Create your own problem! When given descriptions of real-world situations, do students pose and solve modelling problems?

Luisa-Marie Hartmann¹ · Janina Krawitz¹ · Stanislaw Schukajlow¹

Accepted: 13 January 2021 / Published online: 1 February 2021
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Abstract

As problem posing has been shown to foster students' problem-solving abilities, problem posing might serve as an innovative teaching approach for improving students' modelling performance. However, there is little research on problem posing regarding real-world situations. The present paper addresses this research gap by using a modelling perspective to examine (1) what types of problems students pose (e.g., modelling vs. word problems) and (2) how students solve different types of self-generated problems. To answer these questions, we recruited 82 ninth- and tenth-graders from German high schools and middle schools to participate in this study. We presented students with different real-world situations. Then we asked them to pose problems that referred to these situations and to solve the problems they posed. We analyzed students' self-generated problems and their solutions using criteria from research on modelling. Our analysis revealed that students posed problems that were related to reality and required the application of mathematical methods. Therefore, problem posing with respect to given real-world situations can be a beneficial approach for fostering modelling abilities. However, students showed a strong tendency to generate word problems for which important modelling activities (e.g., making assumptions) are not needed. Of the students who generated modelling problems, a few either neglected to make assumptions or made assumptions but were not able to integrate them adequately into their mathematical models, and therefore failed to solve those problems. We conclude that students should be taught to pose problems, in order to benefit more from this powerful teaching approach in the area of modelling.

Keywords Problem posing · Problem solving · Modelling · Real-world situations · Word problems · Teaching approach

1 Introduction

Mathematical modelling is important for students' lives, as it enables them to solve problems in the real world with the help of mathematics (Blum et al. 2007). Accordingly, mathematical modelling is an essential part of curricula all over the world (e.g., National Council of Teachers of Mathematics (NCTM) 2000; Kultusministerkonferenz (KMK) 2004). Modelling problems are real-world problems that can be characterized by diverse features such as their authentic connection to reality and their openness (see, e.g., Maaß 2010). Prior research has demonstrated that students often encounter difficulties in solving modelling problems (Galbraith and Stillman 2006; Blum and Borromeo Ferri 2009;

Kaiser 2017). A promising approach for fostering modelling might be to prompt students to pose problems that are based on real-world situations. Prior research has indicated that problem posing can be a successful teaching approach for fostering students' problem-solving abilities (Chen et al. 2013). As modelling can be considered a problem-solving activity (Blum and Niss 1991), problem posing might also be beneficial for modelling. Surprisingly, there is almost no research on modelling through problem posing.

The present study addresses this research gap by examining what types of problems students pose when they are asked to pose problems that are based on given descriptions of real-world situations and how they solve their self-generated problems.

✉ Luisa-Marie Hartmann
l.hartmann@uni-muenster.de

¹ Department of Mathematics, University of Münster,
Apffelstaedtstr. 21, 48149 Munster, Germany

2 Theoretical and empirical background

2.1 Mathematical modelling

Mathematical modelling is a complex problem-solving process aimed at solving real-world problems with the help of mathematics (Blum et al. 2007; Maaß 2010), and an important topic in mathematics education (Blum and Borromeo Ferri 2009). However, we do not have robust research evidence on which teaching approaches foster the learning of mathematical modelling (Schukajlow et al. 2018). An example of a modelling task is the ‘Fire-brigade’ task presented in Fig. 1.

2.1.1 Modelling activities

Theoretical frameworks often describe the process of modelling in the form of a cycle that is passed through by engaging in different activities (e.g., Blum and Leiß 2007): First, students have to *understand, simplify, and structure* the information to construct a real model of the situation. In the task presented in Fig. 1, the problem solver needs to think about the parking position of the fire engine, the height of the fire engine where the ladder is attached, and how the information given in the modelling task can help him/her solve the problem. A real model can include the house, the distance from the house to the fire engine, and the length of the extended ladder. By *mathematizing*, the real model is transformed into a mathematical model, whereby the student has to recognize that the leg of a rectangular triangle must

be calculated. Then, by *working mathematically*, specifically by applying the Pythagorean theorem, the student can arrive at a mathematical result, which has to be *interpreted* back into the real world as a real result, to end up with the answer that the Munich fire brigade can rescue people with this fire engine from a maximum height of 27.5 m. *Validation* of the mathematical model (i.e., the use of the Pythagorean theorem) and the real result (i.e., a maximum rescue height of 27.5 m) on the basis of the given situation or the given real model may show that the result or the chosen process and models are not appropriate for reflecting reality, and the problem solver will need to go through the process again.

2.1.2 Students’ difficulties in mathematical modelling

Obviously, students often find it challenging to *mathematize* and to *work mathematically*. But many other modelling activities can also be demanding for students and have the potential to represent a cognitive barrier (Galbraith and Stillman 2006; Blum and Leiß 2007; Kaiser 2017; Schukajlow et al. 2018). Major difficulties can occur from the very beginning of the modelling process.

As in real-life, modelling tasks contain information about the real world with superfluous or missing data. If information is missing, students need to make assumptions. For example, in the fire-brigade task, the information about the locations of the fire departments and some of the technical data are not needed to solve the task, and assumptions can be made about the parking position of the fire engine or the height of the fire engine where the ladder is attached. Students often fail to solve modelling tasks because they do not

Fire-brigade

In 2004, the Munich fire-brigade got a new fire engine with a turn-ladder. Using the cage at the end of the ladder, the fire-brigade can rescue people from great heights. According to the official rules, while rescuing people, the truck has to maintain a distance at least 12 m from the burning house. The fire department has several locations in downtown Munich, so that there is a maximum distance of 6 km from a burning house. On average, a truck manages to drive about 40 km/h in Munich city traffic.

