ORIGINAL PAPER



Ethical dilemmas and professional judgment as a pathway to inclusion and equity in mathematics teaching

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Accepted: 15 December 2023 © The Author(s) 2023

Abstract

This study focuses on ethical dilemmas that arise in moments of inclusion and equity in mathematics teaching and how they might be tackled through teachers' professional judgment. Skovsmose's inclusive landscapes of investigation approach was used to design the study and to collect teachers' joint reflections on moments of inclusion and equity in their teaching. Ethical dilemmas and professional judgment were the analytical foci for a qualitative thematic content analysis. Three explorative workshops were held with two teams of teachers from two schools in Sweden. The analysis identified three themes of ethical dilemma, and ways in which these were responded to by teachers' professional judgment: (1) dilemmas of diversity and acting justly; (2) dilemmas of resources and allocating them fairly; (3) dilemmas of values and recognising diversity. We conclude that mathematics teachers' professional judgments involve showing bravery, going outside of the norm, negotiating values and duties, listening to the students, and throughout this, engaging in collegial learning in the best interests of the learner.

Keywords Ethical dilemmas · Equity · Inclusion · Mathematics teaching · Professional judgment · Teacher education

1 Introduction

Ethical aspects permeate research and practice devoted to facilitating high quality mathematics teaching. Nevertheless, studies show the diverse challenges and dilemmas faced by teachers of mathematics in their mission to provide equity and inclusion for every learner (Askew, 2015; Atweh, 2011; Planas et al., 2018; Tan et al., 2019). It is when teachers' responsibilities, obligations, and care for learning clash with certain ethical principles and values that what we label "ethical dilemmas" arise (Bergman, 2009; Bornemark, 2020). These dilemmas are temporal, highly complex and recurrent, which indicates that teachers' professional judgments in response to these are also conditioned by the temporality and complexity of the instantaneous moments

in which they arise. Hence, professional judgment warrants greater research attention (Frelin, 2014).

These moments of tension in which dilemmas arise are especially prominent in relation to aspects of inclusion and equity in mathematics education, as these involve highly complex processes and multitudinous challenges (Atweh, 2011; Kollosche et al., 2019). In this article we use moments of inclusion and equity to refer to moments within the fluid, lived context of teaching in which teachers make in situ decisions, handle frictions, take care of possibilities for, or counteract obstructions to, inclusion and equity. When we refer to inclusion and equity as embedded in these moments, we refer to how teaching affords all students equal opportunities to learn, feel empowered and have agency in mathematics (Bagger & Roos, 2023; Roos & Bagger, 2021). We understand ethical dilemmas during these moments of inclusion and equity as manifestations of how values in education are challenged both at a classroom and a national level (Frønes et al., 2020).

Ethical dilemmas must be met with good judgment and professionalism (Howe et al., 2018) to be resolved. Unresolved ethical dilemmas in the mathematics classroom can threaten students' inclusion and equity in education both in that moment and in the longer term, and at a systematic

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Published online: 12 January 2024



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level. The classroom context, teachers' knowledge, their actions and the choices they make are at the core of this (e.g., Civil & Planas, 2004; Chronaki, 2018). As Biesta (2015) claims, in the field of education it is especially important to explore the conditioning of teachers' possibilities to act on their professional judgment. This study contributes to this by furthering knowledge on how professional judgment in response to ethical dilemmas act as a pathway to inclusion and equity in mathematics teaching. The following research questions aim to fulfill this purpose:

- 1. How are ethical dilemmas conditioned in teachers' talk of moments of equity and inclusion in mathematics teaching?
- 2. How is teachers' professional judgment connected to these ethical dilemmas?

Consequently, we add to the body of research exploring the role of ethics in mathematics education. We do so by adopting the perspective of *ethics of care* and draw on Noddings (2013) thinking in the sense of "how we should meet and treat" (p. x) students in *caring for* ideas and people, something which takes place through caring relations. Core is the recognition of the teacher's attention and response in terms of professional judgment, which we approach with inspiration from Boylans (2016) writings on the ethics as intrinsic to the situational and complex decision making by teachers. Consequently, we frame mathematics teaching as including respect and allowance for a diversity of ways of learning, learners, teachers, relationships, ideas and thinking. As stated by Noddings: schools "can work toward establishing an environment in which caring-for can flourish" (p. 169).

2 Mathematics teachers' professional ethics

In mathematics education there are different considerations of ethics. Scholars following the philosophy of mathematics, sociopolitical or critical approaches are often dedicated to exploring different ethical aspects of teaching (e.g., Athweh, 2011; Ernest, 2018; Radford, 2023; Vale et al., 2016; Atweh & Brady, 2009). In this article we do not pursue mathematics as ethics, instead we focus the ethics *in* mathematics and are interested in the socially constructed, relational and ethical work of the teacher (Müller, 2022). Consequently, we do so without delving into, or analyzing if and how teachers are practicing normative virtue, duty or utilitarian ethics (see Boylan, 2016; Dubbs, 2020).

