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ORIGINAL PAPER

# Exploring a framework for integrated STEM: challenges and benefits for promoting engagement in learning mathematics

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**Abstract**
Advocacy of STEM curricular approaches is based on a concern to engage students in authentic disciplinary and interdisciplinary practices in the STEM (science, technology, engineering, mathematics) disciplines, and the need to promote participation in STEM pathways. The *STEM Academy* professional learning program was developed to support teachers to engage and motivate students by creating real-world, challenging problems. The initiative involved interdisciplinary teams of secondary STEM teachers attending workshops and working with university experts to design, implement and evaluate STEM curricular experiences. This paper focuses on case studies undertaken in two of the twelve schools involved in the initiative, using interview data from teachers and students, to explore the nature of their programs and their experiences. We investigate key features of these two schools' approaches, using a conceptual framework for integrated STEM, and explore the challenges and benefits of different features of integrated STEM that promote different dimensions of engagement in learning mathematics. We explore how the framework characteristics can be reframed into four dimensions that promote mathematics engagement in integrated STEM education - design thinking in authentic contexts, content integration, STEM practices and 21st Century skills, and exposure to professional practice.

**Keywords** Interdisciplinary mathematics curriculum · Integrated STEM curricula · Engagement with mathematics learning · Mathematics in STEM

## 1 Introduction

An increasing international interest in promoting science, technology, engineering, and mathematics (STEM) and STEM education pathways is part of a concern for national wealth creation which is associated with STEM Research and Development (Freeman et al., 2015). Increasingly, interdisciplinary approaches to STEM Education have been advocated (Bybee, 2018) on the basis that these offer a way of more effectively engaging students with these subjects (Honey et al., 2014), and developing the knowledge and skills needed in a fast-changing world. However, questions are regularly raised as to the depth of disciplinary learning that comes from integrated, project based-STEM approaches, and to the longer term disciplinary integrity of a STEM curriculum agenda to represent distinct epistemic processes (Reinholz et al., 2018; Williams et al., 2016). This question of integrity of disciplinary learning has arisen particularly in relation to mathematics (English, 2016; Siemon et al., 2019).

The University of Sydney *STEM Academy* initiative provides professional learning support for STEM practices in secondary schools. Our previous research in Academy schools identified: a range of models for integrating mathematics within STEM; challenges for teachers in planning and implementing mathematics learning in integrated settings; the nature of tasks that engage students with deeper learning of mathematics (Tytler et al., 2019); and dimensions of innovation for Academy schools around this vision of student engagement (Anderson et al., 2023). In this paper, we draw on Roehrig et al.'s (2021, 2023) conceptual framework of integrated STEM to interrogate case studies from two Academy schools to find how different characteristics

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of integrated STEM might contribute to students' engagement with mathematics.

## 2 Literature Review

While STEM thinking and skills are recognised as critical for all students (Bybee, 2018), reports have called for both a greater focus on the teaching and learning of the individual STEM subject areas as well as on integrated STEM approaches (National Council, 2015). Addressing teaching and learning in individual subjects as well as in integrated STEM raises questions about the balance between teaching in the individual subjects (Siemon et al., 2019) or through connected contexts when addressing real-world problems (Bybee, 2018) and how a STEM project can support deep learning of key disciplinary ideas. Roehrig et al. (2021) note "not all science content can and should be taught using an integrated approach" (p. 2) and the same applies to the other STEM subjects. They also indicate that when using an integrated approach, content and connections must be made explicit, and that mathematics must be foregrounded to highlight the important role it plays.

Secondary mathematics classrooms have traditionally focused on the development of skills and procedures with some opportunity provided to apply knowledge to problems (Li & Schoenfeld, 2019). However, many of the problems presented in secondary mathematics textbooks are low level (Jader et al., 2020), and have little connection to real-world applications or genuine mathematical modelling tasks (Geiger et al., 2022), that typically require the use of knowledge from other subject areas. Tight teacher control over the delivery of skills and procedures can cause a lack of resilience, and learned helplessness (Johnston-Wilder et al., 2015) resulting in students' repeated failure in mathematical problem solving, a lack of interest and minimal engagement. Students frequently report that the mathematics they learn is not 'useful' and certainly not relevant to their current lives or to their future career interests (Fitzmaurice et al., 2021).

While attempts have been made to change practices in secondary mathematics classrooms to include greater focus on group problem solving (Williams, 2023), reasoning and mathematical modelling (Geiger et al., 2022), the lack of sufficient embedding of these practices means little evidence of overall improvements in students' attitudes, aspirations, and engagement in mathematics (Attard & Holmes, 2022; Skilling et al., 2021). Given the current promotion of integrated STEM education as a solution to engagement in the STEM subjects (Bybee, 2018; Maass et al., 2019; National Council, 2015), further research is required into whether integrated STEM programs and practices can improve student engagement, particularly in mathematics, and possible implications of this. In a review of mathematics in integrated STEM, Becker and Park (2011, p. 32) argued that while learning gains in mathematics in integrated settings were not large, "the increased student interest in the subject due to seeing its real-world connections, may lay the basis for improved achievement in the longer term".

Engagement is a multifaceted construct that is malleable and responsive to contextual factors. It includes behavioural engagement, emotional engagement and cognitive engagement (Fredricks et al., 2004). While these three types of engagement overlap, behavioural engagement suggests participation, emotional engagement suggests positive and negative reactions and willingness to do work, and cognitive engagement implies investment in effort necessary to understand, master (Skilling et al., 2016) or even develop (Williams, 2023) challenging concepts and skills. Teachers play a key role in supporting the developing and maintaining of students' engagement through their actions, design and delivery of tasks, and organization and management of classroom climates (Fredricks et al., 2004; Skilling et al., 2016). The degree to which students' needs are met, including relatedness, belonging and community, connects directly to engagement and can fluctuate from class to class and teacher to teacher (Fredricks et al., 2004). Skilling et al's. (2016) study found many grade 7 and 8 mathematics teachers focused on the relevance and future value of mathematics, others considered student autonomy, empowerment, and interest, but few considered cognitive engagement. Helme and Clarke (2001) defined cognitive engagement as "the deliberate task-specific thinking that a student undertakes while participating in a classroom activity" (p 136). Task characteristics associated with cognitive engagement included complexity, challenge, familiarity, intrinsic interest and personal meaningfulness (Helme & Clarke, 2001) suggesting well designed integrated STEM tasks might support student engagement, particularly in mathematics (English, 2016; Fitzallen, 2015; Maass et al., 2019). Further, high level cognitive engagement accompanied by high positive affect leads to high quality learning gains in emotionally safe learning environments, where tasks have multiple solutions and/or solution pathways and learners have control over their exploration (Middleton, 2013).

