ZDM – Mathematics Education (2023) 55:1009–1020
https://doi.org/10.1007/s11858-023-01520-4

ORIGINAL PAPER

Instructor perspectives on quantitative reasoning for critical citizenship

Gregory D. Foley[1] · Deependra Budhathoki[2] · Amrit B. Thapa[1] · Harman P. Aryal[3]

Accepted: 17 August 2023 / Published online: 15 September 2023

Abstract
A tertiary course in Quantitative Reasoning (QR) has the potential to develop key practical and intellectual skills for citizenship, such as critical thinking, problem solving, quantitative literacy, and oral and written communication. In this article, we present research conducted on four instructors of such a QR course for students enrolled in a wide variety of nonscience degree programs at a university in the United States. The course used a student-inquiry approach to proportional reasoning, probability, statistical reasoning, and mathematical modeling. The findings are framed by a 5 C model of QR, which entails **C**ritical thinking to link real-world **C**ontexts to mathematical **C**oncepts supported by student **C**ollaboration and QR **C**ompetencies. The research addressed the questions of how university instructors support student development of the skills needed for critical citizenship and how this support relates to the 5 C model. We found that three of the four instructors viewed critical thinking as a central goal of the QR course and as supporting citizenship education. All four engaged students in tasks designed to develop a combination of skills associated citizenship, including critical thinking, self-questioning, collaboration, and communication. The discussion addresses such issues as the course's merits and challenges, student engagement, the relative importance of the five Cs, the importance of instructional autonomy, and recommendations for related professional development and future research.

Keywords Mathematical literacy/numeracy/quantitative literacy · Subjective numeracy · Quantitative reasoning · Critical thinking · Citizenship · Professional development

1 Introduction

Our capacity as humans to understand and use quantities in real-world contexts is essential to thriving in a data-rich "world characterized by escalating technological, economic and social transformation" (Geiger, 2019, p. 929). This human capacity—known alternatively as *mathematical literacy*, *numeracy*, or *quantitative literacy*—is not innate but rather a learned set of competencies linked to language and culture (Everett, 2017). Quantitative literacy enriches a person's life and allows the person to engage critically in the public sphere. Such literacy and the confidence to use it in everyday situations (i.e., subjective numeracy) affect an individual's comprehension, decisions, and outcomes related to health and financial stability (Peters, 2020). Numeracy prepares people to participate thoughtfully as citizens at home, at work, and in society (Briggs, 2018).

By contrast, innumeracy leaves many individuals, demographic groups, and even entire nations vulnerable in our contemporary world (Gal et al., 2020). Thus, there is cause for deep concern that the results of the Programme for the International Assessment of Adult Competencies 2017 and the Programme for International Student Assessment 2018 both show substantial differences across nations and between demographic groups within nations in adult *numeracy* and adolescent *mathematical literacy*, respectively.

✉ Gregory D. Foley
foleyg@ohio.edu

✉ Deependra Budhathoki
dbudhathoki@definace.edu

✉ Harman P. Aryal
harman.aryal@stockton.edu

Amrit B. Thapa
at737719@ohio.edu

[1] Ohio University, Athens, OH, USA

[2] Defiance College, Defiance, OH, USA

[3] Stockton University, Galloway, NJ, USA

To address the problem of innumeracy, over the past three decades and with recent acceleration, many U.S. universities, liberal arts colleges, and two-year colleges have developed entry-level (i.e., *gateway*) courses in quantitative reasoning (QR). Professional organizations—including the American Mathematical Association of Two-Year Colleges, the Mathematical Association of America, and the National Numeracy Network—recommend and support QR as an alternative gateway mathematics course for students not required to take calculus or statistics. Along with Foley and Wachira (2021), we argue that a gateway course in quantitative reasoning can develop 21st century practical and intellectual skills for citizenship, such as critical thinking, problem solving, quantitative literacy, and oral and written communication. The American Association of Colleges and Universities (AAC&U 2020) embraces this view and has developed rubrics for several such 21st century skills, including quantitative literacy.

Nonetheless, these gateway QR courses face many challenges. These courses "have been designed as alternative courses for a general education population, many of whom come to college [or university] with a hostile or fearful attitude toward math" (Piercey & Militzer, 2017, p. 693). Due to the novelty of such courses, many student advisors are uncertain about recommending students to enroll in them. The students who do enroll in QR often not only possess mathematics anxiety, avoidance, or even hostility but also lack prior related experience. Although some QR courses are designed for special majors, such as business, law and humanities, or the social sciences (Frith & Lloyd, 2021; Lloyd & Frith, 2013; Piercey & Militzer, 2017), others are open to a wide variety of degree programs, making it difficult to select real-world contexts that all students find relevant. When taught within a mathematics department, which is often the case, the existing instructors for gateway mathematics courses typically are accustomed to teaching techniques-focused courses, such as College Algebra, rather courses centered on student communication and reasoning. In addition, many QR instructors are unclear about the nature of the mathematical content and the ways to teach students in this nontraditional mathematics course. In a study of eight QR instructors from eight public postsecondary institutions in the state of Ohio, Budhathoki (2022) found that (a) QR instructors generally sought professional support for determining appropriate mathematical content, appropriate instructional strategies, or both in this course and (b) mathematics departments at some institutions require QR instructors to teach this nontraditional mathematics course in traditional ways.

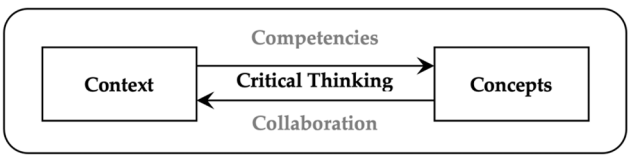

Fig. 1 The Five C Model of Quantitative Reasoning

In this article, we examine instructor perspectives on a relatively new gateway QR course at one U.S. university in Ohio. The course was designed to address the longstanding need for quantitative literacy (Gal, 2000; Steen, 1997, 2001, 2004) by helping students connect real-world contexts to mathematical and statistical content using critical thinking, supported by QR competencies and collaborative learning. In this study, we investigated how the content and methods of the QR course in question were used to advance the quantitative, intellectual, and practical capabilities needed for *critical citizenship*, that is, informed civic engagement at home, at work, and in society that employs critical thinking in quantitative judgments and decisions.