Technical data of the engine:

Engine model:	Daimler Chrysler AG Econic 18/28 LL – Diesel
Construction year:	2004
Cubic capacity:	6374 cm ³
Power:	205 kw (279 HP)
Weight of unloaded engine:	15540 kg
Dimensions of engine:	length 10 m, width 2.5 m, height 3.19 m
Dimensions of ladder:	length 30 m

From what maximal height can the Munich fire brigade rescue people with this fire engine?

Fig. 1 The ‘Fire-brigade’ modelling task (Blum and Borromeo Ferri 2009)

recognize the need to make assumptions and tend to neglect real-world aspects, and these issues can lead to unrealistic answers (Verschaffel et al. 2020; Krawitz et al. 2018).

2.1.3 Teaching approaches that can address students' difficulties with modelling

Different teaching approaches were applied to find out what helps students overcome their difficulties with modelling. For example, some promising teaching approaches addressed the use of cognitive strategies (e.g., by making drawings; Rellensmann et al. 2017) or metacognitive strategies (Vorhölter 2019; Schukajlow et al. 2015). Other teaching approaches investigated the role of self-regulative learning methods for students' modelling competencies. It was found that student-centered teaching methods are beneficial for students' interest and enjoyment in modelling (Schukajlow et al. 2012). Further indications were provided in the study conducted by Schukajlow et al. (2015). In this study, scaffolding students' self-regulated learning by giving them a plan for solving modelling problems was found to improve students' modelling competencies. Another way to foster self-regulated learning in student-centered teaching methods could be by incorporating problem posing activities in classroom activities.

2.2 Problem posing

Problem posing is an important teaching approach in mathematics education. By posing problems, students become the authors of their own problems and thus become actively involved in their own learning processes. All around the world, school curricula acknowledge the importance of problem posing. For example, the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics (NCTM) 2000) stress that students should "have opportunities to formulate [...] problems" (p. 325). Problem posing means "both the generation of new and the re-formulation, of given problems" (Silver 1994, p. 19). A distinction can be made concerning whether problem posing takes place before (pre-solution), during (within-solution), or after problem solving (post-solution) (Silver 1994). A further distinction can be made between problem posing in free situations (e.g., posing a problem without any restrictions), in semi-structured situations (e.g., posing a problem that is based on a given situation), and in structured problem posing situations (e.g., reformulating the given problem) (Stoyanova 2000).

In our study, we focused on the generation of problems before problem solving in semi-structured situations with given descriptions of real-world situations. Giving students a rich situation, such as a real-world situation, provides

students multiple opportunities for learning (English et al. 2005; Bonotto 2013).

To generate a problem that is based on a given real-world situation, students need to understand the situation and organize the given information by distinguishing data that might be relevant for their problem and discovering relations between the given elements (Christou et al. 2005). Thus, through transformation, the problem-posing process leads to a self-made network of connections between individual perceptions of the real-world elements (Silver 1994). On the basis of these mental connections, students create a personal interpretation of the given situation that can depend on their mathematical skills and experiences and lead to the formulation of individual problems (Stoyanova 1997; Bonotto 2011).

Research has repeatedly indicated that problem posing is strongly related to problem-solving activities (Silver and Cai 1996; Cai and Hwang 2002; Chen et al. 2013), as problem posing can stimulate interest in and curiosity about the subject and improve mathematical thinking processes and perceptions of the subject (Silver 1994; English 1997, 1998; Mestre 2002; Bonotto 2011; Cai et al. 2015). Most approaches that have investigated the relationship between problem posing and solving included a problem-posing test and a problem-solving test (Silver and Cai 1996; Chen et al. 2013). First, students posed problems that were based on a situation (e.g., pictures, arithmetic word contexts, mathematical computations) offered in the problem-posing test, and afterwards, they solved other problems from the problem-solving test. The analyses demonstrated a positive relation between students' problem posing and problem-solving abilities. However, as problem posing is considered to be inseparable from problem solving (Leung 2016), it would be interesting to know how students solve their own self-generated problems. Cai and Hwang (2002) assumed that the problem-solving strategies that students typically employed guided the sequence of the problems the students posed. Therefore, Cai and Hwang (2002) suggested that students might also think about possible solution steps while posing a problem. Due to the limited amount of research that has been conducted on students' problem-solving processes when solving their self-generated problems, the question of how students solve the problems they have posed remains open.

2.3 Modelling and problem posing

Also in the context of modelling, problem posing can take place before, during, or after solving modelling problems (Hansen and Hana 2015). In our study, we focused on problem posing before solving modelling problems. For this purpose, we used real-world situations described in modelling tasks.

2.3.1 Problem posing based on given real-world situations

The real-world situations described in modelling tasks can offer rich contexts that can inspire students to pose a variety of problems. For example, the fire-brigade context can lead to more questions than the question about the maximum height from which the Munich fire brigade can rescue people. One could also ask, for example, how long it will take for the fire engine to reach the site or how many litres of fuel are needed to reach the site.

In problem posing, researchers have used different criteria to analyze students' self-generated problems (Silver and Cai 1996; Leung and Silver 1997; English 1998; Chen et al. 2013; Leung 2016; Palmér and van Bommel 2020). As we were interested in connecting research on modelling and problem posing, we classified problems on the basis of criteria for modelling tasks (Maaß 2010) and criteria used in problem posing research (Silver and Cai 1996).

In research on mathematical problem posing, students' self-generated problems were analyzed concerning their mathematical reference. A distinction can be made between mathematical problems, which can be solved using mathematical models (Silver and Cai 1996), and non-mathematical problems, which can be solved without using any mathematics (e.g., What color is the fire engine?). As modelling problems require problem solvers to construct a mathematical model and apply mathematics, among other activities, they can be classified as mathematical problems. The key characteristics of mathematical modelling tasks are their connection to reality and openness (Maaß 2010). Therefore, the posed problems can also be classified according to these characteristics.

