Comparison of the Abitur examination in mathematics in Germany before and after reunification in 1990

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Abstract

In Germany, the Abitur is the highest qualification granted at the end of secondary education after 12 or 13 years of schooling; it provides a general university entrance qualification. Traditionally, written and oral examinations are required to obtain the Abitur. Until 1990, there were mainly decentralized examinations in mathematics in West Germany (Federal Republic of Germany [FRG]), which were taken after 13 school years, and a centralized examination that students took after 12 years of school in East Germany (German Democratic Republic [GDR]). In the unified Germany, examinations are now increasingly set by the 16 individual federal states. This paper has a special focus on changes and permanent features in the written Abitur examination in mathematics in Germany in the context of the social changes caused by the German reunification in 1990. These changes since 1990 are described with regard to the initial situation and framing conditions for the written Abitur examination. Two time periods are considered: (1) the examination situation in the FRG and GDR before 1990 and (2) the changes in the five eastern German federal states (former GDR) under the system change and accession to the FRG after 1990 but before the PISA shock.

Keywords Central examination in mathematics · Comparison of examination tasks · German Abitur · Social changes

1 Introduction

Changes in the education system of a country often become visible in the modifications to the way input and output are controlled. On the one hand, this concerns curricular requirements (input) and, on the other, regulations for educational transitions and, in particular, regulations for final examinations (output). Interesting phenomena and effects of societal upheavals can be studied on the basis of changes to the framework conditions and content requirements of examinations at the end of compulsory education or for gaining admission to higher education (Abitur).

Two events have influenced the development of framework conditions and requirements for the written Abitur examination in Germany in, roughly, the last 30 years:

- the German reunification of 1990 with the accession of the eastern part (former GDR) to the western part (FRG), creating a united Germany
- the PISA shock in 2002 with the strong reaction of the German public and politicians to the results of the first PISA studies (Reiss et al., 2016).

Both events have led to significant but different effects on Abitur examinations in the eastern and western part of Germany.

While the political developments resulting from the accession of the GDR to the FRG in 1990 only significantly changed the educational landscape in the five eastern federal states, the PISA shock resulted in changes to the Abitur, especially in the western states. This can be seen in the trend toward the importance of the centralized part of the written examination and the introduction of centralized educational standards since 2004 and especially to the Abitur in 2012. Questions of the transition from school to university related to mathematics as a subject are also investigated (e.g. Hoppenbrock et al., 2016; Neumann et al., 2017).

Descriptions and comparisons of the school systems in the eastern and western parts of Germany have existed since

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the 1990s (e.g., Geißler, 2011) and ongoing developments and challenges for the Abitur were discussed in overviews (Kramer et al., 2016; Neumann, 2014). However, no studies are known to the author that have analyzed the examination requirements and tasks of the written Abitur examination in the East and West before unification and in the 1990s. This is important, among other things, because the media in the western part claimed at the time of the upheaval in 1990 that the Abitur was easier in the GDR than in the FRG (see SPIEGEL, 1990). These statements have not yet been scientifically scrutinised in relation to the subject of mathematics.

From a didactic perspective it is interesting how the examination requirements for the Abitur were designed under different social framework conditions before 1990 and how they have developed since 1990 and the reunification of Germany.

The following research questions can be derived for both periods:

1. From 1980s to 1990: What were the striking similarities and differences in the framework conditions (decentralized or centralized tasks) and requirements of the written Abitur examination between West and East Germany in the 1980s?
2. From 1990 to the PISA shock: Which structural changes (related to the control of the educational system with consequences for mathematics education) occurred after 1990 in the eastern part of Germany (the former GDR) and what consequences resulted for the written Abitur examination in mathematics?

The structure of this paper is based on the two research questions, starting with a discussion of the study's procedures and foundations. Furthermore, this paper also involves the author's findings (Bruder, 2020) on the traditions and changes in the teaching and learning of mathematics in Germany including the period of the German division between 1945 and 1990.

2 Methodology and foundations

For a balancing of the goals and content of the curriculum for mathematics education for general schools, three perspectives are essentially important, as follows:

... a subject-systematic perspective, a general education-reflection-oriented perspective, and a usefulness- and application-oriented perspective. A distinction between these three perspectives allows for a tendential classification of existing concepts and can thus help to

explain differences in their content concretization and operationalization.¹ (Bruder et al., 2015, p. 112)

A possible starting point for a comparison of examination questions is a look at the typical learning processes in upper secondary mathematics education: "...conceptual and rule learning; acquisition of cognitive strategies (as strategies of self-control of complex mental activities, such as learning, problem solving, argumentation, reasoning, etc.); learning of attitudes (e.g., towards mathematics, problem solving, personal performance, etc.)" (Tietze et al., 1997, p. 5).

These learning processes should reflect the skills necessary for the acquisition of a general university entrance qualification. These categories of learning processes can also be understood as operationalisation of the three tasks of the upper secondary school, namely, in-depth general education, general study ability, and science preparatory education, sometimes also known as the "triad of goals of the upper Gymnasium" (Trautwein & Lüdtke, 2004, p. 327).

The design of the Abitur examination itself can be understood as an instrument of output control. In this context, it is of particular importance to consider how quality assurance of the implementation of the requirements of the Abitur examination takes place. It begins with responsibility for the development of tasks and selection of tasks (decentralized or centrally set tasks as well as options) and extends to the type and level of requirements of the tasks set as well as the aids provided. The last stage of quality assurance is evaluation of the student solutions along with the clarification of corresponding responsibilities. A comparison of the framework conditions is possible due to the respective legal bases.

The nature of the tasks in the written Abitur is of particular interest from the point of view of subject didactics. However, there are various approaches to assessing the quality of test items (e.g. Drueke Noe, 2014; Huntley et al., 2010). My interest in this paper is directed to possible differences in Abitur tasks concerning comparable content. It is not a matter of judging which tasks are perhaps better or worse, but rather of the entire testing setting and the observable structural changes up to any discernible effects.

Possible aspects of comparison of the task requirements in the Abitur examination between the FRG and the GDR before 1990 are as follows:

- a. Consideration of different target perspectives and aspects of action
- b. Breadth of the subject areas shown and objective level of requirements for the tasks, with distinction between compulsory and optional parts as well as task formats.

¹ All translations from German are by the author.

In order to describe the development of the written Abitur from 1990 onwards, the administrative perspectives and the experiences of the people involved are of interest, especially teachers and pupils. In this paper, however, only the administrative perspective and the consequences for the pupils can be dealt with. There are individual media reports on the teachers' perspective (see Pieper, 2018), but hardly any studies so far.²

The analysis in Sect. 4 is based on the examination's (changed) legal foundations, available statistical data, and results of relevant studies as well as statements in the media including reflections in scientific publications and commentary from political institutions. Section 5 addresses the changes to the written Abitur examination and the shift from input to output control connected with the PISA shock, and provides further research questions.