2 Conceptual framework and research questions

There are two established international research traditions in quantitative literacy education. One tradition studies *school mathematical literacy* (Geiger et al., 2015a, b; Goos et al., 2011); the other examines *adult numeracy* (Gal & Geiger, 2022; Gal et al., 2020). Given that our research context is a tertiary-level course in Quantitative Reasoning, our work is positioned between school and adult numeracy. Consequently, our work not only draws on these established research traditions but also builds on a third tradition of emerging numeracy education research at the tertiary level in South Africa, the United Kingdom, and the United States (Boersma et al., 2011; Frith & Lloyd, 2021; Grundy, 2020; Lloyd & Frith, 2013; Piercey & Militzer, 2017; Prince & Frith, 2020; Tunstall et al., 2019).

Figure 1 depicts our conceptual framework, which shows the key elements of a QR course and how these elements interact with one another. Because the five elements in this framework begin with the letter C, we refer to it as the Five C Model of Quantitative Reasoning, or simply as the 5 C model. It combines the Foley and Wachira (2021) model of student engagement in Quantitative Reasoning with findings from Budhathoki (2022).

As shown in Fig. 1, critical thinking is the heart of our approach to QR. *Critical thinking* involves interrogating an object of study to conceptualize and analyze it in order to reach a conclusion or judgment about it. In quantitative reasoning, we start with a real-world situation, make sense of it, *mathematize* it (Freudenthal, 1973), and analyze the resulting mathematical model. Hence, quantitative reasoning is a close cousin of mathematical modeling (Niss &

Blum, 2020). The critical thinking of quantitative reasoning involves "translating the problem situation into mathematics, working within the resulting mathematical model of the situation, and interpreting the mathematical outcomes in the given situation" (Durandt et al., 2022, p. 363). Like mathematical modeling, quantitative reasoning extends beyond theory and calculation. Quantitative reasoning (QR) entails "sophisticated reasoning with elementary mathematics more often than elementary reasoning with sophisticated mathematics" (Steen, 2004, pp. 9–10). QR also involves a "tug-of-war between the desire for mathematical simplicity and the fact of [real-world] complexity" (Pollak, 1966, p. 118). Validation is central to the continual back and forth between the real-world situation being studied and the mathematics and statistics used to make sense of it (Alhammouri, 2018). As students gain experience and expertise in QR, they (a) think deeply about the real-world context, (b) use mathematics and statistics to make sense of it, and (c) make frequent checks on how the context and its model are related to validate their thinking and computations.

The arrangement and shading in Fig. 1 are intended to convey that **C**ompetencies and **C**ollaboration support the **C**ritical thinking that QR students use to connect a real-world **C**ontext to mathematical and statistical **C**oncepts to reach conclusions about the contextual situation. Here, *Competencies* refer to the six interrelated *core competencies* for QR of representation, interpretation, calculation, analysis, assumptions, and communication (AAC&U 2009; Boersma et al., 2011; Ohio Department of Higher Education 2015) as well as allied competencies, such as 21st century skills (e.g., information literacy) and the use of physical and digital representational "tools as mediators of mathematical thinking and action" (Goos et al., 2011, p. 132). *Collaboration* is a pillar of inquiry-based learning (Laursen et al., 2014; Laursen & Rasmussen, 2019), which Budhathoki (2022) found to be a critical element of effective QR instruction.

Although not explicitly part of the 5 C model, *subjective numeracy*—the "confidence in one's objective numeracy abilities" (Peters et al., 2019, p. 19386)—is a goal of the QR course examined in this article. Student confidence is a critically important ally that can be developed in concert with the objective components of QR (Dingman & Madison, 2010). Subjective numeracy—both separately and in interaction with quantitative literacy—affects an individual's engagement and persistence with numeric tasks, so in turn influences the person's quantitative decisions and related behaviors (Peters, 2020). Because they relate to active, informed citizenship, we are interested in both subjective and objective numeracy.

Our prior related research has focused on QR assessment practices across several colleges and universities in Ohio (Budhathoki, 2022). Not only has this prior work helped us to see the interrelationships shown in the 5 C model but also to realize the importance of three other factors associated with effective QR instruction:

1. *Collaborative instructor meetings*. QR instructors reported benefitting from regular meetings with their QR teaching colleagues.
2. *Instructional autonomy* (i.e., instructors being allowed "to decide what content to use, how to teach, and when and how to assess student learning" [Budhathoki, p. 306]) was associated with an instructor's asking students for explanations and with students' providing their reasoning and thinking.
3. *Group projects.* QR instructor use of group projects supported student collaboration and was associated with instructor use of "thought-provoking, reasoning-based questions while students worked on their projects" (Budhathoki, p. 274).

To address the scarcity of research in this area, a case study was conducted at one university that (a) embraced the 5 C model in its QR course, (b) held regular QR instructor meetings, (c) allowed for QR instructional autonomy, and (d) encouraged the use of student group projects. Within such a setting, we examined two research questions,

- *How do university instructors support student development of the quantitative skills needed for critical citizenship, such as critical thinking, problem solving, and oral and written communication?*
- *How does this support relate to the 5 C model?*

For this study, the intellectual and practical skills of critical thinking, inquiry, problem solving, quantitative literacy, and oral and written communication served as indicators of critical citizenship. The 5 C model did not exist at the time of data collection, but we used it as a lens to interpret our results.