A problem has an authentic connection to reality when it poses a question that is important in the described real-life situation and therefore reflects relevant aspects of the real-world situation (Palm 2007). The problem has an artificial connection when it does not reflect on relevant aspects of the real-world situation, and it is considered a problem with no connection to reality if it is a pure mathematical question. For example, the problem in Fig. 1 (i.e., From what maximal height can the Munich fire brigade rescue people with this fire engine?) can be considered a problem with an authentic connection to reality because it is an important real-life question. The problem, how much do one and a half fire engines weigh, represents a problem with an artificial connection to reality. The problem, how many times does 11 fit into 6000, focuses on the mathematical activity, in particular, the division of 6000 by 11, without reference to the context.

Similarly to problems in general (Silver 1995), real-world problems can be open or closed. There are multiple meanings of open problems in mathematics education, for example, openness in terms of the possibility of using different

mathematical models, or openness in terms of an unclear initial state (Silver 1995). As an unclear initial state is characteristic of modelling problems (Maaß 2010), we considered the problems with respect to whether the initial state was unclear or not. The problems we just presented (i.e., how much do 1.5 fire engines weigh and how many times does 11 fit into 6000) are closed problems because all the information needed to solve the problems is presented in the situation. Therefore, the initial state is clear. By contrast, to solve the problem presented in Fig. 1 (i.e., From what maximal height can the Munich fire brigade rescue people with this fire engine?), information about the parking position of the fire engine and the height of the fire engine where the ladder is attached is missing. Therefore, the initial state is unclear. In modelling research, such open problems are called problems with missing information or vague conditions. To solve these problems, students should notice what information is missing and make realistic assumptions about the missing information (Schukajlow et al. 2015; Krawitz et al. 2018). Although students can complete their solution processes without considering the parking position of the fire engine or the height of the fire engine where the ladder is attached, failing to consider these pieces of information would lead to an inappropriate result in the real-world situation, and the solution would not be meaningful in the real world. Therefore, when solving such problems, it is especially important to focus on the construction of an appropriate real model.

Studies investigating problem posing based on given situations (e.g., arithmetic word problems, pictures, diagrams, mathematical computations) have shown that most students pose problems that can be solved with the help of mathematics (mathematical problems) (Silver and Cai 1996; Leung and Silver 1997). Further, research has indicated that students usually pose problems for which sufficient information is presented in the situation (closed problems) (Silver and Cai 1996; Leung and Silver 1997), and rarely pose complex problems that require multi-step solutions (English 1998). However, because there is only a little research on problem posing that is based on given real-world situations as described in modelling tasks, there is not yet any information about how the problems students pose are connected to reality. Therefore, it is necessary to examine the real-world problems students pose with respect to whether they are open and have an authentic connection to reality in order to judge whether they can be considered modelling problems, or whether these characteristics are not fulfilled for self-generated problems, in which case they can be considered word problems but not modelling problems.

2.3.2 Fostering modelling through problem posing

Even though problem posing naturally takes place in a broad sense in modelling activities (Hansen and Hana 2015),

problem posing has rarely been investigated in research on modelling. In processing modelling activities, students need to specify (or reformulate) the question to determine the constraints of the real-world situation. Further, the real-world problem needs to be transformed into a mathematical problem that can be solved by using mathematical procedures. Asking questions about whether the mathematical model matches the real-world situation is an important part of the modelling process (Hansen and Hana 2015). Conducting problem-posing activities before solving modelling problems can also trigger students' capacity for and likelihood of engaging in these activities while modelling, and improve their modelling competencies.

Further, problem posing may be particularly helpful for overcoming the cognitive barriers involved in modelling. During problem posing, students have to deal with the real-world situation in an in-depth manner. They have to understand, filter, and structure the given information (Sect. 2.2), and hence, problem posing may help them identify the relationship between the elements and improve their perception of the real-world situation. Additionally, the use of metacognitive strategies can enhance students' modelling (Sect. 2.1). When students think about possible solution steps while posing a problem (Sect. 2.2), they may consistently use metacognitive strategies by scrutinizing their mental ideas, and may thereby already be engaged in validating their problems while posing them. Consequently, problem posing may serve as an innovative teaching approach that can support students in constructing appropriate real-world models and in validating their solutions, and may therefore help them improve their modelling.

Empirical studies have provided indications that problem posing may help students solve modelling problems. Problem posing has generally been shown to be helpful for problem solving (Sect. 2.2), and student-centered teaching approaches that foster self-regulated learning seem to be beneficial for fostering modelling (Sect. 2.1). Initial results from a long-term intervention on problem posing based on given material from the real world (e.g., supermarket bills, restaurant menus) have indicated that problem posing can help students consider aspects of reality while solving problems connected to the real world (Bonotto 2011). However, due to the lack of research on problem posing from a modelling perspective, it is an open question whether problem posing based on given descriptions of real-world situations can help students solve different types of real-world problems that require modelling activities.

3 Research questions and expectations in the present study

The goal of the present study was twofold. First, we aimed to find out what types of problems students pose when they are asked to generate problems that are based on given

descriptions of real-world situations. Second, we wanted to find out how they solve self-generated problems, especially problems that require characteristic modelling activities. To do so, we examined students' solutions to different types of self-generated problems (modelling or word problems) and students' difficulties when solving these problems. Accordingly, we posed the following research questions:

1. What types of problems do students pose when they are asked to generate problems that are based on given descriptions of real-world situations?
 - (a) Do students pose mathematical problems?
 - (b) What types of mathematical problems do they pose?
2. How do students solve their self-generated modelling and word problems?
 - (a) Are self-generated modelling problems more difficult for the students to solve than self-generated word problems?
 - (b) What difficulties do students have while solving their self-generated modelling and word problems (e.g., difficulties related to constructing a mathematical model, calculating mathematical results, interpreting the results)?