3 From the 1980s to 1990

3.1 Historical tradition and goal of the Abitur in the German education system

In Germany, the Abitur is the national exit examination for secondary schools. The Abitur has a very long tradition and special significance as a general university entrance qualification.

In 1788, the Prussian "Reglement für die Prüfung an den Gelehrten Schulen" (Regulations for Examinations at Schools for Scholars) was the first royal edict to introduce an (oral) Abitur examination at secondary schools (Bölling, 2010, p. 25). It was intended to control university access, which had been unregulated until then, in a performance-oriented manner. With the Abitur regulations under Prussian King Friedrich Wilhelm III in 1834, it was stipulated that one could enroll at university only with a secondary school certificate (Abitur).

The Abitur is the general higher education entrance qualification that (formally) entitles students to study all subjects at all universities and colleges in the country, provided there are places available. In Germany, with a few exceptions, there are no university entrance exams. Passing the Abitur virtually ensures the allocation of a study place in the particularly popular subjects. The Abitur grade is still considered the best predictor of whether or not a student will be successful in a particular course of study (see Schneider & Preckel, 2017).

Ensuring the quality of the Abitur qualification is one of the most important tasks of the Standing Conference of the Ministers of Education and Cultural Affairs of the federal states (KMK). For this reason, in addition to a variety of other quality assurance measures, it issues Uniform Examination Requirements for the Abitur Examination (EPA), which describe acceptable orientation knowledge, subject-specific standards, and basic competencies that go beyond the specific mandate of the respective subject.

The consensus in the general social expectations for the Abitur does not change the fact that there were a whole series of problems and changes in the last years of the gymnasium. On the one hand, there were changes in the image of the gymnasium with diverse reform efforts, and on the other hand, Tietze (1990) was able to show that teachers also differed greatly in their ideas about the curriculum and lesson design. Tietze et al. (1997) provide an overview of details with critical reflections on the corresponding developments in the FRG from the 1980s until the early 1990s.

Similarly, in the GDR there were repeated discussions and a need for action to further develop the pathways to the Abitur in view of the growing need for qualifications (Lenhardt & Stock, 2007). In the GDR, however, there was no research on teachers' ideas about their teaching and the curriculum. The focus was on considerations about the further development and optimisation of teaching.

A striking difference between the FRG and the GDR until 1990, however, was the attitude towards the collection of empirical data to compare the mathematics performance of pupils. There were controversial arguments pro and contra central written examinations in mathematics in the time before the reunification of Germany. While in the GDR central written final examinations were unquestioned as an important part of output control, there were in the federal system of the FRG different regulations and tasks in many of the federal states. In the FRG, the educational science-based view was that quality education could not be effected with centrally imposed test items. The social system of the GDR defined itself as a meritocracy and there were corresponding effects on the education system (see Sill & Sikora, 2007, p. 21).

3.2 West Germany (FRG) before 1990—system of education, organization of the written Abitur examination in mathematics

3.2.1 Initial situation: regulations

General guidelines for the goals and content of mathematics instruction in upper secondary school in the federal system of the FRG were negotiated in the KMK. The legal basis for the Abitur examination in the western part of Germany was regulated by the EPA; the regulations were created in 1971

² For one of the few examples of teacher surveys (from the eastern perspective) about the upheaval in the schools 1989–92, but also not specific to mathematics, see Riedel et al., (1994, p. 38f.).

and were further developed in 1989 and 2002. The EPA provides a framework for access requirements, temporal scope, the organizational design of the written examination, and its assessment. For example, it was agreed nationwide that the tasks for the written examination should be set *or* approved by the school supervisory authority. In most of the federal states, a decentralized written examination was taken in mathematics, but it was not mandatory for all students who wanted to take the Abitur in every federal state.

The following applies to the Abitur examinations (see KMK, 1972a, 1972b/2021, p. 11): At least three written examination subjects and at least one oral examination subject are obligatory. Among the Abitur examination there must be:

- at least two subjects with an increased level of demand
- two of the following: German, foreign language, or mathematics
- at least one subject from each of the compulsory subject groups.

Mathematics belongs to the mathematical-scientific field. It is therefore possible for a student to take an exam in only one scientific subject and not mathematics.

Two-thirds of the Abitur grade consists of performance over the entirety of years 12 and 13. The remaining third is made up of the results of the Abitur examinations. By far the greater weight for the Abitur grade in mathematics is, therefore, given to the grades earned in mathematics classes in grades 12 and 13.

3.2.2 Initial situation: access to the Abitur examination in the 1980s

In 1989, 21.6% of the cohort graduated from general education schools (gymnasiums) (GESIS-ZUMA, p. 136). Another 9.8% of the cohort obtained the general university entrance qualification (Abitur) via a vocational qualification (Bölling, 2010).

In the traditional three-tier school system in the western part of Germany, the educational path via the gymnasium up to the acquisition of a university entrance qualification (Abitur) is considered particularly desirable.

However, this means that a decision must already be made after primary school for one of the three possible secondary schools (Hauptschule, Realschule, or Gymnasium). At that point, the children are usually only 10 years old. The Hauptschule ends after the 9th grade, and the Realschule after the 10th grade. Afterwards, young people can start vocational training or continue school in the gymnasium. The gymnasium concludes after the 13th grade with the Abitur. From the perspective of parents and students, the Abitur is important as a ticket to a successful professional future.

The alternative was a two-tier system (see Hurrelmann, 1988), which was implemented in some of the new eastern federal states after 1990.

3.2.3 Input control: number of hours per week in mathematics and curricular specifications for mathematics teaching (gymnasium)

In the upper level (years 11–13), basic courses were generally taught for 3 h and advanced courses for 5 h per week. In the agreement on the design of the school years 11–13 (KMK, 1972a, b, p. 11), it was stipulated that in the mathematical field of activity the following should be conveyed:

- understanding of the process of abstraction,
- the ability to draw logical conclusions,
- certainty in simple calculations, and
- insight into the mathematization of facts.

The following three subject areas were considered to be of central importance:

- Analysis as the basis of understanding fundamental mathematical terms and methods for describing dependencies and change processes,
- Linear algebra/analytic geometry with its methods for algebraizing objects and for the analytical description of space,
- Stochastics with the possibility of quantitative description of processes that depend on chance and evaluation of their results. (EPA, 1989/2002, p. 3)

Each federal state drew up its own framework plan and also had its own examination responsibilities—within the framework of the EPA. There were no specifications for the didactic-methodical design of mathematics lessons at the gymnasium. In the FRG, on the other hand, there was a wide variety of approved textbooks for teachers to use as a guide.