3 Research context

3.1 USA tertiary QR context

The study took place at a research university in the United States and involved an entry-level, credit-bearing undergraduate QR course taught in the mathematics department. Unlike Grundy's (2020) work in the United Kingdom, the instructors were not social scientists trying to enrich the quantitative skills of undergraduate majors in the social sciences. Nor was this a remedial intervention offered through

a Numeracy Centre at a university to shore up quantitative deficiencies as reported by Frith and colleagues in South Africa (Frith & Lloyd, 2021; Lloyd & Frith, 2013; Prince & Frith, 2020). Our research was situated between these, is in the tradition of the work reported in the edited volume by Tunstall et al. (2019), which presents multiple perspectives on the state of QR in U.S. higher education. Our work centers on the perspectives of instructors accustomed to teaching techniques-driven mathematics despite decades of calls for genuine real-world connections in gateway mathematics courses, especially for students not majoring in science, technology, engineering, or mathematics.

These calls for reform can be divided into two waves. Madison (2001), Madison and Steen (2003), Sons (1994), and Steen (1997, 2001, 2004) launched a *first wave* of the QR movement at colleges and universities in the USA and planted the seeds for the current, *second wave*, which has been undergirded by a call for alternative *mathematical pathways* and is causing a proliferation of QR courses. This second wave of the QR movement has been led by the American Mathematical Association of Two-Year Colleges (2018), the Carnegie Foundation (n.d.), the Dana Center (n.d.), the Mathematical Association of America (Saxe & Braddy 2015), and Transforming Post-Secondary Education in Mathematics (n.d.). This current wave has given numeracy a home in the form of QR courses within college and university departments of mathematics (Madison, 2019).

3.2 QR courses at public higher education in Ohio

The U.S. state of Ohio has a system of 23 public two-year colleges and 13 public universities, many with multiple campuses and centers and even courses offered within high schools for postsecondary credit. During the early stages of the second wave of QR reform in the USA, a committee headed by Leitzel (2014) launched the *Ohio Mathematics Initiative* of the Ohio Department of Higher Education. The related learning guidelines for QR state that "a Quantitative Reasoning course needs to highly emphasize the core mathematical general education outcome [of] critical thinking as its primary objective" (Ohio Department of Higher Education 2015, p. 2). These state guidelines also require the QR course to address proportional reasoning, probability, statistical reasoning, mathematical modeling, and the core competencies of interpretation, representation, calculation, analysis, assumptions, and communication. Each public college or university must demonstrate in writing how its QR course addresses critical thinking and these other learning outcomes. As of March 2023, 27 of Ohio's 36 public colleges and universities offer state-approved QR courses. The enrollment in these courses has grown strikingly from a mere 251 students in 2015–2016 to 9,759 students in 2021–2022 (P. K. Compton, personal communication, 21 March 2023).

Since 2018, members of the author team have conducted research studies on various aspects of QR at the growing number of Ohio public postsecondary institutions that have approved QR courses (e.g., Budhathoki, 2022). But we had not done an in-depth case study at any single institution. Consequently, the study reported herein focused on just one of the 36 public higher education institutions in Ohio.

3.3 QR at a selected university and the content of the course

This study took place at a public university in Ohio. It is a research university situated in a rural area but serves primarily students from urban areas in Ohio and elsewhere in the United States. The university has a limited number of international students as well.

QR was first offered at this university in fall semester 2018, beginning with just one section, one instructor, and nine students. From the beginning, a student-centered, inquiry approach had been used in all sections. Since spring semester 2019, the growing number of QR instructors had held regular (typically weekly) meetings to share student tasks, teaching strategies, successes, and challenges. The university received state-level approval for its QR course in April 2020.

This study took place in fall semester 2021. By this time, the QR course had been established as a regular course. The department chair selected QR instructors based on their interest in teaching this course and their commitment to student-centered instruction and instructor collaboration. These QR instructors exercised individual autonomy in pacing, topics covered, tasks and assessments used, while operating within state and university guidelines. In keeping with these guidelines, the QR course addressed proportional reasoning, probability, statistical reasoning, mathematical modeling, and the QR competencies of interpretation, representation, calculation, analysis, assumptions, and communication. The proportional reasoning unit typically included a group student project on personal financial literacy, such as comparing 15-year and 30-year mortgage options for the purchase of a residence. In some sections, the statistical reasoning unit required a statistical survey designed and conducted by a group of three or four students. Major grades were based on such projects instead of exams. A premium was placed on students explaining and justifying their thinking orally and in writing, including written project reports, and on students making their thinking seen and heard by others through classroom presentations.

There were six sections of QR, with six instructors and 177 students. All these QR students were undergraduates

pursuing bachelor's degrees. They were mostly freshmen, and over half were women. The vast majority of these QR students were majoring in fine arts, communication studies, humanities, social science, or education. Some had other majors, and several had not yet chosen a major.

4 Research methods

To examine how QR instructors use the 5 C model to support students in developing critical citizenship, we conducted a case study of the implementation of the QR course at the selected university in fall semester (i.e., August–December), 2021. The authors of this article served as the research team. The primary data source came from semi-structured interviews of QR instructors. In addition to the interviews, at least one member of the research team attended each of the weekly QR instructor Zoom meetings. This helped the research team to gain perspectives that allowed for deeper interpretation of the interview data than would have otherwise been possible. It also helped the QR instructors to become comfortable in sharing their teaching experiences with the researchers prior to the formal interviews.

4.1 Research design and specific research context

The case involved the QR course over this specific semester, its instructors, and their interactions within the QR instructional context. We chose a case-study design because it allowed for an in-depth analysis of the phenomenon of interest and its participants within a given context (Stake, 1995; Yin, 2018). Our goal was not to manipulate behavior but to uncover the perceptions of the QR instructors concerning the curriculum, the nature of student engagement, and challenges in nurturing critical citizenship.

Due to the COVID-19 pandemic, this was the first semester for in-person classes in more than a year (since March 2020). QR classes were offered in various modes: remote, hybrid, and in person. For in-person meetings, a mask mandate was in effect for students and the instructor.

4.2 Participant selection and characteristics

After receiving approval from the Institutional Review Board, we invited QR instructors to participate in the study. In all, four of the six QR instructors volunteered, gave their consent, and participated in the study.

The participants included two of the four tenured professors and both graduate teaching assistants (GTAs) who were teaching QR that semester. Both professors had decades of teaching experience, but they were teaching QR for the first time. Both GTAs were second-year master's students majoring in mathematics; each had one year of teaching experience and had taught QR once before. None had taught QR during its development phase at the university.

