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ORIGINAL PAPER

Teacher noticing in mathematics education: a review of recent developments

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Abstract
The teacher noticing construct is widely recognized in teacher competence and education research, particularly in the field of mathematics education. This paper surveys recent research on mathematics teacher noticing published between July 2019 and 2022, following an earlier literature review on teacher noticing across different disciplines. The study presented here analyzed 118 English-language articles published in peer-reviewed journals, focusing on conceptualizations, research methods, and relationships with other constructs, including teacher knowledge and beliefs. The findings suggest that the cognitive-psychological perspective on noticing, which emphasizes a set of cognitive processes, remains the predominant conceptualization. Recent research on noticing is characterized by a high proportion of studies based on small samples and qualitative research methods. While several studies have demonstrated the interrelatedness of noticing and professional knowledge, the relationship between noticing and beliefs and between noticing and instructional quality has rarely been addressed. Based on these findings, we highlight noteworthy contributions and critical shortcomings, and suggest directions for future research.

Keywords Teacher noticing · Teacher professional vision · Teacher expertise · Mathematics education · Literature review

1 Introduction

Teacher noticing, which concerns "the specialized ways in which teachers observe and make sense of classroom events and instructional details" (Choy & Dindyal, 2020), has become central in the discourse surrounding teacher expertise and professional competence in recent decades. Given the complex and dynamic nature of classroom instruction, teachers must filter relevant information and make impromptu decisions to ensure effective teaching and learning (Sherin & Star, 2011).

While similar concepts have long been discussed in expertise research, recent studies on teacher noticing in mathematics education, focusing on students' thinking and reform-oriented teaching approaches, have inspired further research (Sherin et al., 2011; van Es, 2011; van Es & Sherin, 2002). Recent theoretical and empirical work has framed teacher noticing as part of teachers' professional competence (Blömeke et al., 2015, 2022; Krauss et al., 2020; Yang & Kaiser, 2022).

However, the terminology and conceptualizations surrounding teacher noticing remain heterogeneous, complicating knowledge accumulation. König et al.'s (2022) recent comprehensive review covered noticing research from 2000 to 2019, and recent developments in noticing research have been evident since then, as demonstrated by *ZDM – Mathematics Education*'s recent special issue on teacher noticing (Dindyal et al., 2021).

Building on König et al.'s (2022) work, this systematic literature review provides a comprehensive review of developments in teacher noticing research in mathematics education, between July 2019 and 2022, focusing on conceptualizations

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of teacher noticing and methodologies in its study. Addressing the research gaps identified by König et al. (2022), the paper includes research concerning the relationship between noticing, knowledge, beliefs, and instructional quality as well as comparisons between novice and expert noticing. Overall, we aim to identify promising new research perspectives and stimulate future research.

2 Theoretical framework

The teacher noticing construct has been the subject of extensive theoretical discussion, with multiple perspectives informing our understanding of its nature, components, and development (see Scheiner, 2021). Four theoretical perspectives have emerged from König et al.'s (2022) and Santagata et al.'s (2021) recent literature reviews, which form the theoretical foundation for our analysis.

Cognitive-psychological perspective: This perspective considers teacher noticing as a set of mental processes, including perception, interpretation, and response, in individual teachers' minds. Research in this area builds on van Es and Sherin's (2002) initial teacher noticing conceptualization, which encompasses identifying significant classroom situations, connecting them to broader principles, and applying contextual knowledge.

Socio-cultural perspective: This perspective highlights teacher noticing as a socially situated activity, shaped by discursive practices and socio-political contexts. Research within this perspective draws on Goodwin's (1994) work on 'professional vision,' referring to socially organized ways of seeing and understanding profession-specific events and highlighting the implicit power relations and ideologies.

Discipline-specific perspective: This perspective focuses on intentionally directing attention and heightened awareness toward specific aspects of one's teaching practice. Research within this perspective largely follows Mason's (2002) discipline of noticing, which outlines practices for developing teachers' sensitivity and presence, aiming for methodical and intentional practice without becoming mechanical or reactive.

Expertise-related perspective: This perspective focuses on how novice and expert teachers differentially perceive, process, and monitor classroom information, informed by Berliner's (1988) and Carter et al.'s (1988) earlier work demonstrating that expert teachers have more extensive classroom knowledge repositories and can more effectively evaluate significant classroom incidents and act adaptively.

To date, all four perspectives have been repeatedly addressed, with the cognitive-psychological perspective clearly predominating (König et al., 2022). In addition, teacher noticing is increasingly salient in teacher competence discourse (Blömeke et al., 2015), which regards perception, interpretation, and decision-making as situation-specific skills that mediate between cognitive and affective–motivational dispositions and professional performance. The competence discourse has raised awareness of the interaction between noticing and other competence-related constructs, such as knowledge, beliefs, and instructional quality, in studies on teaching (Depaepe et al., 2020; Kaiser et al., 2015; Krauss et al., 2020).

3 Relating teacher noticing, competence and expertise

Highlighting the relevance of competence-related constructs, Schoenfeld (2011) emphasized the role of knowledge and beliefs in the study of noticing. Teachers' professional knowledge includes knowledge of various domains relevant to teaching (e.g., content knowledge, pedagogical content knowledge, and general pedagogical knowledge) and is considered a core component of professional competence (e.g., Kunter et al., 2013). Moreover, knowledge affects teachers' ability to make connections and engage in knowledge-based reasoning (Sherin, 2007; van Es & Sherin, 2002). Qualitative studies highlight the relevance of specific knowledge elements for teacher noticing, such as knowledge about learning progressions in mathematics (Dick, 2017; Schack et al., 2017).

In terms of affective-motivational dispositions, teachers' instructional performance is likely to be shaped by their professional beliefs (Blömeke & Kaiser, 2017). While beliefs can be generally understood as "psychologically held understandings, premises, or propositions about the world that are felt to be true" (Richardson, 1996, p. 103), research on mathematics teachers' professional beliefs has focused on beliefs about the nature of mathematics as well as beliefs about teaching and learning mathematics (e.g., Hoth et al., 2022). However, previous research on teacher noticing has primarily addressed the role of beliefs about diversity (Keppens et al., 2021; Roose et al., 2019).

Teachers' noticing skills can also be conceptualized as a prerequisite for providing high-quality instruction, which in turn conditions student learning progress (e.g., Blömeke et al., 2022). Focusing on teachers' observable behavior in the classroom, instructional quality refers to the extent to which teachers succeed in initiating and supporting student learning processes (e.g., Kunter et al., 2013), and is typically described in terms of various dimensions that address both potentially generic and subject-specific aspects (e.g., Schlesinger et al., 2018). However, evidence on the relationship between noticing and instructional quality is limited.