According to previous studies on the posing of problems that are based on various given descriptions (e.g., descriptions of dressed-up word problems, pictures, diagrams) (Sect. 2.3), students will presumably pose mathematical problems that are based on the given descriptions of real-world situations. However, due to the variety of real-world information given in the situations, it might also be possible that students will pose non-mathematical problems. Further, the given real-world situations might give students the incentive to pose problems with a connection to reality. We did not have clear expectations about whether the problems would have an authentic or artificial connection to reality. From previous studies on problem posing, we know that most students pose closed problems that rely on information given in the situation and do not require them to make assumptions about missing data (Sect. 2.3). Hence, students might also pose closed problems based on given descriptions of real-world situations. However, as the descriptions of the real-world situations are adapted from modelling tasks, it is also possible that students will pose open problems. If so, the solutions to these open problems would require them to make assumptions about real-world aspects of the situation.

There is only a limited amount of research on how students solve their own self-generated problems. Due to the theoretical assumption that students are thinking about

possible solution steps while posing problems (Sect. 2.2), they might be able to solve their self-generated problems. However, we know from prior research that students have difficulties solving real-world problems, especially modelling problems (Sect. 2.1). Moreover, there is not yet any research on how problem posing can affect students' modelling. Therefore, we did not have clear expectations about whether their solutions would differ when solving self-generated modelling problems, as distinct from self-generated word problems, and what difficulties they would manifest while solving self-generated problems.

4 Method

4.1 Sample

Our sample comprised 82 ninth- and tenth-graders (49% female adolescents) from 3 classes including high-track (German Gymnasium, 29%) and middle-track (German Realschule, 71%) classes from 2 different German schools. The students were between 14 and 17 years old. According to the teachers, none of the students had previous experience with problem posing.

4.2 Procedure

We offered the students six real-world situations and provided them with the following information:

In this booklet, you will find a number of different situations from the real world. Unlike most of the tasks you are familiar with, there is no mathematical problem for you to solve for these situations because today you will develop the problem yourself. First, read the description of the situation. Then think about a mathematical problem you can pose based on the given situation that can be solved by using information from these situations and write this problem down. Then you should solve your self-generated problem.

Students could take as much time as they needed for problem posing and task processing. To ensure that different problems that were based on the given situations could

be generated, we chose situations from the real world that allowed the students to pose several problems. For this purpose, we used situations described in modelling tasks from earlier studies (e.g., Schukajlow et al. 2015; Krawitz and Schukajlow 2017). We deleted the problem from the modelling task and extended the description by adding more information about the situation in order to allow the development of different problems that referred to various mathematical areas. For example, for the 'Fire-brigade' task (see Fig. 1), we extended the situation by adding the information that on average, an engine manages to drive about 40 km/h in Munich city traffic. The other real-world situations we used can be found in the "Appendix" (see Figs. 9, 10, 11, 12, 13).

4.3 Data analysis

4.3.1 The problems students posed

To answer the first research question, we analyzed the problems posed by the students using Mayring's (2015) content analysis, based on criteria for posed problems as presented in Sect. 2.3. The coding scheme consisted of three main categories with different specifications (see Table 1). All categories were rated by well-trained raters. First, we provided examples of posed problems to teach the raters about the codes they should assign. Second, they rated the posed problems. To test the interrater reliability, 20% of the posed problems were coded by two independent raters. Interrater reliability assessed with Cohen's kappa (Cohen 1960) was at least moderate for all categories for the six given situations (see Table 1).

The problems the students posed were first categorized as mathematical or non-mathematical problems. The assessment of the mathematical reference was based on Silver and Cai's (1996) classification system (Sect. 2.3). Problems that did not require mathematical argumentation or the application of mathematical models were categorized as non-mathematical problems. Many of these problems could be solved directly by selecting information presented in the situation such as the following non-mathematical problem: What distance must the fire engine maintain from the burning house?

For mathematical problems, we further analyzed their connection to reality and their openness (Maaß 2010). For

Table 1 Categories of the coding scheme

Categories of coding scheme	Range of each category			Range of Cohen's kappa
Mathematical reference	Mathematical problem	Non-mathematical problem		0.81–1.00
Connection to reality	Authentic connection to reality	Artificial connection to reality	No connection to reality	0.67–1.00
Openness of a problem	Open problem	Closed problem		0.61–1.00

example, the problem that was posed about the perimeter of the fire engine was categorized with the help of the presented coding scheme as a mathematical problem that had an artificial connection to reality and was closed. For further analyses, open problems with an authentic connection to reality were summarized as modelling problems (e.g., From what maximal height can the Munich fire brigade rescue people with this fire engine?). Self-generated real-world problems with either an artificial connection to reality or with a closed initial state were coded as word problems (e.g., What is the perimeter of the fire engine?), and self-generated problems without a connection to reality were coded as intra-mathematical problems (e.g., How long is the leg of the rectangular triangle?).

4.3.2 Students' solutions to their self-generated problems

The second research question addressed students' solutions to their self-generated problems. To code students' solutions, we differentiated three solution steps, namely, mathematical model, mathematical result, and interpretation. We coded all solutions, whether or not each of these solution steps were completed successfully. A code of 0 was given to an incorrect or missing solution step, and a code of 1 was awarded when the student developed an adequate mathematical model, mathematical result, or interpretation. If the mathematical model or the mathematical result was wrong, the subsequent solution steps were nevertheless scored correct if they were coherent in and of themselves. To estimate the quality of the solutions, the codes for the three solution steps were summarized for each solution. Thus, the scores ranged from 0 (no correct solution steps) to 3 (all solution steps correct). Further, we assessed assumption-making with

a code of 0 or 1 depending on whether assumptions about real-world aspects were made or not. A code of 0 was given if no assumptions were made, and a code of 1 if assumptions were made. As assumption-making is needed only in solving open problems, we considered this aspect separately and did not add it to the solution score.

