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Logica Universalis

The Talmudic Logic Project, Ongoing Since 2008

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Abstract. We describe the state of the Talmudic Logic project as of end of 2019. The Talmud is the most comprehensive and fundamental work of Jewish religious law, employing a large number of logical components centuries ahead of their time. In many cases the basic principles are not explicitly formulated, which makes it difficult to formalize and make available to the modern student of Logic. This project on Talmudic Logic, aims to present logical analysis of Talmudic reasoning using modern logical tools. We investigate principles of Talmudic Logic and publish a series of books, one book or more for each principle. http://www.collegepublications.co.uk/stl/ The series begins with the systematic analysis of Talmudic inference rules. The first book shows that we can present Talmudic reasoning intuitions as a systematic logical system basic to modern non-deductive reasoning, such as Argumentum A Fortiori, Abduction and Analogy. The second book offers a systematic common sense method for intuitively defining sets and claims that this method adequately models the Talmudic use of the rules Klal uPrat. These books also criticize modern Talmudic research methodology. Later books deal with additional topics like Deontic logic, and Temporal logic, Agency and processes in the Talmud and more. The aims of the project are two fold:

1. To import into the Talmudic study modern logical methods with a view to help understand complicated Talmudic passages, which otherwise cannot be addressed.
2. To export from the Talmud new logical principles which are innovative and useful to modern contemporary logic.

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1. Orientation

The Talmudic Project proceeds on three levels of reference.

1. The first level is the authors' overarching project of using (importing) logical theories and formulations arising in fields of formal logic, computer science (CS), artificial intelligence (AI), Linguistic, Philosophy and Law, and applying them to modelling Talmudic concepts and argumentation.
2. The second level is in the opposite direction; namely, formulating Talmudic concepts and argumentations in a logical form and applying (exporting) and/or comparing these new formulations to known systems in CS, AI, Philosophy, Law and Logic itself.
3. The third level of reference deals with the specific Talmudic concepts appearing in the Talmud and devoting a book to each one of them. So far we have thirteen books and we expect 25–30 more volumes. It takes about 10 months to fully complete a book. See [1–13] for the list of published books.

Regarding the first level, the research is timely and of potential importance for the study of the Talmud. It brings about a new comparative formal language, that of formal logic, in which one can analyse Talmudic concept and argumentations. For both the believer and non-believer in God and the Bible, this research shows an amazing cultural contribution of the Jewish people over the last 2000 years.

For the believer there is an additional dimension to this research. The Talmudic reasoning rules were given by God to Moses along with the Ten Commandments. Thus formalising in modern terms the Logic of the Talmud is a step in getting closer to the Logic of God Himself.

The second level, applying Talmudic logic to CS and AI, and exporting to logic itself, constitutes the original and innovative aspect of the project. It applies new ways of thinking into the realm of CS and AI and Logic. New Logical systems need to be developed to model some aspects of the reasoning in the Talmud and this a contribution is made to the future development and evolution of Logic.

To understand the background and prospect of success of both the first and the second levels, observe the following two simple facts:

1. Computer Science and Artificial Intelligence develop devices to be sold and to service the Human and his society in their everyday activity. To be successful CS and AI have to understand and model such activity.
2. The Talmud is practical and solves practical problems on Human Behaviour and Reasoning. Again the Talmud has to understand Human activity and legislate and guide the Human through his problems. Some of the Talmudic assumptions and solutions have a bearing to CS and AI.
3. It may be that newly developed aspects in CS and AI already exist in the Talmud and it may be that the Talmud can give a new twist to modern newly discovered point of views.
4. It may be that CS and AI address now some aspect which has already been dealt with in the Talmud.

Nevertheless, although this is a very original and innovative connection between modern Logic, CS, AI and the ancient Talmud, the question arises as

to the role the Talmud plays in this process: is the Talmud a good source of inspiration and of unique ideas which one can choose from and apply them as satisfactory solutions for certain AI problems? Is there something deeper, such as a few unifying principles summarizing Talmudic logic which should be applied to general logic and AI? We believe there is, as we shall discuss later.

