



How specific can language as resource become for the teaching of algebraic concepts?

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Abstract

Classroom research into mathematics and language has studied issues of context specificity such as cultures of explanation or the impact of language policies on practice. More recently, researchers in the domain have started to study issues of content specificity aimed at performing language-responsive mathematics teaching for the learning of precise mathematical content. Progress in the conceptualization of language as resource for mathematics teaching and learning makes it necessary to strengthen the discussion of the contexts of culture and interaction along with the linguistic demands given by the specificity of the mathematical content at play. In this paper, I introduce a sociocultural framing for a mathematical-linguistic view of grammar as resource with the focus on explicitness in communication. I then report developmental work with two teachers on their teaching of algebraic concepts, and address the question of how to learn to communicate explicit meanings for these concepts in classroom mathematical talk. The structuring principle adopted for this work was to critically distinguish and choose or produce instances of teacher talk that overtly communicated conceptual meaning within the algebra of equations. I conclude with preliminary evidence of the effectiveness of the work with the teachers.

Keywords Mathematics teacher professional development · Classroom teacher talk · Explicitness in communication · Content specificity · Algebra of equations

1 Introduction

We cannot address issues of communication in and for school mathematics teaching and learning in full without working for the understanding and improvement of classroom teacher talk. Mathematics teacher education research particularly needs more systematically to examine features of teacher talk with the potential of opening up the discussion and learning of specific mathematical content. So far, research on how to connect the language as resource orientation to the exploration and refinement of the language of the mathematics teacher in the classroom remains limited in terms of curricular content and educational levels. This is in part due to the prevalent focus, for some decades, on the languages of the diverse learners in this orientation in the field (Planas 2014, 2018). An increasing body of studies is reversing the current research gap with work on mathematics

teaching that is based on findings from classroom research on mathematics and language (e.g., Adler and Ronda 2017; Moschkovich and Zahner 2018; Prediger and Zindel 2017). In complementary ways inspired by a diversity of theoretical-analytical frames, these pioneering studies share views of language as resource for mathematics teaching in classroom talk. The background and quantity of studies in this line of research is on the rise, but more empirical evidence of the didactic possibilities of teacher talk in school mathematics is necessary. In the current study, I seek to make a contribution by conducting developmental work with two secondary school teachers during their teaching of conceptual aspects of the algebra of equations. In this context of practice, I research the question of how to learn to communicate explicit meanings for algebraic concepts in classroom mathematical talk.

My regular work for more than a decade with mathematics teachers in Catalan schools provides the institutional and educational backdrop to this paper. In 2005 I started working with a group of mathematics teachers in what would become a ten-year project (Planas and Civil 2009). The major structuring principle of the practice with teachers at that time was

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to construct, implement and evaluate classroom tasks—with a version for learners and a version for teachers—aimed at promoting explanatory talk in the sense of talk that supported practices of explanation of mathematical concepts and relationships between them in the lessons with the learners (Planas and Civil 2013). At present this principle continues to mediate my work with teachers along with a principle for explicit communication of specific mathematical content in teacher talk. In what follows I explain the steps towards the justification, adoption and shaping of this newer principle. We must go back to the spontaneous conversation in 2018 with Maia and Jana, two mathematics teachers from the same school in Barcelona with several years of teaching experience. They took the initiative to share some difficulties encountered in their teaching of algebraic concepts, and I was then invited to spend time in the school and to visit their classrooms. Once there, I mostly observed pedagogies of whole class teaching in which Jana or Maia presented a rule or procedure, and then routine exercises from the textbook were done to consolidate the rule or procedure. The planning of a progression of goals for teacher learning and classroom transformation was conducive of a professional development intervention (PD) with the initial goal of improving the mathematical talk of the teachers. The structuring principle for the PD was to critically distinguish and choose or produce instances of teacher talk that explicitly communicate conceptual meaning within the algebra of equations. This purpose was supported by practices of noticing mathematically relevant content amongst sentences spoken in lessons, and of zooming in on issues of lexical elaboration for the overt communication of the content in question.

Understanding how teacher talk unfolds for content-specific teaching fits the topic of this Special Issue by expanding the principle that instruction should include comparison of language pieces for raising students' language awareness (Erath, Ingram, Moschkovich and Prediger 2021) into a principle that includes sites of mathematics teacher education. Comparing instances of the language in the classroom can be transferred at the level of work with teachers for raising awareness of the didactic potential of their talk in teaching. While there is agreement on the didactic importance of teacher talk in our research field, much remains to be studied with regard to the instructional practices that create opportunities for learning how to improve this talk in sites of professional development.