3.2.4 Output control: quality assurance of the requirements in the Abitur examination

3.2.4.1 Responsibility for developing the examination tasks In the West German examination reality, the decentralised Abitur examination revealed a very different picture of how the EPA was implemented.

In the federal state of Hesse, for example, the respective examination tasks for the Abitur examination were developed by the teachers of the final classes in two equally weighted examination suggestions. These tasks were checked by the spokesperson of the group of mathematics teachers at each school and presented to the school board for examination. The school board then decided which of the

two suggestions was actually to be included in the examination. Teachers and students found out the decision on the day of the exam.

This was handled differently in the other western federal states. In Bavaria, another federal state and kingdom in the nineteenth century, central examination tasks for the entire state had existed since 1854. The tasks for the central examination were developed by a task commission of the Ministry of Education, in which experienced teachers worked together. These tasks were (later) published³ with solutions, and served as orientation for teachers in the other federal states that did not have their own central Abitur. These publications were also used for training for the next Abitur examination by both students and teachers.

3.2.4.2 Requirements for the examination tasks The EPA requires that an examination set for the written Abitur examination in mathematics should consist of two to five tasks. The examination set must contain at least two of the three subject areas—analysis, linear algebra, or stochastics.

Three requirement areas⁴ are distinguished for the assessment of the difficulty of a task. The focus should be on requirement area II⁵ and the other two should be taken into account, with requirement area I to receive more consideration than requirement area III⁶ (EPA, 1989/2002, p. 14). In the EPA requirements, examples of subtasks for (oral and) written examinations were also used to illustrate the differences between basic and advanced courses (see Appendix I).

3.2.4.3 Performance evaluation The written examination is considered passed if at least 45% of the possible points have been achieved. The grade ‘very good’ is achieved with 90% of the points. Each examination paper is subjected to a first and a second correction. Despite the great importance of the Abitur as the highest school qualification in Germany

and recurrent discussions about the comparability of Abitur grades between the individual federal states, there were major reservations about central examinations imposed at a level beyond the federal states. This is because a central education system or a nationwide central examination in mathematics was seen to contradict the federal system.

3.3 East Germany (GDR) before 1990—system of education, organization of written Abitur examination in mathematics

3.3.1 Initial situation: regulations

With the founding of the GDR in 1949, the classical gymnasium was abolished. Here, the extended secondary school (EOS) was introduced as a four-year community school with stable classes after grade 8, but only 2 years after grade 10 since 1981. It was the higher school in the GDR’s school system and led to the university entrance qualification after the 12th grade. For more details, Baske (1998) described the development of the school and education system in the GDR.

In the GDR, the Law on the Unified Socialist Education System (1965) regulated the possible paths to higher education in §21 and the responsibilities in §22 (until the end of the GDR). The Act contained the following provisions for the upper school:

§16. (2) The content of teaching in the upper level has the following emphases: In mathematics lessons, pupils are introduced to calculus. Geometry and larger mathematical relationships are dealt with. Special emphasis is placed on mathematical deduction and logical reasoning. In accordance with the general tendency towards mathematical penetration of the sciences, pupils are to be enabled to apply mathematical knowledge and methods in other subjects, in vocational training and in practice.

In East Germany a centrally prescribed final examination in mathematics was taken by all students of the EOS at the end of the 12th grade. The result of the written examination was worth 50% of the overall grade in mathematics for the Abitur and thus had a higher weight for the final grade in the Abitur than in the FRG.

3.3.2 Initial situation: access to the Abitur examination in the 80 s

In East Germany, ca. 14% of students obtained the Abitur after 12 years of schooling and corresponding examinations. Around 9–10% of the class cohort could transfer to the EOS after the tenth grade (since 1981), where they could obtain the Abitur. In addition, around 4–5% of the pupils born in

³ The complete examination papers with solutions for the federal state of Bavaria are available for download on the WP Wissensportal GmbH (2020) for all years since 1984.

⁴ Requirement area I comprises the following: the availability of data, facts, rules, formulae, mathematical sentences, etc. from a defined area in a learned context; the description and use of learned and practiced working techniques and procedures in a defined area and in a repetitive context (EPA, 1989/2002, p. 11f).

⁵ Requirement area II comprises the following: independent selection, arrangement, processing and presentation of known facts under given aspects in a context known through practice; independent transfer of the learned to comparable new situations, whereby it can be either about changed questions or about changed factual contexts or about modified procedures (EPA, 1989/2002, p. 12).

⁶ Requirement area III comprises the following: planned and creative work on more complex problems with the aim of independently arriving at solutions, interpretations, evaluations and conclusions; conscious and independent selection and adaptation of suitable learned methods and procedures in novel situations (EPA, 1989/2002, p. 13).

Table 1 Content overview in mathematics class, grades 11 and 12

11.1	Sequences of numbers, the proof procedure of complete induction, combinatorics	35 h
11.2	Limit values of number sequences and functions	25 h
11.3	Differential calculus	66 h
11.4	Integral calculus	24 h
12.1	Vector calculus and analytical geometry	72 h
12.2	Other classes of non-rational functions; their differentiation and integration	58 h

The planning in grade 11 is based on 30 teaching weeks and in grade 12 (until the Abitur examinations) 26 teaching weeks with 5 teaching hours (lessons) each week

the same year had the opportunity to complete vocational training with the Abitur (see Anweiler et al., 1992, p. 156). However, there were also special schools, e.g., with a focus on science, musics or sports, which had a four-year Abitur program and required a change of school after the 8th grade. For all other school leavers after 10th grade, there was a state-guaranteed entitlement to an apprenticeship.

The selection criteria for EOS included performance, social environment (special support for workers' children), and political attitudes (see §2 of the Admission Regulations, 1981).

3.3.3 Input control: number of hours per week in mathematics and curricular specifications for mathematics teaching (EOS)

The mathematics lessons in grades 11 and 12 each lasted five hours per week. There was no distinction between basic and advanced courses.

The basis for the Abitur examination tasks in the GDR, set by a central task commission, was the Curriculum Mathematics Upper Level (1979)—(for a translated extract, see Appendix II). In contrast to the FRG, in the GDR the contents of the lessons for grades 11 and 12 were clearly prescribed with time limits in the syllabus (see Table 1 and more details of the table in Appendix II).

