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An Intensional Formalization of Generic Statements

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Abstract. A statement is generic if it expresses a generalization about the members of a kind, as in, 'Pear trees blossom in May,' or, 'Birds lay egg'. In classical logic, generic statements are formalized as universally quantified conditionals: 'For all x, if..., then....' We want to argue that such a logical interpretation fails to capture the intensional character of generic statements because it cannot express the generic statement as a simple proposition in Aristotle's sense, i.e., a proposition containing only one single predicate. On the contrary, we want to show that typed lambda-abstraction can help us transform the classical, non-simple and extensional expression of generic statements into a new, simple and intensional formalization, through the introduction of an abductively defined operator ALL^*. This new operator allows for the possibility of a single predication, e.g. $fly()$, because it builds, out of a concept like 'bird', a concrete universal, e.g. 'birds', upon which the single predicate can be applied to authentically formalize a generic statement, e.g. 'birds fly'.

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1. The Logical Expression of Generic Statements

A statement is generic if it expresses a generalization about the members of a kind, as in, 'Stones fall to the ground', 'Pear trees blossom in May' or 'Birds fly' [5,6]. Such generalization are fairly common in ordinary language and

For this paper, I am indebted to the guidance of Jean-Pierre Desclés and the helpful comments of Hannes Fraissler.

usually expressed as a relation between concepts or properties; they have the form: 'All N are M,' 'Ns are Ms,' or 'Ns do M.' In what follows, we want to challenge the classical interpretation of such sentences in logic and argue for another kind of formalization. As Tovena [24] remarks about the treatment of generic statements in classical logic, "the literature on the topic classifies generic statements as categorical". While philosophy [23], linguistics [9] and psychology [8,25] make a distinction between generic and categorical statements, classical logic reduces the first to the second by considering that 'Ns are Ms' is equivalent to the universally quantified proposition: 'for all x, x is an N implies x is an M.' But generic sentences are not categorical statements expressing hierarchies of classes like in 'All men are mammals' or 'Squares are rectangles.' Rather, generic sentences are expressing a general property or behavior of members of a kind, as in, 'men desire happiness,' 'Frenchies are arrogant,' 'pear trees blossom in May,' or 'Birds fly to their nests.' For such propositions, the classical universally quantified formalization is inappropriate because the predication doesn't hold for every single extension of the category (e.g. not *all* French people are arrogant). Generic statements are semantically richer than categorical statements so they require a finer and more intensional logical approach.

1.1. Are Generic Statements Simple or Composite Propositions?

Restrained universally quantified statements put a predicate into a certain relation with another predicate. In this sense, they give an accurate formalization of generic statements. Consider the following proposition:

$$(P) \quad \text{`Men desires happiness'}$$

(P) is a proposition in which the predicate 'man' stands in a certain relation with the predicate 'desires happiness.' Symbolic logic can help us achieve a better understanding of this relationship. In classical logic, such a quantified expression is thus formalized:

$$(P_\forall) \quad \forall x\ (man(x) \rightarrow desire(x, Happiness))$$

Where man(_) and desire(_,_) are respectively unary and binary predicates. 'Man(X)' is true if X is indeed a man. 'Desire(X,Y)' is true if X desires Y.

Aristotle [1] tells us that "of propositions one kind is simple, i.e. that which asserts or denies something of something, the other composite, i.e., that which is compounded of simple propositions" (*De Interpretatione*, 5). The expression "Men desire happiness" can rightly be called a proposition because it is indeed a truth-bearer, a *logos apophantikos*: this expression is either true or false. Now, we may ask: is 'Men desire happiness' a simple or a composite proposition? And, if it is a simple proposition: what does it assert of what, i.e., what is the predicate and what is the subject?

At first glance, the proposition (P) 'Men desire happiness'[1] is clearly not a composite proposition; it is a simple proposition. Indeed, it contains a single verb playing the role of the predicate: *desire*. So, to the question "what is asserted of what?" the obvious answer is that *desire* is a binary predicate taking as its arguments *Men* and *happiness*. If we bring verb and object together:[2] it is said of '*Men*' that they '*Desire Happiness*'.

However, we won't recognize a proposition of this form in the classical interpretation:

$$(P_\forall) \quad \forall x\ (man(x) \rightarrow desire(x, Happiness))$$

$(P_\forall)$ is clearly not a simple proposition in Aristotle's sense simply because it contains two predicates: the unary predicate 'being a man' represented by *man()*; and the binary predicate 'desiring' represented by *desire()*. Once the variable x is bound (say to the subject *Socrates*), we can even say that $(P_\forall)$ contains two propositions, 'x is a man' and 'x desires happiness', that are compounded by a conditional. Therefore, what natural language expresses as a simple proposition, classical logic interprets as a composite proposition. Can $(P_\forall)$ genuinely formalize (P) when (P) is a simple proposition and $(P_\forall)$ is not?

One might object that the quantifier unifies two propositional functions $man(x)$ and $desire(x, Happiness)$ into one simple proposition by bounding the free variable x under the quantifier $\forall x$. But the Aristotelian definition of a simple proposition doesn't leave room for any combination of even propositional forms because a simple proposition can only contain one predicate.

Another explanation might come from the fact that in (P) the concepts *man* and *desire* are not on the same level because they don't have the same position in the conditional expression. One is the *categorical concept*, here *man*, because it restrains the category to which the statement applies. The other one is the *predicative concept*, here *desire*, because it is the actual predicate of the sentence: it is what is predicted of the members of the category *man*. But this precision doesn't make the formal expression (P) a valid one with respect to the Aristotelian definition because the categorical concept *man* still appears as a predicate in $(P_\forall)$, i.e. $man(x)$ when it is actually the subject in (P) considered as a simple proposition.

1.2. The Meaning of a Generic Statements and Its Simplicity

From this assessment, a series of questions follows: (1) Does the classical logic representation of this sentence capture its meaning? (2) if not, does it have to do with the lack of *simplicity*[3] of its representation, i.e. the separation of a simple proposition into two propositions linked through a conditional? And

[1] For the sake of clarity and convenience, we shall use (P) as an archetypal example of generic statement. Although we won't always specify it as we expose our arguments, what is said is assumed to be valid for all generic statements. We'll say more on this in the conclusion (See Sect. 4).

[2] See the notion of curryfication in Sect. 1.3.

[3] By 'simplicity' we mean the property of being a simple proposition in the sense of Aristotle's definition in *De Interpretatione.*

(3) if it is indeed the cause, can we restore its simplicity, i.e express it as a simple proposition?

1.2.1. Do (P) and ($P_\forall$) have the Same Meaning? Does it mean the same thing to claim (P) 'Men desire happiness' and to claim ($P_\forall$) 'For all x, if x is a man, then x desires happiness'? No, it doesn't. Actually, there is a double gap between the two propositions: in one sense, (P) says more than ($P_\forall$) and, in another sense, (P) says more than ($P_\forall$).