2 Theoretical basis for designing learning practices on language and mathematics teaching

Following a sociocultural view of teaching as socially mediated by cultural tools such as tasks, practices and languages (Adler and Ronda 2017), in this section I present the more

concrete theory that substantiates the organisation of the developmental work with Jana and Maia. In prior PDs, aimed at creating and enacting classroom tasks as mediators of explanatory talk, part of the mathematical content planned—that is, specified in the teacher written version of the task—often remained hidden or out of focus in both teacher talk and whole class discussion. While the tasks had been designed carefully and practices of explanation were promoted in the lesson experiments, implicitness or ambiguity in content meaning seemed to impose limitations on student learning across lessons of different teachers. For example, in a 2015 lesson regarding plane isometries, the teacher said, “When mathematicians think of angles, they have more in mind than two segments and an end point.” In the lesson talk of that teacher, this sentence did not go with any explicit mention of the rotation concept, which is essential for the construction of the angle concept in the geometry of transformations. In the PD practice, we had created a task based on manipulating materials and having participants interacting with each other, but in its design and enactment, the clear communication of important mathematical meaning was not fulfilled.

There is a wide range of theoretical-analytical frameworks for the study of language, meaning making and communication in mathematics classrooms (see a survey of the theoretical diversity in the domain by Planas and Schütte (2018), but not all of them offer tools for the analysis of mathematical meaning communicated through the resource of grammar. The theory of Systemic Functional Grammar, or SFG in brief (Halliday 1978, 1985), takes the realization of meaning as mediated in practice, and hence fits in well with sociocultural views of teaching and of teacher talk that address appropriateness to the rules of language and of the relevant social context. SFG consists of various linked aspects such as the ideational functioning of grammar in context to construct and communicate objects, relationships and logic, which for the mathematics classroom includes the objects, relationships and logic of school mathematics. Although there is a research basis that suggests the means by which prompting practices of explanation in the teaching mediates school mathematics learning (e.g., Khisty and Chval 2002; Zahner et al. 2012), we do not know much about how specific grammar choices in teacher talk (i.e., choosing which words to use and how to connect them with other words) differently realize meaning, and can offer or fail to offer learners the opportunity to hear and engage with specific mathematical content. A mathematical-linguistic view in which grammar is a process of making mathematical meaning in context can thus enhance sociocultural research on classroom teacher talk.

Halliday (1978, 1985) equates grammar with the indissoluble articulation of syntax and context. Accordingly, the language that functions to make and communicate

mathematical meaning is both syntactically and contextually ruled. In the more linguistically focused mathematics education research literature, some authors have drawn on SFG to study uses of language for a variety of curricular contents and educational contexts. Herbel-Eisenmann and Otten (2011) tracked content-based ideational and interpersonal functions of grammar in lessons aimed at teaching and learning plane areas; Morgan (2006) examined learner written productions to elucidate ideational and textual functions of grammar in the communication of conceptual meaning for triangles and generalization patterns; and Pöhler and Prediger (2015) explored linguistic means in the realization of precise mathematical ideational meaning in lessons on percentages. Taking the aspects of Halliday's theory related to the creation of content-based ideational meaning, rather than interpersonal meaning produced in the interaction or textual meaning more broadly produced to organize coherent messages, Schleppegrell (2007) pointed to the complexity of choosing and combining words for the enactment of meaning that can be recognized as constitutive of school mathematics. She called for research on the ways learners and teachers use resources from the grammar, like the "dense noun phrases that participate in relational processes" (p. 139), to produce academic meaning.

Halliday's concept of grammar as both syntactic and contextual is shaped by a number of analytical tools that connect linguistic forms with the realization of meanings in use; these tools are however mostly developed for the case of the English language. When an investigation deals with sources of data in spoken languages other than English, the adoption of parts of SFG requires some caution. In accordance with Boas and González-García (2014), the SFG considerations for clause processes in spoken English generally apply well to Catalan and to Spanish, the two languages of the data in my research context. Although there are differences, such as the obligation in English of explicitly including the subject of a sentence or clause, in these three languages most often lexical complements to verb forms are not obligatory. This linguistic feature implies that syntactically correct sentences may stay open or vague in meaning and function for a diversity of contexts and registers. For example, if we take the sentence 'she orders the tickets' and do not specify the context of use, the meaning for 'orders' remains vague; it may variably function within out-of-school and school mathematics registers. Moreover, in English, Catalan or Spanish, we find syntactically precise sentences that, in spite of knowing the context in which they are said, may require lexical complements not to stay semantically open. Even in the mathematics classroom, 'She orders the tickets' may be understood as requesting tickets. If the intended meaning is mathematical, lexical complements like 'from the cheapest to the most expensive' are useful.

While clarity in meaning should prevail in teacher talk, this is not always easy. The moment the teacher places words together in a sentence for teaching mathematical content, this sentence enters into a relation with the context, specifically including its relation with other sentences and what has or has not been communicated in precedent talk. Although certain meanings can be left as tacit in some lessons, the use of talk in ways that determine the meanings pursued as content of learning turns out to be problematic. To this end, clarity in meaning through lexical elaboration is challenging due to semantic vagueness of grammar and disassociation between syntactic precision and meaning in use (Halliday 1978, 1985). Vagueness and disassociation are actually intrinsic to communication and essential in order to grasp the complexity embedded in the realization of the didactic potential of teacher talk. Clarifying the semantics of a sentence requires words that can function to indicate objects, relations, attributes, variables... If the grammar is minimal in the sense of lacking lexical complements for objects, relations..., the sentence stays open or vague with the subsequent difficulty in noticing the meaning intended, and the risks of varied interpretations. All of this is connected with the questions as to how grammar in teacher talk can become a resource for the communication of concrete mathematical meaning, and to which degree this can be planned in sites of mathematics teacher education and then made effective in classroom teaching.