For example, in Fig. 2, the solution that was given to the problem presented in Fig. 1 was given a score of 2 because the student built an inadequate mathematical model (i.e., coded 0), but the mathematical result was calculated correctly (i.e., coded 1), and the mathematical result was adequately interpreted in the given situation (i.e., coded 1). Further, the student made additional assumptions about the height of the fire engine where the ladder was attached. Therefore, the solution received a code of 1 for assumption-making. Again, 20% of the solutions were rated by two raters to test for interrater reliability measured as Cohen's kappa (Cohen 1960), which was moderate for students' processed solution steps, ranging from $\kappa = 0.59$ to $\kappa = 1$ for the six given situations.

4.3.3 Statistical tests

We computed a t-test for independent samples on the influence of the independent variable (problem type) on students' solution scores, in order to analyze whether students' solution scores differed when solving modelling problems or word problems (RQ2a). As we were interested in the differences in students' posing and solving different types of problems, we used the number of problems as the unit of analysis.

To answer the last research question (RQ2b), with which we aimed to investigate students' difficulties in solving their

Fig. 2 Example of a solution for a problem that a student posed in the real-world situation 'Fire-brigade'

self-generated modelling problems and word problems, we used the Chi-Square test.

4.3.4 In-depth analysis

As we are especially interested in students' difficulties in solving self-generated modelling problems, we conducted an in-depth analysis of students' solutions to the modelling problem that was posed most frequently. Consequently, we picked the given real-world situation for which the largest number of modelling problems was posed and further focused only on solutions to the modelling problem that was posed most frequently by the students. The goal was to analyze examples of errors that students typically made when solving their self-generated modelling problems.

5 Results

The results are presented in two sections in accordance with the two research questions. The first section analyzes students' posed problems. The second section presents an analysis of students' solutions to their self-generated problems.

5.1 Students' posed problems

To answer the first research question, we analyzed students' self-generated problems that were based on given descriptions of real-world situations. Most of the students (67%; 55 out of 82 students) generated a problem in every given situation; some students (46%; 38 out of 82) generated more than one problem based on at least one situation; and all students posed at least one problem overall. Students posed a total of 495 problems. The vast majority of the posed problems were classified as mathematical problems (98%; 483 out of

495 problems). Half of the students (54%; 44 out of 82 students) posed a mathematical problem based on every given situation, and all students posed at least one mathematical problem overall.

Students' self-generated mathematical problems ($N=483$) were of particular interest, and we further analyzed whether they met the characteristics of modelling problems, such as an authentic connection to reality and openness. The largest amount of diversity in the problems (12 different problems) was found for the real-world situation 'Fire-brigade'. Therefore, we used examples of the problems students posed for this situation to illustrate the results for the mathematical problems students posed.

5.1.1 Characteristics of the mathematical problems students posed

Figure 3 represents the distribution of the characteristics of the self-generated problems.

All of the students' mathematical self-generated problems were connected to reality and could therefore be classified as real-world problems. Of these problems, more of them had an authentic connection to reality than an artificial connection to reality (see Fig. 3). Further, most problems were closed, and 12% were open.

For example, students posed the problems presented in Fig. 4. Most likely the problems presented in Fig. 4a, c were posed with the intention of constructing mathematical tasks. For example, the problem presented in Fig. 4a was probably posed to construct a division task. However, with respect to the given real-world situation, these problems seemed nonsensical; for example, concerning the problem posed in Fig. 4a, it would be unrealistic to put around 600 fire engines on the road. Therefore, these problems are more likely to be considered problems with

Fig. 3 Connection to reality and openness of students' self-generated mathematical problems

an artificial connection to reality. Concerning the problem presented in Fig. 4b, fire fighters might actually face this problem in real life. Therefore, this problem is more likely to be considered as a problem with an authentic connection to reality. To solve the problems presented in Fig. 4a, b, only information described in the real-world situation is needed. For example, to solve the problem presented in Fig. 4a, information about the length of the route and the fire engine is needed, and the length of the route has to be divided by the length of the fire engine. Therefore, the problems are more likely to be considered closed problems. To solve the problem presented in Fig. 4c, additional assumptions about the height and the shape of the fire engine have been made; therefore, this problem is more likely to be considered an open problem. Overall, the problems presented in Fig. 4 are examples of open problems with an artificial connection to reality, closed problems with an artificial connection to reality, or closed problems with an authentic connection to reality. Therefore, these problems can be regarded as word problems.

Other students posed the problems presented in Fig. 5. As fire fighters might actually face these problems in real life (e.g., the problem presented in Fig. 5a, which involved figuring out how often the fire engine must be refueled), these problems can be considered problems with an authentic connection to reality. To find a solution for these problems, students needed some additional information that was not given in the real-world situation. For example, to solve the problem presented in Fig. 5a, information about the average consumption rate of the fire engine is needed, and additional assumptions have to be made. Therefore, these problems

are more likely to be considered open problems and can be regarded overall as comprehensive modelling problems.