2. Scientific Background: Logic

Logic began with Aristotle. He realised that in order to write his books he needed logic as a tool (*organon*). So he wrote his five books on logic. The system is syllogistic logic. Aristotle's logic was refined in later periods and the next significant step came with Pierre (or Petrus) Abelard who worked in the early 12-th century. His treatise the *Dialectica* [14] contained new ideas such as *de re* and *de dicto* modalities. It became possible to apply logic to language, theology and philosophy. New handbooks of logic appeared in later centuries, by Peter of Spain, Lambert of Auxerre and William of Sherwood. Later logicians were William of Ockham, Jean Buridan, Gregory of Rimini and Albert of Saxony. The best known textbook was by Antoine Arnauld and Pierre Nicole *The Port Royal Logic* [15].

Two points to be borne in mind about the development of logic up to the 19th century:

1. It was mainly syllogism with extras.
2. It dealt with human beings, their language reasoning and behaviour (as opposed to pure mathematics).

Modern mathematical logic was developed in the late 19-th century carrying on until the middle of the 20-th century [16]. There were four pillars to mathematical logic: model theory, set theory, proof theory and recursion theory. Emphasis was diverted from the study and application of logic to the humanities to the study and application of logic to mathematics and its foundations [17].

This changed with the rise of computer science, artificial intelligence, computational linguistics etc. There was a strong consumer demand for devices using this new technology and, in turn, there was an urgent need to develop and evolve logic to serve these demands. Emphasis in logic reverted back to the analysis of day-to-day human activity. New logics were developed by diverse non-cooperating non-communicating communities, each driven by the needs of certain types of application or device. The landscape of logic turned into a wild jungle of competing specific logical systems.

Worse yet, the new developments, though also sometimes applied in the humanities area (logic and law, logic an analytic philosophy, logical analysis of language, logic and theology, logic and argumentation and debate), did not include a new unified coherent logical theory.

However, these logics arising from computer science and AI, do offer the components to deal and model with Talmudic reasoning and debate- after all,

as we have mentioned earlier, both discipline deal with the human being and his daily activity.

We now have the tools to embark on the next phase of our human logic evolution and study and model cultural systems of thoughts such as the Jewish Talmudic logical way of thought.

We started our investigations in modelling Talmudic Reasoning. The Jewish Talmud is a body of arguments and discussions about all aspects of the human agent's social, legal, ethical and religious life. It was completed over 1500 years ago and its argumentation and debates contain many logical principles and examples very much relevant to today's research in logic, artificial intelligence, law and argumentation.

The Talmud is thus a unique source in human civilisation for conducting an investigation of humans and their thinking in just about any domain related to human endeavour and enterprise: Law, belief, ethics, etc. and the complex interaction among these domains [18].

In a series of books on initial key topics of *Talmudic Logic*, which we have published since 2009, we have studied some of the logical principles involved in the Talmud, one by one, devoting a volume to each major principle. We discovered principles which we could export to current research in theoretical computer science and AI and Law.

The multi-faceted character of Talmudic law with its myriad of branches in logic produces an appreciation of the conceptual structures that connect the formal jurisprudential requirements with the real-world. Our research plan, based on Talmudic logic reasoning, will enable us to expand and support the various approaches to modes of reasoning and discourse in many areas in computer science and AI as well as general logic theory, and their conceptual infrastructure. Our method of writing books is an incremental showing by examples how more and more complex logics need to be developed in response to modelling more and more features of human behaviour, and how these logics can be developed in a coherent way, following thematic meta-level principles.

We have so far published thirteen books [1–13] and various papers [19, 20, 38–41], and are working on the fourteenth and fifteenth books, both dealing with various aspects of identity and change in time, which has serious applications to AI. Our object is to examine the feasibility of a methodological study of logic in the Talmud, using tools derived from Western Logic. Our conclusion has been that two directions offer great potential: (1) Import of logical tools to use in understanding difficult and obscure issues in the Talmud, (2) Export of logical insights derived from the Talmud to Western Logic and other areas of human thought.

3. Methodological Steps

The following is an algorithmic description of the procedure we have followed in the past, and intend to apply also in the work proposed here.