The elaboration of sentences that are precise in meaning takes even greater significance for sentences made of relational verbs, since these are a feature of academic registers in contrast to sentences made of material verbs suggesting processes of doing and acting. As developed by Halliday (1985), relational verbs are those realized in grammar to perform functions of explaining and connecting attributes and relations. 'Be' is the most typical relational verb and the most ambiguous as well. In the mathematics classroom, it is common to find sentences with this verb conjugated, in which an example is meant to exhaust the larger set: 'A is B' put as ' $3x^2 + 4 = 2$ is the equation' expresses a one-to-one relation of equivalence when what is mathematically involved is a one-to-many relation between the equation at play and one of its representatives. While the choice of more concrete verb words that function as relational, such as exemplify, name, classify, compare, define, and so forth may avoid some ambiguity, openness or vagueness regarding the meaning for the object equation may still remain due to poor lexical elaboration. 'A exemplifies B' put as ' $3x^2 + 4 = 2$ exemplifies the equation' keeps a wide range of possible meanings for the equation concept. In the absence of more lexical detail, it is unclear why $3x^2 + 4 = 2$ is an example of an equation and what this tells us about how examples of the same object should look. If we intend to communicate that $3x^2 + 4 = 2$ represents an equality reasoning, this might

be further elaborated lexically, for example, as ‘ $3x^2 + 4 = 2$ exemplifies an equality that some numbers may satisfy.’

Albeit timidly and not particularly centered on issues of semantic clarity or precision within the algebra of equations, SFG approaches are also present in the study of language in mathematics teacher education and professional development research. In the reflection on language issues in the domain, a major argument is the distinction between the content of mathematics teaching and learning at the level of the classroom and this content at the level of mathematics teacher education and professional development. Prediger (2019) is attentive to the two levels of content in her argument towards strengthening the collaborative research with teachers in the construction of mathematical content-specific knowledge about language for mathematics teaching and learning. She proposes a language focus in design-research work towards the anticipation, discussion and evaluation of explicit explanation and connection of content meaning. Morgan (2014) specifically addresses the question of choice, which is central to the theory of Halliday. In the analysis of means of describing and judging learners’ mathematical texts of utility for ways of organizing the knowing, doing and teaching that are important to teachers and teacher educators, Morgan interrogates, “what might be the differences if other choices had been or were to be made?” (p. 132). It is noteworthy that the choices of teachers and teacher educators about which language to use may not be deliberate, but rather the result of common ways of speaking. In this respect, choices that keep mathematical meaning implicit in the interaction with learners may not be seen as problematic because implicitness and its impact on classroom learning may not be perceived. It is thus very pertinent, in research on developmental work with teachers, to guide the interrogation and noticing of grammar choices for content-based mathematics teaching.

3 Methodology and methods for the developmental work and its study

In this section, I explain the methodology and methods for researching and developing task-based professional learning practices with two secondary school mathematics teachers. Guided by the view of mathematics teaching as making grammar choices, I used an interventionist methodology for simultaneous research and developmental work organized around the performance of tasks that highlight issues of language choice and content-specific talk. By this I mean a methodology that involved preparing and contextualizing the emerging PD practice, from, and within, data drawn from the participant teachers’ talk in lessons that I observed, analysed and selected for the design and implementation of tasks. For the discussion of normative aspects—‘what ought to

be’—these tasks intend to mirror descriptive aspects—the ‘what is’—of classroom data in the form of short instances of talk comprising mention of algebraic concepts. In the next subsections, I particularly discuss the basis and role of the structuring principles in the design and enactment of the PD tasks. In the first subsection, I present the phase around the empirical preparation and theoretical basis of the sessions for developmental practice. In the second subsection, I report details of the implementation and analysis of the work with Jana and Maia in one of the sessions and with regard to one task. At the time of this writing, the analysis of changes in the talk of the two teachers, that seem to be important for the conceptual mathematics learning of the learners in their classrooms, is being done. Some of the changes identified are discussed in the section following this report.

3.1 Preparing developmental practice grounded on lesson data and research findings

In the introductory section, I anticipated the instructional principle that structured the developmental work with Jana and Maia: Critically distinguish and choose or produce instances of teacher talk that explicitly communicate conceptual meaning within the algebra of equations. In the theoretical section, I summarized the rationale for taking the aspects of SFG that deal with content-based ideational meaning. In line with all this, I now provide the details of how the tasks used in the PD practice were thought out and prepared. While it had been possible to work with anonymous instances of teacher talk released from past collaborations with other teachers, my experience of developmental work recommended building the newer collaboration with insights and material from Jana and Maia’s own classrooms. Having in mind the difficulties explained by these teachers in our initial conversation, the decision of working with and from their classroom talk was specified for lessons aimed at formulating, manipulating and solving equations.