Overall, 9% (42 out of 483) of the problems students posed met the characteristics of modelling problems (authentic connection to reality and open). The modelling problems were posed by 44% (36 out of 82 students) of all students. More specifically, 38% (31 out of 82) of all students posed at least one modelling problem, and only 6% (5 out of 82) posed more than one modelling problem. Most of the self-generated modelling problems were based on the real-world situation 'Fire-brigade' (see Fig. 1) (62%; 26 out of 42), and second most were based on the real-world situation 'Chopsticks' (see Fig. 13) (21%; 9 out of 42). No modelling problems were based on the real-world situation 'Sports field' (see Fig. 9).

5.2 Students' solutions to self-generated problems

To answer the second research question, we analyzed students' solutions to their self-generated modelling and word problems.

5.2.1 Differences in students' solution scores

Table 2 presents the means and standard deviations of students' solution scores for the self-generated modelling and word problems.

There was no significant difference between the solution scores for students' solutions to self-generated modelling problems and self-generated word problems ($t(480) = -1.03$, $p = 0.304$, $d_{Cohen} = 0.16$).

Fig. 4 Examples of students' word problems that were based on the 'Fire-brigade' situation

Fig. 5 Examples of students' modelling problems that were based on the 'Fire-brigade' situation

Table 2 Students' solution scores for self-generated modelling problems and word problems

Problem types	Number of problems	Solution score	
		Mean	Standard deviation
Modelling problems	$N=42$	$M=1.86$	$SD=0.84$
Word problems	$N=441$	$M=2.03$	$SD=1.05$

The solution score scale ranged from 0 (min) to 3 (max)

5.2.2 Students' difficulties in solving their self-generated problems

To answer the last research question (RQ2b) about students' difficulties in solving their self-generated modelling and word problems, we analyzed students' written solutions with respect to which solution steps (e.g., constructing a mathematical model) they succeeded in addressing and which they did not. For most of the self-generated open problems (71%, 41 out of 58), students did not manage to make additional assumptions about aspects of the real world. For nearly half of their self-generated mathematical problems (49%, 238 out of 483), students did not manage to build a correct mathematical model; for 28% of the solutions (135 out of 483), they did not manage to arrive at a correct mathematical result due to computational errors; and for 22% (104 out of 483), they did not manage to arrive at an adequate real result because students did not interpret the mathematical result in the given real-world situation. Taken together, students solved less than half of their self-generated real-world problems (41%, 198 out of 483 mathematical problems)

completely correctly, including an adequate interpretation of the result in the real world.

Further, we compared the solution steps for self-generated modelling and word problems. As making an assumption is needed only when solving open problems, we analyzed solutions for two types of problems: the modelling problems and the open word problems. For 33% (14 out of 42) of the modelling problems and for 19% (3 out of 16) of the open word problems, students made additional assumptions about aspects of the real world. A Chi-square test revealed that there was no significant difference concerning assumption-making between the modelling problems and the open word problems ($\chi^2(1) = 0.49, p = 0.484, \phi = -0.098$).

In Table 3, the percentages of students' solutions in which the solution steps were processed adequately are presented for students' self-generated modelling problems and word problems. The percentages refer to the overall number of solutions (e.g., a correct mathematical model was set up in 28% of all modelling problems, the mathematical result was correct in 72% of all modelling problems, and an appropriate interpretation was noted for 84% of all modelling problems).

As presented in the table, building a correct mathematical model was much more difficult for the solution to the self-generated modelling problems in contrast to the word problems ($\chi^2(2) = 11.03, p = 0.001, \phi = 0.159$). Concerning the other solution steps, there were no significant differences (mathematical result: $\chi^2(1) = 0.00, p = 0.957, \phi = 0.003$; interpretation: $\chi^2(1) = 0.30, p = 0.584, \phi = -0.026$).

5.2.2.1 In-depth analysis of students' solutions to their self-generated modelling problems

To gain deeper insight

Table 3 Percentages of students' solutions in which the individual solution steps were processed adequately

Problem types	Correct solution steps		
	Math. Model (%)	Math. Result (%)	Interpretation (%)
Modelling problems	28	72	84
Word problems	53	72	78

into students' difficulties in solving their self-generated modelling problems, we conducted an in-depth analysis of students' solutions to the most frequently posed modelling problem. The most frequently posed problem occurred based on the real-world situation 'Fire-brigade' and addressed the question of the maximal height from which the Munich fire brigade could rescue people with the fire engine (posed by 23 students). In the following, typical errors in students' solutions of this self-generated problem are presented and illustrated with examples of solutions.

In students' solutions, most errors occurred with respect to creating a correct mathematical model. In Fig. 6, solutions are displayed that lack a mathematical model or involve an incorrect mathematical model due to incorrect or missing mathematization.

In the solutions displayed in Fig. 6, the house, the distance from the house to the fire engine, and the length of the extended ladder were recognized as important information in the real-world situation. Therefore, the real model was adequately constructed for the given real-world situation. However, in the solution presented in Fig. 6a, difficulties in structuring the problem can be identified, as an incorrect length (6 m instead of 12 m) was assigned to the distance between the vehicle and the building. Therefore, the student whose solution is displayed in Fig. 6a did not manage to create a correct mathematical model on the basis of his/her

real model. Additional lines in the drawing indicate that the student identified the implicit geometrical figure of a triangle (or at least one angle of the triangle), but he/she did not manage to finish the translation into a mathematical model.

In the solution displayed in Fig. 6b, the important information in the real-world situation was also identified, but the height of the fire engine was not considered. Additionally, the student recognized that the problem had to be solved by calculating the missing leg using the Pythagorean Theorem. However, in the mathematical model, the triangle's hypotenuse and leg were mixed up. Therefore, the student whose solution is presented did not manage to create a correct mathematical model due to a lack of mathematical knowledge.

By contrast, in the solution presented in Fig. 6c, the important information in the real-world situation was identified, and the height of the fire engine was considered in the real model. Additionally, the student recognized that the Pythagorean Theorem had to be used. However, the height of the fire engine was inadequately integrated into the mathematical model. Therefore, the student whose solution is presented in Fig. 6c did not manage to construct the correct mathematical model.