First, ($P_\forall$) says more than (P). Indeed, necessary conditions can be met for (P) to be true while some necessary conditions for ($P_\forall$) to be true are still missing. In general, it is commonly accepted that ($P_\forall$) says more than (P): "Generic sentences are often judged true despite weak statistical evidence" [8]. To show this in the case of (P), it is enough to find one individual who—maybe by accident—doesn't desire happiness.[4] Such a situation necessarily falsifies ($P_\forall$) without necessarily falsifying (P). There is a sense in which it is possible for a specific person not to desire happiness while for 'men desire happiness' to remain true because the first refers to each particular individual while the second is a generic statement. For instance, if this particular case happens by accident, it doesn't immediately make (P) a false statement. It could be true that 'men desire happiness' and still this one man ends up not desiring it at some point. So ($P_\forall$) says more than (P) in the sense that it is a stronger statement: if ($P_\forall$) is true, it is impossible to find such a thing as a man not desiring happiness.

Now, in another sense, (P) says more than ($P_\forall$) precisely *because* it can't include something happening by accident [23]. It is enough that all extensions of *man* to desire happiness for ($P_\forall$) to be true but it is not enough for (P) to be true. It is not enough for the very reason that ($P_\forall$) could still happen to be the case by accident. (P) 'Men desire happiness' says something about what manhood is that ($P_\forall$) doesn't say. Suppose all men that ever existed in the universe happened to desire happiness: does it necessarily make it true that 'men desire happiness'? It makes it highly likely, of course, but not necessarily true. Because it could be *per* accident that such men ended up all desiring to be happy. On the contrary, (P) doesn't leave room for a subject to accidentally fall under a concept. If 'men desire happiness' is true, it means that it is part of the definition of manhood, i.e. of its intension, that its instances desire happiness. Such an intensional meaning cannot be captured by an extensional logic like classical logic. Can any symbolic formalism capture it?

1.2.2. What is the Role of Proposition Simplicity in the Discrepancy? To answer this question, we need first to move to question (2): if there is a gap—actually, as we just saw, a double gap—between (P) and ($P_\forall$), then is this essentially caused by the fact that (P) is a simple proposition and ($P_\forall$) is not? A proposition asserts something of something. The first something is the

[4]This point is even easier to make with more ordinary and perfectly valid examples of generic statements like 'birds fly' or 'pear trees blossom in May' where it is enough to point at a penguin or a dead tree to falsify ($P_\forall$).

predicate, the second is the subject. What is the subject in (P) and what is the subject in $(P_\forall)$? As we just said, (P) says something about '*Men*'. On the other hand, $(P_\forall)$ says something about '*any x*', namely that '*its being a man implies its desiring happiness*', whatever 'x' may be (possibly a man but not necessarily). So $(P_\forall)$ really says something about everything (or anything). And it is precisely where lies the gap: $(P_\forall)$ is stronger than (P) in as much as it is true *for any x* that happens to be man and it is weaker than (P) in as much as it doesn't take '*Men*' as the subject. In both cases, the problem springs from a mistake on the subject. And this is inherent to the extensional nature of classical logic: it cannot take an intensional concept like '*Men*' as a subject, it can only take fully determined extensional objects like 'x'. Indeed, as Frege says in the *Grundgesetze* [20], "The domain of what is admitted as argument must also be extended to objects in general. *Objects* stands opposed to functions. Accordingly I count as *objects* everything that is not a function, for example, numbers, truth-values, and the courses-of-values." So '*Men*' cannot be admitted as an argument of the predicate '*desire*' insofar as it is itself a concept but only through *any object x that happens to be a man*. Indeed, Frege [20] calls "a *concept* a function whose value is always a truth-value" and accordingly doesn't permit it to be the argument of a function or another concept. Only objects can. And Frege is right: how can we make sense of *being a man* desiring happiness? Only an object, *a man*, *this man*, *any man*, *every man*, *this x*, *any x*, and so forth, can desire happiness. So it seems that $(P_\forall)$ is indeed the right logical way to express (P). But then we reached a dead end because this would inevitably leave us with the double gap we just pointed out. Is there any chance that (3) we can formally restore the simplicity of (P) by expressing it differently? We just said that the possibility for a concept to be the subject of another concept was a critical step on the way to formally express (P) as a simple proposition. We are going to show that a simple analysis of $(P_\forall)$ in the framework of λ-calculus opens new perspectives on the possibility for a concept to be the subject of another concept and, therefore, on the possibility of formalizing (P) as a simple proposition.

1.3. Expression of (P) in the Framework of λ-Calculus

We give the following definition:[5]

$$(def_1) \quad ALL \equiv_{def} \lambda P_1.\lambda P_2.(\forall x(P_1(x) \rightarrow P_2(x)))$$

If P_1 and P_2 are unary predicates, we can thus formalize any expression of the form 'Every P_1 is P_2' by applying the operator ALL to P_1 and P_2 in this order:

$$ALL(P_1)(P_2)$$

For instance, the following expression

$$ALL(man)(mortal)$$

[5] In what follows, we assume some background in the theory of λ-calculus developed by Church (see [7]).

leads, by definition of ALL, to the following β-reduction:[6]

$$(\lambda P_1.\lambda P_2.(\forall x(P_1(x) \rightarrow P_2(x))))(man)(mortal) \xrightarrow[\beta]{*} \forall x(man(x) \rightarrow mortal(x))$$

Which yields the classical logical expression of 'Every man are mortal'. Now, in our example 'Every man desires happiness', one of the two predicates is a binary predicate (or relation). But we can still use the definition of ALL if we transform P_2 into a unary predicate. And we can do that insofar as we know the second argument, we know what is desired: happiness. We thus define the corresponding unary predicate *desireH*:

$$desireH = \lambda x.desire(x, Happiness)$$

The unary predicate *desireH* when applied to an object x means that 'x desires happiness'.[7] Now we can use the definition of ALL to express (P_λ):

$$(P_\lambda) \quad ALL\ (man)\ (desireH)$$

Lambda-calculus uses left-associativity as a convention, so this is equivalent to the following expression:

$$(P_\lambda) \quad ((ALL\ (man))\ desireH)$$

which is β-reducible to $(P_\forall)$:

$$(\beta_1) \quad ((ALL\ (man))\ desireH) \xrightarrow[\beta]{*} \forall x(man(x) \rightarrow desire(x, Happiness))$$

$$(\beta_1) \quad (P_\lambda) \xrightarrow[\beta]{*} (P_\forall)$$

In a λ-expression of the form $X\ Y$, X is the operator and Y the operand and $X\ Y$ is the operation of application of X upon Y. When applied to the domain of functions, X would be the function and Y the argument. When applied to the domain of predicate logic, X would be the predicate and Y the subject.[8] Therefore, in (P_λ), $(ALL\ man)$ can be considered as a predicate meaning 'what can be said of all men'. Indeed, according to (P) it can be said that they desire happiness: $(P_\lambda)(ALL\ man)\ desireH$. Through λ-abstraction, we've been able to:

1. get rid of all the free variables $(\forall x..x)$
2. get rid of all the objects in general ('x', 'Happiness')
3. express some complex concepts (operators) as applications (or combinations) of some more elementary concepts (operators)
4. manipulate concepts as argument of other concepts $((ALL^*\ man)$ is applied to $desireH)$

[6] To be more precise, this is a sequence of β-reductions (expressed by the '*'). We shall sometimes talk about a β-reduction when it is actually a sequence of β-reductions.

