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EDITORIAL

Theoretical frameworks in research on and with mathematics teachers

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In her overview of trends in mathematics education research, Anna Sfard (2005) suggests that "the last few years have been *the era of the teacher* as the almost uncontested focus of researchers' attention" (p. 409, emphasis in original). Since then the research interest in teachers has certainly not diminished, as evidenced by the high and growing number of conference papers, journal articles, and monographs in the field as well as by the emphasis on teachers and teacher education in recent handbooks on mathematics education. It is a characteristic feature of most of this research that it is conducted within one of the three relatively distinct fields of teachers' knowledge, teachers' beliefs (or affect more generally), and teacher identity. Despite the unquestionable advances in each of the three fields, there is a somewhat surprising disconnect between them, leading to an incoherent view of the teacher and her or his role in instruction. This seems to be due not only to the different objects of investigation, but at least in part to different theoretical and methodological assumptions leading to qualitatively different units of analysis. Notwithstanding the differences within each of the three fields, research practices among them appear to particularly differ in their ways of dealing with (1) the relationships between individual and social understandings of teacher development and the role of the teacher for classroom practice, and (2) the relationship between theory and practice, especially the expectations with regard to impact on instruction and student learning.

The intention of this issue of ZDM is to promote discussion about the relationships between the theoretical assumptions of research conducted on teachers' knowledge, beliefs, and identity. The authors were asked to write papers that may be empirical and contribute with novel understandings within one of the three fields, but which highlight the underlying theoretical and/or methodological positions and assumptions, especially as they relate to the two dimensions mentioned above, that is, the ones of the individual and/or social emphases and of theory–practice relationships. The intention of the issue, then, is not merely to present yet another collection of papers in one or other of the three fields. Rather it is to initiate a meta-discussion about the relationships among the three lines of inquiry in order to take the general field forward, and investigate the possibilities—or lack thereof—for more coherent approaches to the field.

1 Background

The late 1970s marked a shift of focus in mathematics education research and development. As Bauersfeld (1979) noted, the field was until then primarily interested in investigations of either the mathematical contents per se or in clinical studies of student achievement. These studies dealt with the matter meant or the matter learnt. What was missing, Bauersfeld claimed, was a concern for what he saw as the link between the two, the matter taught, that is, "the influence of the teacher [and] the general context of instruction" (p. 200).

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Since Bauersfeld wrote his piece, teachers and teaching have become pivotal concerns of mathematics education research. The field of beliefs was introduced in the early 1980s to document teachers' meta-mathematical understandings as well as their role in instruction and to support change in teachers' beliefs. Research on teachers' knowledge is older, but changed and gained momentum from the mid-1980s, inspired not least by Shulman's work on a knowledge base for teaching (Shulman 1986, 1987). The research interest in mathematics teacher identity has developed more recently and presents a somewhat broader picture of the teachers' tales of themselves as professionals, that is, one that goes much beyond their knowledge and beliefs about mathematics, although this may be part of it.

As indicated above, there are significant differences between these lines of research, in spite of their common interest in teachers and their role in classroom practice. The differences with regard to the social and/or individual emphases may be phrased in terms of the reliance on participation or acquisition as the dominant metaphor for human functioning; the ones related to theory–practice relationships relate to the expected impact of research on practice.

2 Conceptualizing the individual: acquisitionist and participationist approaches

Most research on teachers' knowledge and beliefs has been influenced by constructivism (von Glasersfeld 1995, 2007), and both knowledge and beliefs are considered mental constructions developed through processes of assimilation and accommodation as a response to new experiential realities. These constructions are expected to determine teachers' perceptions and interpretations of unfolding classroom events as well as their own contributions to these events. Teaching, then, is generally understood as an enactment of pre-existing knowledge and beliefs on the part of the teacher, and the practices of the mathematics classroom are indeed 'the teacher's practices.' This is in line with acquisitionism as a metaphor for learning and human functioning.