Further, some did not manage to calculate a correct mathematical result due to computational errors (Fig. 7).

In the solution displayed in Fig. 7a, an incorrect mathematical result was calculated due to an intra-mathematical computational error. In the solution presented in Fig. 7b, the laws of powers were disregarded, and in Fig. 7c, the root laws were violated.

In the solutions displayed in Fig. 8a, b, a correct mathematical result was calculated and interpreted back to the given real-world situation. However, in the solutions, the height of the fire engine was not considered. If the students whose solutions are presented in Fig. 8 had scrutinized their solution again after task processing, for example, by validating their mathematical model, they might have

Fig. 6 Examples of incorrect or missing mathematical models

noticed that their mathematical model did not adequately represent the given real-world situation. Therefore, we suggest that the students did not manage to validate, or ignored the need to validate, their mathematical model. The student with the solution in Fig. 8a might have noticed that he/she did not take into account the height of the fire

engine because his/her answer refers only to the ladder and not to the house. In the solution displayed in Fig. 8b, the height of the fire engine was included in the drawing of the real-world situation. Therefore, the student considered the information in his/her real model, but he/she did not consider it in his/her mathematical model.

Fig. 7 Examples of incorrect mathematical results due to computational errors

Fig. 8 Examples of a correct mathematical result without considering additional aspects of reality in the mathematical model

6 Discussion

Problem posing is an important teaching approach in mathematics education as it can foster students' problem-solving abilities (Chen et al. 2013). However, modelling has not been previously investigated through problem posing. As the posing and solving of modelling problems are necessary conditions for fostering modelling through problem posing, we analyzed the problems students posed when they were asked to pose problems that were based on given descriptions of real-world situations, and their solutions to their self-generated problems.

6.1 Posing problems that are based on real-world situations

One of the major findings of the study is that students managed to pose modelling problems that were based on real-world situations. Hence, problem posing should be considered for student-centered teaching approaches that aim to foster modelling by increasing self-regulation. However, most of the problems they posed did not meet the criteria for modelling problems. This result highlights the need to include instructional elements that scaffold the posing of open and authentic problems by students. Further, all self-generated mathematical problems were connected to reality. One reason for this finding is that the authentic contexts and data described in the real-world situations triggered students to pose problems that were connected to reality and did not inspire them to generate problems without a connection to reality (i.e., intra-mathematical problems), for example, by using the numbers offered in the description of the situation. About half of the self-generated real-world problems were artificial 'dressed up' word problems and most of the real-world problems were closed and did not require students to make assumptions about missing information or vague conditions. The finding that students posed closed (and not open) problems is consistent with previous findings from problem posing research (Silver and Cai 1996; Leung and Silver 1997; English 1998). This finding confirmed the assumption from the theory on problem posing that students' knowledge of the subsequent problem-solving process may have influenced their problem posing (Cai and Hwang 2002). As these students usually worked on closed problems in their everyday classrooms, they also generated closed problems when asked to pose problems, even when they were offered a rich context (Bonotto 2013). Another explanation of this result might be that students developed their solution to a problem first, and then they wrote their problem down. Along with students' difficulties in making assumptions (Blum and Leiß 2007; Verschaffel et al. 2020; Krawitz et al. 2018) and their lack of experience with open

problems (Zhu and Fan 2006), this may explain the large proportion of closed problems that were generated by students. This work contributes to the theory of problem posing, as it indicates that assumptions about the importance of students' prior knowledge and prior experience in problem solving for problem posing (Stoyanova 1997) also hold for the authentic and rich situations that were offered in our study. Further, the problems students posed differed for the given situations because different contexts require different situational knowledge (Krawitz and Schukajlow 2018) and can also result in different challenges in solving the self-generated problems. Hence, not every given situation may be equally appropriate for fostering students' modelling. One practical implication of our work is that teaching modelling through problem posing should take into account the limitations that result from the characteristics of problems posed by students.

6.2 Students' difficulties in solving self-generated problems

An important part of the problem posing approach we used in our study was that we asked students to solve their self-generated problems. Our analysis of students' solution scores revealed that students showed no significant differences between solving their self-generated modelling problems and self-generated word problems. This finding is not in line with theoretical considerations or empirical results from modelling research that indicated that solving modelling tasks is more challenging for students than solving word problems (Galbraith and Stillman 2006; Blum and Leiß 2007; Kaiser 2017). A possible explanation could be that the act of problem posing (because students thought about possible solution steps while posing their problems) helped students to overcome the cognitive barriers described in modelling research and therefore helped them solve their self-generated modelling problems. This result could be an initial indication of the effect of problem posing on modelling. This is a novel result because, to the best of our knowledge, no previous research has investigated students' own solutions to their self-generated modelling problems.

To gain deeper insights into students' difficulties in solving their self-generated real-world problems, we analyzed students' solutions with respect to the solution steps students managed or did not manage to implement when solving their self-generated problems, and we conducted an in-depth analysis of students' solutions. The results revealed that most of the students already experienced difficulties in creating a correct mathematical model. This finding confirmed evidence from prior studies about students' difficulties in solving modelling problems (Galbraith and Stillman 2006; Krawitz et al. 2018). Students demonstrated a range

of individual differences in difficulties while mathematizing modelling problems. Some students considered aspects of reality, but they had difficulties creating a correct mathematical model, whereas other students created a correct mathematical model but neglected real-world aspects. A possible explanation could be that the more aspects that are considered in the real model, the more complex a correct mathematical model will be. These results are a novel finding as they demonstrate that well-known difficulties and barriers in the modelling process also occur when students work on self-generated modelling problems. One practical implication from this work is the importance of discussions of students' individual solutions, as students' difficulties varied significantly. Additionally, the results were only partly in line with those of Bonotto (2011), who showed that after problem posing, students considered aspects of reality. A possible explanation for the inconsistent findings could be that Bonotto (2011) used a long-term teaching approach to foster problem posing. As modelling research has indicated that students have a strong persistent tendency to neglect aspects of the real world (Verschaffel et al. 2020), dealing with problem posing over a long period of time is needed to encourage students to consider more aspects of reality in their solutions. After computing a mathematical result, most of the students managed to complete their task processing with a correct real result. However, as students did not include the real-world aspects in their solutions and stuck with an over-simplified mathematical model, we conclude that they did not validate their models and results.