[7] This transformation of a binary predicate into a unary predicate is called *curryfication.*

[8] Here, we are greatly indebted to the work of Jean-Pierre Desclés [12,14] and Anca Pascu [16,17]. See also [4].

5. express all the concepts as unary predicates (in the abstract λ-expression $(ALL\ man)$, ALL has one single argument, here man[9]; and, in the abstract λ-expression $(ALL\ man)desireH$, $(ALL\ man)$ also has one single argument, here $desireH$)

The third point is introducing us to the specific approach of λ-calculus and combinatory logic: we're able to express some complex operators as applications of some more elementary operators. In (P_λ), the quantifying operator ALL is applied to the categorical concept man to build a complex operator meaning 'what is true of all men'. When applied to a concept (the predicative concept), here $desireH$, it states that every man is subsumed under this concept. The fourth point is particularly crucial because, as we mentioned before, the impossibility for a concept to be the argument of a concept is at the root of the double gap between (P) and $(P_\forall)$. Here it becomes possible through λ-abstraction.

1.4. Toward a Simple Proposition

To formalize the generic statement

$$(P) \quad \text{'Men desires happiness'},$$

we went from the framework of classical logic,

$$(P_\forall) \quad \forall x\ (man(x) \rightarrow desire(x, Happiness))$$

to the framework of λ-calculus,

$$(P_\lambda) \quad ALL\ (man)\ (desireH),$$

In this framework, it is now possible for the concept $desireH$ to be the argument of the other concept $(ALL\ man)$: $(ALL\ (man))$, as a λ-abstraction, is applied to $desireH$. So we can say that we have the form of a simple proposition in Aristotle's sense.

However we do not yet have the right order between subject and predicate. As we mentioned earlier, (P) says of '*Men*' that they '*desire happiness*'. So *Men* is the subject and *desireH* is the predicate. But this is not the case in (P_λ) where, formally, the concept $(ALL\ man)$ is applied as a predicate to the subject $desireH$. In (P_λ), the predicate is $(ALL\ man)$, 'what can be said of men', and the subject is $desireH$, 'desiring happiness'. Compared to (P), the roles are inverted: the predicate in (P) is 'desire happiness' and the subject is 'Men'. Moreover, (P_λ) is not really a simple proposition because the subject 'desiring happiness' and the predicate 'what can be said of men' are not really two 'somethings' in Aristotle's sense. So we do not yet have the simple proposition we are looking for. But we're very close: we just need to invert the order of application. If 'what can be said of men' can truly be applied to 'desiring happiness', then 'what desires happiness' can truly be applied to 'men'. The inverse application is a simple proposition. Assuming a new operator (ALL^*)

[9] Even though $(ALL\ man)$ is still an unsaturated concept in Frege's sense.

that we yet have to define, we can say that we are looking for the following formal expression, in the right order this time:

$$(P_\lambda{*}) \quad \mathit{desireH}\ (\mathit{ALL}^*\ \mathit{man})$$

This is the simple proposition we are looking for because (1) it contains one single predicate and one single subject and (2) they correspond to the actual predicate and subject in our generic proposition (P). $(P_\lambda{*})$ can be read: *desireH* is predicated of $(\mathit{ALL}^*\ \mathit{man})$.

At this point, we do not have yet defined $(\mathit{ALL}^*\ \mathit{man})$. In what follows, we are going to try to express the inversion process inversion process from (P_λ) to $(P_\lambda{*})$ in the framework of Combinatory Logic.[10]

Remember that we're trying to relate the well defined expression

$$(P_\lambda) \quad ((\mathit{ALL}\ \mathit{man})\ \mathit{desireH})$$

to the incompletely defined expression

$$(P_\lambda{*}) \quad \mathit{desireH}\ (\mathit{ALL}^*\ \mathit{man})$$

To do this, we'll show that $(P_\lambda{*})$ is β-expandable to (P_λ) by introducing some combinators [10]:

1	$\mathit{desireH}\ (\mathit{ALL}^*\ \mathit{man})$	$(\mathrm{P}_\lambda\ {*})$
2	$\mathbf{I}\ \mathit{desireH}\ (\mathit{ALL}^*\ \mathit{man})$	**I**-i
3	$\mathbf{C}\ \mathbf{I}\ (\mathit{ALL}^*\ \mathit{man})\ \mathit{desireH}$	**C**-i
4	$\mathbf{B}\ (\mathbf{C}\ \mathbf{I})\ \mathit{ALL}^*\ \mathit{man}\ \mathit{desireH}$	**B**-i
5	$[\mathit{ALL} \equiv_{def} \mathbf{B}\ (\mathbf{C}\ \mathbf{I})\ \mathit{ALL}^*]$	def_2
6	$\mathit{ALL}\ \mathit{man}\ \mathit{desireH}$	ALL-subst.
7	$((\mathit{ALL}\ \mathit{man})\ \mathit{desireH})$	(P_λ)

Thus, inversely, we have the following β-reduction:

$$(\beta_2) \quad ((\mathit{ALL}\ \mathit{man})\ \mathit{desireH}) \xrightarrow[\beta]{*} \mathit{desireH}\ (\mathit{ALL}^*\ \mathit{man})$$

Before we go further into the analysis, several things are worth noticing in this *inversion process*:

1. Our initial movement takes us from $(P_\forall)$ toward (P). We went indeed halfway in this direction by going from $(P_\forall)$ to (P_λ) but, while the other half would have been expected to go from (P_λ) to $(P_\lambda{*})$ which is assumed equivalent to (P), the β-expansion above goes in the opposite direction: from $(P_\lambda{*})$ to (P_λ).
2. So far, two definitions have been given, def_1 and def_2. In both of them, *ALL* is the *definiendum* but the *definiens* is different. However these *definiens* do not conflict with each other as def_2 is not a complete definition (see point 4)

[10] Here we assume some background in Curry's Combinatory Logic. For useful references, see [4,10,11,15,19].

3. ALL^* is not properly defined. In def_2, ALL^* belongs to the *definiens*, that is to say what defines and not what is defined. More precisely, it belongs to the proximate genus part of the *definiens* I, that is to say the nearest general class to which a thing belongs: ALL is a specific kind of ALL^*.
4. From the preceding point, we can deduce that ALL itself is not completely defined in def_2. However it is completely defined in def_1.