Integrated STEM curricula for school students has the potential to improve engagement and increase participation, but designing quality tasks can be challenging for teachers, particularly tasks that allow for individual subject integrity to be retained (Fitzallen, 2015; Roehrig et al., 2021). Like Li and Schoenfeld (2019), we believe there is a need to problematize the teaching and learning of mathematics if it is to better connect with other STEM subjects and improve student engagement with learning. On the basis of analysis of curriculum frameworks, and of discussions amongst STEM disciplinary experts, Reinholz et al. (2018) concluded that while good problems differed for different STEM disciplines, common features included real-world connections, reinforcement of conceptual understanding, a low floor and high ceiling, multiple solution paths, multiple solutions/products, and building disciplinary dispositions. Researchers have observed that many of the 'real-world' problems available do not require complex thinking in each of the STEM subjects, something that has become increasingly apparent for mathematics (English, 2016). Others have described the 'silence' of the M in STEM (Baldinger et al., 2020; Maass et al., 2019). Baldinger et al., arguing a need to identify models of STEM integration "that allow mathematics to fully voice its disciplinary power" (p. 71), analysed 32 research articles to identify the features of different, mathematically rich models of STEM integration. From this detailed analysis they identified four themes that cut across all studies: mathematical communication and engagement in mathematical practices, task authenticity, inquiry pedagogies, and learning in informal spaces.

Given the diversity of perspectives on integrated STEM education and limited evidence supporting optimal approaches to increasing student engagement in STEM subjects, Roehrig et al. (2021) developed a detailed conceptual framework of integrated STEM from an extensive review of research into STEM education. They identified seven key connected characteristics of integrated STEM, noting a focus on "student engagement in STEM practices rather than broad notions of student-centred pedagogies" (p. 3) (see Table 1). A question arises, however, concerning ways in which such integrated STEM characteristics fit within coherent curriculum experiences, and ways in which teachers and schools can design learning sequences in which these characteristics productively interconnect to enhance students' engagement with learning in mathematics. In this paper we investigate how this might occur, and the usefulness of the framework to identify factors relating to mathematics teaching and learning.

**Table 1** Seven key characteristics of integrated STEM. (adapted from Roehrig et al., 2021, p. 4)

| Characteristic | Brief Description of Student Experiences |
|---|---|
| Focused on real-world problems | • learn through solution of context-based problems with multiple solutions<br>• apply and expand knowledge as agents of change |
| Engagement in engineering design | • engage with full process of design thinking including failure and opportunities to redesign |
| Context integration | • apply STEM disciplinary content through solving context-based problems<br>• situate disciplinary knowledge within wider social contexts |
| Content integration | • make explicit connections amongst STEM subjects<br>• use M and T beyond tools to service S and E |
| Engagement in authentic STEM practices | • engage in self-determined solution pathways<br>• have epistemic agency using cultural and personal knowledge<br>• apply data practices as well as evidence-based reasoning |
| Twenty-first century skills | • participate in collaboration, critical thinking, creativity and higher order cognitive tasks |
| STEM careers | • learn about STEM careers and exposure to role models |

With integrated STEM approaches, many mathematics teachers need support to develop appropriate projects that preserve disciplinary integrity, involving creative extending of mathematical ideas in authentic tasks. To achieve this, science, technology and mathematics teachers can engage in collaborative curriculum design, delivery and evaluation –a focus of the Academy professional learning program. Because teams in each participating school designed a unique integrated STEM approach, we collected data from teachers, students and leaders to identify challenges and benefits for teaching and learning, and to examine factors supporting STEM program development and implementation aimed at increasing engagement with mathematics. For this paper, the research questions are:

1. What are the challenges and benefits for mathematics teaching and learning associated with different features of integrated STEM?
2. What are key school and teacher processes that can effectively support the planning and implementation of STEM programs that enhance student engagement in mathematics?
3. How effective/useful are the characteristics from the Roehrig et al. framework for highlighting the promotion of mathematics engagement when using authentic STEM tasks?

## 3 Methodology

The professional learning program attended by case study teachers, the *STEM Academy*, was designed for teachers of grades 7–10 mathematics, science and technology to enhance teachers' knowledge of content and pedagogy, to inspire them to reinvigorate their classroom practices and improve student engagement in STEM subjects. Academy sessions were facilitated by academic STEM specialists, with some sessions led by teachers. The program began with three days of face-to-face sessions followed by up to two full school terms where cross-disciplinary school teams developed, planned and implemented STEM inquiry-based learning curriculum approaches. Teachers then returned for a further two days at the University to share their experiences, present evidence of teacher and student learning, discuss issues and challenges, and consider future initiatives.

The selection of Kirk and Merri (pseudonyms) as our two case study schools was based on variations in school type, size, and geographical location, progress in development of schools' presented STEM programs at the end of the initial three-day sessions, the potential for mathematics learning to occur within the program, the availability of documents including a final report detailing the STEM program with student work samples, and the schools willingness for their students to be interviewed.

Both case study schools are coeducational with students from communities of above average socio-economic status. Kirk, a Catholic system school located in a regional area, with nearly one thousand students in grades 7 to 12, and Merri, an independent school located in the outer suburbs of a large metropolitan area with just over six hundred students from Kindergarten to grade 12, represented different approaches to curriculum integration but both showed evidence of careful planning of the integrated activity in relation to curriculum requirements. They thus offered the possibility of investigating conditions for approaches to integrated STEM with quality learning in each discipline, that could provide insights into how these conditions were associated with student responses to M in STEM.