Step 1: Allocation of a subject or issue of great logical interest in the Talmud which have a potential of serious application to CS, AI, General Logic and Law.

Step 2: Familiarisation and in-depth study of the subject in the Talmud, including writings and interpretations from later generations. This is done with the help of at least one qualified and knowledgeable Rabbi who has life long knowledge and experience and is able to gather and assess all relevant data. In the first 13 volumes this role was played by Professor Rabbi Dr. Michael Abraham who is also a physist. The current Rabbi is Rabbi Shlomo David. We also rely on many Expositions of the Talmud such as the Schottenstein Talmud [21].

We do not necessarily need a specialized Talmudic/history/text University Scholar to work with us, although their advice can be most valuable. (*This point will be address in the Appendix discussing issue* 1.)

Step 3: Choosing a typical example site from the Talmud and building a logical model appropriate to explaining the deliberations of the sages and later commentaries.

Step 4: Testing and verifying the consistency of the model with respect to the various sites in the Talmud where the subject is discussed.

Step 5: Generalising the logical model.

Step 6: Comparing the logical model with existing logic models in the landscape of current logics.

Step 7: Extracting unifying principles for general logic from the way different existing aspects of logics are unified in the Talmudic models.

Step 8: Go to step 2 for more iterations until satisfied.

Step 9: Go to step 1 and allocate a new subject or issue.

Step 10: Summarise resulting models and applying them to problems in computer science AI general logic, and Law.

4. Examples from Our Past Work

Example 4.1. [Time Logic (conditions on the future)]. This topic was dealt in [1–13, Vol 4]. Consider the simplest flow of time is discrete time: {day 1, day 2, ... }.

Such a flow of time is sufficient to illustrate the conditional:

(*1) [Do $x \rightarrow$ Do y in 10 days]

The semantic/temporal meaning of $\rightarrow$ is: At any day t in which x is done, we should have y done in 10 days afterwards, at $t + 10$.

For example

(*2) [open a file now $\rightarrow$ close it in 10 days from now]

This conditional is of the form $[A \to B]$, where A is earlier in time than B. This is a standard conditional appearing in programs and it is dealt with in program specification and verification using temporal logic with future operators.

However: The Talmud extensively deals with conditionals $A \to B$ where A is in the future of B.

To clarify this type of conditional, let us look at a common practice of insurance policies renewal.

I take an insurance policy Jan. 01, 2016 to Dec. 31, 2016.

I get a letter in Dec. 2016 reminding me to renew with an invoice to pay. Common practice is that if I pay by Jan. 31, 2017, the policy is renewed from Jan 01, 2017 for another year until Dec. 31, 2017.

(*) Note that that the policy is not renewed on Jan 01, but is renewed retrospectively on Jan 01 only if I pay by Jan 31.

We are now ready to describe the Talmudic Future Conditional. To achieve this, let us give the *Talmudic schematic structure*:

(**) Your policy is renewed on Jan 01 but if you fail to pay by Jan 31 then the policy is cancelled from Jan 01

In other words the deal is cancelled retrospectively.

There is a difference between (*) and (**). If you have not paid yet by Jan 15, according to (*) the policy is not valid on Jan 15, but according to (**) the policy is valid. I can take actions based on the fact that I have a valid policy on Jan 15, according to (**), but these actions will be undone if I fail to pay on Jan 31. This causes enormous problems in practice, so we can export a new kind of temporal logic that we have developed, and thus allow for such conditionals in computer programming problems.

Example 4.2. (Merging/Contradiction). This topic was dealt with in [1–13, Vol 8], and [22].

A very interesting modelling of the Talmud with export to modern logic and AI is the case of identity merging:

Some of the rules of behaviour on the Day of Atonement (Yom Kippur) contradict some of the rules of behaviour on the Sabbath. So what happens when the Day of Atonement falls on a Sabbath? There are two bodies of laws relevant to this day: The Yom Kippur laws and the Sabbath laws. They may be in conflict, so how do we resolve them? The Talmud enabled us to define rules of logic to overcome this problem see [1–13, Vol 8], and [22].

