resources in mathematics education" (Trgalová et al., 2018) which especially integrate analog and digital media into the teaching process. Moreover, the focus is now on which digital technology or which analog approach could be suitable from a content-related and pedagogical perspective to initiate the intended mathematical teaching–learning processes. The new question can thus be formulated based on Koehler and Mishra (2009): Which (digital) teaching media can be integrated to support specific mathematical learning processes in a meaningful way?

Therefore, from our perspective, the use of digital and analog technologies should not be seen as fundamentally different but similar from an epistemic point of view. That is why the term "teaching media" will be used to cover both digital and analog media in the MPC-model. This term is rarely used in international journals and is derived from the German-language educational literature. Teaching media take on a mediating role in the classroom between the mathematical content and the student, with the goal that the latter develops mathematical competencies and understands mathematical concepts and relationships (Barzel & Greefrath, 2015). Teaching media include analog tools such as textbooks and worksheets as well as digital technologies such as videos, interactive whiteboards, and math-specific software (e.g., dynamic geometry software or spreadsheets). When choosing a teaching medium in mathematics education, it is crucial to consider different teaching media from the perspective of content and pedagogy according to the group of learners addressed. This integrative approach of digital and analog teaching media in the MPC-model has the advantage that they as a whole are to be referred to as essential elements of teaching.

Accordingly, the MPC-model considers professional digital media competencies as a subset of professional media competencies. Similar to the TPACK model, content competencies and pedagogical competencies are defined as crucial competency dimensions. Content competencies are meant to be the ability, based on the content knowledge, to develop strategies for the teaching of contents and to transfer these adaptive to classroom situations. Pedagogical competencies are the ability to foster and support the students in their learning processes methodologically, e.g., by arranging the learning situation (e.g., the social forms) in an adequate way.

From this we formulate three central competency dimensions, which can be identified as dimensions isolated from one another but also in their relationships to one another. With regard to media competencies, the intersections content-related media competencies, pedagogical media competencies, and content-related pedagogical media competencies emerge. Digital media competencies then are a subset of (general) media competencies with reference to the designated sub-competencies.

2.4 Extension 3: situated experiences as the basis of competencies

The previous explanations of the taken perspective on digital media competencies as a subset of general media competencies requires further specification. Various empirical studies have shown that the appropriate use of digital technology in a given context is not an indicator of the general ability to use this digital technology professionally in a different context (Rosenberg & Koehler, 2015).

This context-specificity phenomenon can be explained from an epistemological perspective based on the concept of subjective domains of experience (Bauersfeld et al., 1983). This concept states that every human experience is acquired in certain learning contexts and is closely related to these. Experiences are stored in subjective domains of experience isolated from each other. The term "subjective domain of experience" is related to the internationally used term "microworld" according to Lawler (1981). He described microworlds as a representation of an experience in memory, that "reflect in little, in the microcosm of the mind, the things and processes of that greater universe we all inhabit" (p. 4).[2] All subjective domains of experience within a person are referred to as the "society of mind" (Lawler, 1981). They exist in a non-hierarchical structure and compete for activation. A subjective domain of experience relates either to a concrete experience (for example, related to a digital teaching medium) or to other subjective domains of experience (provided there is a linked, superordinate subjective domain of experience). A superordinate subjective domain of experience links two or more other ones and, when it is activated, it enables the individual to make a specific (conscious) decision among the perspectives of the linked subjective domains of experiences. In the context of teaching

[2] The term microworld goes back to Minsky and Papert (1974), who used it to describe special learning environments such as "Turtle Geometry". In contrast, Lawler uses the term to describe cognitive structures and thus established it in the field of psychology. However, this use of the term is less common in educational literature.

media, this opens up the possibility of a level of reflection at which an individual can discern between two or more subjective domains of experience (e.g., with reference to digital or analog teaching media and their intended use in mathematics teaching).

The MPC-model describes media competencies on the basis of the set of subjective domains of experience that relate to media. In addition to knowledge about teaching media, the model also incorporates other motivational, affective, or emotional components of subjective experiences related to the use of teaching media. Thus, the use of the subjective domains of experience concept is suitable to describe the concept of competency introduced in Sect. 2.1. Considering the MPC-model, a teacher has a set of subjective domains of experience that are related to teaching media. All subjective domains of experience with reference to these media are considered in the MPC-model as a subset of the society of mind. In our model, the totality of this subset determines the professional media competencies of a teacher. Thus, if a medium is understood in its mediating role as in the MPC-model, the use of digital teaching media and of analog teaching media complement each other. The meaningful support of the intended mathematical teaching–learning processes is what matters.