Over two days was spent in each school collecting data through observations and interviews (20–40 min) with various stakeholders from the school community. Interviews undertaken at Kirk School included a group interview with STEM Academy teachers and school leaders associated with the STEM Project: Jane (Technology and Applied Studies [TAS] teacher), Ann (Learning and Teaching Coordinator), Col (school timetabler and a teacher of technology), Vena (mathematics coordinator), Abby (science coordinator), and Don (physics teacher with engineering background). In addition, a group interview was held with two male and two female students from the grade 10 TAS class who were actively involved in building the Billy Carts in the STEM project. At Merri School, individual interviews were held with Mark (STEM team leader, deputy principal and mathematics teacher), Boyd (Head of Mathematics and Science), Dick (Head of TAS), and Anna (a mathematics teacher). Individual interviews were held with six students (two grade 7 and one grade 8 girl, two grade 8 and one grade 9 boy). At Merri, teachers and students were observed undertaking STEM practices. Leaders' and teachers' interview responses were stimulated with questions about their own STEM history and/or the history of STEM in their school, challenges, opportunities STEM had provided for the school, teachers, and students, outcomes of their school's STEM education, and their feelings about STEM and whether and how these had changed over time. Student responses were stimulated with a general question about their learning in STEM, whether and how it differed to ways they had learnt previously, whether they thought they were learning as much, more, or differently, and how they felt about learning through STEM.

All three researchers read the collection of documents and interview transcripts for each school, discussed these, then individually identified themes, looking for evidence across sources of data. The themes were refined through author team discussions over time. These related to the nature of the STEM integration, school processes supporting program innovation, and student engagement with mathematics related learning. School, teacher and student engagement themes were then related to the characteristics of the Roehrig et al. (2021) framework. Comparisons and further discussions ensued to establish shared understandings of results and raise questions for further data interrogation.

## 4 Results

### 4.1 Case study 1: Kirk School

The choice of Kirk as a case study school was influenced by the sense in early Academy workshops that teachers were well coordinated as a group. The school had applied to participate based on concern about the poor take-up of more challenging STEM subjects in the senior school (grades 11 and 12) and saw it as an opportunity for professional learning of staff. The school had instituted inquiry cycle planning across all subjects, and there was a history of cross curriculum collaboration around STEM projects, largely driven by Jane, with other teachers seeing the potential to promote learning in their subjects through these projects. The STEM team was strongly supported by school leadership and by education district personnel who viewed this initiative as a model for other schools.

The STEM innovation at Kirk was focused around a Billy Cart project involving the design and construction of carts that were evaluated on a range of criteria, judged on a race day. The Billy Cart construction was the responsibility of students in Jane's grade 10 TAS class, but the design and testing regimes were a shared responsibility across this class and the grade 10 mathematics and science classes. Each science and mathematics class included student members from the TAS class. These students brought questions, challenges, and Billy Cart parts to these classes to be considered and worked on, with possible solutions then feeding back to the TAS groups working on each Billy Cart design. Buy-in to the project of all grade 10 students was strategically managed, with the Billy Cart topic chosen as offering potential for both science and mathematics learning and of inherent interest:

> *Jane: It's a good group project, they have to work collaboratively. .... And I knew these kids would take to it well because they'd been asking for it for so long-. and so then when it came to doing the writing and the maths, it wasn't a chore for them because they were interested in it.*

The STEM team worked over the summer holidays prior to the project's start to construct a series of cross-curricular activities around the Billy Cart theme that aligned with TAS, mathematics and science curricula at grades 9 and 10.

#### 4.1.1 Mathematics teaching and learning in integrated STEM project

Mathematics in the project was strategically structured, with an emphasis on an open pedagogy that framed but did not direct students' approaches to solving the problems that arose. The team articulated three core elements of their commitment, with student engagement central:

> *Jane: the core elements that we identified for this unit of work were student engagement, cross-curricular collaboration and critical thinking and problem solving through STEM.*
> *Vena: … it wasn't just about getting kids inside of maths, it's about engaging those kids who you lose. ... they are so excited when they start in [grade] 7 and [grade] 8 and you lose them somewhere in 9 and 10.*

Student project activity in which mathematics was integrated with other STEM domains included:

- Watching a Billy Cart race to identify mathematics involved in the design and construction of the Billy Carts. Exploring mathematics associated with car safety ratings and finding how tests were conducted. Using web data, constructing a method to evaluate the safety of a new car, an older model and their dream car. In doing so, they became aware of the usefulness of mathematics in real-world contexts of interest to themselves (authentic).
- Engaging with investigations prompted by TAS class representatives which included sketching and exploring diameter-circumference relationships, axle-wheel size relationships, revolutions related to distance, and measures of speed of different wheels down ramps of different gradients to recommend wheel and ramp design features (see Fig. 1).
- Identifying the speed of their final Billy Carts (using mobiles) and determining factors affecting speed. Investigations included Billy Cart physics simulation, using mathematical formulae from physics to relate theory and practice, linking mathematical formulae and graphs to authentic situations through Geogebra, and creating a model to investigate projectile motion.
- Investigating changes to car features and their functions in recent decades, including wheels and tyres, body shape, engine size, and cost corrected for inflation during a Vintage Car Road Show by practical measurement and data collection.

Student consolidation of trigonometric ratio procedures opened possibilities for deep mathematical learning as students sought to identify and change features of carts that could be manipulated to increase chances of winning. This exploratory activity linked real-world and theoretical applications of gradient and ratio; mathematical concepts recognized for their conceptual difficulty (Dougherty et al., 2017).

#### 4.1.2 Teachers' experience of the STEM innovation

Academy teachers were enthusiastic about effects of the innovation on student engagement with learning. They were convinced that students, including in mathematics, were thinking in new and deeper ways about STEM problems, and there was no indication of the negativity of some students encountered in normal mathematics classes. Abby talked about the difference from normal teaching and learning of mathematics, and the need to be responsive to what was happening in the other subjects.

> *Yes. And for me, personally when I'm trying to teach now, I'm trying to find ways that make it interesting for the students but still covers the content that I need to do.*

Members of the team were explicit about their close working relationship and their shared commitment to an open, student-oriented pedagogy and about their commitment to enlisting other teachers into this way of working.