This has great relevance to modern times, as can be seen in numerous examples. We shall mention just a few:

1. The Boston Marathon terrorist. On one hand he is a terrorist, and should perhaps be sent to Guantanamo. On the other hand, he is a US citizen and is entitled to the full use of American law.

The export applications here are to Logic and Law, how to handle conflicting rules/laws in such situations. There are also more direct exports to computer science as we see below:

2. Rules may conflict in the Internet of Things (IoT), e.g. in a smart home, or in robotic actions, or in operational AI systems (e.g. self-driving cars), when the law of the land and our human values must both be obeyed (see [23]).

In modelling the Talmud we have developed the Talmudic Calculus of Cancellations. It is an algebra of elements and a relation saying which elements attack/cancel which other elements and using such algebra, we can get the correct results. This is a new mathematical model that can be exported to AI. We shall explain this using an example.

Consider two simple reasonable rules:

1. If a small job need fixing at home, e.g. sink is blocked—save money and do it yourself.
2. If a big job needs fixing at home (e.g. Toilet/drainage is blocked)—don't mess with it yourself, it is too risky, call an expert to do it.

Assume now that both a sink is blocked and a toilet is blocked. Since logically the two rules do not conflict, you will call a plumber to do the toilet and you will do the sink yourself. But this is not common sense. If the plumber is called for the toilet he might as well do the sink! This is not easy to formalise in modern logic, and we need a calculus of cancellations.

There is a surprising application to automata theory. In [24] we show that the number of states of an automaton can be reduced if we allow it to use calculus of cancellation rules.

5. Summary

We published 13 books on Talmudic logic since 2009. Each book analysed a logical principle in the Talmud, and exported it to modern logic, theoretical computer science and AI and Law.

We explained above what we have done in two of these books and showed how we exported results to Computer Science, AI and Logic and Law. Our other books contain similar results.

We have proved since 2009 that our methodology is sound, workable and fruitful. We have published our results in international journals as well as showed the cultural inheritance of the Jewish people. We achieved international recognition for Talmudic Logic. Some departments, such as the computer science department at the University of Luxembourg, make it compulsory for advanced graduate students to learn about Talmudic Logic.

6. Current (2019) Research Activity

Classical logic deals with objects that have no internal structure and deals with predicates applied to them. Temporal logic deals with behavior of such objects and predicates through time. So if our objects are files, we can talk about files being opened or closed by a user. We can write a program specifying

how users can open and close files through time. We can specify how many users can look at any file, and how many files a user can look at, etc.

When we deal with Dropbox we have further problems: Users can change the files. How do we specify and manage that? This is a special case of dealing with change of object through time. We plan to address the topic of 'Change through Time'. This is an ancient central problem in logic and its applications, and appears in all aspects of everyday life all the time. It is therefore important to AI applications. We are working on two books, vols. 15 and 16.

To introduce the problem, assume we start with an object X built up of parts. It could be a program, it could be a pile of sand, it could be a file in Dropbox undergoing continuous change by users. Various predicates and functionalities apply to this object. We slowly change it, step by step. We make it bigger, substitute its components, take away parts, etc.

The question is how does it change? What predicates continue to apply? What new predicates apply? How do we specify, control and verify the process of change? Such processes happen in practice, in philosophy, in law in computer science and AI and in the Talmud.

Let us consider some examples that relate to changes over time of objects:

Example 6.1. [The paradox of the heap (sorites)]. One grain of sand is not a heap. If n grains of sand is not a heap, neither is n+1 grains. Therefore any arbitrarily huge amount of grains of sand is not a heap. This is counter intuitive. It is a known paradox since Aristotle and is addressed by many books and many departments/schools of thought, see [25].

The Talmud has addressed it in terms of mixtures. Drops of non-kosher wine falls into a jug of kosher wine. Is the jug kosher or not? The Talmud has to solve the paradox because it has to tell people whether the wine is usable or not. It is real life for real people. The Talmud adds a component to such dynamics and asks how this heap-candidate was formed? If it was formed grain by grain starting with a small number of grains, the Talmud will say e.g. it is not a heap. However, if it was formed from a huge starting collection of grains by deleting grains one by one, then the Talmud will say it is a heap. If we do not know how it was formed, then we have any default position stipulated.