Acquisitionism is at odds with what Lerman calls *the social turn* in mathematics education research (Lerman 2000, 2006), that is, with the tendency to challenge primarily individualistic, psychological theorizations to account for learning and lives in mathematics classrooms. Such challenges come, for instance, from social practice theory (Lave 1988; Lave and Wenger 1991; Wenger 1998), socio-cultural theory (Vygotsky 1978, 1986; Wertsch 1998), discourse analysis (e.g. Gee 2005), and distributed cognition (Salomon 1993). Between them they challenge acquisitionism by suggesting that humans do not function by enacting individual mental constructions, but by participating in a variety of social practices, either in a broad cultural sense of the social or in a more local interactionist one. These participatory understandings have guided the relatively recent research interest in teacher identity.

Using Lerman's terminology, then, studies on teachers' knowledge and beliefs have generally not 'turned social,' while research on identity to a greater extent has. This situation invites discussion of the relative advantages of acquisitionist and participationist approaches to research on and with teachers and of ways of coordinating (or not) different theoretical frameworks for the purpose of understanding the role of the teacher for the practices of the mathematics classroom. It is one ambition of the present issue to put this discussion on the agenda in mathematics education research.

3 Considerations of impact: understanding the role of teachers and contributing to their further development

Belief research was initially a response to the experience that instructional practice and student learning did not comply with recommendations for reform. To some degree this was blamed on teachers' views of the mathematical enterprise, including their views of the teaching and learning of the subject. The task of the new field of beliefs was to document the situation and remedy the dismal state of affairs by proposing ways of changing teachers' beliefs. This view was widely shared by the mathematics education research community, as evidenced for instance by the second ICMI study, *School mathematics in the 1990s* (Howson and Wilson 1986; cf. Skott 2009). To a large extent it still orients the field.

The situation is somewhat similar for research on teachers' mathematical knowledge. Earlier studies had failed to document a clear connection between teachers' knowledge of mathematics and their students' learning, much to the surprise of the mathematics education community (Begle 1972; Eisenberg 1977). Reluctant to accept that content matter knowledge does not matter in instruction, the search began to identify the significant elements of teachers' content preparation. Focusing on two of the seven categories of knowledge in Shulman's scheme (1986, 1987), the ones of content knowledge and pedagogical content knowledge, led to a reformulation of the contents per se; for instance, in the form of what was later to be known as mathematical knowledge for teaching (Ball et al. 2008), profound understanding of fundamental mathematics (Ma 1999), and the knowledge quartet (Rowland et al. 2009).

In these lines of research, the teacher is often seen as a major obstacle to change and a major problem of

implementation. Consequently, the task for teacher-related research and development work is to solve the problem by changing teachers' beliefs and providing them with opportunities to develop forms of knowledge that are deemed relevant for the profession.

The interest in improved student learning is shared by the field of identity. In general, however, it adopts a less technical and also a less optimistic approach. The more social approach seems at odds with the idea of implementation as conceived traditionally, and it places greater emphasis on understanding learning and lives as they unfold at schools and in mathematics classrooms than on research on the individual teachers' knowledge and beliefs. This is not to say that there is generally no interest in reform, but that the immediacy of impact of research on classroom practice is questioned.

The relationship between theory and practice is a recurrent theme in mathematics education. This is so also in connection with research and development work on and with teachers. Often, however, the underlying expectations are not made explicit. One intention of this issue is to do so.

4 The papers in this issue

The authors of the papers in this issue draw on different theoretical frameworks and have responded very differently to the challenge of highlighting the role of the frameworks for their research on or with teachers.

Potari (2013) uses a participatory framework, Engeström's activity theory, and does so to analyze changes in the professional identities of prospective and practicing teachers as they participate in a master's course in mathematics education. The course requires the teachers to engage in research-like activities, and Potari's emphasis is on the tensions between the activity systems of teaching and of researching as they relate to the development of the participants' professional identities. Building on an analysis of the activities of a group of five prospective and practicing teachers as they progress through the course, she documents how their initial problems with linking research and classroom teaching give way to the development of a shared object of the two activity systems, namely, understanding students' thinking, and a recognition that research literature may inform classroom practice in important ways.