Mathematics education research has explored the teacher's role from different ethical perspectives. For example, Ernest (2018) highlights three aspects of mathematics teachers' ethical obligations. First, the teacher has a duty

of care for the learners. Second, the teacher must strive to teach mathematics effectively to benefit all learners, and third, teachers have a responsibility for the development of the teaching profession itself. The duty of care is also referred to in the literature as "ethics of care" (Dubbs, 2020; Morvan, 2017; Simanjorang et al., 2021), which implies the need for teachers' increased understanding of the learners' perspective, and of their cultural and ethnic background (Hâwera & Taylor, 2017). For example, authentic mathematical problems from the learners' own reality have been proven to develop learners' critical numeracy and thus bring ethical and social issues into the teaching of mathematics (O'Keeffe & Paige, 2021). In turn, such an approach holds the potential to teach and work within the zone of proximal development (ZPD) (Simanjorang et al., 2021). Abtahi (2021) refers to the ZPD in terms of being ethically sensitive to the learners' perspectives and challenges in mathematics.

Collaboration between teachers and learners has been discussed as an ethical form of human collaboration and joint labor in the mathematics classroom (Radford, 2016). Empathetic care has a special role, therefore: it is the care teachers show as they support learners to overcome challenges, with an awareness of how sociopolitical context and experience may impact learners (Matthews, 2020). Hence, mathematics education can be a natural arena for discussing sociopolitical and societal issues from an ethical perspective (Miller, 2022). The sociopolitical turn in mathematics education highlights the need for more ethical discussions in mathematics (Register et al., 2021). A similar concern also informs the more recent spiritual turn in mathematics education research, in which values and ethics are centered and relational understandings of knowing, existing, and healing are embraced (Gutiérrez, 2022). An example can be found in the argument that learners' mathematical knowledge can be strengthened by using ethical problems (Vecchio, 2021). Though, the teaching culture in the subject of mathematics most often identifies mathematics as a non-value-driven subject, leading to difficulties to even consider bringing ethical and societal issues up for discussion in mathematics teaching (Kuusisto et al., 2016; Lyakhova et al., 2019).

2.1 Ethical dilemmas in mathematics education

Ethical dilemmas can be seen in the valuing of logical thinking in relation to valuing learners' self-confidence. Here, the dilemma is seen between mathematical values and the teacher's desire to "nurture" the learners (Bills & Husbands, 2005). This dilemma arises from different perspectives on what mathematics is; however, the teacher has an ethical responsibility to nurture an epistemic diversity in mathematics (Agarwal & Sengupta-Irving, 2019). In relation to this, Abtahi (2022) mentions ethical dilemmas that may



arise when employing the cultural, historical, or linguistic resources of non-Western communities to teach mathematics. Here, the intention may be good, but in practice we must reflect on the "possible harm that mathematics and its learning and teaching could cause for members of different communities" (Abtahi, 2022, p. 16).

Another ethical dilemma appears when priorities relating to external demands, such as performance in school rankings, are in tension with the best interests of the learner. For example, Ingram et al. (2018) describe how teachers exploit rules in systems of examinations to maximize test scores and the school's position in rankings. National school competition pushes teachers into ethical dilemmas and could lead to teachers deciding to teach superficial knowledge before the exams rather than a deep conceptual understanding (Njiru et al., 2020). This 'teaching to the test' training is criticized as an unethical method, as it advocates a mechanical view of knowledge (Moran, 2008) and leads to learners feeling alienated (Dematté, 2022). This alienation is a form of violence that occurs when learners are forced to learn mathematical steps without meaning, creating unethical relationships between teacher and learners. For the relationship to be ethical, the teacher and the learner must engage in an open conversation (Dematté, 2022).

2.2 Ethical dilemmas and professional judgments

In this study, how teachers tackle ethical dilemmas is scrutinized in terms of how teachers advocate for their professional judgment. Teachers face many challenges during a lesson, during which thousands of decisions are made. It is only in the moment of (in)decision, when values clash and no easy or ideal solution presents itself, that the organization, teaching, and teacher are put to the test (Pope et al., 2009). We understand these moments when values clash as instances of ethical dilemmas. If an ethical dilemma is not resolved, it might lead to ethical stress in teachers (Bornemark, 2020; Howe et al., 2018). We emphasize that unresolved ethical dilemmas could contribute to ill health for both teachers and students, especially if it hinders teaching that promotes inclusion and equity. As ethical dilemmas need to be met by good judgment and professionalism (Howe et al., 2018). being able to respond to ethical dilemmas involves training and experience of exercising professional judgments when dilemmas arise (Bornemark, 2020). Teachers' professional judgment entails making momentary but complex decisions without fully knowing what will ensue. These moments exist on a continuum of ethical principles and will continue to resurface in new, unresolved forms (Frelin, 2014). Furthermore, we draw on Boylan (2016) in our understanding of ethical dilemmas as demanding that the teacher's decision making derives from relational awareness, a sphere of action and ethical thought in the care for the students' learning and ideas.