> *Vena: So, what I'd like to do is to encourage other teachers in [mathematics] ... encourage them to bring ideas to it… to think outside the square ... and be a bit more creative in their teaching ... it's about team building and our professional development.*

The promotion of a more student focused, inquiry pedagogy in mathematics initially received resistance from some

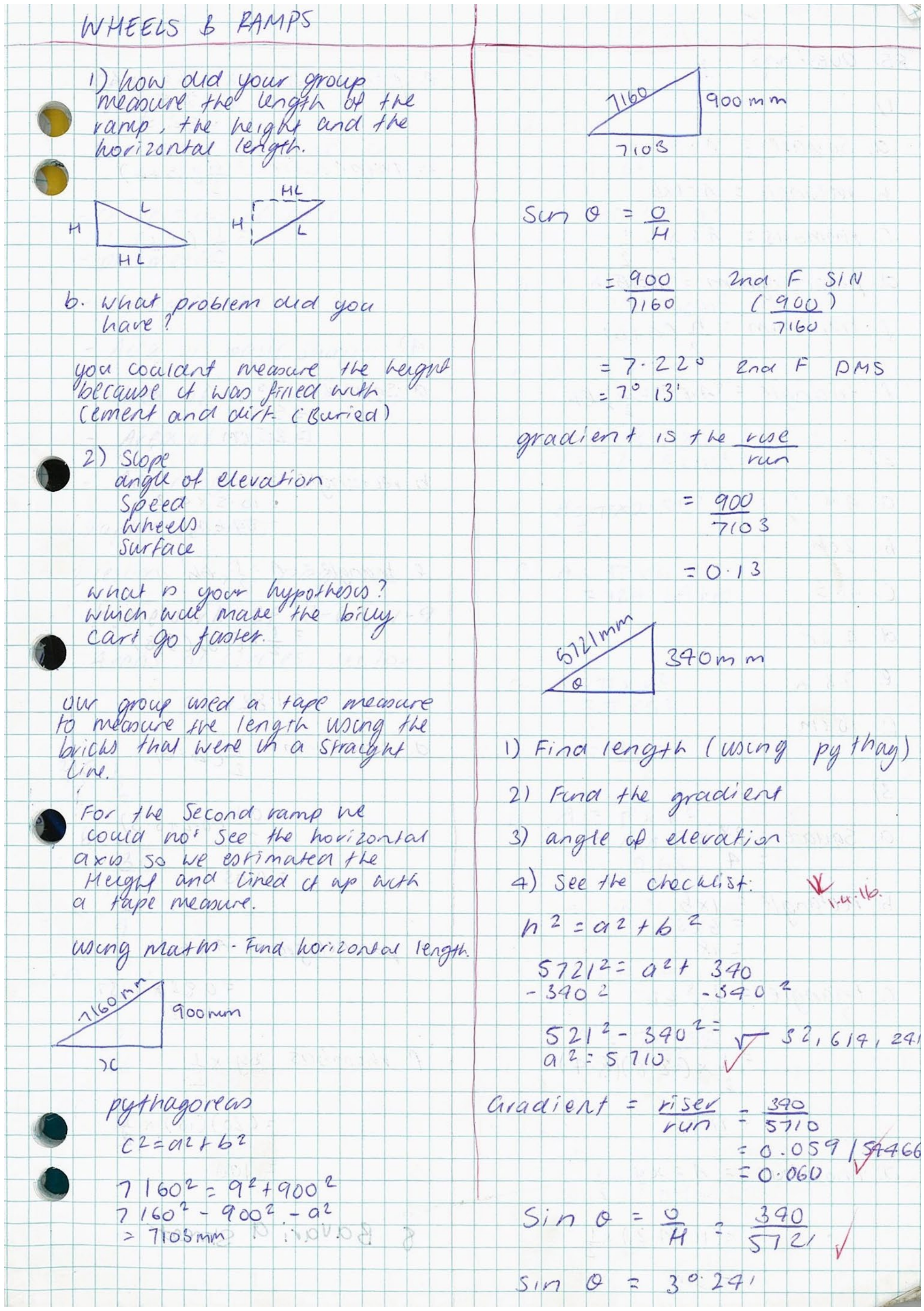


**Fig. 1** Student work sample featuring trigonometric calculations

students who performed well in traditional mathematics classes, who did not accept being asked to take ownership of their learning.

> *Vena: But the other thing is winning the kids over. Because that made me think some classes you go into, and you want to do something differently, and they just go, "Can you just tell us what work we have to do". It's like, "Oh my God, I don't want you to just sit down and do work".*

Interview data point to benefits but also challenges of having mathematics serve authentic STEM practices, which for both teachers and students involved a different epistemological perspective on mathematics from dominant modes of presenting rules and procedures, and very different pedagogy.

### 4.1.3 Students' perceptions of the integrated STEM approach

The group interview was with four highly engaged TAS students with varying degrees of interest in mathematics, ranging from comfortable studying higher level mathematics to disliking conventional mathematics lessons. All four were enthusiastic about the STEM Billy Cart sequence, including very positive assessment of the mathematics and science aspects of this. Three themes emerged strongly related to engagement with learning mathematics – there being a purpose they could subscribe to that drove the mathematics; the 'efficiency' of centering their TAS, mathematics and science subjects around the same project; and their collaboration and support in the design and problem-solving work.

Students were very clear about the importance of the mathematics having a practical purpose associated with the disciplinary integration:

*Rick: Like in one of our maths classes, we got outside to test wheels and that, so it was practical in subjects like maths and science rather than sitting around and sort of doing the plain old book work.*
*Thea: ... you were working toward a project that you're going to be able to see in full scale and how it's going to work in like when your science and your math related - you knew sort of because you were doing the design and technology- you had like the opportunity to really relate to that one project of the billy cart, **which was good**.*

They also talked of the coherence across subjects that comes from working on the common project, and the value of extended collaborative group work around shared problems:

*Thea: Yes, I think working with STEM ... has benefitted my learning because ... for someone who loves [TAS] you are sort of like "Oh, I'm ... going to maths"- you don't really like it - oh but you're going to do the STEM side of things today and you [think], "Oh yay, I'm actually doing work and look forward to it."*
*Gae: People know different stuff and they bring different things to the table and you have more options- it kind of breaks it down for me- so instead of having one subject ... it's ... totally different ... when you go in the maths you just sit down you do your work, that's all we do.*

In terms of influencing their future intentions about mathematics, Ron was clear that the experience had paradoxically changed his view of mathematics and stimulated him to choose higher mathematics in the following year.

