TECHNICAL ARTICLE



# Application of Kaplan–Meier and Weibull Procedures for Qualitative Assessment of Recycled Aluminum Bronzes

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The article describes the application of the Kaplan-Meier and Weibull procedures for the qualitative assessment of aluminum bronzes obtained from the remelting of metals (e.g., copper, aluminum, iron, manganese). The analysis was carried out on the example of selected bronzes, evaluating the influence of chemical composition on fatigue strength. Melting kinetics and method of casting alloys as well as the influence of admixtures on the castings' properties are important for quality assessment of mentioned alloys. The performed analyses of bronze production show that small changes in the technological process can have a significant impact on the content of individual elements in the alloy, thus determining the properties of the cast element. The calculations carried out allowed for the estimation of the "survivability" function, including the probability of failure using fatigue tests on randomly selected samples. The tests carried out showed that the BA1032 alloy withstands a maximum of 4.89 h (out of 5 h assumed during the strength test) of loads with the parameters assumed in the company. The quality of production was improved by applied methods of statistical assessment of chemical composition allowed to obtain the probability of destruction at the level of 6.13%.

# INTRODUCTION

In assessing the quality of castings made of aluminum bronze, both the melting kinetics and the casting method are of importance. For this purpose, various statistical methods are used to support decisions regarding the selection of both the composition of the mixture and the casting technology. It has been shown that various types of admixtures affect the properties of BA132 bronze. The stages of this bronze casting process have been indicated, starting with making the molds and ending with removing the castings from the molds. The methods of producing castings were also classified, as well as the advantages and disadvantages of individual casting methods, and various criteria were considered. The fourth subsection/subchapter presents the concept of survival analysis.<sup>1-6</sup> Basic concepts are discussed, and data censorship is explained. Kaplan–Meier and Weibull procedures are described as examples of statistical methods related to survival analysis.<sup>2,3,5,6</sup> The study focuses on a production company which specializes in making castings from copper alloys.<sup>4-6</sup> The Kaplan–Meier and Weibull procedures have been applied to analyze the company's data.

Copper alloys bearing aluminum not exceeding 10% are classified as so-called special bronzes. This type of bronze belongs to a very interesting group of alloys,<sup>7-13</sup> from both the technical and economical point of view. They can be divided into two groups:

- simple, ordinary (Cu-Al alloys),
- complex (these contain separately or jointly alloying additives, such as: Fe, Mn, Ni).

<sup>(</sup>Received April 5, 2023; accepted October 9, 2023; published online November 9, 2023)

Aluminum-iron-manganese bronze BA1032 (CuAl<sub>10</sub>. Fe<sub>3</sub>Mn<sub>2</sub>) is an alloy that is very resistant to corrosion, variable impact and abrasive loads. It is used for heavily mechanically loaded parts of machines, engines and accessories for elements exposed to corrosion and abrasion under uniform mechanical stress and in the communication, chemical, shipbuilding and aviation industries. Aluminum-iron-manganese bronze BA1032 alloy's hardness is around 120 HB. The percentage of aluminum in this alloy should be 9–11%, iron 2–4% and manganese 1-2%.<sup>14,15</sup> The chemical composition of CuAl10Fe3Mn2 bronze is presented in Table I.

# KAPLAN-MEIER AND WEIBULL PROCEDURES—SURVIVAL ANALYSIS

The survival analysis is, in general, a collection of statistical methods used to study processes in which the key factor is the time during which a given phenomenon occurs. Most of the survival analysis is based on censored data. Each study is performed during a certain time period. Censoring occurs when an incident on a given specimen occurs outside the study.<sup>16,17</sup> In the most cases, the data are censored on the right-side basis.<sup>16,17</sup>

A significant term in the survival analysis is the survival function S(t). This function indicates the probability that the subject will live longer, exceeding the assigned time t. The survival function is the fundamental element of the survival analysis as it contains the probability of the incident's occurrence for various values of t. In theory, the time scope of the function is:  $t \in < 0$ ;  $\infty >$ . Moreover, every survival function:

- Is continuous
- Is non-increasing
- In time *t* = 0, *S*(0) = 1 (no incident occurred for any subject in the beginning of the study; therefore, the probability of survival is equal to 1)
- In time  $t = \infty$ ,  $S(\infty) = 0$  (if the study were to be

performed in infinity, the probability of survival would be infinitely zero).