3 Theoretical underpinnings: inclusive landscapes of investigation

We draw on Skovsmose's (2019, 2022) model for 'inclusive landscapes of investigation', which "are landscapes intended to be accessible for different groups of learners, whatever the differences concerning their abilities or social diversities" (Skovsmose, 2022, p. 186). This informs the overall approach to the research when we gather teachers' experiences of ethical dilemmas in moments of inclusion and equity in mathematics teaching. In other words, we understand the teachers as learners with a diversity of skills, abilities, and social positioning and they too need to be able to access the research as a joint investigation. The landscape of investigation thus governs our approach to the data collection and how to collaborate with the teachers.

Inclusive landscapes of investigation include three major elements: facilitating investigations; accessible for everyone; and facilitating collaborations (Skovsmose, 2019). By using this approach, we invite teachers to explore their own and each other's learning environment, through a joint investigation of the connections between ethical dilemmas and professional judgment in moments of inclusion and equity.

Facilitating investigations refers to how mathematics teaching invites engagement in processes that focus on inquiry or, in our case, how research invites the team of teachers to engage with inquiry into moments of inclusion and equity. To make mathematics education accessible for everyone, joint workshops were held with a focus on exploring how to create an accessible environment for all. Throughout the process, we shared the researchers' analysis to the participating teachers and hence made accounts of ethical dilemmas and professional judgment accessible to them. These workshops thereby facilitated collaboration, as teachers explored how to promote mathematics education that creates possibilities for inclusion and equity by advocating for how they used their professional judgment in response to ethical dilemmas. Here, the differences among learners, teachers, and schools helped teachers unravel, challenge, and cultivate the learning environment. Understanding the importance of shared engagement and how boundaries between differences must be crossed through dialogue were central to the workshops. In boundary-crossing dialogues, "the very notions of normal, not-normal, ability and disability lose significance" (Skovsmose, 2019, p. 81). This is shown in the study by a deliberate facilitation of



meetings in which a diversity of ideas, personalities, knowledge, and understandings were shared.

4 Methodology

Joint exploratory workshops were carried out with two schools and two teams of teachers. This design advocated a critical paradigm of validity procedures where the lens of the researcher is understood in terms of reflexivity and the lens of the study participants is oriented to collaboration (Creswell & Miller, 2000). The exploratory nature of the workshops was necessary because, as Skovsmose (2022) points out, when the communication cannot be expected to follow predictable patterns, it calls for dialogue, which in turn helps to establish an activity. Also, the exploratory nature allowed for reflexivity exploring the role of ethics in mathematics education.

4.1 Selection

Two participating schools, one in the north and one in the south of Sweden, were selected to achieve depth and breadth in terms of diversity of learners and mathematics classrooms. Both schools have a special interest in inclusion and equity and expressed a need to develop mathematics teaching to better include all learners and secure equity.

The southern school is situated in a small, industrial, working-class area, with a low socio-economic profile. In the school, there are some learners with backgrounds other than Swedish and a few newly arrived refugees. Eight teachers from grade F–3 (6 to 9 years old) participated in the study. All the teachers have a general teacher education for primary school. Approximately 450 learners attend this F–9 school (learners from 6 to 15 years old).

The northern school is situated in a mid-sized industrial city with blended socioeconomic neighborhoods surrounding it. There are some learners with backgrounds other than Swedish, but not many. This school has invested in developing good and inclusive learning environments using both digital and physical tools and furnishings. In this school, six teachers from grade F–3 participated in the study. All the teachers have a teacher qualification for pre-school and primary school. Approximately 300 learners attend this F–6 school (6 to 12 years old).

¹ The data in this article is collected within the ethically approved project, Mathematics Education for Inclusion and Equity (MInE).



4.2 Data collection and operationalisation of landscapes of investigation

Six focus group interviews (three at each school) were held in a workshop format over the course of one and a half years. The focus groups were audio recorded and transcribed, lasting for approximately two hours. During the workshops, the way we interacted with the teachers was informed by the inclusive landscapes of investigations approach (see Skovsmose, 2019; 2022), while they explored the learning environment by focusing on moments of inclusion and equity. These workshops and this approach constitute a joint endeavor to explore experiences of teaching mathematics, facing ethical dilemmas, and enacting teachers' professional judgment. In doing so, we got a record of the teachers' relational awareness, sphere of action and ethical thought on ethical dilemmas they faced (see Boylan, 2016).