4.3 Instrument and data collection

The author team developed a set of semi-structured interview questions. Via multiple writing and editing cycles, we refined the questions with the goal of obtaining rich descriptions of each instructor's experiences and perceptions about teaching the QR course, especially in relation to the quantitative skills associated with citizenship. This process resulted in five domains of interview questioning: (a) general perceptions about the course, its nature, the instructional approach, and assessments; (b) quantitative skills related to citizenship; (c) connections to the real world; (d) self-efficacy in social or citizenship roles, empowerment, and anxiety; (e) recommendations for changes in curriculum, pedagogy, and assessment. The semi-structured nature of the interviews allowed for follow-up questions to pursue points of interest.

Three of the team members conducted the interviews, each of which lasted roughly an hour. All four interviews were one-on-one and were done at the participant's convenience during December 2021, at the end of the semester. Three interviews were conducted remotely and recorded using Zoom, and one was done in person and voice-recorded using the researcher's iPhone. As compensation for their participation, we gave each GTA a US$30 Amazon gift card; we did not provide compensation to the two professors.

4.4 Data analysis

The analysis was a multi-step process. Our conceptual framework provided several potential codes and themes: real-world contexts, mathematical and statistical techniques and concepts, critical thinking, QR competencies, subjective numeracy, student collaboration, allied competencies (e.g., other 21st century skills), and citizenship.

The three team members who conducted the interviews used NVivo software to create initial transcriptions and checked these initial drafts against the audio recordings to refine the transcriptions. Then, two of the team members worked together to identify the main codes, which they then organized into the themes of *Learning outcomes related to citizenship, Real-world contexts and applications*, and *Subjective numeracy*. Some codes emerged, such as "Structuring the course," which did not naturally fall within any of the anticipated themes. They shared these codes with the other team members for cross-checking, and we added the theme of *Teaching issues* to accommodate these emergent

codes. Ultimately, we classified all the anticipated and emergent codes into these four themes as shown in Table 1.

5 Findings

To provide a frame for the more detailed results, we first present an overview, which further explains the themes and codes just mentioned. We then briefly describe the four participating instructors in aggregate. Next, we describe them individually and present their individual perspectives on the QR course and how it developed competencies for critical citizenship. We follow this with a comparative and collective analysis of the four instructors and their perspectives.

5.1 Overview of results

As shown in Table 1, the four themes that emerged relate to the five domains of interview questioning detailed in Sect. 4.3. The responses concerning general perceptions about the course generated codes that cut across all four themes. There were reasonably direct connections between questioning domains (b), (c) and (d) and Themes 1, 2, and 3, respectively: Learning outcomes related to citizenship; Real-world contexts and applications; and Subjective numeracy. The responses concerning recommendations for changes and some of the other questioning domains morphed into Theme 4: Teaching issues.

Many learning outcomes related to citizenship and real-world connections emerged by asking the instructors to describe the course in general terms. When asked about recommendations for changes, they mainly focused on what they would try to do differently in future semesters rather than recommending changes at the departmental level for all QR instructors. This finding suggests general satisfaction with the course and its goals as well as a focus on self-improvement, and may be due to their autonomy for decision making related to instruction and assessment.

5.2 Instructors and their perspectives

As noted above, these instructors had the autonomy to select or design their own student activities and choose their own methods and weights for student assessments. Together with one of the other two QR instructors, who had been a course developer, the participants met once a week throughout the semester to share their approaches, successes, and challenges in teaching the course. The instructors—for whom we use pseudonyms—each had their own perspectives on what the course was and what it should be.

5.2.1 Prof. Dr. Art

Art, a PhD and full professor in mathematics, had more than 30 years of teaching experience. He had spent a great deal of time during summer 2021 preparing to teach the QR course, knowing that he would be teaching it for the first time. Art had deeply held thoughts about the QR course and its goals for student learning. For example, Art believed that sense making and critical thinking led to student empowerment and made them less vulnerable:

> *Every day … something quantitative is being presented to [the students]. And they need to make sense of it, really make sense of it. And if they don't, then somebody else will manipulate them, OK? … And that's an important tool to empower them to make sense of the information.*

In Art's mind, students' self-questioning was central to their critical thinking, inquiry, and problem solving:

> *The first thing we ask ourselves is what? What does it really mean? What's going on here? What am I supposed to do? … And the big problem for these students is they start calculating something without asking themselves these questions.*

Consequently, Art worked to get his students to learn how to ask questions to help them make sense of a given real-world situation.

> *It made the process essentially to get them to ask the questions. … The empowering thing is to ask questions, to say what does this really mean? You see a percentage, then percentage of what? … If you see a*

Table 1 Instructor Views

Theme	Codes
1.Learning outcomes related to citizenship	Communication (effective, oral, written); Competencies (analysis, argumentation, assumptions); Critical citizenship; Critical thinking, problem solving, self-questioning; Information literacy; Meaningful learning; Permanent impression; Student collaboration
2. Real-world contexts and applications	Critical consumer of information; Day-to-day life; Political issues as subject matter; Professional life, Sociopolitical and citizenship roles
3. Subjective numeracy	Self-awareness; Self-confidence; Student empowerment
4. Teaching issues	Adopting materials; Group projects; Innovative, nonstandard assessments; Instructor collaboration; Self-designed student activities; Structuring the course; Uncomfortable with politically charged issues

fraction, fraction of what? If you get a number from some calculations, then the question is, what does this mean in the real-world context?

From Art's perspective, self-questioning was critical to achieving the QR core competencies of communication, analysis, and assumptions and thus to making a valid "quantitative argument."

Art actively searched for contexts that would be meaningful to his students and that would challenge them to think deeply about mathematical and statistical concepts. And some of these had implications for critical citizenship: In one task, he challenged his students to explain why, when comparing Ohio to the neighboring state of Kentucky, the mean *family* income is higher in Ohio, but the mean *individual* income is higher in Kentucky. He asked,

Why is that the case? … The students had a hard time explaining.