*oh I think it's definitely ... definitely ... helped a little bit actually with the maths in that ... it's just ... I just don't see it as maths any more- ... I am a bit more engaged in maths and that… just with the stuff we're doing- so from the wheels and ... angles and ….*

The other three students indicated a strengthening rather than a change of intention.

At Kirk then, both teachers and students provided evidence of improved engagement with learning mathematics, and of students undertaking mathematics that is meaningful, involves choices and supports autonomy in exploration. The key features leading to engaging these students with mathematics were – the authentic purposes offered by the problem, and the presentation of knowledge in each subject as being focused around an extended problem, so that both context and content integration is bringing a coherence and an efficiency to learning.

## 4.2 Case study 2 – Merri School

Merri school's student population had been declining over recent years and fewer students were choosing to study the STEM subjects beyond grade 10. Executive staff members were keen to find a new focus to revive community interest, attract more students, and improve engagement in mathematics and science. The Deputy Principal (Mark) had been inspired to investigate "a connected approach to science, technology, engineering, and mathematics". Discussions with the Head of Mathematics and Science (Boyd) and the Head of TAS (Dick) led to the design of a 'STEM-Ed' program which was trialled with one class of more capable students during 2014 and 2015. Dick was keen to use project-based learning (with an emphasis on design thinking) as the pedagogical approach.

With support of the school Principal and parents, the program ran for two years with Dan noting some of the students who participated in that program, now in grade 10, were "far more engaged and ready for what [grades 11 and 12] have to bring next year" than previous grade 10 students. Boyd indicated the students were now "thinking about things a lot more deeply". Students in grade 10 were also expressing interest in pursuing STEM subjects in grades 11 and 12, and so improved engagement and greater aspirations for future study provided sufficient evidence for the three executive team members to expand the program to all students in grades 7 and 8 in 2016.

#### 4.2.1 Mathematics teaching and learning in the integrated STEM project

Extending the program required expanding the team of teachers involved in its implementation. The three team leaders (Mark, Boyd and Dick) as well as three other teachers (Janet – agriculture/geography, Anna – mathematics, and Peter - science) attended the first three days of the Academy program where they refined initial integrated curriculum plans with the support of academic mentors. Using the science curriculum as a starting point, they mapped the content from mathematics and technology onto this to design a scope and sequence. Mapping the content from the mathematics curriculum to each project revealed several topics that did not fit within a project. Thus, some mathematics content was taught separately to the integrated STEM.

Several teacher interviewees drew attention to the STEM project satisfying a need to change from a more traditional approach to the teaching and learning of mathematics:

> *Anna: Where I was teaching before I came here was really traditional rows of class, Very textbook heavy and the kids had no passion no excitement about maths- I think the only children that liked it were the kids that got it right.*

Mark, a more experienced mathematics teacher and administrator, articulated a similar frustration:

> *In my teaching career I began to feel as though traditional approaches to maths teaching were progressively becoming less and less effective-, that I was working hard at being excellent at the traditional model but it was seeming increasingly disconnected from how students liked to learn- I was feeling a generational shift.*

The innovation required convincing parents and students about a different way of teaching and learning mathematics and science. During the trial period, parents regularly attended presentations about the program, viewing student work and supporting where possible.

> *Dick: We had to re-educate the children how to learn as well- a lot of them just wanted the textbook and the answers and to know if they were right or wrong … we started talking about them becoming more involved in driving their own learning … and I think a lot of them struggled early.*

Projects were also designed to assess students' disciplinary learning; marking criteria and rubrics were developed for each STEM subject so that students were clear about project expectations. The rubric for mathematics in a *Landscape Design* project indicated students were required to create a composite figure using three types of geometric shapes and calculate the area of the composite figure. They had already studied area and volume so this task provided opportunities to investigate areas of composite figures. Extra marks could be gained by allocating one group member to the role of "thinking bigger" in each project and to respond to the prompt:

> *Identify another interesting way that mathematics applies to your project. Describe this clearly and show how you made use of this mathematics. Don't forget to think bigger – don't describe maths that should have been mastered in primary school!*

Other group roles included undertaking research and formulating data displays. Building student resilience was considered important and for some projects students needed to seek information and work things out for themselves. Students were sometimes resistant to such struggle.

Teachers were encouraged to make connections wherever possible. Boyd emphasised the importance of using "a common language" across subjects so that students hear similar explanations for the same concepts.

> *[I] try to be deliberate, even more with science teaching where I can actually start teaching some maths concepts as well- draw them out clearer and in the same way that maths has been introduced to them … at the moment they're doing data- I have done more experiments so they get a better range of data and can do more analysis on it.*

For these teachers at Merri, STEM integration offered engagement with meaningful and deeper learning but presented a challenge to some students because of very different presumptions about learning and assessment. Also, as with Kirk, the aligning of the STEM contexts with the mathematics curriculum required careful planning by mathematics teachers.

#### 4.2.2 Teachers' experience of the STEM innovation

Teamwork was essential to the effective implementation of the program as the three 'new' teachers needed ongoing mentoring and support and regular communication was essential. Timetable limitations meant predominantly lessons were taught within regular timetabled subject specific time while others involved team teaching in a larger purpose-built space. Mark's support was important to the success of the program, and he was strategic in his focus for supporting teachers' professional learning:

> *We've taught our staff design thinking- which is a strategy deliberately taught to them to shift the notion of themselves as the providers of content to the designers of educational experiences.*

For teachers, there were challenges in managing the learning and making connections between project requirements and mathematics concepts. Anna, who was new to the school and to STEM, felt she struggled to find ways to connect and said "it's always hard when you're teaching indices or something where you go here's the rule- just do it- versus can we actually apply this to something that we're doing". Anna acknowledged the support from other team members and noted the mathematics learning in STEM projects was more purposeful than in traditional mathematics lessons. Students had more autonomy, asked more meaningful questions, and displayed more interest and engagement. They drew scale drawings of their garden bed designs in their technology lessons, but struggled when asked to draw a similar shape with half the area. This provided a learning opportunity for the mathematics teachers.