Logically the Talmud thus adds another parameter to the paradox of the heap, and thereby solves it.

Example 6.2. (The Theseus ship). We start with an object, a ship X, and slowly, step by step change small parts of it, until all of its parts have been replaced. At each step the intuition is that it is still the same ship. On the other hand if we take all the replaced parts and put them together again we get a reconstruction of the original physical ship. This is an ancient paradox further developed by Thomas Hobbes [26].

The Talmud faces this problem as it appears in many aspects of real life. I steal a computer and upgrade it and try to sell it. I am caught. The owner wants the computer back because the Bible says "return what you stole". The thief says it is not the same computer anymore. It is mine. I shall pay money instead. What do we do? Is it the same computer or not?

The Talmud adds in such cases another component—purpose:

If the object after the change cannot serve the original purpose then it is a new object. We look if the change of the small part still keeps the computer/ship/object fit for it purpose. For example, if we have a mug for drinking tea, and we make a small hole in it so it leaks just very little, we can say it is not the same mug because we are not going to use it. But a garden water bucket with a very small hole in it is the same object, because we won't care about the hole.

The Talmud therefore solves the Theseus paradox as follows: If the purpose of the process is to have a ship to actually sail, then the 'main' ship is the one with the new parts, not the reconstructed old one. On the other hand, if the purpose is to have an exhibit in a historical museum then the 'main' ship is the one made of the original parts.

Similarly, assume some kosher wine has been mixed with non-kosher wine. The Talmud decides whether the mixture is itself kosher or not, by considering the way it was created.

Classical logic dealing with object-change lacks this point of view, and therefore faces paradoxes and problems. We believe it is important to develop logics based on the Talmudic approach. Such logics may then be applied in the development of practical applications in Computer Science and AI.

A practical use of this old Talmudic idea (of including in the properties of an object also the way it was constructed) is actually adopted/rediscovered in modern AI. Consider a robot trying to identify injured people lying in the street as opposed to a sleeping homeless. One can use neural networks with some success, but one can also develop algorithms for constructing people from parts. A robot can then attempt to identify the parts. If some are missing or not identifiable the robot cannot construct the image of a complete person, and the image is therefore of an injured person [27].

A reconstruction method could perhaps also be used to identify a security threat. Suppose an intelligent security camera identifies a man standing next to a suitcase. Identifying his body parts, a program may conclude the suitcase too heavy for him to move, so how did it get there?

Or consider a computer program where somehow errors were introduced. Reverse engineering of the program in order to reach its original state could be based on this kind of logic. Although these ideas appear in the Talmud and some are used in modern AI, no significant logical systems involving change in time in the Talmudic sense have been developed.

Talmudic inspired logical model can be of use to any specification-rich program for handling objects (files, databases), which change through time. Current programs (e.g. Dropbox) attempting to coordinating agents beliefs, temporal database management, disciplines for applicability of legal laws and definitions, etc. deal with formal changes only, without the intelligence of taking into account the logical content of the change. So if I open a file in Dropbox on two computers and on one of them I make a change and then cancel the change, Dropbox will block me saving on the other computers, not realizing that there was no real change. The Talmud is aware of this problem. The Bible

says that if a man steals an object and he is caught, he must return the stolen object to its owner. The man might say I am using this object, I propose to keep it and pay money instead. The answer is no, he must return the object. The Talmud asks, what if the thief disassembles the object and reassembles it again to its original form (like in the Dropbox case where we make a change and cancel it), do we have now a new object? Some Talmudic opinion says yes, this is now a new object and the Biblical rule does not apply.

We believe the time-estimate is reasonable, as it is based on the experience of 8 years of work, 13 books and numerous articles. Each area of the first thirteen books could be further researched and applied further in CS and AI. We have not done that, because we choose to discover and develop new principles/books.

We now indicate how we are going to conduct our research during the two years of the project. More details in the section on Time Schedule and Work-plan.