It was important to establish and agree on the rules for, and the nature of, the conversations in the workshops to facilitate the collaboration and to make it accessible for everyone. The approach in the workshops varied slightly depending on the teachers' understandings, as well as our own individually and jointly. In line with the ethics of care, we adopted the perspective that a diversity of ideas and understandings should be heard, and we did not define in advance or during the session what inclusion or equity should be. The exploration of diverse meanings, knowledge, ideas and understanding of these was instead a joint endeavor.

One initial meeting preceded the workshops in which we presented our understanding of moments of inclusion and equity as the situated, fluent, lived and highly contextual moments in teaching when teachers make in situ decisions, handle frictions, take care of possibilities for, or counteract obstructions, for equal opportunities to learn, feel empowered and to have agency in mathematics (Bagger & Roos, 2023). This understanding leans on the assumption that it is not only what happens in the classroom that is at stake, but also questions of power and democracy at the societal level. Inclusion is affected by structures and processes in society (Halai et al., 2016). This was discussed with the teachers to allow for diverse understandings of inclusion and equity. Since the contextual aspects are key in moments of inclusion and equity, what counts as inclusion or equity is expected to vary in each classroom. This exploration was intended to facilitate the collaboration and prepare for the joint investigation. This means that we also expected and welcomed contradictions and confusion in our joint investigation, as these would allow for diverse ideas and understandings to come to the fore. After the initial meeting the teachers made personal notes on their understanding of moments of inclusion and equity and wrote down examples of these from their mathematics teaching. Following this, in workshop 1 we facilitated *the collaboration* in the *joint investigation* by discussing moments of inclusion and equity that built on a content analysis of their personal notes on *what moments of inclusion and equity could be*, and *how they could be seen in the classroom*. These two questions were then further explored during the workshop.

In workshop 2 we started by displaying themes that derived from a content analysis of the discussions in workshop 1. We also reflected on the conversation and climate during workshop 1 and how they experienced it as a form of member checking to ensure validity and establish credibility (see Creswell & Miller, 2000). The themes were presented as case descriptions of ethical dilemmas. This prompted the teachers to undertake a *joint investigation* of the displayed, and new, ethical dilemmas. We posed questions to reveal both what constitutes the ethical dilemmas and possible ways to handle them in line with professional judgment. Possible questions to prompt reflections on ethical dilemmas include: What values clash? Why is this/is this not inclusion or equity? What was at stake? What in this makes it sensitive? Questions to prompt reflections on professional judgment include: What could you then do? What did you think could have been done? What did you wish that you could have done? What decisions did you make? In workshop 3, the previous focus group interviews were revisited in the same manner as in workshop 2, and new ethical dilemmas were presented. We used the same questions as in workshop 2.

4.3 Principles for analysis

The ethical dilemmas and ways in which they were met by professional judgment was analysed with inspiration from Braun and Clarke's (2006) six steps of thematic analysis: (1) familiarizing oneself with the data; (2) generating initial codes; (3) searching for themes; (4) reviewing themes; (5) defining and naming themes; and finally, (6) producing a report (p. 84).

In step 1, we separately read the transcripts and selected reflections on moments of inclusion and equity in the material. In step 2 we coded for ethical dilemmas by identifying situations in which values or justice were challenged or clashed with one another, and what values were challenged or clashed, which was our definition of an ethical dilemma (see Bornemark, 2020). We coded for professional judgment in accounts of how decisions, actions or arguments were made (see Frelin, 2014). First, we coded separately and then compared initial codes to gain reliability. The codes were named by words describing the challenge/clash or decisions/actions/arguments made. This made some codes alike, although the specific moment of inclusion made them work

differently. In step 3, initial themes were jointly construed from the coded material in terms of connections between ethical dilemmas and professional judgment and how these two were conditioned. After thematizing the ethical dilemmas and teachers' professional judgment in step 4, we validated the analysis by member checking. At this stage the teachers who had participated in the workshops reviewed the themes in order to, as Creswell and Miller (2000) describe, establish the credibility of the analysis. With that in mind, in step 5 we continued to review the initial themes, and to refine and label them.

Examples of the analytic procedure with codes are presented in Table 1. There is some overlap between themes as there are no clean borders.

5 Ethical dilemmas and professional judgments

In the following, we have displayed the account of teachers' professional judgment as they explore ethical dilemmas. Ethics of care has guided the analysis of teachers' decision making in terms of how they could and would have preferred to meet and treat (see Noddings, 2013) their students. Not surprisingly, both justice and fairness were voiced by teachers. Ethical dilemmas often meant that students relative and experienced fairness in the classroom and/or their justice as stated in governing documents and laws, was put at stake (see for example Goldman & Cropanzano, 2015). The themes identified can be understood as social constructions residing in the situational and complex decision making. Three themes were identified through the thematic analysis: (1) dilemmas of diversity and acting justly; (2) dilemmas of resources and allocating them fairly; (3) dilemmas of values and recognising diversity. These show typical ethical dilemmas and how they relate to actions, decision making or approaches in terms of teachers' professional judgment to facilitate inclusion and equity in their teaching: acting/ allocating/recognising. This is followed by a value that was challenged in the dilemma: justly, fairly, diversity.