Weibull distribution of random variable X of continuous type is the distribution of density determined by formula  $1:^{1,17,18}$ 

$$f(x) = \begin{cases} \frac{\delta}{\Theta} (x - x_0)^{\delta - 1} \exp\left(-\frac{(x - x_0)^{\delta}}{\Theta}\right) & \text{dla } x > x_0 \\ 0 & \text{dla } x < x_0 \end{cases}$$
(1)

where  $\delta > 0$  is the parameter of shape,  $\Theta > 0$  is the parameter of scale, x is the probability of destruction casting, and  $x_0$  is the location parameter, also called the threshold parameter of the distribution.<sup>4,17–19</sup>

The Weibull distribution is asymmetrical. The asymmetric measure of distribution can be determined by the asymmetry coefficient, as provided in formula 2:

$$\gamma_1 = \frac{\Gamma\left(\frac{3}{\delta}+1\right) - 3\Gamma\left(\frac{2}{\delta}+1\right)\Gamma\left(\frac{1}{\delta}+1\right) + 2\Gamma^3\left(\frac{1}{\delta}+1\right)}{\left(\Gamma\left(\frac{2}{\delta}+1\right) - \Gamma^2\left(\frac{1}{\delta}+1\right)\right)^{\frac{3}{2}}} \quad (2)$$

The following properties occur for the respective value of  $\delta$ :<sup>4,17–19</sup>  $\gamma_1$  = determined asymmetry coefficient,

$$\delta = 3,60232 \Rightarrow \gamma_1 = 0,$$
  
 $\delta = 3,43938 \Rightarrow E(X) = Me(X),$   
 $\delta = 3,31125 \Rightarrow E(X) = Mo(X),$   
 $\delta = 3,25889 \Rightarrow Me(X) = Mo(X).$ 

The Weibull distribution of a continuous random variable X was analyzed using the formula:<sup>1,11</sup>

$$f(x) = \begin{cases} \frac{\delta}{\Theta} (x - x_0)^{\delta - 1} \exp\left(-\frac{(x - x_0)^{\delta}}{\Theta}\right) & \text{dla } x > x_0 \\ 0 & \text{dla } x < x_0 \end{cases}$$
(3)

Table 1. Chemical composition of bronze $CuA_{10}$ e <sub>3</sub> Mm <sub>2</sub>	Table I.	Chemical	composition	of bronze	CuAl <sub>10</sub> F	e <sub>3</sub> Mn <sub>2</sub>	7-15
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Element	Minimum amount (%)	Maximum amount (%)		
Copper (Cu)	Rest	Rest		
Aluminum (Al)	9.0	10.5		
Iron (Fe)	2.0	3.8		
Manganese (Mn)	1.2	2.0		
Tin (Sn)	0.2	0.2		
Zinc (Zn)	0.5	0.5		
Lead (Pb)	0.1	0.1		
Nickel (Ni)	0.5	0.5		
Silicon (Si)	0.3	0.3		
Phosphorus (P)	0.05	0.05		
Antimony (Sb)	0.02	0.02		
Arsenic (As)	0.05	0.05		
Bismuth (Bi)	0.01	0.01		
Sulfur (S)	0.05	0.05		

where  $\delta > 0$  is the shape parameter,  $\Theta > 0$  is the scale parameter, and  $x_0$  is the position parameter, also called the threshold parameter of the distribution.<sup>1,11,16</sup>