In other words, ALL^* is not being apprehended inductively nor deductively but rather through abduction in Pierce's sense [22]. We formulated a hypothesis for the proper logical form of a generic statement ($P_\lambda{}^*$) which involves a new unknown operator ALL^* and, from that hypothesis, we were able to deduce the well known proposition (P_λ).

In the β-expansion, we introduced the operator ALL^* through the following definition:

$$(def_2) \quad ALL \equiv_{def} \mathbf{B}\ (\mathbf{C}\ \mathbf{I})\ ALL^*$$

From this definition, we have, for any two concepts f and g, the following β-reduction:

$$ALL\ g\ f \underset{\beta}{\overset{*}{\longrightarrow}} f\ (ALL^*\ g)$$

For example:

$$ALL^*\ (man)\ (mortal) \underset{\beta}{\overset{*}{\longrightarrow}} mortal\ (ALL^*\ man)$$

and, in our present case:

$$(\beta_2) \quad ALL\ (man)\ (desireH) \underset{\beta}{\overset{*}{\longrightarrow}} desireH\ (ALL^*\ man)$$

$$(\beta_2) \quad (P_\lambda) \underset{\beta}{\overset{*}{\longrightarrow}} (P_\lambda{}^*)$$

So (P_λ) is β-reducible (β_2) to ($P_\lambda{}^*$) and, inversely, ($P_\lambda{}^*$) is β-expandable to (P_λ). Let us also remind that (P_λ) is β-reducible to ($P_\forall$):

$$(\beta_1) \quad ((ALL\ (man))\ desireH) \underset{\beta}{\overset{*}{\longrightarrow}} \forall x(man(x) \rightarrow desire(x, Happiness))$$

$$(\beta_1) \quad (P_\lambda) \underset{\beta}{\overset{*}{\longrightarrow}} (P_\forall)$$

We thus have the following path of β-conversion from ($P_\forall$) to ($P_\lambda{}^*$):

$$(P_\forall) \underset{\beta}{\overset{*}{\longleftarrow}} (P_\lambda) \underset{\beta}{\overset{*}{\longrightarrow}} (P_\lambda{}^*)$$

in which we can see the peculiar half-way forward/half-way backward approach mentioned in point 1 above.

Now that we seem to have formally expressed, in ($P_\lambda{}^*$), the simple proposition we were looking for, we need to investigate regarding the definition and meaning of ALL^* on which this new generic statement formalization ($P_\lambda{}^*$) entirely depends.

2. What is *ALL**?

As we shall see now, we don't really have, properly speaking, a definition of ALL^*. We thus need to investigate on the identity and the meaning of this new operator. As we mentioned above, ALL^* is defined abductively. This means that we can't formally define this concept through the traditional means of induction or deduction. But we can say quite a bit about it and we can approach it, abductively, through different angles, by under-approximation and over-approximation, to discover what it is and what it is not, what it can be and what it can't be. The different angles of approach will be the following: the absence of definiendum, the combinators, the β-reduction and type theory.

2.1. A Concept Introduced Without a Definition

As it was pointed out earlier, ALL^* has been introduced as the definiens in the definition of ALL. What is here very particular is that a new term is usually introduced by being the definiendum of a definition and not the definiens. If we want to introduce a new term, clarity requires that we say what it is, not just it has as a particular species. We could call it an *abductive definition* [22] because it doesn't give us a full deductive grasp of what ALL^* is but only an abductive grasp of it. The abductive definition is somewhat counter-intuitive: while we usually start from a primitive operator to define more complex operators, here we proceeded in the opposite direction from the definition of a complex operator, ALL, to somewhat *guess* the meaning of a more primitive (or more general) operator ALL^*. Basically, we're given an instance or a species of ALL^* to understand what it is. It's like defining the concept of cat to apprehend the concept of mammal.

Now it is important to point out that the abductive approach of ALL^* fits some of the special features of generic statements like

$$(P) \quad \textit{Men desire happiness}$$

In some sense, the general aspect of such sentence, which is formalized through the ALL^*, is grasped abductively. Indeed, this generic statement itself is not inferred through induction nor deduction. Rather, we observe some evidence about *me*, *you* or *her* before considering its generic version (P) as a valid hypothesis.[11] The generic statement is not as clear and obvious as one of its particular instanciations (eminently, *my desiring happiness as a man* is the most obvious of all) but it is plausible [22]. So, just like the definition and the generality of ALL^* is to be apprehended *behind* the particularity of ALL, the genericity of (P) 'men desire happiness' is to be guessed *behind* the particularity of my desiring happiness.[12] One might say that this is specific

[11] On this particular generic statement, an example of abductive reasoning can be found in [3]. But the point is assumed to be valid for generic statements in general. For generic statements as abduced explanatory hypothesis [22] instead of induced or deduced general rules, see [23].

[12] We're not trying to show that ALL^* is exactly apprehended like a generic statement is apprehended. That would be a categorical confusion. Instead, we're trying to show that they share an analogous, abductive, property with respect to truth and inference. Simply, this

of a sentence like ($P_\forall$) that involves a very subjective (in the contemporary meaning of the word) concept like 'desire'. But it's not true. Other generic statements like 'Frenchies are arrogant' and 'Birds fly' also come to provide explanatory power [22] for observed patterns and evidence.

Over all, we can say that ALL^* has the specificity to be introduced abductively, from the operator ALL, behind which emerges a larger and more fundamental concept.

2.2. A Definition from the Combinators

What is happening in the β-reduction (β_2)? We can say that the definition def_2 of ALL allows for a displacement of the quantifying operator. This displacement is induced by the combinaison of elementary combinators **B (C I)**. In ($P_\lambda{}^*$), the span of ALL^* is limited to the categorical concept *man* and its application builds an entity that is itself in the span of the predicative operator $desireH$. What happens, through β-expansion (β_2), is that ALL^* is somehow extracted from this limited span to be placed at the forefront position, where its span covers, firstly, the concept *man* and, secondly, the concept $desireH$, thus getting us closer—to be precise two lambda-abstractions away—to the classical expression ($P_\forall$). Said differently, the complex combinator defining ALL^*, **B (C I)**, transforms the *natural*[13] span of ALL^* in ($P_\lambda{}^*$) (as a formalization of (P)) into the *classical* or *Fregean* span of ALL in ($P_\forall$).