2.5 Extension 4: level of reflection

To make adequate use of opportunities offered by digital technologies, teachers need to develop professional media competencies (i.e., to have a sufficient number of subjective domains of experience related to digital teaching media). The activation of adequate subjective domains of experience is only possible if a teacher can rely on solid experiences with reference to digital teaching media. For the adequate selection of a teaching medium for a specific learning situation, it is necessary to assess the opportunities and advantages in relation to different teaching media relevant to the context. Such a conscious consideration is, according to the subjective domains of experience concept, only possible if the teacher is able to activate a superordinate subjective domain of experience in which two or more subjective domains of experience with reference to (digital) media are linked. To succeed in this area, a meta-cognitive level of reflection to connect the given situation with the subjective domain(s) of experience is necessary.

This does not mean that the superordinate subjective domain of experience is activated at all times and combines all experiences on media use. It rather means that even a few concrete experiences can be connected and thus enable locally limited reflections (e.g., experience on GeoGebra to introduce the derivative and experience on graphical derivatives with pencil and paper). Moreover, the reflection on the given situation has to be seen in relation to the content, the pedagogy, and the teaching media, and the connection between these three.

3 Case study

3.1 Methodology and case description

As described in the previous section, the MPC-model is an extension of the TPACK model, based on four aspects (see Sects. 2.2–2.5). To explain the MPC-model, this section presents an empirical case based on Yin (2013). The case study serves to explicate the basic assumptions of the MPC-model, there is no claim to representativity connected to it. With this empirical explication, we want to highlight that the extensions may be crucial aspects of the professional media competencies of mathematics teachers and should be investigated in more depth.

In this case study we chose for our investigation the female teacher, Mrs. Heinrich, from a high school in the state of North Rhine-Westphalia, Germany. She participated in the design and research project DigiMath4Edu (Dilling et al., 2022b). In this project, mathematics teachers are supported by a team of the University of Siegen in planning and implementing lessons with digital technologies. Specially trained pre-service teacher students (so-called "digital media teaching assistants") support teachers at a school for one year in planning, using, and teaching with digital media. At the date of the interview Mrs. Heinrich has approximately 15 years of professional experience as a teacher and has also been in contact with digital technologies before the project, as will be explained in more detail in the following section. In the project, she and a digital media teaching assistant planned and implemented a lesson introducing the concept of vectors with the integration of the VR application edVR (Baur, 2019), in which points, vectors, straight lines, and planes can be visualized in a virtual three-dimensional coordinate system by entering mathematical parameters. By wearing VR goggles, students can move around the virtual coordinate system and change the visualization with controllers.

A reflective interview was conducted with Mrs. Heinrich by the authors of this paper after the VR-supported lesson. Further information about the lesson can be found in the following section. The goal of the semi-open interview was to describe a part of her professional media competencies based on the MPC-model. The basis for the interview was a guideline, which included the following questions:

- What previous experience did you have with digital media before the lesson?
- What previous experience did you have with VR?

- What was your motivation to use VR technology in the mathematics classroom?
- Outline the lesson design process from the initial idea to the final implementation.
- Outline the finally implemented lesson.
- What analog or digital media could have been used as an alternative to VR technology in your lesson? What are the probable advantages and disadvantages that would have resulted from that?
- At what points did the digital media teaching assistants support you?

The questions address important aspects of the MPC-model: First, previous experiences and basic beliefs were asked for in order to get a first impression of existing subjective domains of experience in relation to digital media. Focusing on the teacher's explanations of the planning and implementation of the lesson, Mrs. Heinrich's experiences made in relation to VR could be surveyed. The questions about analog or digital alternatives to VR technology in the lesson were used to be able to distinguish subjective domains of experience from another and identify possible superordinate subjective domains of experience in the sense of a meta-cognitive level of reflection. Finally, the last question served to assess the influence of the digital media teaching assistants. On the basis of these statements in the interviews and conversations with the digital media teaching assistants, it can be assumed that the decisions regarding the use of technology in the described lesson were mostly driven by herself. This is also supported by the fact that Mrs. Heinrich makes her beliefs explicit in the following interview and justifies her decisions and evaluations in detail. However, the influence of the digital media teaching assistants, which cannot be excluded completely, does not pose a significant problem for the present purpose of explicating the MPC framework.