Goos (2013) uses two studies to demonstrate how sociocultural theories can contribute to both understanding and reforming mathematics teaching practice. Re-interpreting Vygotsky and Valsiner's constructs of *zone of proximal development* (ZPD), *zone of free movement* (ZFM), and *zone of promoted action* (ZPA) from the perspective of teacher-as-learner, Goos argues that those working to support teachers' development can use these constructs to identify productive interventions. We see how misalignment between a teacher's ZPD and ZFM/ZPA complex and the actions taken by the teacher in response to this can explain and predict his behavior. We also see how understanding a teaching team's ZFM allowed the researchers to promote a ZPA that fit with the teachers' ZPD and led to improvement in their teaching. This work speaks to the importance of understanding teachers *and* their contexts in order to determine how their development might be constrained and provides a mechanism for both understanding teachers' person-environment relationships and identifying opportunities for change.

Like Potari, Gellert, Espinoza, and Barbé (2013) also focus on identity. They present a case of one primary teacher engaged in professional development. The case is discussed from the perspectives of knowledge accumulation, belief adjustment, and identity formation. The authors argue that in order to understand the developmental path of the teacher it is necessary to take the institutional and broader social and political discourse into account. They construct and discuss an extended model of teachers' identities in which processes of introjection, projection, and identification are integrated. This integration makes explicit that mathematics teachers' professional development is, more or less directly, influenced by educational policy. The case presented by Gellert et al. exemplifies how a too fast pacing of reform initiatives may counteract the overall reform intentions.

Beginning with a critique of the concepts and methods of the traditions of belief research, Skott (2013) suggests interpreting teachers' contributions to emerging classroom practices in participatory terms rather than as an enactment of reified mental constructs, beliefs. He builds on social practice theory and symbolic interactionism to do so and subsequently extends the participatory approach to teachers' knowledge and identity, trying to develop a coherent approach to understanding the role of the teacher in emerging classroom practices. In the empirical part of the paper he reports on a case study and seeks to show how the teacher re-engages differently in a range of significant prior practices in different episodes in the same classroom, depending on how the interactions unfold. As a consequence, the learning potentials differ significantly between the episodes.

Brown, Heywood, Solomon, and Zagorianakos (2013) make a very different contribution and draw on contemporary philosophy in order to bring conceptions of the object of mathematics and of the subjects of teacher trainees into a new relation. Their main reference is Alain Badiou's take on objectivity and subjectivity, in the context of epistemology, ontology, and pedagogy of science and

mathematics. They argue in favor of practices of mathematics teacher education in which pre-service teachers engage in what they call *the becoming of mathematics*: teacher education practice, rather than being about the sharing of fixed objects and the reproduction of knowledge, can be regarded as an ongoing adjustment of the relationships of mathematical objects and new subjectivities. These relationships may be located differently, as exemplified by bodily movement exercises in the context of planetary movement. Brown et al. emphasize that the new subjectivities—teachers of mathematics—refer to a developing sensitivity towards the contingent character of mathematical objects. The discursive spaces that are needed to subjectively build objects play an essential role for learners of mathematics at all levels.

Simon (2013) analyses the relationship between theory and practice in an attempt to understand the difficulties involved in reforming teaching–learning practices in ordinary classrooms. Interpreting teaching from a cognitive perspective, he explains the relative lack of success of reform initiatives over the last 25 years as due to teachers' *major assimilatory structures*. These are coherent and comprehensive mental constructions that fundamentally shape teachers' perceptions and interpretations of their experiential reality and have the potential to affect their instructional decisions in significant ways. Simon's main example is what he calls a *perception-based perspective* on mathematics and its teaching and learning. To understand teachers' practice, Simon argues, we need to interpret it in terms of the decisive, assimilatory power of such structures. The concept of major assimilatory structures, however, is also important for our thinking about teacher education and professional development, not least as the important task of challenging them may be a harder task than usually expected that requires more long-term development initiatives than the ones that are generally available.