The time specifications had the advantage of giving the teachers an orientation for the meaning and weighting of the different topics. It was not prescribed how the subject matter was to be taught. However, the teachers were supported by a scientifically-based general didactic-methodological concept for teaching mathematics (e.g., Fanghänel, 1989).

The structure and content of the textbook, written by specialized methodologists from the universities, was adapted to the syllabus. However, there was no textbook selection. "Research showed that mathematics teachers oriented their lesson planning to the textbook and often used it as a source of methodological tools for lesson planning" (Borneleit, 2003, p. 144).

There was a significant difference in the mathematical examination topics in GDR compared to the FRG. For instance, in the GDR there was no obligatory learning content on the subject of stochastics/probability calculation until 1990. Corresponding topics were offered in optional courses (working groups). However, elements of combinatorics, which were also regarded as the basis for quantifying probability considerations, were already integrated from the primary level onwards, (see Sill & Kurtzmann, 2019) and also appeared in the compulsory part of the Abitur examination.

3.3.4 Output control: quality assurance of the requirements in the Abitur examination

3.3.4.1 Responsibility for developing the examination tasks

The tasks for the Abitur examination were set by a central commission of experienced teachers and subject advisors as well as subject didactics experts (see Fanghänel et al., 1990). The examination questions of the Abitur in the GDR for the school years 1979/80 to 1989/90 are available with solutions online in German, (see Polster, 2020).

3.3.4.2 Performance evaluation

The tasks and results for the Abitur were presented annually in the special journal for mathematics teachers 'Mathematik in der Schule' and were also (increasingly) critically discussed there (see Fanghänel et al., 1990). Here is one such example: "In general it must be stated that with regard to the mathematical achievement level in the Abitur level a stagnation has occurred in recent years" (ibid., p. 162). The argumentation was based, among other things, on observed difficulties regarding availability and applicability of mathematical basics among first-year students (ibid.).

Anweiler (1990, p. 40) explained the high sensitivity to all 'disturbances' and the desire to ensure efficiency through increased 'performance and control' with the subordination of the GDR's education system to comprehensive social planning. However, realistic assessments of the educational situation have the advantage that test results such as those from the PISA tests do not lead to big surprises (see also Sect. 4.2).

3.4 Comparing examination tasks in Bavaria and GDR: some examples

A comparison of some of the tasks of the GDR Abitur with some tasks of the West German federal state of Bavaria's examination seems useful, because they were all centrally provided. Also the proportion of Abitur graduates, around 14% in the GDR and slightly under 20% in Bavaria in 1989, is roughly comparable. As an example, for the year 1989, the examination for the basic course in Bavaria is compared with the examination in

Fig. 1 Structure of the written Abitur examination in Bavaria (basic course) and in the GDR before 1990

the GDR. In the GDR there was no distinction between basic and advanced courses, but all students of the EOS had to take the examination. If one compares only the 3-years upper secondary school in Bavaria (basic course) with the 2-years upper secondary school in the GDR up to the Abitur examination, then the 9 weekly hours in the basic course in Bavaria are roughly comparable to the 10 weekly hours in the GDR. Figure 1 illustrates the different structures of the two centrally-written examinations.

In Bavaria, the three content areas of analysis, linear algebra, and stochastics are each tested with an examination task comprising several subtasks. The Abitur assignments for Bavaria each contained two suggested tasks for each subject area, from which the teacher could select one suggestion in advance that better suited her own teaching priorities. For the students in Bavaria, all three content areas were compulsory. This was a stricter requirement than the EPA demanded.

The examination in the GDR consisted of a compulsory part and an optional part. While the compulsory part tended to test basic knowledge and skills in the range of subjects covered, the elective part focused on a local thematic deepening and higher complexity. Students could choose between three tasks with different content in the 'optional' part of the examination (there are two versions of tasks in Bavaria and three versions of elective tasks in GDR, indicated by vertical lines in Fig. 1).

To allow for meaningful comparison, those tasks from 1989 are selected which can be assigned to the area of infinitesimal calculus (or analysis). This topic plays an important role in both exams and cannot be deselected. (One proposal of three examination tasks for the basic course in Bavaria 1989 can be found in Appendix III. The mandatory and elective tasks of the 1989 Abitur examination in the GDR can be found in Appendix IV.)

3.4.1 Consideration of target perspectives and aspects of performance

The sample tasks make it clear: there are striking similarities in the basic layout of the Abitur tasks in Bavaria and the GDR in 1989. These include the strict use of mathematical terminology, a strong focus on calculation, and a coherent approach to the tasks, as well as the basic inner-mathematical structure of the tasks with individual thematic links. Therefore a subject-systematic perspective is adopted. Concrete applications or challenges in the form of unfamiliar interconnections play a subordinate role. However, there are also cross-thematic links in the elective tasks of the GDR, e.g., between analysis and analytical geometry. Each elective task also involves, among other things, an optimisation or extreme value question and distance determinations, and a flexible application ability is required for these concepts. But the main focus in both examination programmes is on conceptual and rule learning as well as the ability to provide (moderate) mathematical justifications.

3.4.1.1 Breadth of the subject areas as well as level of requirements and formats in the compulsory and optional tasks With regard to the subject content of the tasks and the task structure, the examination in the GDR showed a great breadth and variety. The compulsory basic requirements were broad and ranged from the basics of number sequences, coordinate geometry, simple exponential, logarithm, and angle functions with elements of curve discussion or parameter-containing area calculation, to the description of positional relationships between straight lines with analytical geometry, and a combinatorial problem. On the other hand, the Bavarian Abitur required plane equations and various probability calculations that did not appear in the GDR Abitur. As was noted in the section on require-

ment structure, the compulsory part in the GDR consistently contained basic tasks as subtasks with a moderate transfer through partly unusual links with questions from other topics. The latter was especially true for the elective tasks.

The availability of a greater thematic breadth was more characteristic for the GDR exam than for the Bavarian (and that was true of the entire exam), but the Bavarian exam had more local depth. In the Bavarian examination, upper level material was consistently dealt with in the form of standard tasks of medium difficulty. The function types in the Bavarian examination were significantly more complex than in the GDR examination. This resulted in a higher effort being required for calculation and representation in some places. However, the concepts and procedures to be mastered from infinitesimal calculus were almost identical.

All subtasks had a closed task format. The subtasks were independent of each other or a result was given, which could be used for further calculations (Bavaria). Different approaches were conceivable for individual sub-tasks, but as a rule there was something like an optimal solution path. This means that a good result could be achieved in both exams by preparing for the exam and practicing routines and calculations.

The examination results achieved in the GDR in 1989 were discussed in generalised terms by Fanghänel et al. (1990). According to this report, 71% of the cohort achieved good and very good results in this examination.