The statements of Mrs. Heinrich represent self-reflection on the designed lesson, possible alternative approaches, and her professional development. Thus, this is not a quantitative measurement of competencies as is common in the context of competency research. Instead, the study is intended to qualitatively describe the structure of a part of Mrs. Heinrich's professional media competency and thus generate hypotheses in relation to the MPC-model. For this purpose, the basic assumption is made that the description of experiences and the justification of decisions in planning and conducting lessons in the context of reflective interviews can be used to reconstruct subjective domains of experience, which in turn are associated with competencies (see Sect. 2.4). Nevertheless, reflective interviews do not allow to draw direct conclusions about the teacher's handling of the media and interactions with the students in the classroom.

The answers of Mrs. Heinrich were transcribed and analyzed using an interpretive approach according to the systematic-extensional analysis method (Beck & Maier, 1994). In this method, transcripts are first divided into episodes. The division is based on the course of interaction in the interview and also depends on the research interest or questions. Afterwards, specific episodes are selected for analysis. For this purpose, the selection criteria "obvious relevance to the research question" as well as "crisis nature of the episode" according to Krummheuer (1992) were applied. On this basis, five episodes were selected concerning Mrs. Heinrich's prior experiences, her decision to use VR technology, the design of the lesson on VR, and the discussion of alternative analog media or alternative digital media. For readability, the third episode is divided into two segments. The analysis of the first episode is done initially in a less systematic manner so that subjective initial interpretations can be captured. Subsequently, individual statements within an episode, in this case by Mrs. Heinrich, are interpreted extensively. This means, that possible interpretations are discussed, which are then supported or rejected based on further interactions within the episode. Finally, an interpretation of the entire episode is made, which in turn may be supported or rejected by other episodes. To generate scientifically controllable interpretations, it is important to make the theoretical background underlying the analysis as explicit as possible (see Sect. 2 of this article). In order to keep the argumentation within this article consistent, possible interpretations based on the MPC-model are in focus.

We are aware that analyzing the professional media competencies of a single person is of course not sufficient for any concluding remarks and can “only” generate hypotheses. However, the aim of this study was to show that the central components of the MPC-model, in particular the extensions to the TPACK model, can make sense. The aim here is to provide some kind of "proof of existence", which forms a basis for and should be supplemented by subsequent broader qualitative and quantitative studies.

3.2 Case analysis

Part 1: Previous experiences of Mrs. Heinrich

According to the MPC-model, the previous experiences of the participating teacher (Mrs. Heinrich) were first described. The prior experiences of Mrs. Heinrich with digital technologies for mathematics teaching were particularly relevant. The teacher described her experiences as follows:

> Mrs. Heinrich: Well, they [digital technologies] came up from time to time in my lessons. I have already worked with Excel. I also know GeoGebra, but I don't really like to use it in class, because you can actually see a lot on it, but it's so fleeting. Somehow, the infor-

> mation is always immediately gone and I often prefer it when the students really draw a lot of things. [...] I also notice that there are great difficulties in drawing exactly and also in describing what you see and so on. That's why I always prefer it when they really do it.

In the interview excerpt, Mrs. Heinrich describes her previous experiences with digital technologies in mathematics classes. Her user experiences appear rather rudimentary: She explains that she sometimes uses Microsoft Excel. She also seems to have some experience with GeoGebra and has the ability to operate it, but according to her own statements, she often deliberately decides against using it. Mrs. Heinrich reasons that students should learn to draw by hand, without technology, and that experiences with GeoGebra, while visually appealing, are fleeting. This is remarkable in that it is precisely the "fleeting drag-mode" that makes dynamic geometry software a powerful tool for learning mathematical reasoning. Her statement reveals two beliefs that appear to guide the teacher's integration of digital technologies (GeoGebra in particular). The first argument has a clear connection to geometry as well as to instructional methodological decisions, and can thus be assigned to the area of content-related pedagogical media competencies in the subjective domain of experience on GeoGebra. The second argument on fleetingness has only marginal connections to mathematics and belongs to the pedagogical media competencies without relation to content, as she probably does not know how to use the drag-mode functionality productively for mathematical argumentation.