According to the asymmetry of Weibull's distribution, except for the specific case when  $\delta = 3.43938$ , the median and expected value are not the same.<sup>1,4,17-19</sup>

Quantification of a physical process by implementation of the given distribution of random variables requires verifying the coherence of the experimental data with the distribution. It was performed using one of the statistical tests.<sup>5,17–20</sup>

The Kaplan–Meier method is one of the most frequently applied methods to forecast the survival function in the statistical survival analysis. This method returns the non-parameter Kaplan–Meier estimator.<sup>20</sup> While estimating via this method, it is allowed to censor data regardless of the survival time.<sup>1,17–19</sup> The survival curve is the Kaplan–Meier's estimator graph in the time of *t*. The curve is a nonincreasing step function which decreases in time along with the "death" of the subsequent samples.<sup>1,17–19</sup>

Using the Riemann integral over the probability density function by the formula:

$$f(x) = \begin{cases} \frac{\delta}{\Theta} x^{\delta-1} \exp\left(-\frac{x^{\delta}}{\Theta}\right) & \text{dla } x > 0 \\ 0 & \text{dla } x < 0 \end{cases}$$
(4)

the probability of sample failure in a given time was determined.

# DESCRIPTION OF THE COMPANY AND QUALITY ASSESSMENT METHODS USED

The company specializes in the production of nonferrous metal castings.

Its operations include:

- Production of alloy copper castings ingots
- Centrifugal bushing casting
- Manufacturing individual industrial casting up to 700 kg, formed in sand mass
- Art casting
- Purchase and sales of non-ferrous metals (liquidation - rollers, bushing, rods)
- Analytical metal industry services and machining

The company's primary activity is production of industrial castings using reliable technology. The main production line in the foundry is comprised of the load preparation department as well as a set of induction furnaces and a semi-automatic casting machine. The basic raw material for manufacturing products based on casting copper alloys is scrap metal collected and acquired through recycling programs. The company uses a precise approach for raw material sorting. As a result, some materials are being resold as quality scrap certified while the remaining part, upon proper processing, gets to be used in production. The melting process is performed in the induction furnaces. The obtained alloys are then subjected to correction of their chemical composition and are refined. Next, they are transferred into ingots and preserved for further use by the in-house shape foundry manufacturing industrial casting as well as for the sleeve casting by centrifugal mold casting.<sup>5,21</sup> The load preparing operation is a very important stage of the technological course in the production of mold copper alloys. It is a two-stage process. The first allows for obtaining the secondary raw materials and waste, for sorting and classification, which is performed with the use of portable X-MET, ARC-MET 900 type emission spectrometers and electromagnetic separators.<sup>21</sup> One hundred percent of the used materials is subject to sorting. Non-load scrap, which exceeds the approved load size of maximum 350 mm, undergoes cutting. A significant part of the obtained waste, after assortment sorting, and the polluted secondary raw materials are subject to the recovery of non-ferrous metals. A part of the obtained waste and secondary raw materials, in the form of thick electrical cables, undergoes mechanical machining using the so-called scraper. The copper, aluminum scrap is then utilized in the melting department along with the insulating materials waste.<sup>5,21</sup>

The Kaplan–Meier method was used, including the parameter-less Kaplan–Meier estimator, as one of the most frequently used methods for predicting the survival function in the statistical analysis of survival.<sup>1,4,20</sup> The Kaplan–Meier method was chosen because as the number of samples increases, the estimator z(t) approaches a normal distribution with the mean value equal to the true unknown value of S(t).<sup>1,8</sup> The Weibull distribution of a continuous random variable X was analyzed using formula (3). The results were obtained in the MATLAB R2009b environment using a statistical package.

#### THE PRESENT STUDY

# **Description of the Analyzed Material**

The materials used in this study were acquired from a real production company. All 210 samples were from seven measurements of  $CuAl_{10}Fe_3Mn_2$ aluminum bronze, also called BA1032. Table II shows mean data from the respective sample series copied from the control report cards. Table II also includes their standard deviation values.