5.1 The dilemmas of diversity and acting justly

The *ethical dilemmas of diversity* emerge in the friction between how to fit diverse learners into "the norm" of what students often know and how they usually work. This was expressed at both schools as challenges to support learners in severe mathematical difficulties: "You do the best you can, but it's hard because they [the learners] are so extremely weak." These moments concerned both equity and inclusion and were coded with *normality*, *knowledge*, and *exclusion*, indicating that the teachers struggle to include learners who



Table 1 Overview of the analytic procedure, showing a selection of codes and the three themes

Moment of inclusion regarding a girl during collaboration with peers	Codes		Theme
	Ethical dilemma	Professional judgment	
"But that maybe she's not included, because she lacks knowledge. She can't learn, like, she doesn't have the skills, maybe"	Excluded Lack of knowledge Diverse knowledge Challenges to learn		Dilem- mas of diversity and act-
"Everyone is somehow included, because they are given the opportunity to answer a question that I think they know. I think that then, they feel that "wow, I could too. I am also one of those who succeeded now and feel included"		Adapt content Adapt approach Diverse teaching Make students knowledge visible	ing justly
Moment of equity regarding a student who needs a 100 box to be able to count	Codes Ethical dilemma	Professional judgment	Theme
"I sat and did the assessment with some of them when they must count. Then there were some students who could make it with the 100 square, but not without"	Conflict of justice Un/fair support Test validity Students access hindered		
'But in this test she could still have had the 100 square in front of her, because then that test would hopefully have shown that she can do this if she is allowed to use the 100 square"		Decision to give access to aids Actions to protect the child's right	Dilem- mas of resources and allocat- ing them fairly
Moment of inclusion regarding the mathematical learning environment	Codes Ethical dilemma	Professional judgment	Theme Dilem- mas of
"To get such a [learning] environment, you work a lot with values, and mindset also. [] But it is hard practically [] even if you talk to them [the learners] about mindset and learning for their own sake there is always someone who [doesn't take it in]."	Values Mindset Tradition Diversity Learning style Different		values and rec- ognising diversity
"It is important that you have an atmosphere in the class that makes them dare. Dare to ask for help, and it's okay for another student to help me. But it really turned out to be such a moment"		Atmosphere Following Listening	

are unusual in their learning progress, which then threatens students' equity in learning. Another example of this was when another learner with low levels of knowledge could not participate and was referred to as being "included both socially and physically, but not mathematically [...] she is included but excluded [...] she is hard to include. [...] she can be good in different areas. We have often experienced that." This example was coded with exclusion, knowledge and, in addition, social inclusion. Even when recognising and valuing diversity, this teacher nevertheless struggles with how to include the student who is diverse from the norm. This case shows some of the dilemmas teachers face when they aim to work inclusively and to secure students' equity in teaching. A similar fstruggle was displayed during moments of inclusion and equity when students with diverse levels of knowledge are evaluated according to national standards in a uniform way, regardless of their individual prerequisites and whether that learner might need different time, tasks,

instructions or placing in order to learn mathematics. These sections were coded with *protection, low knowledge, unfairness, and harm.* One teacher used a metaphor that resonates with this dilemma of diversity: "Because it's like this, so ... jump as high as you want, but [on] that particular day [of national assessment], you still have to jump 120 [cm]." Furthermore, professional judgments about national tests in mathematics in relation to the struggling learner often contained arguments about the responsibility to do no harm, and to instead protect the student: "...it is not fair to expose her to the test and that it would break her".

A dilemma of diversity was shown in relation to language, such as when teachers talked about a newly arrived girl who could not speak Swedish, in relation to a moment of equity. This example was coded with *diversity*, *adaptations*, *access*, and *representations* as teachers described the way they worked with number sense, trying to approach mathematics through a diversity of representations in order



to adapt and provide access, by "[...] work[ing] in so many ways. You nag about it in so many ways. It's both on the outdoor day, and then we go down to the canteen and you do it, and then you do it in the [mathematics] book, and then you play a game with it, so it has many ways." This example also displays a common theme in the professional judgments to act justly, which in this specific case could resolve the dilemma. This professional judgment was often coded pedagogical exclusion, fairness, justly. These kinds of dilemmas were often resolved by the professional judgment to act justly by listening to the learners and then drawing on the teachers' knowledge about mathematics and the student in the decision making regarding what to teach them: "that you kind of ask her" and trust yourself as a teacher when presenting mathematical strategies not to "teach them a lot of different strategies, it's [nevertheless] not good for those who are weak. [Instead] you simply must see here, which strategy is best [for the individual learner]. We interpreted teachers' choices and suggestions of approaches as a strategy in teachers' professional judgment to take diversity as a point of departure x in considering how to teach and act justly. This was also coded as a moment of inclusion regarding a learner experiencing severe mathematical difficulties:

She doesn't get included when we do it [in groups in the classroom]. She can't [join] ... it doesn't work, and the others in the group go crazy over [are not pleased with] her as well.