2.3. A Definition from the β-Conversion

The double gap that we pointed out earlier between (P) and ($P_\forall$) is actually analogous to the double gap we can find between a $\beta - redex$ and its $\beta - contractum$. Curry [10] gives the following definition:

> "We call an ob which can form the left side of an instance of one of the rules (β), (η), or (δ) a *redex* of the corrresponding type ; the right side of the same instance will then be called de *contractum* of the redex. A replacement of a redex by its contractum will then be called a *contraction* (...). Thus (...) a $\beta - redex$ is an ob of the form $(\lambda x.M)N$, its contractum is $[N/x]M$; and a replacement of an instance of $(\lambda x.M)N$ by $[N/x]M$ is a β-contraction." (p. 93)

It is clear, then, that a β-redex is not strictly equivalent to its β-contractum. We want to argue that, just like it is between ($P_\forall$) and (P), the β-redex is extensionally weaker than the β-contractum but, in same time, the former is intensionally stronger than the latter.[14] Let us look at a simple example in λ-calculus:

$$(\lambda x.x^2)3 \xrightarrow{\beta} 3^2 = 9 \tag{2.1}$$

Footnote 12 continued
shared property is further evidence that the intension behind ALL^* matches that of generic statements.

[13] In the sense of *natural language* or *natural reasoning*.

[14] For references on the inverse relation between intension and extension of concepts, see [2,14,16,21].

The β-redex '$(\lambda x.x^2)3$' exhibits an operation that partially disappear in the β-contractum '3^2' and completely disappeared in the expression '9'. The former contains more *meaning* or *intension* than the latter, because it doesn't only say *what* we're calculating but also *how* we're calculating it. A good way to see it more clearly is to compare it to another β-reduction leading to the same result or extension but coming from a different intension. In this example, we arrive at 9 through a different operation:

$$(\lambda x.x * x)3 \xrightarrow{\beta} 3 * 3 = 9 \tag{2.2}$$

In this example, we even arrive at the same 3^2 through a different operation:

$$(\lambda x.3^x)2 \xrightarrow{\beta} 3^2 = 9 \tag{2.3}$$

While the extension, the number 9, can be the same, the intension, $(\lambda x.x * x)3$, $(\lambda x.3^x)2$ or $(\lambda x.x^2)3$ can be different, thus showing that the β-redex is indeed intensionally stronger than the β-contractum. On the other hand, it is easy to show that the β-redex is extensionally weaker than the β-contractum because it is one step further away from the β-normal form which is the form of purely extensional expressions. An expression is in β-normal if it cannot be β-reduced. It is the case for '9'. 9 is both the extension of $(\lambda x.x^2)3$ and 3^2 but while it takes two β-reduction steps to get to 9 from the former, it only takes one step from the latter. The former being the β-redex of the latter, we can say it is further away from their extension. Therefore, the β-redex is extensionally weaker than its β-contractum.

We thus see that we can recognize in the relation of β-reduction the double gap that emerged in the relation between (P) and $(P_\forall)$. In the meantime, we have suggested $(P_\lambda{}^*)$ as the best-fitting logical formalization of (P). If this is right, we should also be able to identify a similar double gap between $(P_\lambda{}^*)$ and $(P_\forall)$.[15] According to what we just demonstrated, this would simply mean showing that $(P_\lambda{}^*)$ is β-reducible to $(P_\forall)$. But so far, the only β-relation that exists between $(P_\lambda{}^*)$ and $(P_\forall)$ is not standard because it goes through a β-expansion (see Fig. 1) followed by a β-reduction:

[15]Or, at least, being able to do so would be one more evidence in favor of our hypothesis, in the sense of the abduction.

1	$desireH\ (ALL^*\ man)$	(P_λ *)
2	$\mathbf{I}\ desireH\ (ALL^*\ man)$	**I**-i [β-exp.]
3	$\mathbf{C}\ \mathbf{I}\ (ALL^*\ man)\ desireH$	**C**-i [β-exp.]
4	$\mathbf{B}\ (\mathbf{C}\ \mathbf{I})\ ALL^*\ man\ desireH$	**B**-i [β-exp.]
5	$[ALL \equiv_{def2} \mathbf{B}\ (\mathbf{C}\ \mathbf{I})\ ALL^*]$	def$_2$
6	$ALL\ man\ desireH$	ALL-subst.
7	$((ALL\ man)\ desireH)$	(P_λ)
8	$[ALL \equiv \lambda P_1.\lambda P_2.(\forall x(P_1(x) \rightarrow P_2(x)))]$	def$_1$
9	$(((\lambda P_1.\lambda P_2.(\forall x\ (P_1(x) \rightarrow P_2(x))))\ man)\ desireH)$	ALL-subst.
10	$((\lambda P_2.(\forall x\ (man(x) \rightarrow P_2(x))))\ desireH)$	λ-elim. [β-red.]
11	$(\forall x(man(x) \rightarrow desireH(x)))$	λ-elim. [β-red.]
12	$\forall x(man(x) \rightarrow desireH(x))$	($P_\forall$)

However, we know that the completely extensional expression of $(P_\lambda{}^*)$ is $(P_\forall)$. So $(P_\forall)$ is the β-normal form of $(P_\lambda{}^*)$. Indeed, it is clear that there is no more β-redex in $(P_\forall)$. On the contrary, we can't claim that $(P_\lambda{}^*)$ is a normal form. First, to be the case, ALL^* would need to be completely and extensionally defined. But we don't really *know* the operator ALL^*: it is a new entity without any particular equivalent in classical logic. Second, the Church-Rosser theorem and its corollaries [?] state that an expression can only be β-reducible to one β-normal form at most. If $(P_\forall)$ is the β-normal form associated with (P_λ), $(P_\lambda{}^*)$ can't also be the β-normal form associated with (P_λ). We've established that $(P_\forall)$ is a normal form. Therefore $(P_\lambda{}^*)$ is not a β-normal form. Now, if we make abstraction of the fact that ALL has two different definitions,[16] the Church-Rosser theorem and its corollaries also imply that $(P_\lambda{}^*)$ has to be β-reducible to $(P_\forall)$. Otherwise, if $(P_\lambda{}^*)$ was β-reducible to another normal form, say (P_2), then (P_λ) being β-reducible to $(P_\lambda{}^*)$, it would also be β-reducible to the normal form (P_2), thus having two normal forms which contradicts the theorem. Thus, we can give (see Fig. 2) a more complete representation of the relations of β-reduction between (P_λ), $(P_\forall)$ and $(P_\lambda{}^*)$.

Hence, $(P_\lambda{}^*)$ is indeed β-reducible to $(P_\forall)$ and, for that reason, in a double gap relationship (both intensively stronger and extensively weaker) with $(P_\forall)$ that is analogous to that which (P) holds with $(P_\forall)$.[17] So the operator ALL^*, although not completely defined, gets us closer to the generic intension in (P).

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[16]Considering the two different definitions, one could object that $(P_\lambda^{def_1})$ differs from $(P_\lambda^{def_2})$ with respect to β-reduction because they use two different definitions of ALL. However, such differentiation isn't relevant for the present argument.

[17]As we mentioned in the previous note, the differentiation between $(P_\lambda^{def_1})$ and $(P_\lambda^{def_2})$ isn't relevant here as the double-gap relationship holds simply because $(P_\lambda{}^*)$ is both extensionally equivalent to $(P_\forall)$ and at a higher level of λ-abstraction than $(P_\forall)$.