Each of the sample series presented in Table II has a slight deviation within the percentage amount of the respective elements because the standard deviation is different than zero. To display it in a clearer visual manner, the data are presented in Figs. 1 and 2.

The respective elements have been presented in separate graphs as their large values decreased the Table II. Percentage composition of elements in various samples of the BA1032 allow

No. of series	Cu	Zn	Sn	Pb	Si	Mn	Fe	Ni	Al.	Р	As
1.	85.270	0.419	0.160	0.073	0.012	1.219	3.225	0.459	9.151	0.012	0.000
2.	84.608	0.497	0.196	0.085	0.181	1.438	3.194	0.424	9.353	0.023	0.001
3.	84.686	0.408	0.191	0.083	0.170	1.343	3.342	0.491	9.262	0.022	0.002
4.	84.353	0.452	0.159	0.059	0.170	1.332	3.122	0.466	9.873	0.012	0.002
5.	84.779	0.494	0.184	0.078	0.190	1.465	3.274	0.436	9.078	0.020	0.002
6.	84.343	0.432	0.141	0.082	0.229	1.449	3.339	0.440	9.526	0.016	0.003
7.	84.490	0.498	0.174	0.095	0.242	1.546	3.115	0.492	9.331	0.016	0.001
*	84.647	0.457	0.172	0.079	0.171	1.399	3.230	0.458	9.368	0.017	0.002
**	0.319	0.039	0.020	0.011	0.075	0.108	0.094	0.027	0.266	0.004	0.001



Fig. 1. Range of variation and dispersion for Zn, Sn, Pb, Si, Ni, P and As in BA1032 alloy samples.

visibility of the remaining ones. Table II displays the desired chemical composition of the  $CuAl_{10}$ - $Fe_3Mn_2$  bronze. It makes it easier to see that copper, aluminum, iron and manganese allow for some deviation. Comparing these data with Figs. 1 and 2 shows that all the elements listed in the samples fit within the assumed range. The amount of aluminum and manganese is closer to the lower limit whereas iron is closer to the upper limit. All the remaining elements display rigid values. Their amount in the alloys has been selected so that it does not exceed the value from Table I. However, there are significant disproportions between the amount of arsenic in the samples and the ideal composition of the BA1032 alloy. The amount of the elements contained in the samples shows slight standard deviation; thus, undoubtedly, it does have a certain impact on the physical properties of the alloys. To develop a better analysis, a coefficient of variation of the amount content of the elements in the alloy has been calculated based on the following formula 5:

$$V = \frac{s}{\overline{x}}, \quad \overline{x} \neq 0, \tag{5}$$



Fig. 2. Median, range of variation and dispersion of Mn, Fe in Al (left) and Cu (right) in BA1032 alloy samples.

where: s = standard deviation,  $\overline{x} = average value$ .

This is a classical measurement of differentiation of trait distribution, and it depends on the value of arithmetic mean, which means that the coefficient of variation is the absolute measure, contrary to the standard deviation, which is determined by the absolute differentiation of the trait. These values are shown in Table III.

As Table IV shows, the coefficient of variation of some of the elements is very high. The highest values are observed in arsenic, silicon and phosphorus. In the case of arsenic, the coefficient of variation equals > 56%, which is caused by its small content in the alloy—average amount of 0.002%—while the spectrometer used on the samples indicates values exactly up to 0.001%. Therefore, such slight variations cause significant changes to the standard deviation and to the coefficient of variety.