The teachers' conclusion was that it is ethical and just to adapt the teaching so that it suited a diversity of students, which could resolve dilemmas arising from diversity. This kind of judgment takes the standpoint that even when moments of inclusion and equity are highly challenging, it is the teacher's professional responsibility to find a way to adapt and adjust the content so that all learners can access the tasks: "It [support] can mean different things to different learners." The *professional judgment* to act justly also included the need to empower students, who, in the best case, should also learn about and be able to advocate for their support needs. The learner's participation also means that "the learner gets to feel that 'I actually can take responsibility for how I myself [need to work] ... which support suits me'."

Although there seems to be emphasis on diversity from the norm, sometimes the teachers make *professional judgments* that instead take diversity as a point of departure. For example, one teacher talked about her professional judgment in terms of changing direction in the way she thought about moments of inclusion and equity in mathematics by not focusing on the methods but rather the diverse needs of the learners: "Instead of thinking about methods, maybe

one should think about what diverse needs to include in the planning in order to include as many people as possible." Furthermore, the teachers talked about the importance of not rushing their mathematics teaching and being attentive to pace and intensity.

If you have gone too fast, you haven't been specific enough [in the teaching], or [you haven't explained] some word they don't understand, instead take it easy and take those parts. Then they will get it right away.

We conclude that ethical dilemmas of diversity and the corresponding professional judgments can be summarized as being a matter of protecting the learners' knowledge and development in mathematics teaching through listening to, and taking the learners perspective, but also to possess both subject knowledge and knowledge of teaching methods.

5.2 The dilemmas of resources and allocating them fairly

Teachers talked about *ethical dilemmas of resources*, such as, for example, time, space, and organisational factors, as being important for supporting students' inclusion and equity. Here experiences of moments of equity were mirrored by frictions when individual needs and the needs of the group were in tension and teachers felt that they were not enough. The *ethical dilemma* relating to resources was also connected to tensions between giving support for learning to learners with and without specific educational needs. Thus, the value put to the test in this dilemma was fairness. This was coded with: *(un)fairness, individuals' needs, groups' needs, support, alone, time, rooms, class,* and *insufficient.* For example, teachers addressed these frictions in relation to teaching mathematics to a full class alone:

One would actually have to sit next to [a learner]. But then you have the rest [of the students] in the classroom. How do you do it? How do you solve it? Because I think it's very difficult. [...]

Central to this dilemma were the organisational factors, rooms, spaces, and number of teachers: There "needs to be several [teachers] in the beginning [when the learners start school] to make it work. [...] It can be organized if you want. [...] You can 'redistribute' your resources in some way." Embedded in this dilemma is the understanding that the teacher and her professional judgment serve as outpost and gatekeeper for students' equity and inclusion in teaching. This example was visible in statements on how to create time and space for learners' learning, such as when one teacher stated that "when you have gone through and started



working, you know who you need to go to. [...] You must make time to sit with them." Here, the teachers struggled with being able to meet all the needs of the learners in the mathematics teaching and felt torn.

To deal with the dilemmas of resources, *professional judgments* on how to allocate these were put to the fore. We coded the *professional judgment* with, *contesting, solutions, fair, ethical, just* and *doing good*. This judgment was displayed as teachers reasoned on how to find time to meet the needs of every learner in the class, and how to set up the mathematics teaching so that it worked for the class: "It is difficult to adapt to individuals when you are alone, so then you have to do something like this with groups, I think." Key in this decision making was to pose self-regulating questions about how learners might feel in the setting, what they need to participate and when they can do so. The teachers stressed that to answer these questions, it is necessary to have time to observe and get to know all learners well.

The teachers argued for and against different standpoints and contested and tried different solutions to the dilemmas. One example was when fair, ethical and just *professional judgment* depended on the willingness to go against the rules. For example, when a teacher knows that the learner has knowledge that can be displayed during a test. The test required learners to write the number following 11 on the number line. The learner knew that it was 12 but could not write 12. The teacher made a *professional judgment* to use a mediating tool, even though the test instruction did not allow it, "and with the help of that, [the student] could find her way to showing her knowledge, namely, writing '12'."