Mark's experience as a mathematics teacher meant he extended and challenged the students through inquiry projects. For a project on designing and delivering a shelter for someone living on Mars, students grappled with the length of lunar years compared to earth years, and decided on the numbers of seasons in a lunar year. Mark wanted his students to "have a good understanding of how units of time work" and "to think about how time works as a mixture of representing physical events in space and … the nature of the planet you're on". In his interview Mark noted

> *I think we're in for the long haul. I think we've seen a substantial change in educational culture at the school in probably about a four-year period. It's not just down to the STEM program, but I think the STEM-Ed program is an important part of it.*

#### 4.2.3 Students' perceptions of the integrated STEM approach

As with Kirk, a key response of students involved opportunities they were afforded to make choices in the projects, rather than be told, or shown what to do and how to do it. They associated this with the creativity enabled by open-ended design problems, and the enjoyment of grappling with meaningful challenges.

> *Grade 7 girl: In science in primary it was like you need to learn this - if it's not that- you're wrong- but now it's more like you get to choose- you have more freedom and creativity ... it's really enjoyable.*
> *Grade 8 boy: It tries to use your brain more because you try to discover how to do it by yourself- (It's a better thing) because it challenges me more in school.*

Another student mentioned the connected, purposeful nature of the STEM program in its focus on knowledge-in-use.

> *Grade 8 girl: I just like the whole STEM program- because you don't just like learn stuff in a classroom and then not do anything with it… it's just very interesting ... we used to usually just learn the stuff and then write it down- but I didn't enjoy it as much as I enjoy STEM because we didn't actually do anything with the information.*

Engagement with other organisations and increased understanding of STEM career options contributed to some students changed aspirations. A visit to a university for a STEM-Ed camp improved one student's understanding of engineering:

> *Grade 7 girl: I wasn't really thinking about being an engineer in primary school, but then when we had the STEM Ed camp and we got to go and visit the university and we got to see different kinds of engineers and what projects they do- it really inspired me.*

### 4.3 A structured comparison of the two schools' approaches

Table 2 interrogates each school's integrated STEM learning sequence through the lens of the Roehrig et al. (2021) framework to highlight commonalities and differences.

The Roehrig framework provides good coverage of key features of the program at each school, while accommodating variation in detail (see Table 2). Below we draw on the case studies to identify the challenges and benefits associated with these distinct characteristics of STEM, based on the framework. We focus first on school and teacher perspectives, before interrogating the impact of the different characteristics of the programs on student engagement with mathematics.

**Table 2** The Roehrig et al. (2021) Framework applied to the two schools' STEM program

| Characteristic | Kirk school learning sequence feature | Merri school learning sequence feature |
|---|---|---|
| Focused on real-world problems | Task chosen with considerations of student engagement with meaningful challenges | Projects designed to be both real-life and meaningful to students |
| Engagement in engineering design | Design thinking about Billy carts, central to STEM subjects served as consistent focus across unit | Each project included strong design element, science experimentation and some links to mathematics (with varying degrees of integration). |
| Context integration | Carefully chosen design project with scope for mathematics and science related to the core curriculum pursued in some depth | Planned units of work enabled significant technology and science disciplinary content to be central to tasks. Mathematics tasks were often focused on the same broad context but indirectly connected to design needs. |
| Content integration | Connections between disciplines structured through assignment of relevant topics and tasks to each subject reinforced by teachers' close collaborating | Units of work, based around science themes chosen to ensure coverage of technology and some mathematics curriculum. Areas of mathematics not covered through STEM were dealt with separately. |
| Engagement in authentic STEM practices | Open ended design problem required groups of students to follow individual solution paths. Students as 'experts' commissioned science and mathematics class problem-solving encouraging data and evidence-based exploration. | Project-based approach encouraged students to work in groups, make design decisions and create and use mathematics to support this. Problem solving was a strong feature of the project design work. |
| Twenty-first century skills | Small group work required collaboration: critical and creative responses feeding into design solutions. | Working collaboratively in groups, decision making and critical and creative design work central to the vision. |
| STEM careers | While no explicit discussion of STEM careers, use of mathematics in billy cart design processes, and tasks analysing car safety standards and vintage car show analyses highlighted use of mathematics in a range of professional engineering/technology contexts. | Projects did not explicitly include discussion of STEM careers, but tasks linked to creative professional work in STEM fields. Wider STEM program, included STEM camp and special activities involving engineering students. |

#### 4.3.1 Challenges, design responses and features of student engagement associated with integrated STEM features

In identifying the challenges and design responses associated with the two schools' STEM innovations, we found it useful to package the Roehrig framework characteristics into four broad dimensions: authentic design tasks as context; modes of content integration; focus on wider 'skills'; and career engagement. This also yielded some clarity concerning distinctive aspects of student engagement emerging as themes from the student interviews (see Table 3) and discussed below.

*Focused on real-world problems; Engagement in engineering design; Context integration* This first dimension speaks to construction of authentic tasks, set in contexts designed to be meaningful to students, that lend a purpose to engaging with mathematics practices. While both programs were framed around design challenges that would be meaningful to students, the structures around this differed. For Kirk, the Billy Cart design and text was the major challenge around which a narrative was constructed that took in multiple themes such as vehicle safety and historical design developments used to generate mathematically rich tasks. For Merri, the problems and design tasks differed in the way they involved mathematics and science, with science generally the driver of the theme and design tasks stemming from this, with differences in the extent of mathematics integration and variety in the depth of mathematical practices engaged with.

Both schools spent significant planning time choosing the context and ensuring it could support quality learning across the STEM subjects, framed within the wider project purpose. This involved extended discussion between teachers of different subjects, to ensure that curriculum requirements were met. This was particularly challenging for mathematics given the structured nature of the mathematics curriculum around the systematic building of mathematical conceptual knowledge and competencies, such that at Merri, mathematics was programmed both through and separate from STEM-Ed.