As the expertise-related perspective emphasizes, teacher noticing overlaps with teacher expertise. Both competence and expertise denote knowledge and skills within a specific

domain. However, while competence emphasizes the skills needed to meet specific requirements (Weinert, 2001), expertise research highlights consistent high performance in representative tasks supported by long-term experience, commonly drawing on expert–novice comparisons (Ericsson & Towne, 2010). Research indicates that experts have higher noticing skills than novices (e.g., Gold & Holodynski, 2017; Meschede et al., 2017); however, few studies have focused on mathematics education (Bastian et al., 2022; Jacobs et al., 2010). Again, the evidence is limited and further insights are required.

4 Research aims and questions

This paper builds on König et al.'s (2022) review of 181 papers published between 2002 and June 2019, which provided an overview of conceptualizations, research approaches, and findings on learning to noticing across different disciplines. Considering the increase in relevant publications, this paper examines developments in research on teacher noticing in mathematics education between 2019 and 2022.

By conducting a systematic review of articles published between July 2019 and 2022, we examine how teacher noticing has been conceptualized and investigated in the recent literature compared with work published between 2002 and June 2019 with reference to König et al.'s (2022) findings and data. The following research questions guided our analysis:

RQ1: How have researchers conceptualized teacher noticing in mathematics education between July 2019 and 2022, in terms of the theoretical perspectives and the multifaceted nature of the construct?

RQ2: How have researchers studied teacher noticing in mathematics education between July 2019 and 2022, regarding the methodological approaches, data collection methods, and samples?

Although noticing is increasingly recognized as intrinsic to teacher competence, König et al. (2022) highlighted the need for further research on the relationship between noticing and other competence-related constructs as well as the differences between expert and novice noticing. Therefore, we investigate the extent to which recent research has addressed these gaps, posing the following additional research question:

RQ3: What new insights does the recent literature on teacher noticing in mathematics education offer regarding (a) the relationships between noticing and competence-related constructs, i.e., teacher knowledge, teacher beliefs, and instructional quality, and (b) expert-novice comparisons?

Finally, we review and discuss the most relevant studies that address these questions, highlighting their contributions and implications for future research.

5 Method

Our review approach adopts König et al.'s (2022) methodology, adapted to focus on teacher noticing in mathematics education. We followed the guidelines of the PRISMA statement (Page et al., 2021) to ensure rigor and transparency. Below, we detail our literature search, selection procedure, and coding process.

5.1 Literature search

In February 2023, we conducted a systematic search for 'teacher noticing', including 'teacher professional vision' as an alternative construct. Five major online databases (ERIC, PsycINFO, ScienceDirect, Scopus, and Web of Science) were searched for titles, abstracts, and keywords using the search terms "teacher* AND math* AND (noticing OR professional vision)".[1] The search was restricted to items published between 1 July 2019 and 31 December 2022, excluding publications in press or early access. The search yielded 222 publications following the removal of duplicates, and the references were exported to EndNote version 20.

5.2 Inclusion and exclusion criteria

The following inclusion criteria were applied: (1) the publication appeared in a peer-reviewed journal; (2) the publication was written in English; (3) the publication explicitly addressed teacher noticing or teacher professional vision; and (4) the publication's content was considered relevant to the discourse on teacher noticing in mathematics education.

Criterion 1 ensured the inclusion of high-quality publications exclusively. Publications ($n = 17$) not published in a journal were excluded (e.g., book chapters and conference proceedings), because it was unclear whether they had been subjected to peer review, an accepted criterion for ensuring scholarly quality. Criterion 2 identified highly accessible publications: those ($n = 9$) in languages other than English were excluded. Criterion 3 ensured that only publications relevant to this review's purpose were selected, excluding publications

[1] Using a truncation symbol at the end of the search terms (*), the databases' search algorithms considered all word endings, including plural forms or alternative spellings (e.g., teacher or teachers; maths or mathematics). The terms 'noticing' and 'professional vision' were chosen rather than 'notic*' or 'vision*' because the latter terms were too broad and yielded too many irrelevant results.

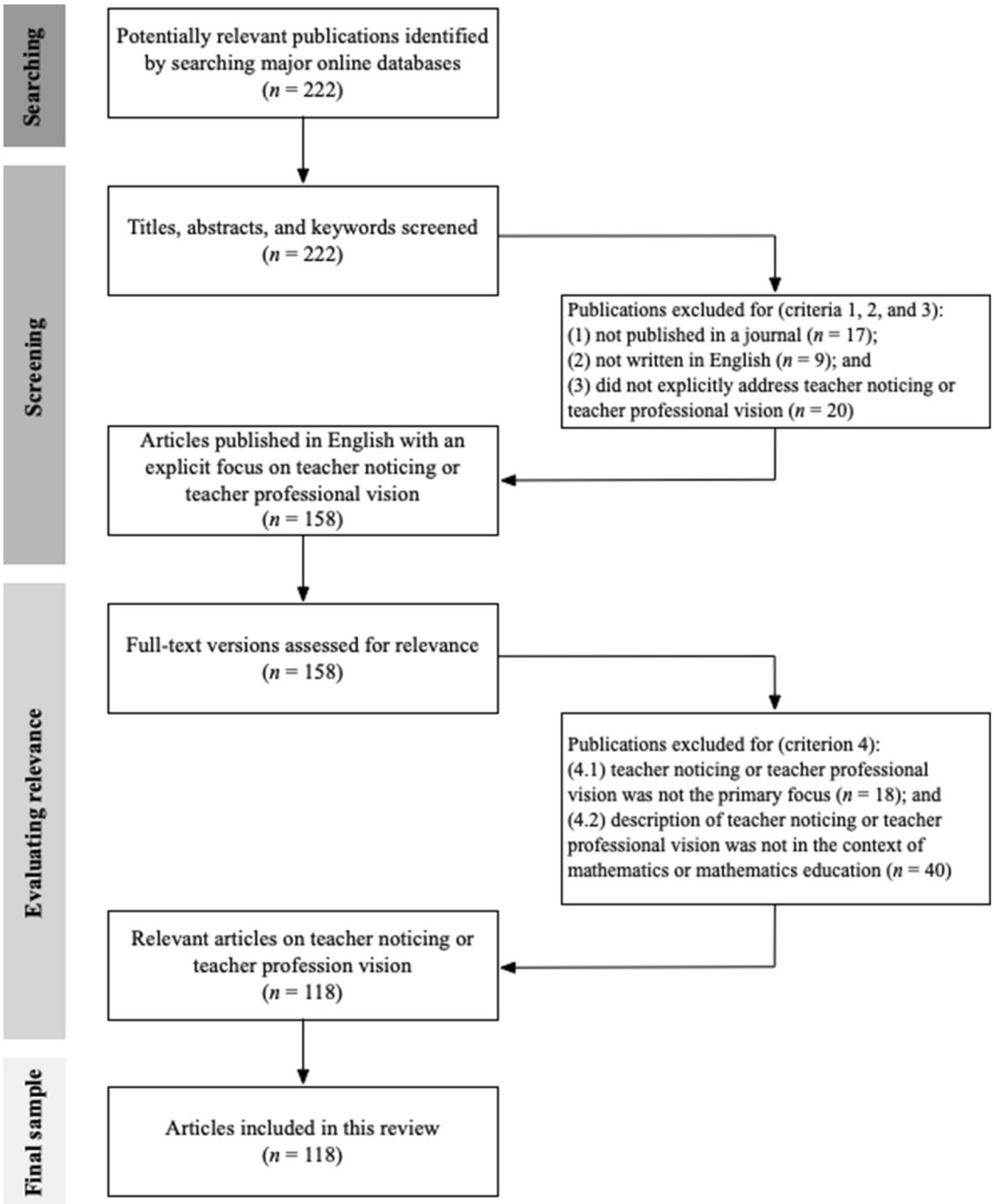

Fig. 1 Search and selection process flowchart

($n = 20$) that did not explicitly address teacher noticing or professional vision. Theoretical and conceptual analyses were considered in addition to empirical studies. Book reviews, commentaries, and editorial notes published in journals were excluded. A total of 158 publications were identified and appeared to meet the selection criteria based on their titles, abstracts, and keywords.