Part 2: The decision using VR

After this excerpt, Mrs. Heinrich also explained that she had no previous experience with VR before the lesson examined in this article. She reports why she decided to use VR:

> Mrs. Heinrich: [...] I assumed that it could help imagine geometric objects in space and, in this way, really experience how points are located in space, how arrows appear in contrast to them, how straight lines appear, and how they can be located. That is very difficult for many students to imagine, even though we live in space. I usually like to work with pens, but then, you quickly reach your limits. You often need a fourth and fifth hand. Somehow, this doesn't work anymore, and I imagined that this could be quite helpful.

The excerpt shows that the motivation of Mrs. Heinrich for integrating VR into her teaching is subject-specific and clearly relates to problems previously identified for teaching this topic. Mrs. Heinrich is aware of the limitations of the analog approach used so far, especially at the methodological and organizational levels. She thus decided on VR out of her subjective domain of experience on visualization with pens in analytic geometry and mentions arguments from the field of content-related pedagogical media competencies. It is interesting that she does so out of pure imagination about what VR might possibly show. It is worth mentioning at this point that Mrs. Heinrich's belief regarding the fleeting nature of the experience with GeoGebra is disregarded in the case of VR goggles, even though the experience with the goggles can be fleeting as well and this is striking even more as she has not had any experience with VR before as a user. This may indicate that the fleetingness belief is specific to the subjective domain of experience on GeoGebra and that the two subjective domains of experience are clearly isolated from each other.

Part 3: The design of a VR lesson

Mrs. Heinrich decided to use VR technology by considering the potential (imagined) benefits and limitations of various digital and analog teaching media in the context of the specific teaching situation in analytic geometry. In a later part of the interview, Mrs. Heinrich describes more concretely how she designed a lesson using VR:

> Mrs. Heinrich: I have thought about what the focus of my lesson is, what I would like to achieve at the end, what is the goal, and what should they learn. And now, in relation to the first lesson, it was just that I wanted it to be quite clear, first of all, that there is a difference between a point and a vector, and I wanted to show what a vector actually is. That it's not just an arrow or a line somewhere or something that maybe has a direction, but that a vector has an infinite number of arrows, that it's an arrow class. Yes, and then I thought, perhaps one would have to draw a point, then the associated location vector, and then different starting points, so that one can really see it in space: quite a lot of arrows belong to this one vector. They all look similar in a certain way, they all point in the same direction, and they all have the same length, but they are all in different places. So, I tried to build up this worksheet.

The main goal of the lesson was to familiarize the students with the concept of vectors. In this respect, it was important for Mrs. Heinrich to explain the differences between vectors and points, as well as between vectors and arrows. For this purpose, she used a geometric interpretation to the concept of vectors, in which they are depicted as arrow classes. From the teacher's point of view, vectors seem to be geometric objects, which are described by the arithmetic representation as tuples.

Mrs. Heinrich used the advantages of the software edVR to achieve her lesson goal by positioning points and different arrows with the same length and direction at different positions in space. Among the activities in the lesson, the students plotted the points and vectors by entering parameters and could then recognize that "quite a lot of arrows belong to this one vector." Such a focus on a concrete mathematical goal that is addressed by the integration of digital technology is only possible with a content-related media competency in relation to VR technology associated with a subjective domain of experience on VR. However, the data does not reveal whether this is a subjective domain of experience isolated from other ones activated in the planning and implementation of lessons in analytic geometry in other contexts (e.g., in relation to the use of pens for visualization).

In another interview excerpt, Mrs. Heinrich highlighted the interaction between the students and elaborated on the methodological design of the lesson:

> Mrs. Heinrich: Then, they always worked in pairs and one student explained the procedure to the other one: Enter the point, enter the vector, what do you see? They were allowed to switch, of course, but the task was that one student enters it and also verbally describes what the image shows. The students were also supposed to record these descriptions on paper. It was intended in such a way and it also took place in such a way that everyone then took a look through these VR glasses at the end and everyone also entered something, but the first task was to really describe in one's own words to the partner what one actually sees there. That was the goal and is also very difficult. [...] So, if someone just says, I see a line, then the partner doesn't know what is meant by it, because the picture does not appear in front of their eyes. So, they were forced to use a more or less precise language.