# Analysis According to the Kaplan–Meier Procedure

To perform the Kaplan–Meier procedure, the data related to survival are needed.<sup>14,15</sup> The samples have been subjected to the sinusoidally variable load. Seven series of samples have undergone the

Table III. Coefficient of variation of the chemical composition of the BA1032 alloy samples

Element	Coefficient of variation (%) mass		
Cu	0.35		
Zn	7.91		
Sn	10.67		
Pb	13.09		
Si	40.93		
Mn	7.15		
Fe	2.68		
Ni	5.39		
Al	2.63		
Р	24.15		
As	56.38		

Table IV. Time of survival of the respective samples subjected to sinusoidal alternating loads

No. of series	Survival time (h)		
1.	03:45:03		
2.	01:57:36		
3.	04:53:24		
4.	02:48:36		
5.	03:26:24		
6.	04:31:48		
7.	02:37:12		

material fatigue test. This means the stress cycle is described by the following parameters.:

- Minimum stress  $\sigma_{min} = -100$  MPa, Maximum stress  $\sigma_{max} = 100$  MPa,
- Period of variation T = 1 s.

The tests were carried out on a testing machine, obtaining a Wöhler diagram by loading with  $\sigma_m$  and  $\sigma_a$  cycles until the limit number of  $N_G$  cycles was exceeded ( $N_G = 100 \times 10^6$  cycles). For the bending fatigue test, round specimens up to (according to PN-67/H-04326) with a constant cross-section, dimensions: d = 7.5 mm, R = 7.5 mm, were used. Ten samples from each series were tested. The material for fatigue tests was put into metal molds. During one period of changes in T, there was one full cycle of load changes from  $\sigma_{\min}$  through  $\sigma_{max}$  to  $\sigma_{min}$ . The test was performed for 5 h. If the sample was not damaged by the force, the information was considered right censored. The samples for the tests were taken with an uncontrolled random method (via a statistical method). The study results are presented in Table IV.

To draw the survival curve, one has to put the time values in ascending order and transform the time into decimal numbers so that they can be easily

Table V.	Sorted and	transformed	survival	times	for
respectiv	ve samples				

No. of series	Survival time (h)	Transformed time $t_i$ (h)
1.	01:57:36	1.96
2.	02:37:12	2.62
3.	02:48:36	2.81
4.	03:26:24	3.44
5.	03:45:03	3.75
6.	04:31:48	4.53
7.	04:53:24	4.89

marked in the abscissae. These data are presented in Table V.

To draw the survival curve, for each time  $t_i$ , one needs to indicate the number of specimens in the risk group  $n_i$  as well as the probability of survival  $p_i$ and the value of the estimator  $\hat{S}(t)$ . These data are presented in Table VI.

Once the estimator's values in given time intervals have been determined, one can begin to draw the survival curve for the BA1032 alloy. It is presented in Fig. 3.

This curve has been produced based on seven sample series with one sample series being destroyed in each time interval. Hence, the curve has seven steps. The more studies performed, the closer the estimated curve will be to the real one. On this basis, we can estimate the probability that a given item will survive less than in a given time. The studies performed on the seven sample probe indicated that BA1032 alloy may endure a maximum of 4.89 h of stress with the parameters provided in the beginning of the subchapter.

# **Analysis of Weibull Distribution**

The study was performed using the same data as in subchapter 3.1. The results were obtained in the Matlab R2009b (v 7.9) package environment using the statistical R175 package (for data analysis in the R environment to perform reproducible statistical and numerical calculations with the SciPy/ NumPy library). Based on these, Fig. 4 was generated.

Figure 4 shows a line graph well matched with the samples, which means the data originate from Weibull distribution. The parameters of this distribution are presented in Table VII.

Based on the parameters of scale and shape, one can determine the density function and draw its graph, as provided in Fig. 5.

One can determine the probability of destruction of a sample in a given time using the Riemann function on density probability. Used data collected from the existing production company were allowed perform calculations to estimate the "survival" function of the alloy depending on the effect of sinusoidal alternating load.