Criterion 4 assessed the relevance of the 158 publications. The authors retrieved and reviewed the articles' full texts and collectively determined whether each publication's content was relevant. We applied the following exclusion criteria: (4.1) teacher noticing or teacher professional vision was not the article's primary focus; and/or (4.2) the description of teacher noticing or teacher professional vision was not in the context of mathematics or mathematics education.

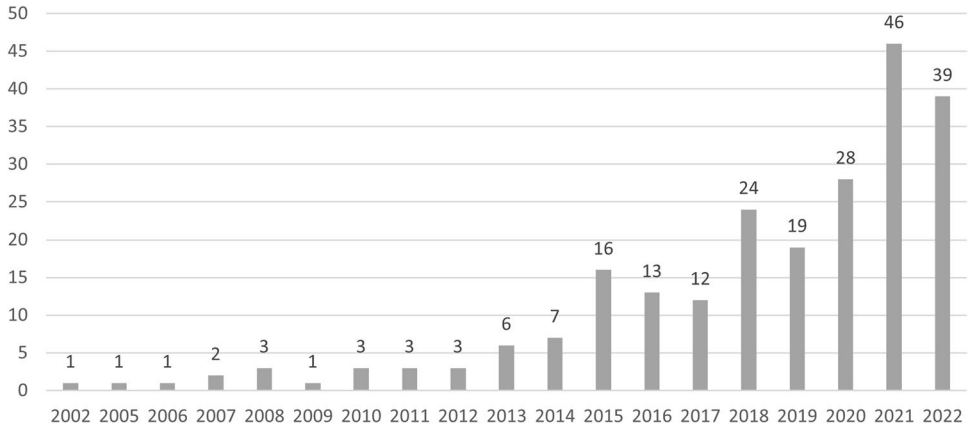

Fig. 2 Number of publications on teacher noticing in mathematics education between 2002 and 2022

Criterion (4.1) excluded publications ($n = 18$) that merely mentioned teacher noticing or teacher professional vision or addressed the constructs in a marginal or overly generalized manner. Criterion (4.2) excluded publications ($n = 40$) that discussed teacher noticing or teacher professional vision in contexts other than mathematics or mathematics education.

A final database of 118 articles was compiled. Figure 1 summarizes the search and selection process and details how many publications were excluded for each criterion.

5.3 Coding process

We adapted König et al.'s (2022) coding scheme to highlight recent advancements in research on teacher noticing in mathematics education. Our modified scheme focuses on two categories: (1) the conceptualization of teacher noticing and (2) the methodological approach. Electronic supplementary material 1 details the modified coding scheme.

The teacher noticing conceptualization included categories for theoretical perspectives (i.e., cognitive-psychological, socio-cultural, discipline-specific, and expertise-related), noticing facets (e.g., perception, interpretation, and decision-making), and other constructs (i.e., knowledge, beliefs, and instructional quality). The methodological approach included categories for the paradigm (e.g., quantitative, qualitative, both methods), data collection method (e.g., interviews, questionnaires, written reports, or video recordings), and sample (e.g., sample size, expertise group investigated). Except for a few categories that were mutually exclusive (e.g., qualitative, quantitative, and mixed methods), multiple categories of a topic area could be applied to the same paper. For example, multiple theoretical perspectives could be employed within a single paper.

To ensure our coding scheme's reliability, the first 25 papers (i.e., approximately 20 percent) were independently coded using two raters. Interrater agreement was good ($M_{Kappa} = 0.84$, $SD_{Kappa} = 0.144$, $\text{Min.}_{Kappa} = 0.58$, $\text{Max.}_{Kappa} = 1$), and discrepancies were resolved through discussion. Codes for which one rater was unsure were flagged and checked by the other rater.

By this means, we identified publications referring to teacher knowledge, beliefs, and/or instructional quality as well as comparisons between teachers of different expertise levels. Based on the coding results, we present and contextualize key findings from these selected papers.

6 Results

6.1 Basic characteristics of articles

Electronic supplementary material 2 contains all references of the final selection encompassing 118 publications. Most papers (113 articles) were classified as empirical, with some identified as review papers (4 articles) or exclusively theoretical (1 article). This distribution resembles that between 2002 and June 2019. Of the 118 papers, 52 included an intervention designed to promote noticing.

When König et al.'s (2022) literature survey is considered, it is evident that publications on teacher noticing have increased steadily since 2002, with peaks in 2021 and 2022 (see Fig. 2), demonstrating the prominence of teacher noticing in mathematics education research. Most studies focused on pre-service teachers, with few examining in-service teachers and only a small percentage of studies including both groups (see Table 1), consistent with König et al.'s (2022) initial literature review.

6.2 Theoretical foundations and conceptualizations of noticing

Following König et al. (2022), we classified the articles' theoretical perspectives, including cognitive–psychological, socio–cultural, discipline–specific, and expertise–related approaches (see Sect. 2). The classification of each article can be found in electronic supplementary material 3. As shown in

Table 1 Samples in empirical studies on teacher noticing in mathematics education

Pre-service teachers	In-service teachers	Other	2002–June 2019 (mathematics only)		July 2019–2022	
			n	%	n	%
x			56	50.9	60	53.1
	x		37	33.6	36	31.9
		x	2	1.8	8	7.1
x	x		7	6.4	4	3.5
x		x	3	2.7	1	0.9
	x	x	3	2.7	3	2.7
x	x	x	2	1.8	1	0.9

Table 2 Articles addressing different theoretical perspectives on noticing

Perspectives	2002–June 2019 (mathematics only)		July 2019–2022	
	n	%	*n*	%
Cognitive-psychological	95	86.4	111	94.1
Socio-cultural	15	13.6	23	19.5
Discipline-specific	20	18.2	24	20.3
Expertise-related	14	12.7	11	9.3
Total number of articles	*110*		*118*	

Note. Several perspectives could be adopted in each of the publications. Therefore, the sum of the codes exceeds the number of articles

Table 2, the distribution of perspectives was similar between 2002–June 2019 and July 2019–2022. Most papers (111 articles) adopted the cognitive–psychological perspective, while the other three perspectives were used less frequently. In addition, as shown in Fig. 3, in both selections, the three less common perspectives were often addressed in addition to, rather than in place of, the cognitive–psychological approach.