7 Limitations and future directions

In the present study, students were asked to pose mathematical problems based on real-world situations that could be solved by using information from these situations (Sect. 4.2). We selected these instructions to ensure that students knew what to do because they did not have any previous experience with problem posing tasks. However, the posed problems might have been different (e.g., concerning the mathematical models and procedures or the openness of the problems) if a different introduction had been used.

We scored the correctness of students' written solutions by computing a solution score with respect to certain steps in the modelling process. By using this score, we were not able to include every single step in the modelling process, as it is difficult to differentiate, for example, among simplifying and structuring and mathematizing, or to infer whether students validated their models or results. This is an important limitation of the present study. To address this shortcoming, we performed an in-depth analysis of written solutions based on the drawings some students constructed in their solutions.

Laboratory studies are necessary to gain a more detailed differentiation in assessing students' solutions, in order to clarify how problem posing can affect certain modelling steps when students solve their self-generated problems. Future research should uncover the mechanism behind the effects of problem posing on modelling and address how problem posing affects modelling activities while students solve self-generated problems.

The current study combined two lines of research, modelling and problem posing, to consider the potential of fostering modelling through problem posing. To gain deeper insight into the connection between the processes of problem posing and modelling, future issues might be to investigate whether and how the processes of problem posing and modelling interact with each other and whether posing their own problems influences students' modelling performance. Further important open questions that should be the focus of future studies are whether the problems students pose differ when students are instructed to pose as many problems as they can, or when they are asked to pose problems with different levels of difficulty based on the given real-world situations, and whether posing problems based on authentic real-world situations influences students' affect in posing and solving mathematical problems.

8 Conclusion

A main conclusion of our work is that students are able to pose and solve mathematical problems that are related to reality. Therefore, problem posing based on authentic situations from the real world seems to be a promising approach for fostering modelling. However, students showed a strong tendency to generate closed problems that did not require problem solvers to make assumption or to structure the real-world situation. A few students who generated modelling problems either neglected to consider aspects of reality or considered aspects of reality but did not integrate them adequately into their mathematical models and therefore did not solve these problems. The results of our study offer new insights into students' posing of problems that are based on given real-world situations and provide initial indications about students' difficulties while solving self-generated word and modelling problems. These findings should be taken into account when self-generated problems are used in learning environments designed to teach mathematical modelling.

Appendix

Real-world situations used in this study:

See Figs. 9, 10, 11, 12, 13

Sports Field

The sports field shown is 100 m long and 50 m wide. Bernd sprints from the top left corner diagonally across the entire sports field to the bottom right corner in 13.1 seconds. His coach wants to spare already heavily used grass and runs along the outer edge of the field from the top left to the bottom right. The athletic club's speed-measuring system shows the coach running at a speed of 25.4 km/h.

Fig. 9 The given real-world situation “Sports Field”

Detour

Christine lives in Schoenbeck and travels every day by car via the L 523 to Kausberg, where she works. She usually needs about 7 minutes to cover this route. Because of construction, the speed limit has been adjusted to a maximum of 30 km/h for the route. Alternatively, she can take a detour at a maximum speed of 50 km/h: So first she drives to Riesand, which is 3.5 km away, and from there she drives another 2.7 km to Kausberg.

Fig. 10 The given real-world situation ‘Detour’

Salt Mountain

In the Middle Ages, salt was obtained from the evaporation of sea water. Nowadays it is mainly obtained by mining. The salt (1.2 t per m^3) is piled up into high salt mountains by means of 1.2 m wide conveyor belts. Later, it is transported away by lorries.

In the Figure, you see a huge salt mountain. The length of its edge is $c = 20 \text{ m}$, and its diameter is $d = 30 \text{ m}$. This salt mountain weighs about 3740 tons.

The technical data for the truck are:

Truck loading platform:	5.11 m x 2.3 m x 1.8 m
Power:	125 kw (175 HP)
Cubic capacity:	4249 cm^3
Maximum load capacity:	26.8 t

Fig. 11 The given real-world situation ‘Salt Mountain’ (Schukajlow and Krug 2014)

Cable Car

The cable car at *Luftseilbahn Engelsberg* is to be completely rebuilt in order to reach the top of the mountain more quickly.

Luftseilbahn Engelsberg:

Horizontal difference:	1041 meters
Max. gradient:	100%
Carrying capacity:	2*45 + 1 person
Conveying capacity:	585 people/ hour
Speed:	9 m/s
Cable length:	1192 meters

Fig. 12 The given real-world situation 'Cable Car'

Chopsticks

Lisa is doing a student exchange in China for half a year. Since her mother likes to eat Chinese, Lisa decided to buy chopsticks for her mother. The chopsticks are 28 cm long, on average 0.5 cm thick and 0.5 cm high. Two chopsticks cost 63 Yuan. 1 Yuan is 0.13 €. She also finds a nice storage case on the market.

Fig. 13 The given real-world situation 'Chopsticks'

Funding Open Access funding enabled and organized by Projekt DEAL.

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