$\frac{\textbf{Time}}{t_{ij}}$	Specimen in the risk group, <i>n<sub>i</sub></i>	Specimen deceased in time $(t_{i,} d_{i})$	$\frac{\text{Censored in time}}{(t_i), c_i}$	$\frac{\textbf{Survivability}}{1-\frac{d_i}{n_i}}$	$\begin{array}{c} \textbf{Estimator} \\ \hat{S}(t), \\ t_i < t < t_{i+1} \end{array}$
0.00	7	0	0	1.00	1.00
1.96	7	1	0	0.86	0.86
2.62	6	1	0	0.83	0.71
2.81	5	1	0	0.80	0.57
3.44	4	1	0	0.75	0.43
3.75	3	1	0	0.67	0.29
4.53	2	1	0	0.50	0.14
4.89	1	0	0	0.00	0.00





Fig. 3. Estimated survival curve for BA1032 alloy.



Fig. 4. Weibull function grid for two-parameter distribution.

The probability density function for the Weibull distribution was also determined. The tests carried out have shown that BA1032 alloy can withstand a maximum of 4.89 h of loading with the parameters given at the beginning of subsection 3.1. Using the Riemann integral over the density function of the

# Table VII. Values and confidence intervals for estimated parameters

Estimated variable	Parameter value	Confidence interval at 95% level for the parameter		
Parameter of shape $\delta$	3.7942	3.1159	4.6200	
Parameter of scale $\Theta$	3.9729	2.2028	7.1653	



Fig. 5. Graph of probability density function for  $\delta$  = 3.7942, i  $\Theta$  = 3.9729.

probability of sample destruction in < 1.5 h, the probability of failure was determined as 6.13%, which was considered a very good result in the company's production conditions (so far approximately 15%). The previous failure of 15% reported by the company was recorded using the statistical method (MLR), and the successful Kaplan-Meier and Weibull procedures used by the present technique are now implemented in the company.

Currently, further control studies are being collected, thanks to which it will be possible to fully identify the covariates affecting the registration of significant differences (15%-6.4% = 8.6%) between the statistical results and the company's results. Similar discrepancies and methodology are also indicated by Frank Emmert-Streib and Matthias Dehmer<sup>4</sup>; therefore, further analyses will be continued in accordance with their guidelines.

Skillful use of Kaplan-Meier and Weibull procedures allows to study the influence of various factors on the quality of the alloy, which has already been signaled in the work of Serrano-Munoz, J.-Y. Buffiere and C. Verdu<sup>5</sup> and J. Z. Yi, P. D. Lee, T. C. Lindley and T. Fukui.<sup>6</sup> Their regular use is highly recommended because with the passage of time and the availability of production data, including for different possible combinations of the alloy composition, the production process itself may change to optimize the quality requirements of the manufactured castings. Thanks to such tools of the Kaplan-Meier and Weibull procedure, it is possible to flexibly design the process so that the final product meets even the highest customer requirements, just as suggested by J. Szymszal, A. Gierek and J. Kliś in their work.

#### CONCLUSION

The analyses carried out using Weibull's formulas showed that small changes in the casting process can have a significant impact on the concentrations of individual elements in the alloy, thus determining the properties of the cast element. This is particularly important for metal byproduct smelting, because sometimes it is difficult to determine the chemical composition of individual charge elements in charge. Therefore, the key objective should be the selection of the appropriate casting method.

The Kaplan-Meier and Weibull procedures make it possible to compare different alloy compositions or the influence of casting type on casting quality. Plotting survival curves allows visual and analytical comparison of sets of samples that have passed the same test. Fatigue tests performed on samples of the analyzed material provided valuable information on the durability of the alloy, fully confirming the indications from analyses according to Kaplan-Meier and Weibull methods.

# CONFLICT OF INTEREST

On behalf of all authors, the corresponding author states that there is no conflict of interest.

# ACKNOWLEDGEMENT

This research was funded by Polish National Center for Research and Development, grant number POIR.01.02.00-00-0113/19.

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