Our analysis suggests that subdivision into subprocesses (facets) is gaining importance when compared to a holistic perspective (Table 3). Moreover, since July 2019, the facet of responding or decision-making has been addressed more frequently. Among the articles that distinguish between different noticing facets, two common conceptualizations can be identified in both literature selections, namely (1) the combination of attending/perceiving and interpreting/reasoning, and (2) the triad of attending/perceiving, interpreting/reasoning, and responding/decision-making (see Fig. 4).

Several authors have proposed extending noticing to incorporate teachers' instructional enactment. Van Es and Sherin (2021) introduced 'shaping', which concerns the teacher's attempts to obtain further information about student thinking through purposeful inquiry. Amador et al. (2021) included 'enactment' as a behavioral facet, distinguishing between decision-making and implementation.

6.3 Methodological approaches

Of the 113 empirical papers included in this review, 60 applied qualitative methodology, 7 used quantitative methodology, and 46 adopted both approaches, consistent with the data available for 2002–June 2019 (see Table 4). Most studies relied on data collection approaches with low standardization, such as written reports and video recordings, while standardized testing was rare (Table 5).

6.4 Noticing in the context of competence and expertise

Table 6 provides an overview of recent empirical noticing studies, accounting for competence-related constructs (teacher knowledge, beliefs, and instructional quality), and studies conducting expert–novice comparisons. While knowledge was studied frequently, beliefs and teaching quality were rarely considered. Few studies compare experts and novices. Electronic supplementary material 4 indicates which publications address each construct. Highlighting noticing as a part of teacher competence and expertise, we report key findings from these studies below.

6.4.1 Noticing and knowledge

This review identified 27 empirical articles examining both teacher knowledge and teacher noticing. However, not all explored the relationship between these two constructs. Recent research has emphasized that—when noticing and reasoning regarding classroom situations—(pre-service) teachers rely on specific knowledge types, including episodic and (pedagogical) content knowledge (Gegenfurtner et al., 2020), specific knowledge for teaching mathematics (Picado-Alfaro et al., 2022), and technological (pedagogical) knowledge (Ng & Park, 2021). Hino and Funahashi (2022) noted that teachers' elaborate decision-making is informed by subject knowledge and pedagogical content knowledge. The theoretical link between knowledge and noticing is widely accepted, as corroborated by intervention studies

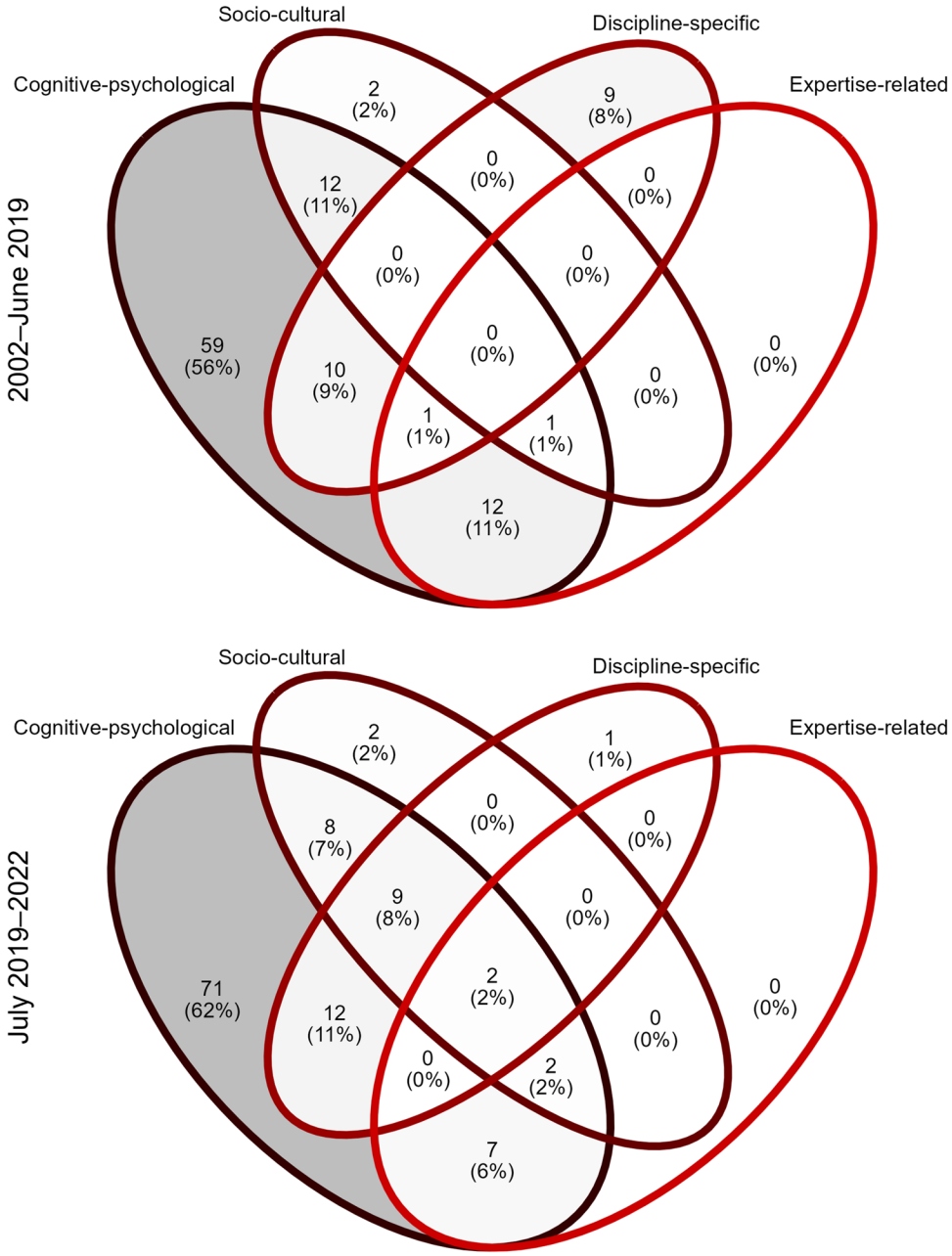

Fig. 3 Venn diagrams depicting the distribution of theoretical perspectives for both literature selections (Venn diagrams have been generated using the R package "ggVennDiagram" (Gao et al. 2021)

considering both knowledge and noticing (Güler et al., 2020; Namakshi et al., 2022).

Despite the theoretically strong link between knowledge and noticing, correlation analyses mostly reveal weak-to-moderate associations (Table 7). Investigating Chinese mathematics teachers, Yang et al., (2021a, 2021b) assume that the low correlations are due to culture-specific characteristics of teacher education in China. Jong et al. (2021) observed knowledge to predict decision-making but not attending and interpreting, which may however be due to specific features of their research design.