We conclude that the ethical dilemmas of resources and the corresponding professional judgments to allocate these fairly can be summarized as being a matter of knowing the individual's and group's potential and challenges. Core in this effort is being able to take learners' feelings into consideration. Furthermore, professional judgment in the dilemma of resources was connected to rearranging resources to better provide for inclusion and equity in mathematics teaching. This demands that teachers have the courage to make decisions for the individuals and the common good, regardless of rules or restrictions.

5.3 The dilemma of values and recognising diversity

The *ethical dilemma of values* concerned both the values omitted in the teaching, but also the values connected to providing an inclusive learning environment and approach in the classroom. These dilemmas were coded with *values, mindset, tradition, diversity, learning style* and *different.* For example, in connection to moments of inclusion, teachers

at the southern school talked about values and recognising diversity as a challenge:

To get such a [learning] environment, you work a lot with values, and mindset also. [...] But it is hard practically [...] even if you talk to them [the learners] about mindset and learning for their own sake ... there is always someone who [doesn't take it in]. And even if you know their [the learners'] different learning styles, you might want them to do it [the mathematics] in different ways. [...]

The ethical dilemma of values represented here lies in the friction between working with established or traditional values in mathematics teaching and to thereby govern ways of doing things, and at the same time to teach learners to value a diversity of ways of thinking and doing mathematics. A difference between the schools in this dilemma was that the teachers at the northern school spoke about values in relation to collaborative work and the learners as a community, which was not as apparent in the southern school. For the teachers in the northern school, moments of inclusion and equity that held these frictions occurred because teachers wished to secure participation for all in the joint problemsolving activity, although it did not work for all students. This indicates a balancing act between the value of protecting learners and the value of supporting their participation in collaborative activities.

When the teachers make *professional judgments* about dilemmas of values, they stress the importance of listening to the learners and to follow the learners' paths to reach a better judgment on how to proceed:

Just this in delving into how the learners think. Equally, ask these questions then, as you always do, 'What were you thinking there?' ... not [focus on whether] the answer is wrong or if it's right, but you get so much out of asking. [...] just to delve into how someone else thinks—that can be an aspect of equity.

The professional judgment connected to these dilemmas circled around the valuing of diversity and was coded with teaching methods, monitoring, understanding, following, and listening. Expressions such as "everyone's on board" and "hallelujah moment" were used to explain how the teacher knew that every learner understood the method and task during moments of inclusion. To resolve these dilemmas teachers had to recognise diversity, including ways to work in mathematics teaching, to value and hold on to the vision of inclusion and equity: "We need to hold on, endure". [...] "You really have to believe in your idea" [...] because "without inclusion, no equity".



A teacher at the northern school pointed out: "I am thinking of all the personalities that come into play when we do this kind of task. We have everything from one-two-threefour-five-six ... [counts rapidly] to one ... two ... three ... four... [counts slowly]." This statement was followed by a discussion among the teachers about learners' pace and intensity as something concerning not only diversity in knowledge, but also personal traits that are important to respect. This indicates that among younger learners, it is central to professional judgment to recognise diversity so that the teacher can balance how to group learners not only according to knowledge and relations in the group but also according to the learner's pace and personality when planning for collaborative work. Another phenomenon in this dilemma was tensions regarding valuing all learners' knowledge. This was challenged as learners compared their knowledge to others: "We talk a lot about being different, and [having a] different pace and time ... and that you really do not need to learn exactly the same thing at the same time. That, yes, for me, it takes longer than for my friend to learn this. I need to keep trying." The professional judgment to recognise diversity here demanded that the teacher recognize the lack of knowledge or areas the learner needs to develop, but also is considerate in showing the learner what he or she knows: "Then they become proud and feel that 'I know this' instead of leaving with the feeling of 'I did not know this yet again".

In sum, the *ethical dilemmas of values* relate to frictions between values concerning social status, personal traits, and levels of knowledge or learning styles. There is no neat way to balance this. It comes down to the teacher's *professional judgment to recognise diversity*, to develop insights and be ready to make a qualified guess, and thereafter revise decisions on the go. We conclude that it is a matter of valuing the learner's voice, way of learning and understanding ideas, and for the teacher to have the stamina and will to stand by the belief in inclusion and equity in their mathematics teaching.

6 Discussion

The purpose of this study was to contribute to knowledge on professional judgment and ethical dilemmas as a pathway to inclusion and equity in mathematics teaching. To fulfill this purpose Skovsmose's (2019, 2022) inclusive landscapes of investigation was used in the framing and data collection. Ethical dilemmas and professional judgments were used as an analytical lens. The three dilemmas identified bring to the fore the complexity of factors at play in teachers' professional judgment during moments of inclusion and equity in

mathematics teaching, where the ethical obligations of the mathematics teacher reside (see Ernest, 2018).