The source data were collected in my visits to the classrooms across the 2018 school year, and the PD was then planned to start in October 2019. I finally chose two lessons, one per classroom and teacher, which had been audiotaped and turned into transcripts limited to the instances of teacher talk. None of the lessons were part of a prepared design experiment or of a pedagogic innovation. They reflected the everyday dynamics of whole class teaching and controlled exercises from the textbook, and in both instruction was centered on the routine of manipulating quadratic equations towards producing representative items for application of the formula. In the second year of the PD, the reach of the project was expanded and moved on to the purpose of also working on the choice or design and enactment of innovative tasks for teaching algebraic concepts through combination of verbal and graphical

tools. We kept the same curricular topic and left for future studies the intuition that professional deepening inside the language demands of teaching equations will support the learning of how to improve the teaching of other topics. The following is an instance of Jana's talk to the whole class in the lesson selected, followed by an English version (shifts between Catalan and Spanish in the original are not marked):

Podem resoldre una equació quadràtica amb fórmula. Modificarem una mica l'equació escrita inicial. Anar fent canvis pas a pas és bàsic. Canvieu cada equació per la següent i obteniu una seqüència. Cada ecuación la cambiáis un poco. Tenéis que utilizar las reglas de transposición. Vais asociando una forma escrita con otra hasta llegar a la fórmula general de la pizarra.
We can solve a quadratic equation with a formula. We will modify the written initial equation a bit. Changing step by step is key. You change each equation into the following and get a sequence. Every equation, you change it a bit. You have to use the transposition rules. You go mapping one written form to another up to the general formula on the board.

The final material for the PD came from a second round of data selection so that the units of attention that could bring grammar choices into focus in the work with the teachers were relatively short. I looked into the transcripts of teacher talk for sentences with the potential of constructing meaning for algebraic concepts. With the ultimate objective of selecting material for the PD practice, I did not pretend to do an exhaustive tracking. I identified sentences of a relational type, and hence with the potential of supporting functions of explaining and connecting. Throughout this process, relevant research literature provided me with criteria with which to identify some of the most significant concepts and relations constituting the semantics of school algebra. Herscovics and Kieran (1980) and Pournara, Sanders, Adler and Hodgen (2016) documented areas of difficulty regarding the algebra of equations, and indicated the relevance of teaching equivalence (between expressions obtained by procedural manipulation) through expanded meanings for the equal sign as a relation, and not just an operator. Kieran and Drijvers (2006) specifically pointed to difficulties encountered by learners in understanding algebraic equivalence as well as in dealing with structural properties of operations involved in connecting representatives or examples. Accordingly, I considered that relational processes of equivalence and of exemplification were crucial to be intended in classroom teacher talk aimed at prompting the learning of concepts such as algebraic equivalence and quadratic equations. Since these processes cannot be omitted or left unclear at the level of the classroom, I also considered its discussion to be crucial at the level of the PD. I compiled critical sentences from

teacher talk suggesting lack of clarity in meaning around the following relational processes:

- i. Equating representatives. One-to-one processes in which the two objects are representatives of an equation, e.g., “you change each equation into the following.”
- ii. Exemplifying equations. One-to-many processes in which one of the objects is the equation and the other is a collection of representatives, e.g., “the equation looks a bit different after each transposition rule.”

The processes above were therefore taken as relevant content of teaching in lessons on the topic of equations, as well as relevant content of professional development work on this topic with the teachers. Drawing on the reading of the lesson transcripts, it was evident that the potential of relational processes for fostering reasons and connections did not always imply the communication of precise mathematical meaning. I found several sentences suggesting relational processes that were not further elaborated towards the construction of algebraic concepts but rather went together with sentences functioning to make learners notice rules or procedural steps of the resolution method. In this way, the explanations initiated were not lexically elaborated to communicate what remained invariable with regard to the equation concept and, more generally, the criterion for belonging to a class of representatives. The following are two examples of this finding:

You change each equation into the following and get a sequence. Now pay attention to how far you are from the general formula.
This final equation is the equation from the beginning. But now the equation equals zero as we wanted and can stop.

As visible in the examples above, most relational sentences appeared paired in the talk of Jana and Maia with the lexical elaboration of complements suggesting material processes of doing and acting. The recurrent objects of explanation ('*explanandum*' in Erath, Prediger, Quasthoff and Heller 2018) were overall the procedural steps of the resolution method and the transposition rules. Across the two lessons, there was not explicit talk about mathematical reasons underlying the steps in the method or about connections between these reasons and the transposition rules to be applied. As a result, exemplifying equations and equating representatives were relational processes whose communication was initiated in teacher talk and interrupted when paired with material processes of algebraic manipulation. It was not less revealing that very few teacher sentences in both lessons contained the words 'equivalent' or 'equivalence': “These are equivalent, and these too”, “Not the same but are equivalent equations”, “Equivalence in fractions is

similar”, and “Transformations that keep the equivalence.” Collectively all these findings guided the selection of some moments of teacher talk over others for the practice in the PD, but also confirmed the adequacy of the SFG approach to the interpretation and study of the didactic potential of the language of the mathematics teacher in the classroom.