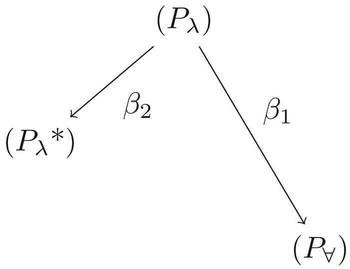

FIGURE 1. A representation of the relations of β-reduction between (P_λ), $(P_\forall)$ and $(P_\lambda{*})$

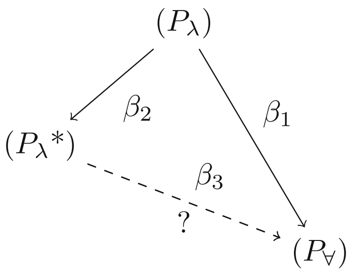

FIGURE 2. A representation of the relations of β-reduction between (P_λ), $(P_\forall)$ and $(P_\lambda{*})$ including the inference from the Church-Rosser theorem

2.4. A Definition from the Functional Types

The definition of ALL^* can also be given through type theory. If we consider Curry's two primitive types J and H for, respectively, individuals and propositions, then concepts like *man* have the type FJH because they are, like Frege defines them, functions taking an individual (denoting an object) as an argument and building up a proposition (denoting a truth-value) out of it. We write

$$X : \alpha$$

to signify that the combinatorial expression X has the type α.

We will now use Curry's [10] "stratification technique" (p. 282) to find out what the functional type of ALL^* is. To make it more concrete, we'll stick to our example but what follows holds for the general case. We have the following application tree[18]:

[18] As usual with Combinatory Logic, we assume left associativity.

$$\dfrac{\dfrac{1.\ ALL \quad 2.\ men}{3.\ ALL\ men} \qquad 4.\ desireH}{5.\ ALL\ men\ desireH}$$

We call η_i the type of the expression (i) in the deduction. $\eta_1 \equiv \mathbf{F}\eta_2\eta_3$ $\eta_3 \equiv \mathbf{F}\eta_4\eta_5$ $\eta_5 \equiv H$ because:

$$(5) \equiv (P_\lambda)$$

and

$$(P_\lambda) \xrightarrow[\beta]{*} (P_\forall)$$

and $(P_\forall)$ is a proposition. Therefore (5) is a proposition.

As we said before, we also know that: $\eta_2 \equiv \eta_4 \equiv \mathbf{F}JH$
Therefore we have:

$\eta_3 \equiv \mathbf{F}(\mathbf{F}JH)H$
$\eta_1 \equiv \mathbf{F}(\mathbf{F}JH)(\mathbf{F}(\mathbf{F}JH)H)$

Now, we will use def_2 to find out the type of $ALL*$:

$$\dfrac{\dfrac{6.\ \mathbf{B} \quad 7.\ \mathbf{C}\ \ \mathbf{I}}{8.\ \mathbf{B}\ \ (\mathbf{C}\ \ \mathbf{I})} \qquad 9.\ ALL*}{10.\ \mathbf{B}\ \ (\mathbf{C}\ \ \mathbf{I})\ ALL*}$$

By def_2, we know that $\eta_{10} \equiv \eta_1$
Curry already gives us the following types (p. 308):
$\eta_6 \equiv \mathbf{F}_2(\mathbf{F}\beta\gamma)(\mathbf{F}\alpha\beta)(\mathbf{F}\alpha\gamma)$
$\eta_7 \equiv \mathbf{F}_2\delta(\mathbf{F}\delta\epsilon)\epsilon$
And we know that we necessarily have: $\eta_6 \equiv \mathbf{F}\eta_7\eta_8$
Therefore we deduce:
$\eta_7 \equiv \mathbf{F}\beta\gamma$ and $\eta_8 \equiv \mathbf{F}(\mathbf{F}\alpha\beta)(\mathbf{F}\alpha\gamma)$ and thus:
$\beta \equiv \delta$ and $\gamma \equiv \mathbf{F}(\mathbf{F}\delta\epsilon)\epsilon$
so that:
$\eta_8 \equiv \mathbf{F}(\mathbf{F}\alpha\delta)(\mathbf{F}\alpha\mathbf{F}(\mathbf{F}\delta\epsilon)\epsilon)$
$\eta_8 \equiv \mathbf{F}(\mathbf{F}\alpha\delta)(\mathbf{F}_2\alpha(\mathbf{F}\delta\epsilon)\epsilon)$
In the same way:
$\eta_8 \equiv \mathbf{F}\eta_9\eta_{10}$
leads to:
$\eta_9 \equiv \mathbf{F}\alpha\delta$ and $\eta_{10} \equiv \mathbf{F}_2\alpha(\mathbf{F}\delta\epsilon)\epsilon$
We also said that
$\eta_{10} \equiv \eta_1$
So we deduce that:
$\alpha \equiv \mathbf{F}JH$
$\delta \equiv J$
$\epsilon \equiv H$

So, the type of ALL^* is:
$\eta_9 \equiv \mathbf{F}\alpha\delta \equiv \mathbf{F}(\mathbf{F}JH)J$

Thus we see that ALL^* is a complex operator which, applied to a concept (of type $\mathbf{F}JH$), transforms it into an object (of type H). Indeed ALL^* *man* is the subject of *desireH* in (P_λ*). From the concept *man*, ALL^* built the subject of ‘men desire happiness’ in (P_λ*), that is to say ‘men’.

3. Illative Logic with *ALL**

Like Curry [10], we have made the choice here to distinguish what he calls “pre-logic” (*Urlogik*), which deals with formal ojects and their definitions independently from the concepts of the propositional domain (truth, predicate, proposition,...), from what he calls “illative logic”, which deals with truth conditions, propositions and inference. Now that we have attempted to define, or rather surround, the concept ALL^* in the *urlogik*, we will briefly consider its role and application in the illative side of things. The illative behavior of the classical formalization of generic statement is very well known:

$$\dfrac{\dfrac{(P_\forall)\ \forall x\ (man(x) \to desireH(x))}{man(Socrates) \to desireH(Socrates)} \quad man(Socrates)}{desireH(Socrates)}$$

And so is well known the logical difficulty that arises when we encounter an exception like a man, say Achilles, who doesn’t desire happiness:

$$\dfrac{\dfrac{\dfrac{(P_\forall)\ \forall x\ (man(x) \to desireH(x))}{man(Achilles) \to desireH(Achilles)} \quad man(Achilles)}{desireH(Achilles)} \quad \neg desireH(Achilles)}{\square}$$

This logical difficulty further demonstrates the inadequacy of the classical account with respect to natural language and natural reasoning. Indeed, generic statements are common place and they do allow for atypical instances or exceptions [8,17,23]. People say things like ‘men desire happiness’, ‘birds fly’ or ‘pear trees blossom in May’ all the time. Nevertheless, it is thinkable that some man doesn’t desire happiness, it is a fact that penguins don’t fly and it may be that such pear tree won’t blossom in May. Nevertheless, this doesn’t make the original generic statement a false statement with respect to ordinary language and natural reasoning.