Thames and Van Zoest (2013) also consider theory–practice relationships. They suggest using what they call a *practice-based approach* to studying teacher characteristics, that is, their knowledge, beliefs, and identity. They argue that whichever teacher characteristic one choses as the focus of research, it is important to study how teachers use that characteristic in their work and how the work of teaching is shaped by that use. Thames and Van Zoest use four examples to both elaborate on what they mean by a practice-based approach and to illustrate how such approaches could contribute to coherence between research and practice. They identify important commonalities among practice-based studies and argue that by their nature such studies require work on conceptualizing practice, formulating questions about practice, and developing methods for studying it. This work provides important mechanisms for developing productive coherence across research that draws on different theoretical perspectives. Thus, taking a practice-based approach, no matter what one's theoretical perspective, has the potential to provide valuable insights into ways of improving teaching and learning.

Like Skott, Barwell (2013) adopts a somewhat critical perspective on acquisitionist approaches to research on teachers. He discusses mathematical knowledge for teaching and suggests that it implies an understanding of teachers' knowledge as "categorizable, measureable, and represented in the teacher's mind." Further, he says, the categories of MKT are part of a research discourse that does not do justice to the way knowing is constructed by the participants in classroom interaction. As an alternative he builds on discursive psychology to engage in fine-grained analysis of two classroom episodes, one of which has previously been used by Hill et al. (2008) to investigate the relationship between MKT and the quality of teaching. The point is to understand how what it means to be knowledgeable is construed by the students and teachers in classroom interaction. This is in contrast to understanding teaching as an enactment of previously acquired knowledge.

Schoenfeld (2013) takes a somewhat different look at the dialectic between theory and practice that occurred as his research team endeavored to turn a theory on teacher decision-making into a usable teacher-focused classroom observation scheme. In the interest of illuminating what typically remains hidden, Schoenfeld chronicles the complexities involved in such work. This chronicle lays bare the challenges of making theory practical and the absolute necessity of doing so. The resulting TRU Math scheme is presented with the theoretical claim that its dimensions may be necessary and sufficient to analyze effective mathematics instruction. The development of the scheme is used as a site to reflect on the role of various theoretical constructs and research methods in the enterprise of studying teaching.

In his commentary, Lerman (2013) structures his thoughtful comments on the individual papers in this issue under the headings of 'Socio-cultural theories' (the first five papers), 'Piagetian theory' (Simon's paper), and 'Learning from practice' (the last three papers). In the final section, he emphasizes the need to explicate our theoretical lenses, whether the intention is to develop coherent approaches or to engage in serious discussion of the differences among them. However, whatever theoretical lens is used, Lerman warns that expectations of coherence across research approaches and wide-ranging 'impact' on teaching–learning practices may be overly optimistic, as dynamic social and cultural traditions differ together with the interpretations made of educational research.

Mathematics education research is generally meant to serve the two functions of understanding and contributing to the practices of mathematics teaching and learning. Lerman's remarks suggest an emphasis on the former of these two intentions, or at least they imply that a sober recognition is needed of how contexts, in a variety of different senses of 'context,' may transform any suggestion for the improvement of practice, in any sense of 'improvement.' One of the two intentions with the present issue is to raise the question of the implications of different theoretical lenses for the view of the relationship between theory and practice. The other is to invite continued discussion of the relationships among the theoretical lenses themselves.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: what makes it special? *Journal of Teacher Education, 59*(5), 389–407.

Barwell, R. (2013). Discursive psychology as an alternative perspective on mathematics teacher knowledge. *ZDM - The International Journal on Mathematics Education, 45*(4), this issue. doi: 10.1007/s11858-013-0508-4.

Bauersfeld, H. (1979). Research related to the mathematical learning process. In ICMI (Ed.), *New trends in mathematics teaching. Volume IV* (pp. 199–213). Paris: UNESCO.

Begle, E. G. (1972). *Teacher knowledge and student achievement in algebra. SMSG Reports no. 9*. Stanford: School Mathematics Study Group.

Brown, T., Heywood, D., Solomon, Y., & Zagorianakos, A. (2013). Experiencing the space we share: rethinking objectivity and subjectivity. *ZDM - The International Journal on Mathematics Education, 45*(4), this issue. doi:10.1007/s11858-012-0478-y.

Eisenberg, T. A. (1977). Begle revisited: teacher knowledge and student achievement in algebra. *Journal for Research in Mathematics Education, 8*(3), 216–222.

Gee, J. P. (2005). *An introduction to discourse analysis. Theory and method* (2nd ed.). London: Routledge.