As in earlier research within the area of inclusive education in mathematics (e.g., Scherer et al., 2016), the results indicate that teachers need to know the learners, their learning needs, and the mathematics subject knowledge itself to be able to exercise their professional judgment to facilitate inclusion and equity. This is especially visible in the dilemmas of diversity and the dilemmas of resources, as the teachers repeatedly refer to tensions between normalizing processes and the need to derive from diversity in preparing the teaching. The professional judgment that facilitated inclusion and equity in these dilemmas is connected to Noddings' (2013) approach to ethics of care in education. This is seen as this professional judgment can be understood as an act of care for and professional decision making founded in the teachers' faith and commitment to the learner. In mathematics education this encompasses the cognitive and affective dimensions in teaching (see Hackenberg, 2010). For example, it was key to balance the students' cognitive functions and feelings about themselves in mathematics, but also to recognize how that related to their peers' cognitive and affective settings. Also, the teachers' desire to 'nurture' the learners (see Bills & Husbands, 2005) was seen in these dilemmas.

Furthermore, earlier research has also shown that what is displayed in the *dilemmas of resources* regarding fairness is often being confused with sameness. What can then happen is that everyone gets the same amount of support, kind of support, or style of support regardless of need (see Frønes et al., 2020). This is equality at work rather than equity. In relation to this we join Ahmed (2007) in her reflection of how the language of diversity can shadow specific learning needs and individuals' rights or challenges. Following, the results show that striving for equity rather than equality is to be brave as a teacher, work outside the norm, and negotiate and maintain a good dialogue among colleagues concerning what is in the best interests of the individual learner in the specific context of teaching. To value diversity in learners' knowledge is seen in the dilemmas of values, and demands a professional judgment that includes care, empathy, listening, and a great deal of uncertainty in terms of whether they have done what is right. This is what scholars have labeled as being ethically sensitive to the learners' challenges in mathematics (Abtahi, 2021) and understanding the learners' background to be able to teach within the zone of proximal development (ZPD) (Simanjorang et al., 2021). Through this attention towards students' prerequisites and needs in learning, the ethics of care can be understood as an integrated part of teachers' professional judgments (e.g., Simanjorang et al., 2021), meaning that the teachers often



highlighted the understanding of the learners' perspective and individual needs.

Empathetic care (Matthews, 2020) was evident in all three themes as, in their professional judgment, teachers emphasised recognizing the diversity of students and valuing struggling learners in their aim to facilitate inclusion and equity through their teaching. This also implies teachers' intrinsic relational understandings of students' knowing, existing, and healing, as they exercise ethical reflexivity (see Gutiérrez, 2022; Register et al., 2021). This can be understood as a part of the spiritual turn in mathematics education (see Gutiérrez, 2022). This focus is especially visible in the dilemmas of values and recognising diversity. In these themes and on the topic of ethical reflexivity, there are interesting differences between the schools. In the northern school, with a quite homogeneous group of learners, it was considered important to group learners in collaborative work, which then generated tensions at times. The same was not seen in the southern school, in which group work was not stressed as highly valuable or challenging. Also, in the northern school, learners' comparison of their knowledge was more apparent than in the southern school that sometimes led to a discreditation of one's own or others' knowledge, which the teachers then had to handle by recognising and valuing diversity. This can be understood as a tension in inclusive education between encouraging participation and protecting learners from feeling less knowledgeable (Norwich, 2014). In mathematics, we claim this is even more enhanced, as the learners get instant feedback on their participation in the social aspect, in the learning, or both. Roos (2023) has conceptualized this as 'having access to the learning of mathematics', and here, we add 'having access to the community of learners in mathematics'.

We claim that there is and needs to be a diversity regarding how teachers practice their professional judgment to facilitate inclusion and equity in mathematics teaching. It is for these reasons important to facilitate collaboration between teachers around these issues to, in the long term, be able to have joint labor with the learners in the mathematics classroom (Radford, 2016). Therefore, mathematics teachers and student-teachers should be given time to discuss, explore, and involve societal issues from an ethical perspective (Miller, 2022).

The ethical dilemmas in the study give a glimpse of the complexity of inclusion and equity, and how contextual, instantaneous, and always novel teachers' professional judgment is and needs to be (see Biesta, 2015; Frelin, 2014). The study implies that policy work and research need to better respect and derive from the tacit, fine-tuned, momentary and lived aspects of ethical reflection in teachers' professional judgment. These aspects of teaching will not allow themselves to be standardized or easily demarcated. Therefore,

teachers' professional judgment and ethical dilemmas need to be further researched, which demands a refined conceptual framework to do so. The framework developed in this article is a first step in that direction. For this to be sustainable, it needs to be developed in close collaboration with a variety of schools and teachers.

Funding Open access funding provided by Malmö University.

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