Contrary to the classical interpretation of generic statements, the ALL^* account leaves room for natural reasoning because it doesn’t require a universally quantified proposition. Indeed, We just saw in the previous section (See Sect. 2.4) that, in (P_λ*), *desireH* is applied to an object (ALL^* *man*). Thus there is no universal quantification. At least, it is clear, given what we know so far, that no contradiction can be inferred from the conjunction of the following propositions:

$$(P_\lambda{}^*)\ desireH\ (ALL^*\ man)$$

and

$$man(Achilles)$$

and

$$\neg desireH(Achilles)$$

The absence of contradiction lies in the fact that $ALL^*\ man$ is an object that is not fully determined. It's an abstract object, here an abstract man, that inherits the intension of the concept 'being-man', including 'desiring-happiness'. Now once this is said, it is possible for a man to not fall under all the non-essential intension of 'being-man'. The abstract object (the abstract man) carries with it a whole series of meaning and intension that doesn't universally applies to all men. So the ALL^* operator leaves room for an intensional and non-universally quantified interpretation of generic statements. A rough formalization of such interpretation[19] is given in the following inference tree where $typMan$ is the predicate 'typical Man' (more on this in Sect. 4.2):

$$\dfrac{\dfrac{(P_\lambda{}^*)\ desireH\ (ALL^*\ man) \quad man(Achilles)}{desireH(Achilles) \rightarrow typMan(Achilles)} \qquad \neg desireH(Achilles)}{\neg typMan(Achilles)}$$

Unlike the classical case, no contradiction may be derived from the intensional formalization.

4. Conclusion

4.1. $(P_\lambda{}^*)$ as the logical expression of (P)

The classical expression of generic statements $(P_\forall)$ fails to capture the intensional character of (P) because it doesn't formally express the *simplicity*[20] of the proposition. Through lambda-abstraction and combination of operators, and via the creation of a new operator ALL^*, we were able to transform $(P_\forall)$ into a new intensional formalization of (P):

$$(P_\lambda{}^*) \quad desireH\ (ALL^*\ man)$$

After a careful analysis of $(P_\lambda{}^*)$ and of its key operator ALL^* we have several reasons to think that $(P_\lambda{}^*)$ is a particularly suited formalization of (P):

[19] For a complete account on this topic, see the Logic of Object Determination [4,13,16,17].
[20] The property of being a simple proposition in Aristotle's sense.

- $(P_\lambda{}^*)$ is intensionally stronger and extensionally weaker than $(P_\forall)$. This 'double gap' also exists between (P) and $(P_\forall)$ and was the starting point of our inquiry because it exhibits the inadequacy of classical logic with respect to generic statements. Given that, as it has been demonstrated above, $(P_\lambda{}^*)$ manifests this same 'double gap' in relations to $(P_\forall)$, we can say that it capture very well the intensional (and non-extensional) character of (P).
- $(P_\lambda{}^*)$ is a simple proposition. We have shown that $(P_\forall)$ is not a simple proposition in Aristotle's sense. On the contrary $(P_\lambda{}^*)$ is a simple proposition because it contains one and only one predicate $desireH()$. Its argument, $ALL^*\ man$, is yet to be investigated but, in the meantime, it appears very clearly that, in $(P_\lambda{}^*)$, only one predication is made, something is said of something: that it desires happiness ($desireH$) is said of men ($ALL^*\ man$).
- $(P_\lambda{}^*)$ respects Frege's constraint on unsaturated concepts. Frege limits the arguments that can saturate a predicative concepts to objects only, i.e. anything but functions (and concepts in particular). This point could appear as a major hindrance to the possibility of ever formalizing (P) into a logical simple proposition because it (P) seems to relate predicatively two concepts. However, it happens that $(P_\lambda{}^*)$ is a simple proposition and does respect Frege's constraint on the saturation of concepts. In $(P_\lambda{}^*)$, the argument of $desireH$ is indeed an object. We just saw that ALL^* is a complex operator which, applied to a concept, transforms it into an object. Therefore $ALL^*\ man$ is an object and not a concept. So the unique predicate $desireH$ of this simple proposition is rightly applied to an object.
- $(P_\lambda{}^*)$ leaves room for exceptions or atypical instances like the use of the generic statements in ordinary language does. Unlike in classical logic, we saw that the proposition $(P_\lambda{}^*)$ could coexist with a man that doesn't desire happiness without *de facto* generating a contradiction.

4.2. What is $ALL * \ man$?

A crucial point in our inquiry was to find a logical expression of (P) that was a simple proposition because that would mean finding a way to express 'what desires happiness', i.e. 'men', as an object. As we saw at the beginning, this is not possible in an extensional and classical setting. Now that we found a way to logically express this object, it is just as much crucial to investigate what it is. If $(P_\lambda{}^*)$ is the right logical expression for (P), then its object $ALL^*\ man$ is the right logical expression for 'men' in 'men desire happiness'. Let us briefly recapitulate what has been deduced from our preceding investigation:

- ALL^* is abductively defined from one of its species: ALL.
- ALL^* is intensionally richer and extensionally weaker than its classical equivalent ALL.
- The span of the operator ALL^* is reduced to the categorical concept (here man) only.
- ALL^* builds an object out of this concept.

The object *ALL* man*, built out of the concept *man*, designates men in general. Not each and every man that exists or existed, in the extensional manner, but *men in general* or *the Man*. Neither is *ALL* man* the unsaturated concept 'being-a-man', because we know it's an object and subject of a predicate. But what object exactly? If it is an object, I should somehow be able to point at it. Referring to what have been said before, *ALL* man* is the object hidden, to be *guessed*, behind the concept *man*. It is *the Man* or *a typical man*. This definition rightly fits (P) as 'men desire happiness' can rightly be re-formulated as 'man desires happiness' or 'a typical man desires happiness'. The plural of natural language in (P) prevents us to see the nature of 'men' as a single object built from the concept *man*. Again, *ALL* man* is an object and not a concept. Which means I represent it mentally. And indeed, this is what I do when I say 'men desire happiness' or 'men have two legs': I represent to myself an object, the typical man, as having such or such property. As a matter of fact, some men don't have two legs and this is perfectly fine because *ALL* man* is not a fully determined extension.

4.3. *ALL* man* is a Concrete Universal

The notion of concrete or self-predicative universal comes from Plato's theory of forms. Ellerman [18] gives the following definition of concrete universals:

> Philosophy has long contemplated another type of universal, variously called a self-predicative, self-participating, or concrete universal. The intuitive idea of a self-participating universal for a property is that it is an object that has the property and has it in such a universal sense that all other objects with the property resemble or participate in that paradigmatic, archetypal, canonical, iconic, ideal, essential, or quintessential exemplar. Such a universal μ_F for a property F is self-predicative in the sense that it has the property itself, i.e., $F(\mu_F)$. It is universal in the intuitive sense that it represents F-ness is such a perfect and exemplary manner that any object resembles or participates in the universal μ_F if and only if it has the property F.