Art thought that becoming "critical consumers of information" was important and connected to being critical in the political sphere:

[Students] need to understand how politicians manipulate them … and how you can lie with statistics. … It will make them more critical, I hope. … They need to be critical consumers of information.

Art believed that in his instructional role it was important to be neutral in political matters and not to advocate for a particular position. Ultimately, Art wanted his QR students to become lifelong quantitative reasoners.

5.2.2 Prof. Dr. Bob

Like Art, Bob was a PhD and full professor in mathematics. Bob had more than 20 years of teaching experience but was teaching QR for the first time. He was the only QR instructor who taught in a hybrid format: some class meetings were face-to face; others were online. Bob was assigned to teach QR just a few days before the semester began. Thus, in sharp contrast to Art, Bob had little time to prepare to teach the course and for the first several weeks relied heavily on the approaches and tasks developed during the previous 3 years by the QR instructors who had developed the course.

I was … teaching the course for the first time, … feeling my way through it.

These circumstances may have influenced his wish that the course be more structured and easier to teach:

I would like [the QR course to be] more structured and [have] less instructor flexibility. … Especially [given] the issue of scaling this up and having instructors come in, it needs more structure.

Bob had taught other lower-division mathematics courses using inquiry-based learning strategies, and was comfortable having students work in small, collaborative groups. But Bob found that the details of what he had done previously required substantial adaptation to the QR setting. In addition, Bob had used innovative approaches to assessment, and he thought that group projects were a good way to assess learning in the QR course:

I like the project format better than tests for some courses. And I think this is one of them. [Some] quick quizzes might make sense, but as a small part of the course.

Bob emphasized student collaboration in teaching QR:

So, I do a lot of group work, and I have done that in other courses to varying extents… I try to have a lot of time when they are talking in their groups, working problems in their groups, doing investigations in their groups, and not lecturing much.

Due perhaps to his prior inquiry-based learning experience, Bob's major student projects were open ended; however, his in-class activities were less open ended but still involved group collaboration.

As the semester progressed, Bob increasingly developed his own activities for students. Some of these were related to the theme of critical citizenship:

We did … misleading graphs and misleading statistics. And I thought that was useful for … not being misled or being skeptical about strange claims, which will help, I guess.

5.2.3 GTA Char

Char was an international master's student in mathematics who had earned a bachelor's degree in the United States. A year earlier, in fall semester, 2020, Char had observed two QR instructors and started attending the weekly QR instructor meetings in preparation for teaching QR in spring semester, 2021, which she did. So, when Char was interviewed, she was finishing her second semester as a QR instructor. Still, she was uncertain about some QR course content and teaching strategies. For example, Char's interview revealed

that the topic of probability, and especially conditional probability, was unfamiliar to her, and she was unsure about how to teach it effectively. In addition, she indicated that she was not comfortable using nonstandard assessments or orchestrating group work. Nonetheless, Char had clear goals for her QR section, which were much like those expressed by Art:

> *The main thing that I'm trying to teach them is critical thinking and maybe questioning … What are you trying to get? How are you going to get there? What questions do you need to answer before you get to your answer?*

Char valued self-awareness and self-confidence as goals for her QR students. She also included information literacy as a course goal even though it was not required by the state or university QR guidelines.

Char mentioned an important challenge that was tacit among the other participants:

> *With the QR, … you are working with people who are very far from math. … People who are very involved in math are trying to teach people who are very, very far from math.*

Although the other instructors understood that the QR students were not mathematics, science, or engineering majors, Char was especially aware of this and worked to connect with her students, but this was difficult, and often it was not clear how best to help them.

Nonetheless, Char recognized the importance of engaging her students in challenging tasks with real-world connections. She had worked with Doug to develop some demanding student projects with real-world contexts. Char especially valued students being able to articulate their thinking and saw this as an area of significant growth among her students:

> *Because, in the beginning of the semester, they would just answer a question with … one sentence, but at the end of the semester, I think they were more elaborating and putting their personal thinking into it.*

Acting as a possible barrier to developing students' critical citizenship, Char was uncomfortable delving into politically charged issues because—in her words—she was "international" and did not wish to get "too personal." This is a concern as QR scales up in Ohio and elsewhere because many U.S. graduate students in mathematics are international students.

5.2.4 GTA Doug

Doug was from the USA. Like Char, he was a mathematics master's student and a GTA. And like Char, during fall semester, 2020, Doug had observed two QR instructors, and attended the weekly QR instructor meetings. And Doug had taught QR in spring semester, 2021. So, when Doug was interviewed, he was teaching QR for the second time. Like Char, Doug valued student self-awareness, but he placed a stronger emphasis on issues related to citizenship and mentioned the importance of processing the ideas of other persons:

> *One part of being an active citizen is the ability to be quantitative in some regard. But another piece is being able to be articulate not only how you feel, what you think, what you believe, but also in processing others—listening, understanding, and being patient. … But then the third piece, I think, is really kind of a self-awareness.*

Doug's biggest goal for the course was critical thinking:

> *But if you teach them how to think, how to research, what's important, what's not. … that's a lifelong skill.*

Importantly, Doug noted, if the instructor can demonstrate the importance of learning from mistakes, students are much more open to learning from their own mistakes. Consequently, Doug was able to leverage his inexperience to positive effect.

5.3 Comparative and collective analysis of the instructors

The participants had a wide range of experience and expertise. At one extreme, Prof. Dr. Art had taught mathematics for 30 + years and was a practitioner and researcher in mathematical modeling for 20 + years. At the other extreme, GTAs Char and Doug were master's students in their second years of teaching. These findings reveal that QR instructors can have diverse levels of experience and expertise and consequently provide different learning opportunities for their students. Moreover, the type of professional support needed can vary greatly across QR instructors.