The problem we are addressing is Change of Objects in Time. As we already said, this problem is central in CS, AI, Philosophy and Law and also central in the Talmud. All the above disciplines attempt to regulate and deal with practical human behaviour and change.

So based on our past experience (thirteen earlier books on Talmudic Logic and export), we start with identifying difficulties and attempted solutions in CS, AI, Philosophy and Law. We have already identified at least two main types of change:

1. Mixture/Merging
2. Object Modification and Transformations.

We check throughout the Talmud how such topics are addressed on the main practical examples. We then check the available logical models CS, AI, etc., and see how to modify them or invent variations of them capable of addressing the Talmudic features. We then check through successive approximations how we can model all opinion/aspect of Talmudic argumentation. Once we get our new Talmudic logical systems, we use them to systematically try to improve solve the problems existing in AI, CS, Philosophy and Law.

We have already identified two additional components the Talmud uses: 1. How mixture/merging was made and 2. What is the purpose/use of an object. These features can offer solutions to temporal change puzzles and problems. There is however a lot of detail to investigate in the projected two years project time.

There is one more export feature we can do here. We will get new logical systems/temporal models. We can look at these and export to the general theory of logics. One immediate idea can be used as illustration. We are talking about objects changing their structure in time. Classical predicate logic deals with atomic objects. It does not allow for internal composition of objects. It allows for predicate properties of objects (which properties) can change in time but not the object themselves. In philosophical logic there is a discussion of objects like the king of France but not a proper predicate logic of changing objects. So we can develop logics where the main thrust of model theory

and proof theory deals with internal object change, and the properties and relationships among of the objects are derived from their internal structure. Predicate logic deals with atomic objects and stipulates their properties. This is completely different!

7. Conclusion

The Talmudic Logic project includes two major components:

1. Modelling Talmudic logic
2. Exporting logics to Computer Science, AI, Law and General Applied Logic.

The first component is in itself of value, as the Talmud is a World Cultural Heritage. We show it is an enormous body of coherent legal argumentation regarding how to regulate human behaviour. Its connection with computer science and AI is that both the Talmud, CS and AI deal with human activity and so logics used in CS and AI can be modified and used to model the Talmud and in turn, be exported back into CS and AI and offer new ideas and solutions there.

So this is not just a project modelling change for CS and AI. This is a project modelling the Talmud, with applications/export to CS and AI! The Talmud is a practical and coherent body of laws developed logically to address human behaviour just like AI and computer science. In this it differs from many other bodies of laws which are stipulative, statuary and less flexible, and not as interconnected and coherent. Hence the serious export from the Talmud to AI, CS and Law.

Appendix A: Discussion of Some Methodological Issues

Issue 1: We have had comments from four referees. Two of them raised the current Issue 1 which we describe as follows:

Do we need to cooperate with a university researcher specializing in analyzing Talmudic Text and in tracing the evolution of Talmudic thought and Talmudic argumentation through the centuries as opposed to the study and interpretation of the Talmud as done today by Rabbis in Yeshivas, Schools and synagogues?

We quote the words of one of the referees (referee 4):

> The author discusses at some length methodological questions that concern the logic part of our research. However, the whole project crucially depends on reading and interpreting Talmudic text. Here the proposal is silent on the methodology required for such interpretation. One can expect that a research of the Talmudic text will make use of methodologies developed in academic study of the Talmud. Indeed, it is not clear how it is possible to cope academically with the meaning of the Talmud text without reference to the complexity of the text which was created and edited by many people in different places over a stretch of several hundreds of years. A discussion of these methodologies should have been added at least to explain why they are irrelevant. And if indeed they are, what alternative methodologies are adopted.

Our Response to Issue 1: In 2008, when we were starting our project we asked several Talmudic professors to join us. The consensus was that we need a Rabbi well versed in the Talmud and not a researcher as described in Issue 1. Indeed Rabbi Dr. Michael Abraham joined us and we very quickly in the period 2008–2015 produced many books and research papers.

Talmudic reasoning and text is available to the general reader and even to school children through many books of variable depth and difficulty such as [21,28], in Hebrew as well as in English.