In the next subsection, I present some events of the third PD session centered on the discussion of the task in Fig. 1. Options 1 and 2 in this task expose Jana and Maia to critical instances of their own talk in the lessons analysed. These are instances in which the communication of conceptual mathematical meanings is initiated and then interrupted by a change in focus from the identification of representatives of a particular equation to the indication of the procedural actions to be taken. A consequence expected from the discussion and comparison of Options 1 and 2 is the creation or design of newer options of talk that more overtly explain and connect precise mathematical meanings. A total of fifteen tasks were developed as progressive towards the discussion, comparison and production of diverse options in languages of teaching that explicitly highlighted important concepts and mathematically relevant meanings in the algebra of equations.

3.2 Enacting and analysing teacher participation and learning in developmental work

The first year of the PD practice with Maia and Jana was designed to develop and support professional learning of features of teacher talk at the time of their participation in a single module of five 90-min workshops, one per month in the school site. All the workshop tasks were planned to be completed in collaboration during the time of the sessions, and they were all centered on issues of explicitness in

communication of conceptual meaning within the algebra of equations. As already explained, the tasks had emerged from my analysis of the teachers’ talk in two lessons the year before. Since it was I who had identified aspects to be improved in their languages of teaching and hence in their teaching, it was necessary to engage the teachers themselves in noticing the didactic content-specific possibilities embedded in their talk. To accomplish this aspect, each workshop consisted of the iteration of three tasks: one of interrogation of instances of these teachers’ talk (like the example in Fig. 1), and two of lexical elaboration. The process for each session was tunnelling the attention of the teachers towards the practice of producing linguistically small but mathematically significant changes in their talk when teaching. All this was to be done without much talk on my side though I flexibly played the role of facilitator.

The five workshops always started with the reading and interrogation of two short pieces of language, one from each lesson transcript, to engage Jana and Maia in examining their talk in the classrooms. The pieces occupied approximately one page and represented about three of four minutes of teacher talk. This was the material in focus from which to find evidence in order to construct a shared argument on the importance of explicitness in mathematical communication. The instruction for the five first tasks was as follows: Underline the sentences in which you talk about concepts of significance in the algebra of equations, and specify the concepts. Jana and Maia faced this instruction with no prior preparation as a way to simulate the way teachers typically encounter their own talk or that of their colleagues in classroom teaching. In the discussion, both teachers struggled to find sentences in which meanings for specific concepts were explicitly communicated, and hence came to notice that important mathematical content remained under-specified

Fig. 1 Task given in the third session of the 2019 module

Task. In a lesson aimed at teaching the method of solving quadratic equations with the formula, the teacher is writing on the board the expressions below.

$$\begin{aligned}x^2 &= 3x + 10 \\x^2 - 3x &= 10 \\x^2 - 3x - 10 &= 0\end{aligned}$$

Then the teacher stops writing and talks to the whole group.

Option 1. *You change each equation into the following and get a sequence. Now pay attention to how far you are from the general formula.*

Option 2. *This final equation is the equation from the beginning. But now the equation equals zero as we wanted and can stop.*

Is the talk in Options 1 and 2 communicating meanings of algebraic equivalence? Can these instances convince learners that the expressions on the board represent the same equation?

Can you think of one or more Options 3 in which the teacher talk in the class explicitly communicates the relation of equivalence between the expressions of the equation?

Options 3. _____

within the algebraic register. Whilst in the first PD session Jana and Maia justified implicitness and ambiguity in meaning by raising some risks of using too long sentences in the teaching, as the module evolved, they progressively associated explicitness and exactness with the possibility of introducing small changes in classroom talk.

To make teachers notice the range of didactic possibilities embedded in talk, and working with the material provided for each first task, the instruction for the five second tasks was as follows: Propose changes in sentences in order to communicate explicitly conceptual meanings of significance within the algebra of equations. Table 1 shows extended texts jointly created by Jana and Maia in collaboration with me over the course of the workshops. For each sentence, the curricular content to be made explicit was discussed, and lexical elaborations that could contribute to this end. Interestingly, most of the sentences used as material for the set of third tasks of the sessions were also in the group of sentences underlined or selected by Jana and Maia as critical. The concept of algebraic equivalence and the related issue of how to teach the expressions that can and those that cannot be regarded as equivalent were particularly brought up by the teachers in all the workshops. We agreed on the affordances of the procedural definition of algebraic equivalence in terms of equations that have the same solutions, but concluded that learners should also be given conceptual reasons. To support the further elaboration of newer sentences, in my role of facilitator I mentioned linguistic devices to be used such as: ‘which means’, ‘which is to say’, ‘that is’, ‘in other words’, ‘X means Y’, ‘by X we/I mean Y’, or ‘X is the same as Y if/because.’ The strategy was to make noticeable that some additional text would support the communication of meaning otherwise implicit. These devices functioned to support explanations and connections while combining or reworking entire sentences, and changing or adding words. Another fundamental outcome of the discussions was the

consideration of disciplinary content knowledge, through the unpacking and noticing of mathematical facts to be made explicit in the lessons, such as: (1) each algebraic equation is a class of equivalent representatives; (2) representatives are equivalent if they have the same solutions; and (3) operating with the same numbers or expressions on both sides of an equation produces an equivalent equation, except for the case of multiplication or division by zero. We also had to work on misconceptions such as the necessity of finding the numerical solutions of the equations in order to test their equivalence.