During the VR activities, the students worked in pairs. One student put on the VR glasses and carried out various tasks. In each case, the image in the VR glasses was described to the partner. Interestingly, the teacher does not see the fact (because of the number of available equipment) that not everybody could use VR glasses at the same time as an issue, but as a methodological opportunity. She highlights that the students are forced to use precise language so that the other students can understand the description without seeing the image. Therefore, Mrs. Heinrich demonstrates well-developed pedagogical media competencies in relation to VR, because she constructively pushes the limits of visualization with the VR goggles, which can only be used by one person, to foster the communication competency. Furthermore, she connects this to the content goal of developing the concept of vectors (content-related media competency). In addition to creating opportunities for conversations about mathematical objects, monitoring the interactions between students is also an essential task of the teacher, as for example Mariotti (2009) found out in the context of dynamic geometry software activities. However, reflective interviews do not allow us to draw conclusions about the teacher's concrete methodological decisions during the lesson, e.g., spontaneous intervention in student conversations.

The student to whom the representation from the VR goggles is explained should record the descriptions on paper. This relates to the problem already indicated in the preliminary discussion on GeoGebra, that the experiences of students with digital technologies can be fleeting. The approach of noting down experiences and findings (developed here for the use of VR in the classroom) could also be applied to GeoGebra. However, with regard to GeoGebra the teacher makes the conscious decision not to use it, which could indicate isolated subjective domains of experience.

Part 4: Reflection on alternative analog teaching media

A major concern of the interview with Mrs. Heinrich was the reflection on the use of other digital or analog teaching media to introduce vectors. With regard to analog media, Mrs. Heinrich stated:

> Mrs. Heinrich: Yes, pens. Pens, but it becomes difficult with the points. So, I need someone for the coordinate system, and I have to hold pens. I had also thought about it, but I have never done that. I got myself some modeling clay so that I can fix my pins in the clay so that at least I have a coordinate system. That might work, but as I said, it's a bit difficult with the points. Then, you have to use small balls, but they don't stay in the same place. Then, it changes again, but I think that this is still helpful as a supplement, because that's something they can also use in exams. Nobody forbids them to make a coordinate system and to hold pencils somehow. That's why I tell them in class: "If you can't imagine it now, then try to work with it" (indicates pens with hands). I do that anyway. I don't leave that out, because they don't necessarily have VR glasses at home either, where they can just look at something.

Mrs. Heinrich compares the VR app and the physical model made of pens with regard to the technical visualization possibilities, where she sees more limits for the physical model. It is noticeable that the specifics of using VR that Mrs. Heinrich mentioned in other parts of the interview do not play a role in the comparison with the approach using pens. For example, with regard to VR goggles, Mrs. Heinrich explains in detail how she deals with the fact that not every student can view the image in the VR goggles at the same time. She sees this sensing imbalance as an opportunity for communication between the students. However, this aspect does not come up when comparing the two approaches, although it is a notable difference (the pen model can be viewed by more than one person at a time, presumably developing different interaction dynamics). This again illustrates the general isolation of the different subjective domains of experience.

One possible interpretation of this observation is that Mrs. Heinrich is not making the comparison from the perspective of the subjective domain of experience on VR, but from another one in which there is no direct access to all experiences related to VR. For example, it would be conceivable that the comparison is made from the subjective domain of experience on the visualization with pens. Our previous interpretation suggests, that the motivation for engaging with VR also developed in this subjective domain of experience. Accordingly, the comparison of the two approaches turns out to be more superficial than it would have been expected with reference to the rest of the interview and focuses on organizational advantages.

Part 5: Reflection on alternative digital teaching media

On this instructional and rather superficial level, Mrs. Heinrich also argues in the interview for the combination of analog and digital approaches. Students do not have VR goggles at home and are not allowed to use them in exams, so they should also learn about visualization with pens.

In response to a question from the interviewer, Mrs. Heinrich also stated the following concerning alternative digital teaching media for the lesson:

> Mrs. Heinrich: Yes, you could probably do it with GeoGebra, I think. I'm not sure at all whether the graphing calculators that the students have, whether they can maybe do that. I think they can also, if they are not in exam mode, they can also theoretically draw geometric stuff, but then of course you are not in the space, but it is always just a top view. That's a bit different and I imagined that you have to move to see something from another perspective. It's actually a bit more authentic than if you only move the system [...] The graphing calculator is too small and it's not colored or anything, and I can't really operate it that well. I would have to really familiarize myself with it. It would take me far too much time to do that. GeoGebra, I think, is actually also an alternative. Apart from the one disadvantage that I just mentioned compared to VR, I don't know if that would be an option. Yes, it is. In the meantime, I would say, I don't know. So, you could do that.