The criticism

> it is not clear how it is possible to cope academically with the meaning of the Talmud text without reference to the complexity of the text which was created and edited by many people in different places over a stretch of several hundreds of years

is not valid. Any competent Rabbi with many years of traditional Talmudic study under his belt can do the job of collecting the relevant material that the project needs.

To explain this methodological point to the international reader who may not familiar with the Talmud, let us look at a similar example from Shakespeare. Consider the Tragedy of Othello, and consider the villain Iago who has two features of interest to modern logic

1. He manages to mislead the Moor into believing that his wife is unfaithful by planting circumstantial evidence
2. He firmly hates the Moor and keeps on putting forward excuses justifying his hatred.

The analysis and modelling of item 1 above falls in the realm of Non- Monotonic logic, see [29].

The analysis of Item 2 above falls into the new research into Reasoning Distortions, see [30].

We do not need a Cambridge professor of English Literature to help with the Logical Modelling of Items 1 and 2. There are many books on Shakespeare with annotations and explanations for the general reader, see for example [31,32], and what is needed is a Shakespeare enthusiast (which may be a school

teacher with many years' experience of teaching Shakespeare, corresponding to our Talmudic Rabbi) who can understand and collect for us all similar instances of Items 1 and 2 throughout Shakespeare's plays.

The perceptive reader might ask why is this criticism then put forward in 2017, when the Talmudic project has already published 13 books and is ongoing? The answer is simple—in 2017 we submitted a project proposal asking for support. The referees were reviewing our project proposal. We were competing for limited funds with the very Talmudic community of university researchers.

Issue 2: What do we mean by "exporting into AI and or Logic"?

Response to Issue 2: We explain our approach by considering several examples.

(E1) Consider our book on the Talmudic Temporal Logic.

Namely:

> Abraham M., Belfer I., Gabbay D., Schild U. (2011) Studies in Talmudic Logic, Volume 4: Temporal Logic in the Talmud (in Hebrew and English), College Publications, London, 2011, 588+70 pages.

This book gave rise to two research papers in applied logic, one on future determination of entities (i.e "the x who will be president of the USA in 2025" stated today at 2017) [19] and one on future conditionals [33] . Those papers just presented the systems and motivated and discussed them . The systems were not developed theoretically, sematically, proof theoretically, etc, etc. In the mind of the first author Dov Gabbay, there is enough material there for several PhD theses and a Third volume in the authors series of books on Temporal Logic published by OUP.

We did not go this route we concentrated on our next Talmudic logic book, bringing out more principles, namely the next example

(E2) Consider our book

> Abraham M., Gabbay D., Schild U. (2011) Studies in Talmudic Logic, Volume 5: Resolution of Conflicts and Normative Loops in the Talmud, (in Hebrew and English) College Publications, London, 2011, 280+25 pages.

Here too we could take the logic route and deal with loops in formal argumentation, liar paradox, loops in action logic, etc., etc., but we did not do that just published a basic paper in the logic of loops [34] and continued with The next book on Talmudic Logic And so on.

(E3) Another example is a new paper modelling the Talmudic idea of future abandonment, see [35] for background.

If a person finds an object in the street, there is a question of whether he can take it for himself or whether he needs to try to find the owner or give it to the Lost and Found department. The underlining principle is to look at the object and if it is clear that the owner has given up on it (say you drop a pound coin, you are not going back to look for it), then the one who finds the object can take it.

In modern action logic terms we have an action T of taking the object which requires the precondition Y of the owner having given up ownership of the object called (Ye'ush in Hebrew). In symbols $Y \Rightarrow T$.

The problem arises when we ask or suspect what if the owner did not yet realise he lost the object and so he has not yet given up on it. The proposed Talmudic logical principle to consider is that since the owner will eventually realise he lost the object and surely will give up on it when he discovers he lost it , we can consider it as if he has already given up on it now. In action logic symbols we are considering the principle

$$(\text{In all possible future paths } Y \text{ holds}) \rightarrow Y \text{ holds now})$$

This principle is considered only for certain atoms, not for arbitrary formulas. Here is what [35] says about this, we quote:

> Normally we think of Ye'ush as a person giving up hope on his object. This understanding leads us to a very significant problem. How can we have a "Ye'ush shelo mida'as"—a 'giving up hope' without knowing?
>
> According to Abaye, this object is not disconnected completely from its owner, and therefore it would be forbidden to take it. According to Rava, the fact that we know he will dissociate from it as soon as he finds out gives us the liberty to say that there is already a complete disconnection even now, since we already have a disconnection from the perspective of the object itself.