All five sessions ended with a third task made of sentences of teacher talk for which the inclusion of lexical complements was sufficient to allow the explicit communication of conceptual meaning of relevance in the algebra of equations. Throughout the study of the lesson data, I found some sentences rather easy to turn into mathematically relevant talk for conceptual meaning making; these were the sentences chosen as material for the third five tasks. Figure 1 reproduces an English version of the group of instructions in one of these tasks, with the ultimate instruction to Think of different ways of explicitly communicating the relation of equivalence between expressions of an equation. The teachers worked together around the sentences in Options 1 and 2. For example, in Option 1 “You change each equation into the following and get a sequence”, the criterion of change and what remains unchanged are not overtly said. Further lexical elaboration could mathematically reinforce the sentence with respect to the semantics of algebra and algebraic equivalence. Here, I facilitated the discussion of clarity in conceptual meaning with respect to the teaching and learning of quadratic equations, as well as the discussion of how small changes in talk can function for meaning disambiguation and explicitness. In the third PD session, the Option 3 jointly produced by the teachers and thought of as an improvement compared to Options 1 and 2 (see Fig. 1), was as follows:

Table 1 Examples of lexical elaboration by the teachers

Selected sentences	Lexical elaborations
We can solve a quadratic equation with formula	We can solve a quadratic equation with formula. <u>That is, we can obtain the numerical values for x that solve the equation</u>
We will modify the written initial equation	We will modify the written initial equation. <u>In other words, we will look for ways of writing the same equation for the final application of the formula</u>
Get a sequence	Get a sequence, <u>which is to say, get a sequence of equivalent equations, or equations with the same solutions</u>
Every equation, you change it a bit	Every equation, you change it a bit. <u>By changing it a bit, I mean adding, subtracting, multiplying or dividing both sides with the same numbers so that the solutions do not change</u>
You have to use the transposition rules	You have to use the transposition rules. <u>That is, the rules for the generation of equivalent equations</u>
You go mapping one written form to another up to the general formula on the board	You go mapping one written form to another up to the general formula on the board. <u>All the equations will be the same because the same numerical values solve all them</u>

You change the place of some terms for each equation, but the solutions do not change. $x^2 = 3x + 10$ is the same as $x^2 - 3x - 10 = 0$ if we look at the numbers that solve them.

By asking Jana and Maia to choose and produce improved instances of teacher talk, I had progressive access to what counted for them as appropriate mathematical communication with the learners in the classroom and could reorient some of the later discussions as well as slightly modify the content of tasks prepared for the remaining workshops. All this process opened opportunities for these teachers to learn about language in mathematics teaching, but also opportunities for mathematics learning by learners in their classrooms.

4 Initiating methods for the evaluation of the developmental work

The first year of the PD was not intentionally designed with the goal of finding an impact of teacher talk on the school learning of algebraic concepts. I had the idea that the participation in the PD practice would affect the teachers' professional growth and learning but would only reach learners in their classrooms indirectly. So, when three weeks after the ending of the 2019 module I observed their teaching in lessons aimed at manipulating and solving equations, I was not looking for evidence of impact at the learner level but rather for little differences and improvements in teaching at the level of explicitness and precision in mathematical communication. The intention was to see whether some professional learning about explicitness in content-based communication was visible in the language choices of the teachers once back in the classrooms. On this occasion, a research assistant videotaped and transcribed the totality of classroom talk over one week of lessons because that would be source material for the 2020 module. Despite the short exposure to the PD practice, the reiteration of the curricular topic as the teaching and learning content was an occasion for teachers to experiment, and for me to observe any innovation in their talk. The observation of the individual teaching was thus the way in which I had planned the evaluation of the effectiveness of the PD. What I found were many opportunities for learners to hear sentences semantically situated within the algebra of equations. This allows me to provide a preliminary evaluation of the effectiveness of the PD from the dual perspective of professional learning and changes in teacher talk, and opportunities for mathematics learning created in the classroom.

What follows are two short lesson episodes, one from each classroom, for illustrating some evidence of mathematically relevant choices in the teacher talk with an effect on the responses of the learners in the interaction. Both Jana and

Maia are explaining solving methods through a language that makes conceptual mathematical meaning explicit in the teaching. Jana overtly renames transposition rules into rules of equivalent equations, and Maia clarifies that calling something equivalent does not clarify the mathematical meaning of this word. In either case, teacher talk seems crucial in offering learners access to explanations for mathematically relevant word meanings, reasons and connections. Still, we could discuss the fact that routine exercises remain central aspects of the teaching pedagogies in the two classrooms. In the 2020 PD module with the five mathematics teachers in the school, work on inquiry pedagogies has been incorporated. One of the strengths of the initial module, and of how the collaboration with the teachers was sensibly approached avoiding too many changes and goals at a time, is thus the successful role played in building continuation and expansion of the developmental work project.