Mrs. Heinrich mentioned GeoGebra and the graphing calculator as two possible digital alternatives. Both of these can be used to create three-dimensional representations in analytical geometry in a two-dimensional projection. However, in both cases, only an external two-dimensional view of the representation is possible, whereas, with the VR glasses, one is "in the space" and can move around in 3D in the constellation. In this, she imagines a substantial advantage of the representation with the VR glasses.

A possible interpretation of the way in which Mrs. Heinrich comes to this conclusion can be given with the subjective domains of experience concept. The teacher sees a substantial problem in the spatial imagination of her students. This has led her to make visualizations with pens. Her experiences on this are stored in the subjective domain of experience on pen-visualizations and make her able to handle analytic geometry lessons. With this subjective domain of experience activated, she has not found GeoGebra and the graphing calculator to be good alternatives for achieving the goal of concept building in analytical geometry, maybe because they represent the spatial mathematical objects only in two dimensions as parallel projections. However, VR technology is considered a good alternative (see the previous interview section). Consequently, the comparison between VR technology and the other digital teaching media within this subjective domain of experience is also again relatively clear in favor of VR technology—although she has only minor experience with it.

Regarding the comparison with the analog approach, organizational and technical reasons are added as arguments that do not focus on content or pedagogical aspects. For example, the graphing calculator is too small, she has too little operating competence with it, and a detailed familiarization with the technology takes too much time. This estimation is striking, as she probably would have to invest at least the same amount of time to be capable of using the VR technology. Mrs. Heinrich regards GeoGebra as an adequate alternative for analytic geometry and only mentions the difference in representation as a disadvantage. At this point, it is interesting that she does not address the fleetingness of the GeoGebra experiences mentioned at the beginning, which could be an advantage of the analog alternative. This fleetingness is part of the subjective domain of experience on GeoGebra; however, it is obviously disregarded when comparing the teaching media in relation to the concrete lesson but perhaps rather occurs out of the subjective domain of experience on pen-visualizations.

3.3 Discussion

In the previous section, a description of the self-reflections of the participating teacher (Mrs. Heinrich) was carried out based on the MPC-model. In doing so, key assumptions of the model were found in the interview. In the MPC-model, it is assumed that a teacher's professional knowledge cannot be considered in isolation from other aspects of professional competencies, such as beliefs, but should be viewed holistically. This has been illustrated by the case study by describing beliefs, e.g. regarding the fleetingness of digitally gained knowledge or the teacher's self-efficacy beliefs regarding the operation of graphing calculators, which clearly determined the planning and implementation of the lesson.

Another paradigm of the MPC-model is the integrated consideration of digital and analog approaches under the umbrella term “teaching media”, which assumes a mediating role between the students and the mathematical content. Nonetheless, this does not preclude digital teaching media from providing features for mathematical learning processes that differ from the opportunities provided by analog media. This aspect is also touched in the reflective interview with Mrs. Heinrich. The teacher sees the motivation for her engagement with VR technology in the perceived limitations and challenges of her previous experience with an analog tool (visualization with pencils). She compares the potentials of both media from the perspective of her previous experiences and concludes (resp. imagines) that VR technology may hold some advantages. The decision to use VR as a teaching medium can thus not be adequately described separately from reflective processes regarding other digital or analog teaching media.

The third basic assumption of the MPC-model is the situatedness of subjective experiences in relation to teaching media and teaching in general. Situatedness can be found at various points in the interview with Mrs. Heinrich and can be described in the MPC-model by specific subjective domains of experience. In the interview, the teacher activates at least three subjective domains of experience that can be distinguished throughout the analysis, which relate to GeoGebra, the visualization with pencils, and the use of VR. At various points in the interview, we can assume that these are partly distinct subjective domains of experience.

As a fourth basic assumption of the MPC-model, the detailed comparison and the associated adequate selection of a teaching medium for a learning situation is only possible from the perspective of a superordinate networking subjective domain of experience. Without such a subjective domain of experience, the competencies in dealing with different teaching media cannot be completely activated at the same time. This was also observed in the case of Mrs. Heinrich. Although she was able to compare different teaching media on a superficial technical level, she did not include some of the features she previously highlighted as crucial with respect to VR technology—an in-depth level of reflection addressing M, P and C is clearly missing. This can be explained by the fact that the comparison seems to be conducted from the perspective of the subjective domain of experience on pen-visualizations. Nevertheless, she conducts a reasonable selection of VR as a teaching medium.