There were two distinct aspects of student engagement that emerged from the interviews that we can associate with

**Table 3** Themes around student engagement, linked to features of the integrated STEM program

| Integrated STEM dimensions (based on Roehrig, 2021) | Engagement theme |
|---|---|
| *Focused on real-world problems, Engagement in engineering design, Context integration* | Engagement with a sense of purpose and meaning through project design needs<br>Appreciation of a practical purpose for/usefulness of the mathematics they were learning for informing design |
| *Content integration* | Appreciation of connected nature of learning across the three subjects, of coherence to curriculum experience, and of time to pursue learning more deeply |
| *Authentic STEM practices, 21st century skills* | Appreciation of:<br>- collaborative nature of problem-solving process and benefits of sharing expertise<br>- freedom to learn creatively through open nature of STEM processes<br>- agency afforded by collaborative design work<br>Engagement with deeper levels of thinking associated with design-led problem-solving |
| *STEM career awareness* | Awareness of usefulness of mathematics beyond 'school learning'<br>Changed aspirations towards further mathematics and STEM related fields or confirmation of STEM related aspirations |

this dimension of STEM: a sense of purpose and meaning for mathematics through the project design needs; and appreciation of the practical usefulness of the mathematics.

*Content integration* The introduction of mathematics relevant to the design problem occurred in different ways, for instance by students grappling with investigation of wheel size that simultaneously employed mathematics science, engineering, and technology ideas. There was a commitment to bringing out and developing disciplinary knowledge through the design process. At Merri, mathematics was explicitly identified within the tasks, and assessment rubrics were used to emphasise disciplinary knowledge in constructing project reports. We can see examples of multidisciplinary relations where mathematics relevant to a task is taught, and interdisciplinary arrangements where mathematics and engineering design are developed side-by-side, as with Kirk's wheel investigation. Relations between the subjects was in each case framed by teacher interactions and the school's physical and temporal subject arrangements.

There were clear dimensions of student engagement associated with linking of mathematics with other subjects around a common problem or task, with students appreciating: the connected nature of learning across the distinct subjects; the coherence of their curricular experience; and the time this allowed to pursue learning more deeply.

*Engagement in authentic STEM practices; and Twenty-first century skills* The open-ended nature of the design challenges that encouraged and validated individual solution paths represents a radical departure from commonly employed mathematics teaching and learning practices. It is here that mathematics teachers, and to some extent students, experienced a two-fold challenge. The first was a challenge of adopting a pedagogy that was more student-driven and responsive to the needs of the design challenges. Teachers needed to be encouraged and supported by core members of the team to be more open and creative. At Kirk the device of having technology students come to the class with problems that were shared and worked on collaboratively seemed to have encouraged more open pedagogies. The other challenge was to design mathematics tasks that were appropriate to the design challenge and led to new and robust mathematical thinking and learning. The teams described how they relied on expertise of teachers with particular strengths to support planning and knowledge. Teachers also emphasised the importance of cross disciplinary communication for ensuring the appropriate representation of disciplinary knowledge in the STEM problem. This authentic STEM practice characteristic challenged mathematics teachers in calling for a reinterpretation of their expertise allied to pedagogical and epistemological innovation.

Students were explicit in their response to the focus on these STEM/21st Century skills central to these design tasks, that were a departure from normal mathematics classroom practice including: appreciation of collaborative nature of problem-solving process and benefits of sharing expertise; of freedom to learn creatively through the open nature of STEM processes and the agency this allowed; and engagement with deeper levels of thinking associated with design-led problem-solving.

*STEM careers* In neither school was there a strong and explicit focus on STEM careers. However, we can see in these case studies both implicit and explicit links to careers that involve mathematics: e.g. Merri STEM camp and links with engineers; Kirk linking mathematics with car safety and design; in both programs, implicit messages of the relevance of mathematics to authentic problems that engage professionals (e.g. landscape designers). Interviews provided clear evidence of student engagement associated with this dimension, including: an awareness of how mathematics could be used in professions beyond 'school learning'; and changed or confirmed aspirations to proceed with further mathematics and STEM.

## 5 Discussion

### 5.1 Challenges and benefits for mathematics teaching and learning associated with different features of integrated STEM

In responding to our first research question we deal first with challenges to teachers, then benefits expressed by both students and teachers.

#### 5.1.1 Challenges for teachers and schools

Two major challenges for mathematics teachers related to curriculum design and teacher knowledge, and the rethinking of pedagogies and epistemology implied by these STEM practices. Real-world problem focus and engagement with engineering design were distinctive innovations for both schools; they represented significant challenges to prevailing mathematics pedagogies, and were a central plank of STEM innovation in both schools. It is clear from the data that content integration posed significant challenges for schools and teachers in planning links between authentic STEM problems and the different disciplines (see also English, 2016; Li and Schoenfeld, 2019). At Merri, it was considered that integration was not entirely possible through STEM-Ed and a separate, stand-alone mathematics subject was designed to meet these additional mathematics curriculum requirements.

Further challenges, for mathematics teachers, in aligning the discipline with integrated STEM topics were epistemological and pedagogical; how to draw significant mathematical ideas out of the STEM context, and how to ensure disciplinary rigour in the less structured curriculum environment (consistent with Fitzallen, 2015; Roehrig et al., 2021). Challenges for teachers and schools in focusing on authentic STEM practices and 21st century skills related to innovating to produce ill-structured mathematics tasks and associated responsive pedagogies. It has been argued (Lehrer et al., 2000; Williams et al., 2016) that the development of mathematical disciplinary thinking in engaging with exploratory, real-world contexts can better expose the nature of mathematics than structured within-mathematics practices.

#### 5.1.2 Benefits for teaching and learning mathematics

While teachers at each school referred to initial challenges in engaging students in less structured mathematical processes, positive outcomes arising from these integrated STEM programs for students' behavioural, affective / emotional and cognitive engagement can be seen in teachers' testimony (student enthusiasm/commitment, deeper thinking about mathematics, increased intentions to continue with STEM) and students' interviews (raised awareness of purposes for mathematics learning, enthusiasm, intentions to continue with mathematics). The findings are consistent with evidence of task features leading to cognitive engagement (Helme & Clarke, 2001).

Table 3 displays distinctive engagement themes associated with particular interdisciplinary STEM features. These themes can be seen through Fredericks et al's (2004) characterization of three types of engagement: behavioural (students' task participation), emotional (appreciation of/enthusiasm for mathematics) and cognitive (investment in the problem-solving process). While students in interviews did not explicitly refer to behavioural engagement, it was clear from their responses and teachers' testimony that the cohorts were invested in the STEM design processes, collaborating on the design problems in small groups or in whole classes. Emotional engagement is explicit in student interviews, with students exclaiming about their appreciation of the STEM program, often contrasting it to their normal mathematics. The more significant aspect of these themes, however, is the interlinking of these with cognitive engagement (see also Middleton, 2013).