It can be said that there were many similarities between the Bavarian and the GDR Abitur. However, there was also a conceptual difference, so that basic requirements in a broad range of topics with selective problem-oriented networking in the elective area in the East contrasted with pronounced requirements in dealing with calculi and schemes in the three subject areas in the basic course in Bavaria.

4 From 1990 to the PISA shock

4.1 Changes to the Abitur in the eastern part of Germany after the 1990 reunification

4.1.1 Perspective of the education administration

The accession of the GDR to the FRG in 1990 opened up scope for shaping education policy in the new eastern federal states. Those states were given opportunities to make changes to the forms of their schools, their curricula, and their final examinations, but had to remain within the framework of the EPA.

For the western federal states there was no reason for structural changes at that time. The EPA requirements (1979) had just been adopted in 1989 after a reform of the upper level of the gymnasium.

A central question in the East was the adjustment of the number of school years leading to the Abitur. While the eastern German states Saxony and Thuringia consistently adhered to the previous regulation of 12 school years, the three other eastern German states adapted to the West German model of 13 school years. A particular challenge was the adjustment of the time tables in mathematics and of the curricula. Until 1990, the total number of hours per week from grade 5 to the Abitur in the GDR was 41 h compared to 30–36 h in the West German gymnasium (in the basic course) from grade 5 in the FRG (see Schiemann, 2013). In the upper level, a distinction now had to be made between basic and advanced courses. This required a reduction of learning content for the basic course in grade 11 for the 1990/1991 school year as well as additions and deepening for the advanced course (see, e.g., proposals by Linke, 1990).

After 1990, four of the five eastern federal states opted for the centralized Abitur familiar from the GDR; only the federal state of Brandenburg changed to the decentralized Abitur.

The examinations for basic courses of the four eastern states with a central Abitur were published (Weber eds., 1997). Differences in the earlier examination in the GDR, particularly those having to do with the elective options, were now apparent. For example, the tasks of the exam in the federal state of Mecklenburg-Western-Pomerania still had a reduced compulsory part and an extended optional part from which students could choose one task each from the two subject areas of calculus and linear algebra/analytical geometry. The structure of the tasks of the federal state of Saxony, on the other hand, was similar to that of Bavaria with three major examination tasks in the 1995/96 school year. The subject teacher was supposed to select in advance from the following: one of each of the two given tasks from the sub-areas (A) analysis and (B) analytical geometry and linear algebra; and one of several tasks based on the optional compulsory curriculum topics from the course semester 12/II (sub-area C). For example, in addition to a stochastic task, Part C also included a task on complex numbers and on conic sections. This variation did not last long, because just 1 year later the variety was significantly reduced and stochastics also became a compulsory subject. Students were now allowed to choose for themselves between two offerings in parts A, B, and C. The federal state of Saxony-Anhalt also followed the Bavarian model of three examination tasks, which the teacher chose in advance. However, for the 3rd area there was a choice between a task from analysis, analytical geometry, or probability, so that stochastics did not have to be learned and examined; this complied with the EPA. The situation was similar in the federal state of Thuringia.

In these written examinations in East Germany, nothing had changed with respect to the format of the tasks. There

Fig. 2 Percentage of Abitur graduates among school leavers in a given school year in western Germany (left bar) and eastern Germany (right bar) from 1988 to 2007 (see Rudnicka, 2020)

were still purely inner-mathematical contexts and questions, and the focus was almost exclusively on calculations, i.e., applications of calculi. The previously dominant perspective of science orientation for teaching in the upper grades and also for the examination was not questioned.

4.1.2 Perspective of students

For students in the eastern part of Germany, new opportunities arose in terms of access to the Abitur, which students used intensively (see Fig. 2). Wolter (2016) described developments in Abitur access.

In the national and international performance comparisons at the middle school level, the performance advantage of the eastern part is still evident today. All five of the eastern federal states also scored (in some cases significantly) above the national average in the 9th grade mathematics test in the 2012 National Education Trend Report (see Pant et al., 2013). There have been fewer comparisons for the upper level so far, but in the TIMSS/III study, for example, the following was shown: „Rather, the east German part achieve above-average results despite broader mathematics courses attended by 27% of an age cohort—compared to about 20% in the other state groups“ (Baumert et al., 1998, p. 111).

The school system and its teachers continued to be accepted by the majority of students and parents in the eastern part of Germany. This was also expressed in various survey results in the 1990s after a short period of irritation (see, e.g., Meyer, 1997). And Martens (2020) wrote as follows:

But the upheaval had not directly led to greater opportunities for co-determination in the student perception. In addition, they missed the increased leisure activities that had been common in the GDR. Conflicts with female teachers occurred frequently, which were also

addressed as disciplinary problems as revealed in interviews from this period. Finally, the fact that there had been a general improvement in the schools was viewed rather skeptically on the part of the students.

Tillmann, a well-known West German pedagogue, summarized the eastern students' view at the time: "Basically, nothing had changed, the lessons were the same, the teachers were the same, only the books were new" (Tillmann, 1994, p. 266). About 89–90% of the GDR's teaching staff continued to work in the school system after 1990 (Martens, 2020).

4.2 PISA shock and current discussions on the further development of the Abitur examination in Germany

About 10 years after the reunification of Germany, a paradigm shift in the control of the federal education system became apparent with the introduction of central educational standards. There was a change of perspective away from almost exclusive input control toward greater attention to the outcome of the educational provision, especially in mathematics. This was triggered by the unsatisfactory results of the international comparative studies TIMSS/III and especially PISA 2002, because this was the first time that a comparison of performance between the federal states was published (see the reasons for the study's major impact in Germany by Trautwein et al., 2018).

Since then, a rethinking of the previous control mechanisms in the German education system has taken place, which, however, is still accompanied by critical discussions—especially in the western part of Germany. This development was not new for the people in the east of Germany, but it was for the western part.

In recent years, attempts have been made to adapt the graduation assignments in several federal states more closely to the intentions of the educational standards that were introduced in 2012. This has led to some changes in the tasks, changes that have been strongly criticized by the three associations—DMV (German Mathematics Association), GDM (Society for Didactics of Mathematics) and MNU (Association for the promotion of mathematics and science teaching). The criticism concerns the nature of the dressings of mathematical contexts as well as the scope of the text in some tasks. In order to improve the comparability of the mathematics Abitur in the individual federal States, KMK introduced a joint task pool for Abitur tasks in 2016. The 16 federal states could then submit proposals for the examination tasks.