4 Conclusion and outlook

The MPC-model takes essential basic assumptions from the TPACK model and suggests extensions on a theoretical level for a more holistic perspective on the use of (digital) teaching media in mathematics classrooms. Although the TPACK model can provide an adequate basis for describing the knowledge required for the subject-specific use of particular digital technologies (e.g., Drijvers et al., 2013), the strengths of the MPC-model is the focus on comparisons and selections of teaching media, e.g. the similarities and differences perceived by the teacher in a special classroom situation or concerning a special learning goal. Furthermore, it is the addressing of competencies instead of knowledge. These competencies can be described with the help of subjective domains of experience. The findings demonstrate the importance of concrete situated teaching experiences for the development of professional media competencies. The extension to MPC also advocates for including a reflective level, which seems essential for the professional selection of a viable teaching medium for a specific teaching situation. Therefore, the MPC-model may provide a useful descriptive qualitative framework for subject-specific professional media competencies. It may also enable a more holistic view of the network of individual components of the media competencies of teachers. Concerning this aspect, the MPC-model may be seen as an important step further than the *DigCompEdu* (Digital Competence of Educators; Punie & Redecker, 2017) model, which also lists explicitly required competencies when dealing with digital media.

The reflective interview with the mathematics teacher Mrs. Heinrich illustrates how the proposed MPC framework might be able to depict the complexity of professional teaching activities in the course of digital transformation and the constantly growing challenges to the design of mathematical teaching–learning processes. The statements of Mrs. Heinrich indicate that professional media competencies can be regarded as highly situated. Her experiences with VR refer exclusively to a very specific application in the field of analytical geometry and are, to a large extent, isolated from other experiences with digital technologies (e.g., GeoGebra) and analog approaches (e.g., visualizing spatial geometric constellations with pens). Mrs. Heinrich compares possibilities and limitations of the teaching media on a rather superficial organizational and technical level, although elsewhere in the interview she reports very profoundly and professionally on well-founded VR experiences. The MPC-model explains this by the fact that she could not develop a superordinate and networking subjective domain of experience and, thus, could reach a reflective level only to a certain extent. Nevertheless, Mrs. Heinrich makes a meaningful decision and possibly can design meaningful teaching–learning situations with VR—in specific situations.

The MPC-model may function as a meaningful basis for describing the professional media competencies of mathematics teachers and seems feasible to be used for qualitative descriptions of context-specific and reflection-based elements of these competencies. The interview with Mrs. Heinrich explicated that interview questions on the comparison of different media for specific applications in mathematics teaching can make the reconstruction of subjective domains of experiences possible. However, we should emphasize that many of Mrs. Heinrich's statements in the interview represent self-reflections and, therefore, have to be cautiously classified, as it was done when analyzing the case. In particular, they do not allow to draw direct conclusions about her use of the media and interactions with students in the classroom. Further limitations arise of course from the fact that only one individual's teaching unit was considered. Nevertheless, the detailed look at this specific situation should help us to explain the MPC-model and its application to the reader. However, this of course does not provide an empirical validation of the basics of the MPC-model. In contrast, the findings of this case study have to be seen as a basis for the construction of hypotheses for up-coming empirical investigations. These hypotheses arise on how the levels of reflection in the context of professional media competencies can be measured (qualitatively or quantitatively), taking into account the situatedness challenge, and by which factors the emergence of such levels of reflection in the professional development of teachers is affected. A possible starting point for operationalization could be the description of potentials and challenges of certain digital or analog media in given teaching situations within questionnaires or reflection diaries. In general, the context-specificity of professional competencies, as well as the explicit and implicit reflective processes on the use of digital technologies, should be considered more strongly in future mathematics education research—further future research to validate the proposed MPC-model could deliver good occasions to do so.

Funding Open Access funding enabled and organized by Projekt DEAL. The Funding was provided by Ministerium für Kultur und Wissenschaft des Landes Nordrhein-Westfalen, Ministerium für Schule und Bildung des Landes Nordrhein-Westfalen.

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