Gellert, U., Espinoza, L., & Barbé, J. (2013). Being a mathematics teacher in times of reform. *ZDM - The International Journal on Mathematics Education, 45*(4), this issue. doi:10.1007/s11858-013-0499-1.

Goos, M. (2013). Sociocultural perspectives in research on and with mathematics teachers: a zone theory approach. *ZDM - The International Journal on Mathematics Education, 45*(4), this issue. doi:10.1007/s11858-012-0477-z.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., et al. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: an exploratory study. *Cognition and Instruction, 26*(4), 430–511.

Howson, A. G., & Wilson, B. (Eds.). (1986). *School mathematics in the 1990s. An ICMI-study*. Cambridge: Cambridge University Press.

Lave, J. (1988). *Cognition in practice*. Cambridge: Cambridge University Press.

Lave, J., & Wenger, E. (1991). *Situated learning. Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Westport: Ablex.

Lerman, S. (2006). Cultural psychology, anthropology and sociology: the developing 'strong' social turn. In J. Maasz & W. Schlöglmann (Eds.), *New mathematics education research and practice* (pp. 171–188). Rotterdam: Sense.

Lerman, S. (2013). Theories in practice: mathematics teaching and mathematics teacher education. *ZDM - The International Journal on Mathematics Education, 45*(4), this issue. doi: 10.1007/s11858-013-0510-x.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah: Lawrence Erlbaum.

Potari, D. (2013). The relationship of theory and practice in mathematics teacher professional development: an activity theory perspective. *ZDM - The International Journal on Mathematics Education, 45*(4), this issue. doi:10.1007/s11858-013-0498-2.

Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2009). *Developing primary mathematics teaching. Reflecting on practice with the knowledge quartet*. Los Angeles: SAGE.

Salomon, G. (Ed.). (1993). *Distributed cognitions. Psychological and educational considerations*. Cambridge: Cambridge University Press.

Schoenfeld, A. (2013). Classroom observations in theory and practice. *ZDM - The International Journal on Mathematics Education, 45*(4), this issue. doi:10.1007/s11858-012-0483-1.

Sfard, A. (2005). What could be more practical than good research? On mutual relations between research and practice of mathematics education. *Educational Studies in Mathematics, 58*(3), 393–413.

Shulman, L. S. (1986). Those who understand: knowledge growth through teaching. *Educational Researcher, 15*(2), 4–14.

Shulman, L. S. (1987). Knowledge and teaching: foundations of a new reform. *Harvard Educational Review, 57*(1), 1–22.

Simon, M. (2013). Promoting fundamental change in mathematics teaching: a theoretical, methodological, and empirical approach to the problem. *ZDM - The International Journal on Mathematics Education, 45*(4), this issue. doi:10.1007/s11858-013-0486-6.

Skott, J. (2009). Theoretical loops in mathematics education: theory-practice relationships revisited. *Asian Journal of Educational Research and Synergy, 1*(2), 74–88.

Skott, J. (2013). Understanding the role of the teacher in emerging classroom practices: searching for patterns of participation. *ZDM - The International Journal on Mathematics Education, 45*(4), this issue. doi:10.1007/s11858-013-0500-z.

Thames, M., & Van Zoest, L. (2013). Building coherence in research on mathematics teacher characteristics by developing practice-based approaches. *ZDM - The International Journal on Mathematics Education, 45*(4), this issue. doi:10.1007/s11858-013-0491-9.

von Glasersfeld, E. (1995). *Radical constructivism. A way of knowing and learning*. London: Falmer Press.

von Glasersfeld, E. (2007). *Key works in radical constructivism. (Edited by Marie Larochelle)*. Rotterdam: Sense.

Vygotsky, L. S. (1978). *Mind in society. The development higher psychological processes*. Cambridge: Harvard University Press.

Vygotsky, L. S. (1986). *Thought and language*. Cambridge: The MIT Press.

Wenger, E. (1998). *Communities of practice. Learning, meaning, and identity*. Cambridge: Cambridge University Press.

Wertsch, J. V. (1998). *Mind as action*. New York: Oxford University Press.