This development now affected the east and the west of Germany equally. The task pool was launched in 2017, and the IQB (institute for quality assurance in education), as the scientific institution of the federal states, was commissioned to coordinate the development of the pool. The IQB's central Task Pool contains two examination sections: examination part A (45 min without aids) and examination part B (225 min for the advanced level and 180 min for the basic level). Examples of IQB tasks are given in Appendix V.

The German examination also requires Area III⁵ in the part without aids in addition to the fundamental requirements. This no longer means mastering multi-step calculations, as was the case until the nineties, but rather a deeper conceptual understanding (e.g., of the derivation concept as a local rate of change). Of course, students can still be coached in the completion of such tasks.

Serious changes with regard to the differences in the focus of the Abitur examination (e.g., operating versus modelling) in the west and east German federal states have not yet become apparent even with the government efforts to achieve greater comparability since 2012 using educational standards, or since 2017 with the establishment of the IQB task pool. Changes in the federal states are unlikely to take place without significant external pressure because it is not possible to change the practice and traditions of the Abitur examinations in the individual states quickly.

5 Conclusion and outlook

Since the early 1990s, the gymnasium has become the strongest type of school in Germany at the lower secondary level (Neumann, 2014). Currently (as of 2019), the portion of school leavers with a general university entrance qualification in Germany averages 40% (Rudnicka, 2020). For the eastern German federal states, this development has meant a considerable increase in the proportion of school leavers with an Abitur qualification since 1990. In view of

this development, the question arises for the Abitur examination as to whether the previous level of requirements can be maintained, because, “With different selectivity of the upper level, different performance results can also be expected on average” (Baumert et al., 1998, p. 108).

In recent years, however, the discussion about the subject-specific level of the Abitur from the perspective of secondary educational institutions, and especially for mathematics-related courses of study, has again intensified (see the Open Letter, 2017 to the KMK and the communication of the scientific community with responsible persons in educational policy in the individual states, e.g., Greefrath et al., 2018).

In view of the unsatisfactory results of the PISA studies in Germany and the growing pressure from colleges and universities, efforts were expanded to implement the (previous) uniform examination requirements. With this increased level of requirements, there is now a compulsory written examination in every federal state. In terms of the basic uniform requirements, however, they are still enacted only in 10 of 16 states.

The traditions of the central examination in Bavaria, and also the arrangement of the examination in two parts as in the former GDR, were used in the construction of the auxiliary part of the written final examination.

There are, however, discernible trends in the tasks that were triggered on the one hand by the PISA studies (with the literacy concept in the background), but also by the availability of digital tools (graphic calculators and CAS) and their potential for teaching mathematics. The examination questions from the advanced courses of the 1999/2000 school year in the four east German federal states with central examination were published in 2000 (Bossek et al., 2000). They show the influence of digital tools. However, the tasks remain true to the previous trends: they are fundamentally aligned with mathematics and now also contain tasks concerned with probability calculation or statistics. All tasks are still formulated in a closed format, have many arithmetic components and require mathematical justifications and evidence.

However, developments in the Abitur examination have not kept pace with the striking changes brought about by the introduction of educational standards and the associated competence orientation.

The question arises as to who the lawyer and stakeholder representative is, of those involved and affected by the issue of the Abitur examination. A look back in history shows that the question of what mathematics lessons should be taught to students before they take the Abitur has been the subject of frequent and controversial discussion, from Felix Klein (1908) to Behnke (1965) in the FRG, right up to the brand letter discussions, particularly on the transition from school to university, which have been the subject of great media attention since 2017.

In these discussions of the transition from school to university, the perspective of those students who do not aim for a mathematics related course of study is missing. And the perspective of the other 60% of students, those who do not take the Abitur, is also missing.

The current international discussions about twenty-first century skills (Bellanca & Brandt, 2010; Trilling & Faden, 2009) have not really arrived in Germany yet. Such questions are, however, somewhat controversial in Germany. There has been conflict, for example, concerning whether to teach term-conversion skills, which are considered a necessary prerequisite for successful STEM studies, in the middle school. Further negotiation processes will be necessary here, but it would seem useful to clarify possible perspectives on the contents of lessons and examinations leading up to the Abitur.

Against this background, the three traditional tasks of the upper secondary school mentioned above appear to represent a far-sighted consensus. However, the question remains open as to how these categories were and are concretely designed and implemented.