Art's extensive teaching experience, expertise in mathematical modeling, and 3 months of preparation time allowed him to use the QR tasks developed other instructors over the previous 3 years as a starting point for creating his own QR materials, which he then shared with the rest of the QR instructional team. Another QR instructor (who we will call Ed), who was not interviewed, had been involved in

3 years of prior QR course development. Art and Ed were able to offer Bob, Char, and Doug help and support during the weekly QR team meetings and in between these meetings as needed.

In keeping with the 5 C model, the data show that Art, Char, and Doug highly valued student critical thinking, communication, and collaboration and viewed these as central goals in the QR course. They all connected critical thinking to citizenship. Art and Char viewed self-questioning as the centerpiece of student inquiry.

Bob, who had prior experience with inquiry-based and project-based teaching, struggled to adapt these pedagogies to this new audience and new course at first, but made progress in this regard after his intial struggles. Though prompted, he never talked deeply about critical thinking or citizenship. Bob did give open-ended final projects that allowed his students to investigate citizenship-related issues that mattered to them.

6 Discussion

6.1 Addressing the research questions

Our findings suggest that a gateway course in QR offers promise for developing quantitative skills that support critical citizenship—provided that adequate mechanisms and resources are in place to support instructors in using contexts and collaboration to help students develop critical thinking and reasoned communication. This finding speaks directly to our research questions and is especially noteworthy because the goal of the course was not critical citizenship. The instructors in this study saw this course as one focused on reasoning, thinking, and communication about real-world contexts using mathematical and statistical tools as opposed to conventional mathematics focused on procedural fluency, conceptual development, and problem solving "detached from societal implications" (Maass et al., 2019, p. 991). Based on the interview data, we contend that the attention to reasoning, thinking, and communication about real-world contexts through mathematical and statistical lenses led naturally to developing quantitative skills that support critical citizenship.

It is important to note that all four participants in this study chose to teach QR and embraced student-centered teaching methods. These instructors had committed to take on this challenging work, which involved weekly meetings and more preparation and grading than a typical gateway course. The four QR instructors had made this choice because each of them valued the importance of QR in some way. For example, Art, Char, and Doug valued helping students develop critical thinking and communication skills, which they saw as beneficial to the students. Bob, who jumped on board just before the semester began, took a while to *feel his way* into the course but ultimately engaged his students in open-ended collaborative projects.

If instructors who had little background, interest, or orientation to QR had been assigned to teach this course, the results likely would have been different. As the enrollment in this course continues to scale up and the QR teaching workforce spreads to a broader collection of instructors, additional mechanisms and resources will be needed to support these instructors not only to focus student attention on reasoning, thinking, and communication about real-world contexts but also to help instructors understand and value the QR course and its goals.

6.1.1 Reasons for successful student engagement

For the most part, the QR instructors in this study were able to engage their students in tasks to develop quantitative skills for critical citizenship. A combination of factors led to this type of student engagement: instructor commitment, existing QR resources, weekly meetings, instructional autonomy, a student-centered approach, and the use of group projects. These results align with and reinforce the findings of Budhathoki (2022), who conducted his research on other QR instructors at postsecondary institutions throughout Ohio.

Moreover, we contend that Char and Doug were greatly aided by having an opportunity to observe QR classes and participate in the QR instructor learning community meetings prior to first teaching the course and that Art benefitted from taking an entire summer to prepare to teach the QR course. This finding supports our argument that if instructors are provided with an opportunity for QR-focused professional development and adequate preparation to teach this course, they can engage their students in tasks to develop quantitative skills for critical citizenship—but this will be difficult to sustain over time as enrollment in QR courses continues to increase.

6.1.2 Instructional challenges

As Art noted, students struggled to make sense of some situations, such as his Kentucky–Ohio task. As Char pointed out, the QR students were *very far from math*, which made helping them to reason and communicate in mathematical ways especially challenging.

The contrast between Art's and Bob's experiences shows the benefits of advance preparation time to teach QR well. Again, our findings reinforce those of Budhathoki (2022), who found that mathematics instructors generally sought professional support for determining appropriate content and instructional strategies for teaching QR. Given the

diversity of the teaching workforce for gateway mathematics—including inexperienced GTAs—who are used to techniques-centered courses, there is a clear need for professional development to teach QR courses, which have as priorities student reasoning and communication as well as in-context mathematics. We say more about this in Sect. 6.3 below.

6.1.3 Relative importance of the five Cs in our QR model

Our results indicate that some of the instructional elements of the 5 C model are more central to developing citizenship than others: The most salient elements we will call *big Cs*; the others, *little c's*. Critical thinking and QR competencies, especially, communication emerged as the most influential components—or biggest Cs—in the 5 C model. Critical thinking appeared in the instructor data as student self-questioning and articulation of their thinking. Argumentation and articulation of thinking are forms of *reasoned communication*, which Geiger (2019) calls "using mathematics as evidence to support critical reasoning and enquiry" (p. 929). These forms of reasoned communication are high-level QR competencies that are central to the QR course and to developing citizenship. The finding of student self-questioning is novel and noteworthy, especially with regard to postsecondary QR.

Consistent with Budhathoki's (2022) findings, collaboration emerged as another "Big C." All four instructors used student collaboration and group projects. Budhathoki found that student collaboration and especially collaborative projects were a key indicator of high-quality QR instruction.

Contexts were important, especially to Art. Our findings support the claim that the level of student engagement is directly related to the genuineness and relevance of the context being studied. In keeping with Elrod (2014), we contend that contexts and their connection to student's life world are important in the effectiveness of QR instruction. AMATYC (2018) and Saxe and Braddy (2015) support this view.

Perhaps surprisingly, the *littlest c* was that of mathematical and statistical content. Bob discussed the content more than the other participants and did not mention critical thinking. Art, Char, and Doug indicated that the primary drivers for developing citizenship were critical thinking—including self-questioning—and student communication. For them, mathematical concepts were linked to developing citizenship to the extent that these concepts were used as cognitive tools to think critically about real-world scenarios.

Therefore, based on the present study, we conclude that content is the "the little c" and less important than the other four Cs in our model. Furthermore, even though prior researchers and research traditions have presented different models for QR instruction, we argue that our 5 C model has merit for the nature of the gateway QR course at the postsecondary level.