Using qualitative approaches, recent studies examined the relationship between teachers' ability to solve mathematical problems and their ability to notice and interpret student solutions to comparable problems. The findings suggest that higher content knowledge is positively associated with higher noticing ability (Cabral et al., 2021; Lee & Lee, 2021). Conversely, a lack of specific knowledge—particularly mathematical content knowledge—can prevent teachers from noticing students' thinking processes (Sevinc & Galindo, 2022). However, some studies have shown a misalignment between noticing and knowledge, suggesting that some pre-service teachers made sense of students' solutions while struggling to solve comparable problems themselves (Buforn et al., 2022). Márquez et al. (2021) showed that the ability to solve a specific mathematical task does not

Table 3 Conceptualization of teacher noticing

	2002–June 2019		July 2019–2022	
	n	%	*n*	%
Noticing (as a holistic facet)	19	17.3	10	8.5
Attending or perceiving	91	82.8	99	83.9
Interpreting or reasoning	84	76.4	93	78.8
Responding or decision-making	50	45.5	67	56.8
Making connections	9	8.2	12	10.2
Other	0	0	3	2.5

Note. Several noticing facets could be addressed in each of the publications. Therefore, the sum of the codes exceeds the number of articles

necessarily imply that students' errors in the same domain are adequately interpreted.

Misalignment between noticing and knowledge may be further attributable to pre-service teachers' difficulties in connecting knowledge and instructional sequences (Warshauer et al., 2021). Similarly, when analyzing instructional practice, teachers may struggle to identify the knowledge elements applicable to the specific situation (Shin, 2021).

6.4.2 Noticing and beliefs

A close association may be assumed between noticing and beliefs. Wallin and Amador (2019) revealed that the development of teachers' noticing and beliefs was deeply interrelated while participating in video club sessions. Highlighting the role of constructivist beliefs, Cross Francis et al. (2022) observed that specific beliefs regarding mathematics teaching strategies may impede or promote teachers' attention to students' mathematical thinking.

Three studies apply standardized testing to examine the noticing–knowledge–beliefs interplay. Larrain and Kaiser (2022) focus on noticing students' errors and report moderate correlations with beliefs regarding mathematics as an inquiry process ($r = 0.367$) and beliefs regarding mathematics learning as an active and student-centered process ($r = 0.436$). Hoth et al. (2022) found constructivist beliefs about mathematics teaching to be moderately correlated with noticing ($r = 0.35$). Remarkably, the effect of knowledge predicting noticing was no longer significant when beliefs were included in the model. In Jong et al.'s (2021) pre–post design, neither teachers' attitudes (i.e., positive feelings about teaching mathematics) nor dispositions (i.e., the intention to adopt a constructivist teaching style) predicted noticing.

6.4.3 Noticing and instructional quality

Only three studies in our selection examined the empirical relationship between noticing and instructional quality. Wallin and Amador (2019) focused on teachers' development when participating in video club sessions and highlighted the noticing–beliefs–instructional practice interrelatedness. However, focusing on the frequency of specific forms of student reasoning, Melhuish et al. (2020) found that what teachers noticed and reported about their own classrooms was only weakly associated with the researchers' ratings of the classroom using the Mathematical Quality of Instruction (MQI) instrument, thereby highlighting the subjectivity of teacher noticing.

Cross Francis et al. (2022) assessed six teachers' post-instructional noticing through interviews and rated video clips of the teachers providing instruction using the MQI instrument. The authors reported alignment (i.e., both noticing level and instructional quality are either high or low) for three teachers and misalignment (i.e., high instructional quality but low noticing level) for the other three teachers. This controversial finding was attributed to teachers' professional identity and beliefs, which may or may not facilitate attendance to students' mathematical thinking.

6.4.4 Expert and novice teachers' noticing

Only five studies included expert-novice comparisons, reinforcing that the expertise-related perspective on noticing is neglected. As anticipated, these studies show that experts outperform novices. For example, Gegenfurtner et al. (2020) found that two expert groups—in-service teachers and school principals—achieved higher levels of knowledge-based reasoning than pre-service teachers when analyzing photographs of mathematics instruction. Cai et al. (2022) demonstrated that expert teachers (but not pre-service teachers) attended to all relevant aspects of the students' solutions to a mathematical modeling task and responded by asking questions rather than issuing instructions.

Studies based on standardized testing have also yielded ambiguous findings. Comparing in-service and pre-service teachers, Friesen and Kuntze (2021) found significant effects of teaching experience on noticing in the domain of functions but not fractions, suggesting that the impact of expertise is context-specific. Bastian et al. (2022) showed that beginning and experienced in-service teachers outperformed pre-service teachers. However, experienced teachers (on average 19.6 years of teaching practice) did not perform better than beginning in-service teachers. By contrast, using a translated version of the instrument by Bastian et al. (2022) with a Chinese sample, Yang et al. (2021b) found that experienced teachers (15–36 years of teaching experience) clearly outperformed pre-service and early career