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References

- Anweiler, O. (1990). *Wissenschaftliches Interesse und politische Verantwortung: Dimensionen der vergleichenden Bildungsforschung [Scientific interest and political responsibility: Dimensions of comparative educational research]*. Leske & Budrich.
- Anweiler, O., Fuchs, H.-J., Dörner, M., & Petermann, E. (1992). *Bildungspolitik in Deutschland 1945–1990 [Education policy in Germany 1945–1990]*. Leske & Budrich.
- Admission regulations. (1981). <http://www.drkoerner-web.de/DDverordn/bbaeosbma.htm>. Accessed 12.03.2021.
- Baske, S. (1998). Allgemeinbildende Schulen [General education schools]. 1945 bis zur Gegenwart In C. Führ & C.-L. Furck (Eds.), *Handbuch der deutschen Bildungsgeschichte* (Vol. VI, pp. 159–201). C.H. Beck.
- Baumert, J., Bos, W., & Watermann, R. (1998). *TIMSS/III Schülerleistungen in Mathematik und den Naturwissenschaften am Ende der Sekundarstufe II im internationalen Vergleich. [TIMSS/III Student performance in mathematics and science at the end of upper secondary school in international comparison]*. Max-Planck-Institut für Bildungsforschung.
- Behnke, H. (1965). *Die Pflichten der Universität gegenüber dem Gymnasium*. [The duties of the university towards the Gymnasium]. *Mathematisch-Physikalische Semesterberichte*, 11(1965), 1–19.
- Bellanca, J., & Brandt, R. (Eds.). (2010). *21st century skills: Rethinking how students learn*. Solution Tree.
- Bölling, R. (2010). *Eine kurze Geschichte des Abiturs [A brief history of the Abitur]*. Schöningh.
- Borneleit, P. (2003). *Lehrplan und Lehrplanerarbeitung, Schulbuchentwicklung und-verwendung in der DDR [Curriculum and curriculum development, textbook development and use in the GDR]*. *ZDM*, 35(4), 134–145.
- Bossek, H. (Ed.). (2000). *Abiturprüfung Leistungskurs 1999/2000 [Abitur examination advanced course 1999/2000]*. Paetec.
- Bruder, R., Feldt-Caesar, N., Pallack, A., Pinkernell, G., & Wynands, A., et al. (2015). Mathematisches Grundwissen und Grundkönnen in der Sekundarstufe. In W. Blum (Ed.), *Basic mathematical knowledge and skills in upper secondary education* (pp. 108–124). Braunschweig.
- Bruder, R. (2020). Traditions and changes in the teaching and learning of mathematics in Germany. In A. Karp (Ed.), *Eastern European mathematics education in the decades of change* (pp. 45–74). Springer.
- Curriculum Mathematics Upper Level. (1979). Berlin: Volk und Wissen. <http://www.hechtnetz.de/index.php/abitur/hinweise/ddr-lehrplan-mathematik?start=1>. Accessed 11.03.2021.
- Drueke Noe, C. (2014). *Aufgabenkultur in Klassenarbeiten im Fach Mathematik. Empirische Untersuchungen in neunten und zehnten Klassen [Task culture in class tests in mathematics. Empirical studies in ninth and tenth grades]*. Springer.
- EPA (1989/2002). *Einheitliche Prüfungsanforderungen in der Abiturprüfung Mathematik [Uniform examination requirements in the Abitur examination in mathematics]*. https://www.kmk.org/fileadmin/Dateien/pdf/PresseUndAktuelles/Beschluesse_Veroeffentlichungen/allg_Schulwesen/epa_mathematik.pdf. Accessed 06.06.2021.
- Fanghänel, G. (1989). *Wie gelingt es, das neue didaktisch-methodische Konzept des Mathematikunterrichts umzusetzen?* [How is it possible to implement the new didactic-methodical concept of mathematics teaching?]. *Mathematische Schule Berlin*, 27(10), 677–687.
- Fanghänel, G., Liesenberg, G., Wallner, M., & Weber, K.-H. (1990). *Einige Gedanken zum Abschlussniveau mathematischer Allgemeinbildung—Schlussfolgerungen aus den Ergebnissen der schriftlichen Abschluss- und Reifeprüfungen* [Some thoughts on the final level of Mathematical General Education—Conclusions from the results of the written final and Abitur examinations 1988/89]. *Mathematische Schule Berlin*, 28(2/3), 146–165.
- Geißler, G. (2011). *Schulgeschichte in Deutschland [School history in Germany]*. Peter Lang.
- GESIS-ZUMA. Abteilung für soziale Indikatoren. https://www.gesis.org/fileadmin/upload/dienstleistung/daten/soz_indikatoren/Schlusssendindikatoren/B004.pdf. Accessed 11.03.2021.
- Greefrath, G., Hoffmann, M. & Koepf, W. (eds.) (2018). *Mathematik in Schule und Hochschule—wie groß ist die Lücke und wie gehen wir mit ihr um?* [Mathematics in school and university—How big is the gap and how do we deal with it?]. *Der Mathematikunterricht* 64(5).
- Hoppenbrock, A., Biehler, R., Hochmuth, R., & Rück, H.-G. (Eds.). (2016). *Lehren und Lernen von Mathematik in der Studieneingangsphase—Herausforderungen und Lösungsansätze*. Springer Spektrum.