Each of the themes in Table 3 has a strong cognitive element, reflecting engagement with the learning processes central to the tasks. For instance, a sense of purpose is associated with learning applied to real-world tasks and the application of mathematical practices to solve design problems. Appreciation of coherence across subjects associated with content integration implies attention to learning more deeply with time to focus on meaningful problems. Students do not talk about liking to work in groups but refer to the power of sharing ideas and drawing on intersecting expertise. Creativity is couched in terms of the freedom to invest their own ideas. We thus argue that for these integrated STEM programs, unlike others that have been criticised for superficial representation of mathematical thinking and practice, engagement is not restricted to behavioural and emotional forms, but also closely bound up with cognitive engagement with the learning tasks. Evidence of improved engagement with significant mathematical thinking comes from the students and from the teachers.

### 5.2 School and teacher factors supporting planning and implementation of STEM programs that enhance student engagement in mathematics

The framing of the case studies in the histories through which schools and teachers came to their commitments to integrated STEM, and the experience of teachers and leadership in establishing this as a coherent and valued curricular experience, provides us with an opportunity to consider

(a) the core commitments and beliefs underpinning schools' implementation of integrated STEM, and (b) how a coherent program was planned and implemented.

At both schools the STEM initiative was driven by a tangible concern about the quality of learning in the STEM subjects and their take-up at grades 11 and 12. A concern for deeper learning in mathematics was associated with the view that students in traditional mathematics pedagogies do not engage with thinking for themselves but depend on being given rules to follow. The construct of engagement, therefore, was central for both schools; seen in terms of commitment to such dispositions/competencies as curiosity, interest, critical thinking and collaborative problem solving. Again, we see the close intertwining of emotional and cognitive engagement.

Across the two schools, recognition, trust and some form of tangible support from school leadership was crucial, even though the issues and settings were different. For Kirk particularly there was a convincing sense of the inner team developing a strong shared understanding of the overall thrust of the initiative and of the open pedagogy involving student collaboration and autonomy that was the hallmark of the initiative. Similarly Merri interviews captured a growing commitment to design thinking across the school as a way of shifting the pedagogy away from teacher delivery. In both schools, there was a need to build in support for mathematics and science teachers unaccustomed to working with these project-based pedagogies.

### 5.3 Usefulness of the Roehrig framework from a mathematics perspective

The Roehrig et al. (2021, 2023) framework proved useful in articulating the different aspects of the two schools' STEM innovations. Each of the characteristics was generative as a lens through which the programs could be conceptualized, and the variations in approach identified. From the point of view of identifying productive approaches to STEM that support deeper mathematical practices and student engagement with learning, the framework characteristics could be more powerfully organized into four dimensions.

The first, *design thinking in authentic contexts*, involved schools choosing contexts that were meaningful to students and enabled full engagement of disciplinary practices in pursuing design solutions. When done well, students engaged through an increased sense of mathematical purpose. For mathematics teachers, shaping mathematical practices to these authentic settings was challenging. While technology design was the dominant feature in these cases, a wider framing of 'design thinking' would open the possibility of mathematics-led design problems as a key STEM focus (English, 2016).

The second, *content integration*, was critical for mathematics teachers in challenging them to create mathematical tasks and practices outside their normal task focus and pedagogy, that gave full scope for mathematical thinking allied with the other disciplines. This was a key focus for schools in planning the matching of tasks to curriculum requirements. The payoff, for student engagement with learning, was a sense of curriculum coherence and time for deeper learning.

The third, *STEM practices and 21st Century skills*, challenged mathematics teachers to adopt open, student-focused pedagogies, aligned with contemporary moves towards problem solving. This was associated with students' engagement through feelings of agency and freedom to create and think deeply through collaborative design processes, enabling a benefit of STEM integration for mathematics teaching and learning, but also a significant challenge to traditional pedagogies.

The fourth, *exposure to professional practice*, we see as wider than a narrow career focus. In these case studies students were exposed to mathematics as practiced in professional situations (safety standards, wheel design and testing), alerting students to a variety of situations in which creative mathematical practices are valued.

While the Roehrig framework characteristics proved useful for descriptive purposes in framing integrated STEM, we argue that for supporting mathematical integrity in integrated STEM settings it is useful to package these into four key dimensions. Each of these is distinctive in speaking to challenges and opportunities for mathematics teaching, and features of students' engagement with mathematics learning.

## 6 Conclusion

The study confirmed the possibility of establishing key distinctive features of integrated STEM programs that attend to disciplinary knowledge and practice, with the Roehrig et al. (2021, 2023) framework useful for identifying (1) the key characteristics of quality STEM programs and the variation that can exist with these, and (2) the challenges and benefits for engagement with mathematics learning that separately attend to key features based on these characteristics. For addressing key foci for mathematics teaching and learning, we propose the framework can be productively collapsed into four dimensions associated with key challenges for planning and teaching, and key features of student engagement with mathematics; 'design thinking in authentic contexts', 'content integration', 'STEM practices and 21st Century skills', and 'exposure to professional practice'.

The evidence from this study points to the substantial commitment required by schools to introduce a coherent and effective STEM program that accounts for learning needs in the individual disciplines. A feature that stimulated this commitment was disenchantment with students' disengagement with learning. Features that supported this commitment were strong leadership committed to change; rigorous and time-consuming team planning processes; and a coherent pedagogical vision and support of teachers to implement this.

Balancing disciplinary with interdisciplinary learning depends on the imagination and thoroughness of planners of STEM projects who are dedicated to integrating mathematics into the project in ways that engage students. Different features of integrated STEM stimulated distinct aspects of student engagement, with careful curricular structuring and pedagogical innovation leading to a productive integration of behavioural, emotional and cognitive engagement (Fredericks et al., 2004) with mathematics learning.

Concerns about the quality of mathematics learning within these integrated settings can be substantively addressed with carefully crafted interdisciplinary collaboration and curriculum design. This does however involve a re-orientation of traditional mathematics pedagogies towards design thinking, support for mathematics teachers to generate and engage with productive mathematical problems in these interdisciplinary settings, and support for some students to accept the challenge of developing mathematical thinking to solve complex problems rather than follow scripted procedures.

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