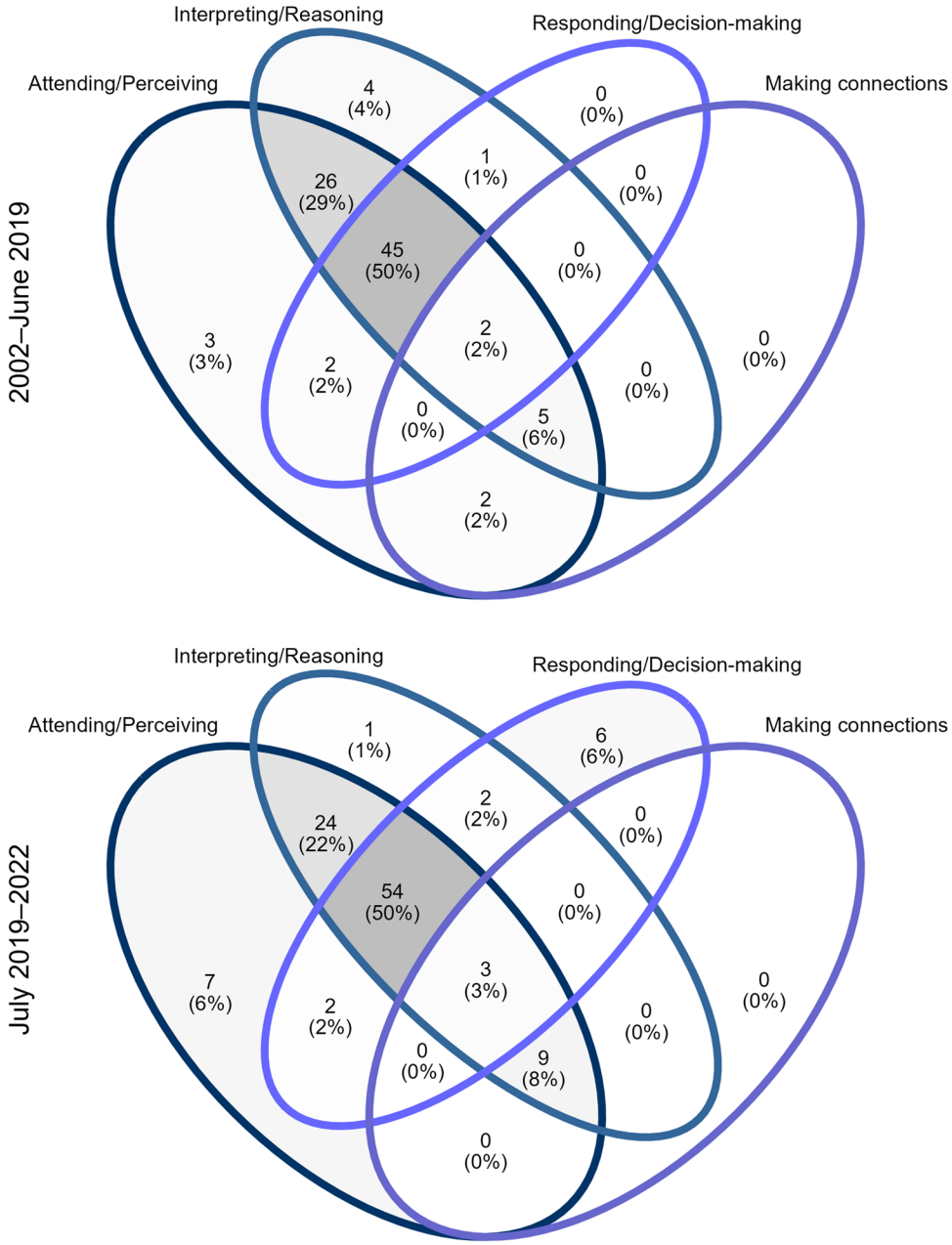

Fig. 4 Venn diagrams depicting the distribution of noticing facets in the individual articles (noticing as a holistic construct is not included)

Table 4 Methodological paradigms and sample sizes

	2002–June 2019 (mathematics only)						July 2019–2022					
	n	Median	Mean	SD	Min	Max	n	Median	Mean	SD	Min	Max
Qualitative	56	7	17.11	25.653	1	126	60	7	16.90	22.925	1	95
Quantitative	13	129	142.38	89.981	17	296	7	131	183.86	145.69	34	457
Both methods	40	27	64.41	81.46	1	321	46	29.5	64.89	99.975	1	496
Total	109	19	50.19	73.374	1	321	113	16	47.04	85.222	1	496

teachers. Moreover, differential item functioning revealed group-specific strengths and weaknesses of pre-service, early career and experienced teachers depending on the test items' specific contents. Overall, these results suggest a complex relationship between noticing and expertise that must also consider the cultural context and specific domain.

Table 5 Data collection approaches differentiated by methodological paradigm

	2002–June 2019 (mathematics only)					July 2019–2022				
	Qualitative	Quantitative	Both methods	Papers	%	Qualitative	Quantitative	Both methods	Papers	%
Written report	26	3	21	50	46%	38	1	31	70	62%
Video recording	32	0	14	46	42%	22	1	12	35	31%
Interview	20	0	11	31	28%	27	0	11	38	34%
Test	0	12	9	21	19%	1	5	10	16	14%
Observation	1	0	1	2	2%	7	0	2	9	8%
Questionnaire	0	0	0	0	0%	0	0	1	1	1%

Table 6 Studies on teacher noticing in the context of competence and expertise (July 2019–2022)

		Knowledge	Beliefs	Instructional Quality	Expert–Novice
Qualitative	n_{studies}	13	1	1	0
	$M_{\text{sample size}}$	26.92	3	3	-
	$SD_{\text{sample size}}$	31.613	-	-	-
Quantitative	n_{studies}	4	2	0	3
	$M_{\text{sample size}}$	155.25	128.50	-	235.33
	$SD_{\text{sample size}}$	93.457	3.536	-	198.500
Both methods	n_{studies}	10	2	2	2
	$M_{\text{sample size}}$	61.90	68.50	42.50	239.50
	$SD_{\text{sample size}}$	66.435	88.388	51.619	279.307
Total	n_{studies}	27	5	3	5
	$M_{\text{sample size}}$	58.89	79.40	29.33	237.00
	$SD_{\text{sample size}}$	70.284	68.413	43.039	198.014

Note. This table refers only to empirical studies in the literature selection, while two literature reviews that also refer to competence/expertise are not included

Table 7 Studies reporting correlations between noticing and knowledge

Publication	Noticing construct	Knowledge type	Coefficient
Copur-Gencturk and Tolar (2022)	Content-specific noticing skills	PCK	$r = 0.62$
		CK	$r = 0.42$
Hoth et al. (2022)	Ability to identify students' errors	PCK	$r = 0.26$
		CK	$r = 0.30$
Jong et al. (2021)	Professional noticing of children's mathematical thinking	CK and PCK combined	
	Attending		$\beta = 0.15$ (n.s.)
	Interpreting		$\beta = 0.06$ (n.s.)
	Decision-making		$\beta = 0.24$
Larrain and Kaiser (2022)	Diagnostic competence in error situations	Mathematical knowledge for teaching	$r = 0.33$
Yang et al., (2021a, 2021b)	Perception, interpretation, decision-making (mathematics instructional aspects)	PCK	$r = 0.34$
		CK	$r = 0.08$ (n.s.)
		GPK	$r = 0.22$

Note. PCK = Pedagogical content knowledge, CK = Content knowledge, GPK = General pedagogical knowledge, n.s. = not significant

7 Discussion

This paper provided a systematic review of recent research on teacher noticing in mathematics education, focusing on conceptualizations and methodological approaches. Highlighting noticing in the context of competence and expertise, we also presented recent findings on noticing in the context of knowledge, beliefs, and instructional quality, further accounting for expert–novice comparisons.

7.1 Central findings and implications

7.1.1 Conceptualizations and methodological approaches

First, it is worth acknowledging the recent significant increase in articles on teacher noticing in mathematics education, highlighting the importance of understanding the construct.

Concerning RQ1, it is notable that the cognitive–psychological perspective, which views noticing as a set of cognitive processes, remained dominant in research from July 2019 to 2022, as previously. This raises the question of different lines of research emerging within this perspective. For example, the well-received work by Blömeke et al. (2015) can be seen as a starting point for a competence-based research perspective on noticing that, on the one hand, emphasizes the role of noticing in competence acquisition and, on the other hand, aims to model the influence of noticing on instructional practice. Moreover, recent work has extended the noticing concept to teachers' instructional enactment (e.g., Amador et al., 2021; van Es & Sherin, 2021). Against this background, the ecological-embodied approach foregrounded by Scheiner (2021) can be highlighted as an emerging perspective, wherein teachers are seen as active agents who engage with their environment to create noticing opportunities (see Scheiner, 2021). In terms of conceptualizing noticing, the most common approach in both literature selections was to consider three facets of noticing, namely (1) attending/perceiving, (2) interpreting/reasoning, (3) responding/decision making. This finding highlights the continuing influence of the professional noticing framework (Jacobs et al., 2011) and the PID model (perception, interpretation, and decision making; Blömeke et al., 2015; Kaiser et al., 2015).