- Huntley, B., Engelbrecht, J., & Harding, A. (2010). A model for measurement: The quality of mathematics question. *Far East Journal of Mathematical Education*, 5(2), 141–171.
- Hurrelmann, K. (1988). *Thesen zur strukturellen Entwicklung des Bildungssystems in den nächsten fünf bis zehn Jahren* [Theses on the structural development of the education system in the next five to ten years]. *Die Deutsche Schule*, 4, 451–495.
- Klein, F. (1908). *Elementarmathematik vom höheren Standpunkt* [Elementary mathematics from a higher standpoint: Arithmetic, Algebra, Analysis]. Teubner.
- KMK. (1972). *Vereinbarung über die Neuordnung der gymnasialen Oberstufe in der Sekundarstufe II vom 7. Juli 1972* [Agreement on the Reorganisation of the Grammar School Upper Level in Secondary Level II of 7 July 1972]. https://www.gew.de/index.php?eID=dumpFile&t=f&f=23166&token=7a5bb5d644a69ca917bb71bfb93cf6da9e890e09&sdownload=&n=KMK_Vereinbarung_1972.pdf. Accessed 14.02.2021.
- KMK. (1977/1978). *Empfehlungen für die Arbeit in der gymnasialen Oberstufe* [Recommendations for the work in the upper level of Gymnasium]. *Bildung und Erziehung*, 31, 561–574.
- KMK. (1972/2021). *Vereinbarung über die Organisation der gymnasialen Oberstufe und der Abiturprüfung* [Agreement on the organisation of the upper level of Gymnasium and the Abitur examination]. https://www.kmk.org/fileadmin/veroeffentlichungen_beschluesse/1972/1972_07_07-VB-gymnasiale-Oberstufe-Abiturpruefung.pdf. Accessed 14.03.2021.
- KMK. (2012). *Bildungsstandards im Fach Mathematik für die Allgemeine Hochschulreife* [Educational Standards in Mathematics for the General Higher Education Entrance Qualification]. https://www.kmk.org/fileadmin/Dateien/veroeffentlichungen_beschluesse/2012/2012_10_18-Bildungsstandards-Mathe-Abi.pdf. Accessed 13.03.2021.
- Kramer, J., Neumann, M., & Trautwein, U. (Eds.). (2016). *Abitur und Matura im Wandel* [Abitur and Matura in transition]. Springer.
- Law on the Unified Socialist Education System (1965). ND-Dokumentation 24. März 1965. <http://www.verfassungen.de/ddr/schulgesetz65.htm>. Accessed 14.02.2021.
- Lenhardt, G., & Stock, M. (2007). *Bildung, Bürger, Arbeitskraft: Schulentwicklung und Sozialstruktur in der BRD und der DDR* [Education, citizens, workforce, school development and social structure in the FRG and the GDR]. Suhrkamp.
- Linke, H.-P. (1990). *Vorschlag für eine Inhaltsskizze für den Mathematikunterricht der Klasse 11 im Schuljahr 1990/91* [Proposal of a content outline for mathematics lessons in grade 11 in the school year 1990/91]. *Mathematische Schule Berlin*, 28(7/8), 530–534.
- Martens, B. (2020). *Die Wende in den Schulen* [Federal Agency for Civic Education. Dossier, 7.9.2020]. <https://www.bpb.de/geschichte/deutsche-einheit/lange-wege-der-deutschen-einheit/47305/wende-in-den-schulen>. Accessed 13.03.2021.
- Meyer, H. (1997). *Bildung in der DDR* [Education in the GDR]. In *Ansichten zur Geschichte der DDR*, Bd. VIII, 7–99. Berlin.
- Neumann, M. (2014). *Das Abitur in Deutschland* [The Abitur in Germany—An overview of current developments and challenges]. In F. Eberle, B. Schneider-Taylor, & D. Bosse (Eds.), *Abitur und Matura zwischen Hochschulvorbereitung und Berufsorientierung*. Springer. https://doi.org/10.1007/978-3-658-06090-9_14
- Neumann, I., Deeken, C. & Heinze, A. (2017). *Mathematische Lernvoraussetzungen für das Studium—Delphi-Studien mit Hochschullehrenden. MaLeMINT* [Mathematical learning requirements for STEM studies—A Delphi study with university teachers. MaLeMINT]. <https://www.ipn.uni-kiel.de/de/das-ipn/abteilungen/didaktik-der-mathematik/forschung-und-projekte/malemint>. Accessed 12.03.2021.
- Open letter (2017). *Mathematikunterricht und Kompetenzorientierung—Ein offener Brief* [Mathematics teaching and competence orientation—An open letter]. Tagesspiegel. <https://www.tagesspiegel.de/downloads/19549926/2/offener-brief.pdf>. Accessed 18.2.2021.
- Pant, H. A., Stanat, P., Schroeders, U., Roppelt, A., Siegle, T., & Pöhlmann, C. (Eds.). (2013). *IQB-Ländervergleich 2012* [IQB comparison of countries]. Waxmann.
- Pieper, K. (2018). *Die Ostdeutschen lachen sich scheckig* [The East Germans have a good laugh]. <https://www.n-tv.de/leben/Die-Ostdeutschen-lachen-sich-scheckig-article20682707.html>. Accessed 12.03.2021.
- Polster, S. (2020). *Aufgaben und Lösungen der schriftlichen Reifeprüfung Mathematik der Erweiterten Oberschulen der DDR* [Tasks and solutions for the written examination in mathematics of the extended secondary schools of the GDR]. https://mathematikalpha.de/wp-content/uploads/2020/09/DDR_Abitur.pdf. Accessed 03.06.2021.
- Reiss, K., Sälzer, C., Schiepe-Tiska, A., Klieme, E., & Köller, O. (Eds.). (2016). *PISA 2015. Eine Studie zwischen Kontinuität und Innovation* [PISA 2015. A study between continuity and innovation]. Waxmann.
- Riedel, K., Griwatz, M., Leutert, H., & Westphal, J. (1994). *Schule im Vereinigungsprozeß. Probleme und Erfahrungen aus Lehrer- und Schülerperspektive* [School in the process of unification. Problems and experiences from the perspective of teachers and pupils]. Peter Lang.
- Rudnicka, J. (2020). *Anteil der Schulabsolventen/-innen mit Abitur in den Bundesländern 2019* [School leavers with Abitur in German federal states]. <https://de.statista.com/statistik/daten/studie/255393/umfrage/share-der-school-leavers-with-high-school-diplomas-in-germany-by-federal-states/>. Accessed 06.06.2021.
- Schiemann, S. (2013). *Mathe = Mathe? Mathematik in den 16 Bundesländern* [Maths = Maths? Mathematics in the 16 federal states]. In: *Mitteilungen der Deutschen-Mathematiker-Vereinigung*, 4, 226–232.
- Schneider, M., & Preckel, F. (2017). Variables associated with achievement in higher education: A systematic review of meta-analyses. *Psychological Bulletin*, 143(6), 565–600.
- Sill, H.-D., & Kurtzmann, G. (2019). *Didaktik der Stochastik in der Primarstufe* [Didactics of stochastics at primary level]. Springer Spektrum.
- Sill, H. D., & Sikora, Ch. (2007). *Leistungserhebungen im Mathematikunterricht: Theoretische und empirische Studien* [Performance surveys in mathematics education: theoretical and empirical studies]. Franzbecker.
- SPIEGEL (1990). *Driiben war es leichter!* [It was easier over there. The mass influx of GDR high school graduates to West German universities is to be prevented] 26.3.1990. <https://www.spiegel.de/spiegel/print/d-13499353.html>. Accessed 10.01.2021.
- Tietze, U.-P. (1990). *Der Mathematiklehrer an der gymnasialen Oberstufe—Zur Erfassung berufsbezogener Kognitionen* [The mathematics teacher at the upper level of Gymnasium—On the recording of profession-related cognitions]. *Journal Für Mathematikdidaktik*, 11(3), 177–243.
- Tietze, U.-P., Klika, M., & Wolpers, H. (Eds.). (1997). *Mathematikunterricht in der Sekundarstufe II* [Mathematics teaching at upper secondary level]. Springer Vieweg.
- Tillmann, K.-J. (1994). *Von der Kontinuität, die nicht auffällt: Das Schulsystem im Übergang von der DDR zur BRD* [Continuity that is not noticeable: The school system in the transition from the GDR to the FRG]. *Zeitschrift Für Pädagogik*, 32, 264–266.
- Trautwein, U. & Lüdtke, O. (2004). *Aspekte von Wissenschaftspropädeutik und Studierfähigkeit* [Aspects of academic propaedeutics and study skills]. In O. Köller et al. (Eds.), *Wege zur Hochschulreife in Baden-Württemberg*. Opladen: Leske & Budrich.
- Trautwein, U., Köller, O., Lehmann, R., & Lüdtke, O. (2007). *Schulleistungen von Abiturienten. Regionale, schulformbezogene und soziale Disparitäten* [School performance of high school graduates. Regional, school type-related and social disparities]. Waxmann.

- Trautwein, U., Klenk, J. & Schmid, J. (2018). PISA-Studie. <https://wirtschaftslexikon.gabler.de/definition/pisa-studie-53740/version-276808>. Accessed 11.03.2021.
- Trilling, B., & Faden, C. (2009). *21st century skills. Learning for life in our times*. Wiley.
- Weber, K.-H. (Ed.). (1997). *Abiturprüfung Grundkurs 1995/96 & 1996/97 [Abitur examination basic course 1995/96 & 1996/97]*. Paetec.
- Wolter, A. (2016). Gymnasium und Abitur als „Königsweg“ des Hochschulzugangs: Historische Entwicklungslinien und institutionelle Transformationen [Gymnasium and Abitur as the “royal road” to higher education]. In J. Kramer, M. Neumann, & U. Trautwein (Eds.), *Abitur und Matura im Wandel*. Springer.
- WP Wissensportal GmbH (2020). *Aufgaben und Lösungen zu den Abiturprüfungsaufgaben Bayern im Fach Mathematik* [Tasks and solutions for the Abitur examination tasks in Bavaria in Mathematics]. <https://www.abiturloesung.de/>. Accessed 06.06.2021.

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