4.1 Lesson episode with Jana: "Why do I call them the rules of equivalent equations?"

Learner 1: Okay, then we stop here. Ready for the formula.

Jana: What is the same with them [$x^2 + 4 = 1 - 2x$; $x^2 + 2x + 3 = 0$]?

Learner 1: Same equation.

Jana: How do you know this? They look different...

Learner 1: Well, not so different. They are quadratic and have two solutions.

Jana: $x^2 = 4$ [on the board] is also quadratic, and also has two solutions.

Learner 2: But this is different.

Jana: How do you know this?

Learner 2: You cannot apply a rule and make two x disappear.

Learner 1: You cannot find a rule that goes from here [$x^2 + 4 = 1 - 2x$] to here [$x^2 = 4$].

Jana: So, if you can find a rule... What did I call transposition rules? Rules of equivalent equations? Why do I call them rules of equivalent equations?

Learner 1: Because they are equivalent.

Jana: And how do you know this? What is the same with them?

Learner 1: Same solutions?

Jana: Right! These two equations are equivalent because they have the same solutions. How can we check this?

In this example, the attention in the teacher talk to word use (i.e., "transposition rules", "rules of equivalent equations") and to meaning explanation (i.e., "how do you know this?", "what is the same with them?", "these two equations are equivalent because they have the same solutions") is obvious. Learners are given the opportunity to hear and learn how the teacher talks about algebraic equivalence, which in turn generates mathematical talk about this concept

amongst learners. A theme of discussion in the PD was word use in naming procedures, specifically the possibilities of supporting conceptual meaning by renaming terminology related to procedures and drawing on it for elaboration of explicit talk about the reasons of equivalence between algebraic expressions. We also discussed that making a concept clear is not realized at the level of word use but requires the communication of relational processes of explanation and connection in the teaching. We precisely discussed the open semantics of transposition in “transposition rules”, and brought up the possibility of referring to them as “rules of equivalent equations.” In the episode above, Jana does not abandon the common name as generally termed in the local textbooks, but draws on the potential of “rules of equivalent equations” to make learners notice what remains invariable in the expressions across the application of such rules.

4.2 Lesson episode with Maia: “What makes them equivalent is not calling them equivalent”

Maia: Let’s think of fractions. Some fractions are equivalent, okay?

Learner 3: Yes.

Maia: What do we know about fractions? What makes them equivalent? Can you tell me when two fractions are equivalent?

Learner 3: One half and two fourths.

Maia: Okay, one half and two fourths. What makes them equivalent?

Learner 3: Yes, when you make the division, you get point five.

Maia: Okay, so they are equivalent because of the decimal. Let’s go back to equations. What do we know? Some equations are also equivalent. We call them equivalent, but why? What makes them equivalent? Two equations are equivalent if...

Learner 3: Like fractions.

Maia: Now it’s about equations. What makes them equivalent is not calling them equivalent. By equivalent equations, we mean...

Learner 3: Can you divide equations?

Maia: Good question! Yes, you can divide them, but what makes them equivalent is not about division and decimals. It’s about having same solutions.

Learner 3: Then you don’t know with the calculator.

Maia: Well, it’s about finding the numerical values that solve them.

Learner 4: Not finding the decimals.

Learner 3: No, that’s for fractions. She was just putting this example first.

Maia: Look at the board again. What about all these equations? Let’ see. What do all of them have in common? Which is to say what makes them equivalent?

Learner 3: The solutions!

This second example brings us back to discussions in the PD sessions about the idea that the words “equivalent” and “equivalence” were not often said in the teaching of algebra, and about the implications of omitting not only taken-as-known words but also their explanation in the talk with learners. In the workshops we concluded that without deliberate attention to the equivalence relation through explicit grammar in teacher talk, opportunities for learning the notion are limited. We can see connections between this conclusion in the PD and what Maia says to her learners in the episode above. She explains that calling the word equivalent does not make its meaning clear. In a situation in which learners may not have identified equivalent as a mathematically relevant word, the teacher elaborates on its meaning in the algebra of equations and does it by explicit contrast with the meaning within the arithmetic of fractions. The talk of the teacher continues to connect the word to explanations immediately after the interrogation of meaning (“what makes them equivalent?”), which denotes the kind of changes practised with the selected sentences in the PD sessions. In a sense, when Maia elaborates on word use and content meaning, she offers what she learned in the workshops to her learners.

4.3 Classroom teacher talk as demonstrated evidence of professional learning

While being cautious and aware that no systematic or comparative analyses have been done with the lesson data from the participating teachers, good choices for mathematical meaning making in the language of teaching can be appreciated. The year before, Maia had resorted to the analogy with fractions but had not explicitly addressed the construction of distinct meanings and structures for numerical equivalence and algebraic equivalence. The mention of equivalent fractions had not been elaborated in her classroom talk to function for disambiguation and concretion of specific mathematical meaning with respect to algebraic equations. Although the data above allow us to be optimistic, it is not clear whether the participation in five workshops oriented the talk of Jana and Maia in the episodes shown. We cannot be certain that the (transformed) teaching practice illustrated is due or partially due to participation and learning in the PD. To test the relationship between participation in such a short PD site and changes in the teaching talk is rather complex, and this is also true and even more complex for the relationship between teacher professional learning and changes in the learning opportunities offered to the school learners. Regarding the first relationship, I asked Jana and Maia to reflect on their experiences in this regard. Both were clear about how much they have gained awareness of the importance of being explicit in the communication of

mathematical meanings in classroom talk with learners. As said by Jana: “Now I cannot stop thinking... did I explain what I want them to learn or did it get taken for granted?” The PD practice thus seems to have mediated processes for teachers to become reflective on the importance of supporting access to mathematical meaning through explicit talk. As I continue the analysis of the post-PD classroom teacher talk in collaboration with Jana and Maia, it becomes more apparent that these teachers have much to say about their language choices in teaching.