Regarding RQ2, the recent literature includes numerous qualitative studies and studies that combine both qualitative and quantitative methods. By contrast, purely quantitative approaches based on large sample sizes are infrequent. Recent large-scale studies by Bastian et al. (2022), Copur-Gencturk and Tolar (2022), Copur-Gencturk and Rodrigues (2021), and Yang et al. (2021b) are noteworthy exceptions. Moreover, many studies utilized unstandardized survey formats, such as written reports or videos, while standardized tests were seldom employed. Although this approach is associated with limited generalizability, it offered detailed insights into how teachers notice and how their noticing develops, as evidenced by studies investigating the relationship between teacher noticing and other competence-related constructs.

7.1.2 Teacher noticing in the context of competence and expertise

Several publications explored the relationship between noticing and other competence-related constructs—knowledge, beliefs, and instructional quality (RQ3a). Positive correlations between teacher noticing and teacher knowledge were identified, albeit with varying effect sizes, consistent with previous research (e.g., Dreher & Kuntze, 2015; Kersting et al., 2012; Sánchez-Matamoros et al., 2019). To better understand the relationship between knowledge and noticing, it is crucial to identify variables that moderate this link, such as the specific operationalizations of teacher knowledge and noticing, the teachers' expertise levels, and aspects of the measurement methodology (Müller & Gold, 2023). Furthermore, domain-specific knowledge may also be relevant: teacher noticing in the context of student mathematical thinking may relate particularly to knowledge about the development of student mathematical knowledge (Dick, 2017; Schack et al., 2013). Notably, recent studies have explored the promotion of teacher noticing using hypothetical learning trajectories, which may provide knowledge specific to this domain (Callejo et al., 2022; Moreno et al., 2021; van den Kieboom, 2021).

Teachers' beliefs can moderate the relationship between teacher noticing and knowledge, as highlighted by Hoth et al. (2022). The limited evidence suggests that constructivist beliefs and student-centered orientations are associated with more elaborate teacher noticing. Consequently, teachers' beliefs may be as relevant as their knowledge. Unfortunately, the relationship between teacher noticing and beliefs and the link between noticing and instructional quality remains underexplored.

Given that teacher noticing can be further developed and manifested in practical teaching situations, we explored recent studies reporting expert–novice comparisons (RQ3b). Although studies commonly demonstrate differences in expert and novice teachers' noticing, evidence suggests that experts do not necessarily outperform novices and that it is important to consider the study's focal domain and cultural context (Bastian et al., 2022; Friesen & Kuntze, 2021; Yang et al. 2021b). These findings raise questions about how expertise in teacher noticing should be conceptualized and empirically traced.

7.2 Limitations and perspectives

This review has several limitations. The decision to limit the search to peer-reviewed English-language journals may have excluded relevant book chapters or articles in other languages. Therefore, the findings must be interpreted cautiously, and future reviews should consider including publications from outlets beyond journals and in languages besides English to reduce the potential for bias and promote inclusivity.

Moreover, the search may have missed relevant publications that did not explicitly use the terms 'teacher noticing' or 'teacher professional vision' or did not label them as such in their titles, abstracts, or keywords. For example, studies focusing on 'expertise' or 'situation-specific skills' were not included despite denoting comparable phenomena (e.g., Blömeke et al., 2022; Stahnke et al., 2016). This relates to so the called "jingle-jangle fallacy" (Gonzalez et al., 2021), which refers to erroneously distinguishing the same construct in two constructs based on different naming, or equating different constructs based on the same naming. To overcome this issue and facilitate knowledge accumulation, future research should provide clear definitions while also addressing terminological inconsistencies within noticing research.

Another limitation arises from the fact that the current state of research on teacher noticing is extensive and multifaceted. This review focused on specific topics: conceptualization, methodology, and noticing as part of competence and expertise. However, the topics covered represent only a small portion of the available scope. Recent literature on teacher noticing reveals a rich diversity of research directions, including the use of technological tools to promote teacher noticing (Kosko et al., 2021; Lee, 2021), intercultural perspectives (Damrau et al., 2022; Dreher et al., 2021), and equity-related factors, such as racial and linguistic diversity in mathematics classrooms (Crespo et al., 2021; Renick et al., 2021; Shah & Coles, 2020; van Es et al., 2022). Recognizing the relevance of these new perspectives, below, we highlight 10 studies of particular interest for the noticing discourse that, among others, account for the emerging topics mentioned above.

The multifaceted nature of teacher noticing research underscores the need for ongoing exploration and investigation of this complex construct. Future research should continue to expand on the existing literature and explore new avenues of inquiry, considering diverse contexts and populations to yield a more comprehensive understanding of teacher noticing and its implications for effective teaching and learning. Highlighting the expertise-related perspective, future research that further explores expert teachers' noticing and its characteristics is likely to be particularly relevant. Finally, future research should focus more on the complex noticing–knowledge–beliefs interplay, paying particular attention to how these facets of competence affect actual classroom behavior. Further syntheses of existing research and further empirical work on the issues raised can provide a comprehensive knowledge base. On this basis, we can better equip teachers with the knowledge and skills required to promote student learning and success in a rapidly changing educational landscape.

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The studies included in this review are marked with an asterisk (*) in the reference list. Papers of particular interest, published within the period of review, have been highlighted as ** of special interest

* Amador, J. M., Estapa, A., Kosko, K., & Weston, T. (2021). Prospective teachers' noticing and mathematical decisions to respond: Using technology to approximate practice. *International Journal of Mathematical Education in Science and Technology, 52*(1), 3–22. https://doi.org/10.1080/0020739X.2019.1656828

** Bastian, A., Kaiser, G., Meyer, D., Schwarz, B., & König, J. (2022). Teacher noticing and its growth toward expertise: An expert–novice comparison with pre-service and in-service secondary mathematics teachers. *Educational Studies in Mathematics*, *110*(2), 205–232. https://doi.org/10.1007/s10649-021-10128-y. This is one of the few studies that use sophisticated testing instruments to compare pre-service, early career, and experienced teachers. The results not only demonstrate that perception, interpretation, and decision-making are measurable as distinct facets but also provide evidence that in-service teachers outperformed pre-service teachers. Moreover, the finding that experienced teachers did not perform better than early career teachers may inspire future research on conceptualizing teacher expertise.