ALL man* is not a fully determine extension but we claim that it is a concrete universal in Plato and Ellerman's sense[21] because:

- *ALL* man* is an object
- *ALL* man* represents a paradigmatic, archetypal, canonical, iconic, ideal, essential, or quintessential instance of man. What is expressed in $(P_\forall)$ by the plural 'men'.
- *ALL* man* is self-predicative in the sense that *man* (*ALL* man*) is analytically true[22]. Indeed, we have shown that the following β-reduction

[21] It is not the point of the present paper to demonstrate that *ALL* man* is a concrete universal builder. We shall present that argument in a subsequent paper.

[22] We could had that it is also self-predicative in the sense that adding a determination 'being-a-man' to the typical object *ALL* man* doesn't modify the object in any way. In other words, *ALL* man* is a fix point of the operation that adds the determination

Table 1. How ALL^* operates on a concept to build a subject of a generic statement

Concept	Concrete universal	Meaning	Generic statement
man	ALL^* *man*	'men', 'a typical man'	'Men desire happiness'
bird	ALL^* *bird*	'birds', 'a typical bird'	'Birds fly'
stone	ALL^* *stone*	'stones', 'a typical stone'	'Stones fall to the ground'
pear Tree	ALL^* *pear Tree*	'pear trees', 'a typical pear tree'	'Pear trees blossom in may'

holds: $man\ (ALL^*\ man) \overset{*}{\underset{\beta}{\rightarrow}} \forall x\ (man(x) \rightarrow man(x))$, which is a tautology.

Thus ALL^* can be defined as a *concrete universal constructor* because it builds a concrete universal out of a concept. This notion of concrete universal is implicitly present in the plural or generic form of the subject of a generic statement, e.g. 'men', 'birds', 'stones', 'wood', 'fire', etc. But one can also express this notion of concrete universality by using the singular indeterminate qualified as *typical*[23]: 'a typical man', 'a typical bird', 'a typical stone', 'a typical peace of wood', 'a typical fire', etc. In Table 1, we can see how ALL^* transforms a concept to make it an appropriate subject of a generic statement.[24]

Footnote 22 continued
'being-a-man' to an object. In the formalism of the Logic of Object Determination [4,16,17], $\delta(being\text{-}man)\ (ALL^*\ man) \equiv (ALL^*\ man)$

[23] Without this qualification, one could think we're talking about just one specific but unknown individual.

[24] Showing how those different examples of generic statements all fall under the same category that can be formalized via ALL^* goes beyond the scope of this paper and will be the object of a subsequent article. In short, those generic statements, like RÖdl shows, are all similar in that they "give rise to an asymmetric contrast of rule and exception, and they explain statements that exemplify them." [23].

References

[1] Aristotle, Barnes, J.: The Complete Works of Aristotle: The Revised Oxford Translation. Princeton University Press, Princeton (1984)

[2] Arnauld, A., Nicole, P.: La logique: ou L'art de penser: Contenant outre les règles communes, plusieurs observations nouvelles, propres à former Le jugement. Flammarion, Paris (1978)

[3] Augustine, Haddan, A. W., Shedd, W.G.T., Böer, P.A.: On the Trinity. Veritas Splendor Publications (2012)

[4] Bergier, H.: Vers une logique du mouvement. Ph.D. thesis, Sorbonne Université (2016)

[5] Carlson, G.N.: Reference to Kinds in English. Ph.D. thesis, University of Massachussetts, Amherst (1977)

[6] Carlson, G.N., Pelletier, F.J.: The Generic Book. University of Chicago Press, Chicago (1999)

[7] Church, A.: The Calculi of Lambda-Conversion. Princeton University Press, Princeton (1941)

[8] Cimpian, A., Brandone, A.C., Gelman, S.A.: Generic statements require little evidence for acceptance but have powerful implications. Cogn. Sci. **34**(8), 1452–1482 (2010)

[9] Cole, P.: Pragmatics. Academic Press, London (1978)

[10] Curry, H.B., Feys, R., Craig, W., Hindley, J.R., Seldin, J.P.: Combinatory Logic. North-Holland Publishing Company, Amsterdam (1958)

[11] Curry, H.B.: Outlines of a Formalist Philosophy of Mathematics. North-Holland, Amsterdam (1970)

[12] Desclés, J.-P.: L'implication entre concepts: la notion de typicalité. Travaux de linguistique et de littérature **XXIV–I**, 179–202 (1986)

[13] Desclés, J.-P.: Langages applicatifs, langues naturelles, et cognition. Hermés, Paris (1990)

[14] Desclés, J.-P.: Une analyse non frégéenne de la quantification. Travaux de linguistique et de littérature **XXIV–I**, 179–202 (2005)

[15] Desclés, J.-P., Guibert, G., Sauzay, B.: Logique combinatoire et lambda-calcul: des logiques d'opérateurs. Cépaduès-éditions, Toulouse (2016)

[16] Desclés, J.-P., Pascu, A.: Logic of determination of objects: the meaning of variable in quantification. Int. J. Artif. Intell. Tools **15**(06), 1041–1052 (2006)

[17] Desclés, J.-P., Pascu, A.: Logic of determination of objects (ldo): how to articulate 'extension' with 'intension' and 'objects' with 'concepts'. Log. Univers. **5**(1), 75–89 (2011)

[18] Ellerman, D: Category Theory and Set Theory as Theories about Complementary Types of Universals. Logic and Logical Philosophy. https://doi.org/10.12775/llp.2016.022 (2016)

[19] Fitch, F.B.: Elements of Combinatory Logic. Yale University Press, London (1974)

[20] Frege, G.: Grundgesetze der arithmetik: begriffsschriftlich abgeleitet. H. Pohle, Jena (1893). (Translated in Frege, G. The foundations of Arithmetics: A Logico-Mathematical Enquiry into the concept of number. Blackwell, Oxford (1953))

[21] Frege, G., Beaney, M.: The Frege Reader. Blackwell Publishers, Oxford (1997)

[22] Peirce, C.S., Hartshorne, C., Weiss, P.: Collected Papers of Charles Sanders Peirce. Thoemmes Press, Bristol (1998)

[23] Rodl, S.: Categories of the Temporal: An Inquiry into the Forms of the Finite Intellect. Harvard University Press, Cambridge (2012)

[24] Tovena, L.M.: The Fine Structure of Polarity Sensitivity. Garland Pub, Spokane (1998)

[25] Yamauchi, T.: Labeling bias and categorical induction: generative aspects of category information. J. Exp. Psychol. Learn. Mem. Cogn. **31**(3), 538–553 (2005)

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