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CORRIGENDUM TO "BRAUER ALGEBRAS OF SIMPLY LACED TYPE"

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Recently, Ivan Marin informed us of an error in the proof of Proposition 5.4 in the paper [2] entitled *Brauer algebras of simply laced type*. He also mentioned this observation in his preprint [3]. Here we give an improved argument. The result is exactly the same as for our incorrect argument and so other results using the proposition remain valid. We are grateful to Ivan Marin for pointing out the error.

Proposition 5.4 states "The irreducible representations obtained in Proposition 5.3 are not equivalent." For the proof, we adopt the notation of [2] and start by recalling the notions involved in the statement of Proposition 5.3. Let Q be a simply laced Dynkin diagram. We write W for the Weyl group of type Q and Φ for the corresponding root system. The Brauer algebra of type Q, denoted $\mathrm{Br}(Q)$, is an algebra over $\mathbb{Q}(\delta)$, where δ is an indeterminate, generated by $2n$ generators which are labelled r_i and e_i for $1 \leq i \leq n$ subject to relations given in [2] Table 1. The subgroup of the multiplicative group of $Br(Q)$ generated by $r_1, \ldots, r_n$ is isomorphic to W. We identify W with this subgroup.

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We fix a subset C of nodes of Q in the following way: a subset B of Φ consisting of mutually perpendicular roots is called admissible if, whenever α is a member of Φ perpendicular to three members of B, there is a fourth member of B perpendicular to α. In [1] a partial order on admissible sets was defined. Each orbit of an admissible set under the Weyl group has a unique maximal element with respect to this partial order. We fix such a maximal admissible set B_0. Then C is the set of nodes of Q whose corresponding roots are perpendicular to every root of B_0.

The subgroup of W generated by all r_i for i in C is denoted $W(C)$. Let U_1 be an irreducible invariant subspace of the regular representation of $W(C)$ over $\mathbb{Q}(\delta)$. According to Propositions 5.2 and 5.3 of [2] there is an irreducible representation V_1 of $Br(Q)$ over $\mathbb{Q}(\delta)$ with basis $\xi_B u$ for B running over the $W(C)$-orbit of B_0 and u over a basis of the $\mathbb{Q}(\delta)[W(C)]$-module U_1.

In order to prove Proposition 5.4 we need to show that two $Br(Q)$-representations V_1 and V_2 constructed in this way from inequivalent $W(C)$-representations U_1 and U_2 (where V_2 is obtained from U_2 in the same way as V_1 is obtained from U_1) are inequivalent. We will use the element e_0 of $Br(Q)$ which is the product over all e_α for $\alpha \in B_0$, where $e_\alpha = we_iw^{-1}$ whenever α is equal to the image under an element w in W of the fundamental root corresponding to the node i of Q. By Corollary 4.6 of [2], for each element B of the W-orbit of B_0, the image $e_0\xi_B$ under e_0 of ξ_B lies in $\xi_{B_0}U_1$, while $e_0\xi_{B_0}$ is nontrivial. This implies that $e_0V_1 = \xi_{B_0}U_1$. Moreover (for instance by Lemma 4.3(iii)), each element of $W(C)$ commutes with e_0, so e_0V_1 is invariant under $W(C)$ and, as a $\mathbb{Q}(\delta)[W(C)]$-module, isomorphic to U_1. In particular, for V_1 and V_2 as above, the $W(C)$-invariant subspaces e_0V_1 and e_0V_2 are inequivalent $\mathbb{Q}(\delta)[W(C)]$-modules, so V_1 and V_2 are inequivalent $Br(Q)$-modules. This proves Proposition 5.4.

References

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[2] A. M. Cohen, B. Frenk and D. B. Wales, *Brauer algebras of simply laced type*, Israel Journal of Mathematics **173** (2009), 335–365.

[3] I. Marin, *Truncations and extensions of the Brauer–Chen algebra*, `https://arxiv.org/abs/1901.06133`.