There is research to be done here. To model Rava's opinion, we need to develop temporal logics where for certain formulas A holds now if at any path into the future A becomes true.

This calls for some sort of Intuitionistic Temporal Logic. It seems to be connected with the Miners Paradox [36].

(E4) Promising export to Machine Ethics: Recently, (2018–2019) Professor Dov Gabbay together with the research group of Professor Leon van der Torre, at the Computer Science and Communications Research Unit, University of Luxembourg, were engaged in addressing the question if replacing Human Taxi drivers by Robots. Put differently it is the question of putting the right intelligence int a robotic taxi driver, See [37]. The question arose What Machine Ethics to put into the intelligence of the Robot driver. The team examine the explosive published literature on the subject of Machine Ethics and the following has become evident to the team:

It seems that, for whatever reason, many of the researchers in Machine Ethics think that AI technology is solved, or will soon be solved. This is false for so many reasons, for example, the gap between vision and legal reasoning is huge. To decide whether a piece of paper is money is the easy thing, to decide whether this money is abandoned and therefore it can be picked up or just left in place for later use is hugely complex (this question is extensively addressed in the Talmud).

The Talmudic Ethical theories and discussion is what is needed here as a first candidate for Robotic adaptation and simplification. Talmudic Ethical reasoning is quite different from the well known multitude of philosophical ethical theories (Utilitarian, Deontic, Virtue, etc) which are not suitable and rather hinder any successful practical deployment of Robotic Ethics.

This also touches on the problem of identification. Robots must be able to identify objects , not mathematically, but in a Human oriented way, namely how only small changes in the objects still leave it as the same object. The Talmud has an extensive theory on this topic, see our book [22].

To sum up, the Talmud potential to export to AI is huge. It is not a matter of a few applications, but the value of doing centuries of relevant discussion of the same problems in the context of Human Behaviour.

Issue 3: What do we mean by "exporting to Philosophy"?

Response to Issue 3. The previous Issue 2 presented the Talmud as a good source of logical inspiration and of unique ideas, for Logic itself, This Issue 3, deals with the specific Talmudic concepts of "identity and change".

We offered a first attempt at Talmudic Approach solutions to the Paradox of the Heap and to Philosophical Problems of Identity through Time. However, Philosophy has its own way of thinking. What exactly are we exporting/solving in Philosophy?

Are we confusing the philosophical issue of *identity* with the practical issue of *identifying* objects?

Our answer is that already in this preliminary stage of research in progress, we can see that we need models which can deal with both issues. This is best explained by an example.

Consider 3 atomic components say, a, b, c. Our objects are composed of these components and assume we have only one constructor, namely set union. So our objects are all the subsets of $\{a, b, c\}$. We have classical predicates applying to objects and the history of their construction. For example Start with $\{a, b\}$, replace b by c and get $\{a, c\}$, then replace c back by b and get $\{a, b\}$. We now have two objects candidates from the Talmudic point of view:

1. $O_1 = [\{a, b\}$, no history]
2. $O_2 = [\{a, b\}$ with history $(\{a, b\}, \{a, c\}, \{a, b\})]$

Talmudic Question: Assume that we are given an object O_1 which may be built up of atomic components, for example a laptop or a wine glass. We also have predicates $P_1, ..., P_k$ which may or may not hold for object O_1. The predicates can describe properties of the object and /or of its components, as well as its relationships to other objects. We now change some of these predicates. We get a possibly new object O_2.

Question: Do we consider that $O_1 = O_2$? For example if the laptop software is updated or the glass get chipped, are we getting new objects or are they the same objects still?

Different Rabbis would follow different principles.

This is the subject for our next book on the Talmudic logic of Identity through time.

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