6.2 Instructional autonomy vs. course structure

As cited above, Bob, who had been assigned to teach QR a few days before classes began, spoke against autonomy and for structure. Nonetheless, the four instructors in this study—including even Bob after some initial struggles—were able to thrive due their access to existing QR materials previously developed by other instructors and the weekly QR instructor meetings that provided mutual support. Thus, consistent with Budhathoki's (2022) findings and the arguments of Elrod (2014), we conclude that there need not be a rigid structure in what and how QR is taught and that instructional autonomy and flexibility support the 5 C approach to QR and the development of skills for citizenship.

6.3 Designing and investigating professional development for QR instructors

To achieve a level of student engagement in teaching gateway QR comparable to that observed in this study across Ohio and elsewhere, we argue that it would be useful to create mechanisms and resources analogous to those that existed among the instructors who participated in this study. The study participants used authentic assessments in the form of group projects; they met on a weekly basis to support each other with long-term professional development, mutual collaboration, and sharing of instructional resources; and they maintained the autonomy to make individual decisions concerning the details of student learning activities and assessments. Wherever such courses are taught at postsecondary institutions, we recommend the creation of state- or provincial-level QR professional development networks that encourage these forms of practice. We further recommend that these networks and related instructional resources employ the 5 C model of QR and that research be conducted to measure their effect on instructional quality and student learning.

6.4 Subjective numeracy

This case study revealed some findings related to subjective numeracy, but further research is needed in this arena. Art spoke of student empowerment and making students less vulnerable to outside manipulation. Bob helped his students to become aware of misleading information and to be "skeptical about strange claims." Both Char and Doug emphasized student self-awareness. However, deeper, student-focused research is needed to plumb the depths of subjective numeracy and the interactions between objective and

subjective numeracy. Thus, we recommend future studies in these areas and specifically studies to investigate whether and in what ways a QR course can serve as an intervention for postsecondary students who avoid, fear, or dislike mathematics to gain confidence in doing and using mathematics.

Funding The research reported herein was supported by the Ohio University research incentive fund, by the Patton College graduate study and educational research fund, and by National Science Foundation award no. 2216197. The findings and conclusions are those of the authors and do not necessarily represent those of the funding agencies.

References

Alhammouri, A. M. (2018). *Professional development in mathematical modeling: Teacher engagement, teacher knowledge, and classroom implementation* (Doctoral dissertation, Ohio University). http://rave.ohiolink.edu/etdc/view?acc_num=ohiou1527243404147734.

American Association of Colleges and Universities (2009). *Quantitative literacy VALUE rubric.* https://www.aacu.org/initiatives/value-initiative/value-rubrics/value-rubrics-quantitative-literacy.

American Association of Colleges and Universities (2020). *Overview of the Valid Assessment of Learning in Undergraduate Education (VALUE) initiative.* https://www.aacu.org/initiatives/value.

American Mathematical Association of Two-Year Colleges (2018). *IMPACT: Improving mathematical PROWESS and college teaching*. https://cdn.ymaws.com/amatyc.site-ym.com/resource/resmgr/impact/impact_links.pdf.

Boersma, S., Diefenderfer, C., Dingman, S. W., & Madison, B. L. (2011). Quantitative reasoning in the Contemporary World, 3: Assessing student learning. *Numeracy*, *4*(2), Article 8. https://doi.org/10.5038/1936-4660.4.2.8

Briggs, W. (2018). Quantitative reasoning and civic virtue. *Numeracy*, *11*(2), Article 7. https://doi.org/10.5038/1936-4660.11.2.7

Budhathoki, D. (2022). *Formative assessment in postsecondary Quantitative Reasoning courses* (Doctoral dissertation, Ohio University). http://rave.ohiolink.edu/etdc/view?acc_num=ohiou165903392100886.

Carnegie Foundation for the Advancement of Teaching. (n.d.). *Carnegie math pathways*. https://www.carnegiemathpathways.org.

Dana Center, University of Texas at Austin. (n.d.). *Dana Center mathematical pathways*. https://www.utdanacenter.org/our-work/higher-education/dana-center-mathematics-pathways.

Dingman, S. W., & Madison, B. L. (2010). Quantitative reasoning in the Contemporary World, 3: Assessing student learning. *Numeracy*, *3*(2), Article 4. https://doi.org/10.5038/1936-4660.3.2.4.

Durandt, R., Blum, W., & Lindl, A. (2022). Fostering mathematical modelling competency of south african engineering students: Which influence does the teaching design have? *Educational Studies in Mathematics*, *109*(2), 361–381. https://doi.org/10.1007/s10649-021-10068-7.

Elrod, S. (2014). Quantitative reasoning: The next across the curriculum movement. *Peer Review*, *16*(3), 4–8.

Everett, C. (2017). *Numbers and the making of us: Counting and the course of human cultures*. Harvard University Press.

Foley, G. D., & Wachira, P. W. (2021). From gatekeeper to gateway: The role of quantitative reasoning. *Ohio Journal of School Mathematics*, *87*, 29–36. https://library.osu.edu/ojs/index.php/OJSM/article/view/8339/6125.

Freudenthal, H. (1973). *Mathematics as an educational task*. D. Reidel.

Frith, V., & Lloyd, P. (2021). Investigating alignment in a quantitative literacy course for social sciences students. *Numeracy*, *14*(2), Article 5. https://doi.org/10.5038/1936-4660.14.2.1384

Gal, I. (Ed.). (2000). *Adult numeracy development: Theory, research, practice*. Hampton Press.

Gal, I., & Geiger, V. (2022). Welcome to the era of vague news: A study of the demands of statistical and mathematical products in the COVID-19 pandemic media. *Educational Studies in Mathematics*, *111*, 5–28.

Gal, I., Grotlüschen, A., Tout, D., & Kaiser, G. (2020). Numeracy, adult education, and vulnerable adults: A critical view of a neglected field. *ZDM–Mathematics Education*, *52*(3), 377–394. https://doi.org/10.1007/s11858-020-01155-9.