Berliner, D. C. (1988). *The development of expertise in pedagogy*. American Association of Colleges for Teachers

Blömeke, S., Gustafsson, J.-E., & Shavelson, R. J. (2015). Beyond dichotomies: Competence viewed as a continuum. *Zeitschrift Für*

Psychologie, 223(1), 3–13. https://doi.org/10.1027/2151-2604/a000194

Blömeke, S., Jentsch, A., Ross, N., Kaiser, G., & König, J. (2022). Opening up the black box: Teacher competence, instructional quality, and students' learning progress. *Learning and Instruction, 79*, 101600. https://doi.org/10.1016/j.learninstruc.2022.101600

Blömeke, S., & Kaiser, G. (2017). Understanding the development of teachers' professional competencies as personally, situationally and socially determined. In J. D. Clandinin, & J. Husu (Eds.), *The Sage Handbook of Research on Teacher Education* (pp. 783–802). Sage. https://doi.org/10.4135/9781526402042

* Buforn, À., Llinares, S., Fernández, C., Coles, A., & Brown, L. (2022). Pre-service teachers' knowledge of the unitizing process in recognizing students' reasoning to propose teaching decisions. *International Journal of Mathematical Education in Science and Technology, 53*(2), 425–443. https://doi.org/10.1080/0020739X.2020.1777333

* Cabral, J., Oliveira, H., & Mendes, F. (2021). Preservice Teachers' Mathematical Knowledge about Repeating Patterns and their Ability to Notice Preschoolers Algebraic Thinking. *Acta Scientiae*, *23*(7), 30–59. https://doi.org/10.17648/acta.scientiae.6302

* Cai, J., LaRochelle, R., Hwang, S., & Kaiser, G. (2022). Expert and preservice secondary teachers' competencies for noticing student thinking about modelling. *Educational Studies in Mathematics, 109*(2), 431–453. https://doi.org/10.1007/s10649-021-10071-y

** Callejo, M. L., Pérez-Tyteca, P., Moreno, M., & Sánchez-Matamoros, G. (2022). The use of a length and measurement HLT by pre-service kindergarten teachers' to notice children's mathematical thinking. *International Journal of Science and Mathematics Education*, *20*(3), 597–617. https://doi.org/10.1007/s10763-021-10163-4. This study focuses on the use of hypothetical learning trajectories to promote pre-service teachers' noticing of students' thinking, which is a major concern of noticing research. The results of this qualitative study with pre-service kindergarten teachers suggest that learning trajectories provide participants with necessary concepts and support them in interpreting children's thinking, formulating learning goals, and generating adequate responses.

Carter, K., Cushing, K., Sabers, D., Stein, P., & Berliner, D. (1988). Expert-novice differences in perceiving and processing visual classroom information. *Journal of Teacher Education, 39*(3), 25–31. https://doi.org/10.1177/002248718803900306

Choy, B. H., & Dindyal, J. (2020). Teacher noticing, mathematics. In M. A. Peters (Ed.), *Encyclopedia of Teacher Education.* Springer. https://doi.org/10.1007/978-981-13-1179-6_241-1

* Copur-Gencturk, Y., & Rodrigues, J. (2021). Content-specific noticing: A large-scale survey of mathematics teachers' noticing. *Teaching and Teacher Education, 101*, 103320. https://doi.org/10.1016/j.tate.2021.103320

** Copur-Gencturk, Y., & Tolar, T. (2022). Mathematics teaching expertise: A study of the dimensionality of content knowledge, pedagogical content knowledge, and content-specific noticing skills. *Teaching and Teacher Education*, *114*, 103696. https://doi.org/10.1016/j.tate.2022.103696. This study is among the few large-scale studies with in-service teachers that capture noticing and knowledge as facets of teacher expertise. The results of this methodologically sophisticated study support the assumption that pedagogical content knowledge, content knowledge, and noticing are three distinct expertise domains. In particular, the authors conclude that specific learning opportunities are required to promote noticing.

** Crespo, S., Bowen, D., Buli, T., Bannister, N., & Kalinec-Craig, C. (2021). Supporting prospective teachers to notice and name student language resources as mathematical strengths. *ZDM – Mathematics Education*, *53*(2), 461–473. https://doi.org/10.1007/s11858-020-01205-2. This study embeds noticing within instruction with multilingual students. The results demonstrate that with specific learning opportunities, prospective teachers can progress from a deficit-oriented language toward a resource-oriented one. However, explicit support seems necessary to recognize the value of linguistic diversity in the mathematics classroom.

** Cross Francis, D., Eker, A., Liu, J., Lloyd, K., & Bharaj, P. (2022). (Mis)alignment between noticing and instructional quality: The role of psychological and cognitive constructs. *Journal of Mathematics Teacher Education*, *25*(5), 599–632. https://doi.org/10.1007/s10857-021-09509-0. This study highlights the value of qualitative approaches in noticing research and considers the complex interplay between post-instructional noticing, beliefs, knowledge, and instructional quality. In particular, the results suggest that high instructional quality is not necessarily correlated with high level noticing. Teachers may orchestrate adequate learning opportunities without shifting their attention to students' thought processes after instruction. This misalignment may be due to other factors, including beliefs and professional identity.

* Damrau, M., Barton, D., Huget, J., Chan, M. C. E., Roche, A., Wang, C., Clarke, D. M., Cao, Y., Liu, B., Zhang, S., & Peter-Koop, A. (2022). Investigating teacher noticing and learning in Australia, China, and Germany: A tale of three teachers. *ZDM – Mathematics Education*, *54*(2), 257–271. https://doi.org/10.1007/s11858-022-01361-7

Depaepe, F., Verschaffel, L., & Star, J. (2020). Expertise in developing students' expertise in mathematics: Bridging teachers' professional knowledge and instructional quality. *ZDM – Mathematics Education*, *52*(2), 179–192. https://doi.org/10.1007/s11858-020-01148-8

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** Dreher, A., Lindmeier, A., Feltes, P., Wang, T.-Y., & Hsieh, F.-J. (2021). Do cultural norms influence how teacher noticing is studied in different cultural contexts? A focus on expert norms of responding to students' mathematical thinking. *ZDM – Mathematics Education*, *53*(1), 165–179. https://doi.org/10.1007/s11858-020-01197-z. This study considers that the characteristics constituting good teaching differ across cultures, which also affects teacher noticing. The authors specifically compare how German and Taiwanese researchers respond to a norm breach when reading a text vignette. The results demonstrate that researchers from both cultures evaluate the norm breach negatively but give different reasons for doing so, suggesting that the cultural context considerably impacts noticing and noticing research.

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diagram. *Frontiers in Genetics, 12*, 706907. https://doi.org/10.3389/fgene.2021.706907

** Gegenfurtner, A., Lewalter, D., Lehtinen, E., Schmidt, M., & Gruber, H. (2020). Teacher expertise and professional vision: Examining knowledge-based reasoning of pre-service teachers, in-service teachers, and school principals. *Frontiers in Education*, *5*, Article 59. https://doi.org/10.3389/feduc.2020.00059. This study compares different expertise groups with respect to their knowledge-based reasoning when viewing classroom photographs. The results suggest that in-service teachers draw more frequently on specific knowledge elements than pre-service teachers. In-service teachers further expressed more metacognitive verbalizations (e.g., self-monitoring), validating metacognition as a component of expertise. The study's methodology is also noteworthy, as it encompasses a systematic variation of presentation time and complexity of the stimulus material.

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