5 Possible paths for how to move on from here

I have illustrated a very small-scale, unfinished project of collaborative developmental work with two secondary school mathematics teachers aimed at improving their talk in teaching for the enhancement of content-based learning. I do not have large-scale, long-term results as evidence of teachers taking up practices in a wide, systematic way following participation in professional development programs that are cyclical (e.g., the lesson study reported in Adler and Alshwaikh 2019). Despite the limitations of the study at this stage, I have argued my point on how grammar features of teacher talk can be interrogated, noticed and practised as effective ways of improving mathematics teaching and of offering learners opportunities to hear mathematically relevant conceptual content. That said and in line with the literature about the difficulties of lexical elaboration for semantic accomplishment (Halliday 1978, 1985; Halliday and Martin 1993), we cannot expect teachers to realize the didactic potential of their classroom talk without sustained educational programs that allow them to develop this kind of professional learning. Lexical elaboration for explicitness in mathematical communication, as object of professional learning and kind of professional expertise, can be placed at the intersection of mathematical and pedagogic knowledge for mathematics teaching.

Some opportunities of mathematics learning in the classroom rest upon a number of grammar choices in the teaching and the overt provision of mathematically precise sentences in teacher talk. In this respect, the attention to talk in teaching should become an object of explicit attention in sites of mathematics teacher education and professional development. Practices of lexical elaboration through, for example, adding explanations that give a clue as to the meanings of important concepts are being very productive also in the second year of our PD with the participant teachers completing “you find an equivalent equation” with “which means one with the same solutions.” Not only lexical elaboration of explanations, but also clarity in talk between the explanation and what is explained are of outmost significance

in the teaching and in teacher education. This significance particularly applies to classrooms with learners who are in the process of learning the language of instruction, and who are more often than not disadvantaged by being offered simplified, conceptually poor versions of mathematical language that hinder their understanding (Barwell et al. 2016). Throughout the paper I have briefly mentioned that the talk of the teachers was bilingual, but I have not referred to the various home languages other than Catalan and Spanish in their classrooms. In spite of not having dealt with the multiple languages and the multilingual practices in the lessons observed, or with the fluid bilingualism that operates in the PD context, it is important that the argument for lexical elaboration be further examined in articulation with the argument for flexible translanguaging in teacher talk. Most talk of Jana and Maia in their classrooms mixes Catalan and Spanish in ways that those holding linguistically purist perspectives of language would not identify as precise or adequate. We may thus need to be clearer about interpreting syntactic precision in teacher talk with regard to the mathematics, not the language of instruction. But even so, the unpacking of what precision implies for the use of language in the linguistically diverse mathematics classrooms is more complex than it may seem. Just as for all learners and classrooms, classroom researchers on mathematics and language recommend not interpreting precision separately from the didactic potential of informal languages in communication and learning. While the use of precise talk in mathematics teaching is key, studies in the field also tell us that low levels of explicitness may function to connect the informal languages of learners with the processes of making precise mathematical meaning (Moschkovich 2008), and that some level of vagueness is inherent in language and communication in all situations of mathematics teaching and learning (Rowland 2000). Precision and explicitness are hence not absolutes in the sense of being always desirable or reachable norms to be pursued or satisfied in classroom practice. Important learning difficulties may arise due to low exposition to precise mathematical talk, but also due to restrictive use of informal everyday meanings for the communication of the essential diverse ideas behind specific mathematical concepts.

I cannot finish without a final expression of relativeness and caution. The body of research on knowledge for mathematics teaching and mathematics teacher education tells us that effective teaching for the learning of specific content is not always informative if other contents are implied. This fact brings up several questions. Did/could the developmental work begun with school algebra have a side impact on Jana and Maia’s everyday teaching? Did/could Jana and Maia connect their developmental and teaching experiences around the concept of algebraic equivalence to the understanding of the teaching of other content? What we

know so far is that, along with the variability introduced by the context of culture and of interaction, content variability increases the challenge of articulating language-responsive content-based studies on mathematics teaching and mathematics teacher education. It is not easy to move among the linguistic demands of teaching and learning different mathematical content in a diversity of school and classroom cultures. This is not a limitation though, but a hint that additional studies continue to be necessary to shed light on the articulation of content-based research findings. Future research will have to show results from the study of how to learn to communicate explicit meanings in classroom teacher talk that articulate mathematical content and contexts of practice.

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