Geiger, V. (2019). Using mathematics as evidence supporting critical reasoning and enquiry in primary science classrooms. *ZDM–Mathematics Education*, *51*(6), 929–940. https://doi.org/10.1007/s11858-019-01068-2.

Geiger, V., Forgasz, H., & Goos, M. (2015a). A critical orientation to numeracy across the curriculum. *ZDM–Mathematics Education*, *47*(4), 611–624. https://doi.org/10.1007/s11858-014-0648-1.

Geiger, V., Goos, M., & Forgasz, H. (2015b). A rich interpretation of numeracy for the 21st century: A survey of the state of the field. *ZDM–Mathematics Education*, *47*(4), 531–548. https://doi.org/10.1007/s11858-015-0708-1.

Goos, M., Dole, S., & Geiger, V. (2011). Improving numeracy education in rural schools: A professional development approach. *Mathematics Education Research Journal*, *23*(2), 129–148. https://doi.org/10.1007/s13394-011-0008-1.

Grundy, S. (2020). The past, present and future of Q-Step—A programme creating a step-change in quantitative social science skills. *Numeracy*, *13*(1), Article 2. https://doi.org/10.5038/1936-4660.13.1.2

Laursen, S. L., & Rasmussen, C. (2019). I on the prize: Inquiry approaches in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education*, *5*, 129–146. https://doi.org/10.1007/s40753-019-00085-6.

Laursen, S. L., Hassi, M. L., Kogan, M., & Weston, T. J. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. *Journal for Research in Mathematics Education*, *45*, 406–418.

Leitzel, J. R. (Chair). (2014, March). *Rethinking postsecondary mathematics: Final report of the Ohio Mathematics Steering Committee*. Ohio Department of Higher Education. https://transfercredit.ohio.gov/static/files/math/MATH-REPORT_FINAL_4.22.14.pdf.

Lloyd, P., & Frith, V. (2013). Proportional reasoning as a threshold to numeracy at university: A framework for analysis. *Pythagoras*, *34*(2). https://doi.org/10.4102/pythagoras.v34i2.234.

Maass, K., Doorman, M., Jonker, V., & Wijers, M. (2019). Promoting active citizenship in mathematics teaching. *ZDM–Mathematics Education*, *51*(6), 991–1003. https://doi.org/10.1007/s11858-019-01048-6.

Madison, B. L. (2001, August/September). Quantitative literacy: Everybody's orphan. *MAA Focus*, *21*(3), 10–11. https://www.maa.org/sites/default/files/pdf/pubs/augsept01web.pdf.

Madison, B. L. (2019). Quantitative literacy: An orphan no longer. In S. L. Tunstall, G. Karaali, & V. Piercey (Eds.), *Shifting contexts, stable core: Advancing quantitative literacy in higher education* (pp. 37–46). Mathematical Association of America.

Madison, B. L., & Steen, L. A. (Eds.). (2003). *Quantitative literacy: Why numeracy matters for schools and colleges*. National Council on Education and the Disciplines.

Niss, M., & Blum, W. (2020). *The teaching and learning of mathematical modelling*. Routledge.

Ohio Department of Higher Education (2015, December). *TMM011—Quantitative Reasoning*. https://transfercredit.ohio.gov/static/files/transfer/documents/OTM/TMM011%20Quantitative%20Reasoning%20FINALIZED%20v2-%2012-21-2015.pdf.

Peters, E. (2020). *Innumeracy in the wild: Misunderstanding and misusing numbers*. Oxford University Press.

Peters, E., Tompkins, M. K., Knoll, M. A. Z., Ardoin, S. P., Shoots-Reinhard, B., & Meara, A. S. (2019). Despite high objective numeracy, lower numeric confidence relates to worse financial and medical outcomes. *Proceedings of the National Academy of Sciences*, *116*(39), 19386–19391. https://doi.org/10.1073/pnas.1903126116.

Piercey, V., & Militzer, E. (2017). An inquiry-based quantitative reasoning course for business students. *PRIMUS*, *27*(7), 693–706. https://doi.org/10.1080/10511970.2016.1251519.

Pollak, H. O. (1966). On individual exploration in mathematics education. In E. G. Begle (Ed.), *The role of axiomatics and problem solving in mathematics* (pp. 117–122). Conference Board of the Mathematical Sciences (published by Ginn).

Prince, R., & Frith, V. (2020). An investigation of the relationship between academic numeracy of university students in South Africa and their mathematical and language ability. *ZDM–Mathematics Education*, *52*(3), 433–445. https://doi.org/10.1007/s11858-019-01063-7.

Saxe, K., & Braddy, L. (Eds.). (2015). *A common vision for undergraduate mathematical sciences programs in 2025*. Mathematical Association of America. https://www.maa.org/sites/default/files/pdf/CommonVisionFinal.pdf.

Sons, L. R. (Ed.). (1994). *Quantitative reasoning for college graduates: A complement to the standards*. Mathematical Association of America. https://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/quantitative-literacy/quantitative-reasoning-college-graduates.

Stake, R. E. (1995). *The art of case study research*. SAGE Publications.

Steen, L. A. (Ed.). (1997). *Why numbers count: Quantitative literacy for tomorrow's America*. College Entrance Examination Board.

Steen, L. A. (2004). *Achieving quantitative literacy: An urgent challenge for higher education*. [MAA Notes No. 62]. Mathematical Association of America.

Steen, L. A. (Executive Ed.). (2001). *Mathematics and democracy: The case for quantitative literacy*. National Council on Education and the Disciplines & Woodrow Wilson National Fellowship Foundation. https://www.maa.org/sites/default/files/pdf/QL/MathAndDemocracy.pdf.

Transforming Post-Secondary Education in Mathematics. (n.d.). *Vision*. https://www.tpsemath.org/about.

Tunstall, S. L., Karaali, G., & Piercey, V. (Eds.). (2019). *Shifting contexts, stable core: Advancing quantitative literacy in higher education* [MAA Notes No. 62]. Mathematical Association of America.

Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). SAGE Publications.