Complex Anal. Oper. Theory (2013) 7:1427–1428
DOI 10.1007/s11785-012-0246-6

**Complex Analysis and Operator Theory**

ERRATUM

# Erratum to: Geometry Behind Chordal Loewner Chains

**Manuel D. Contreras · Santiago Díaz-Madrigal · Pavel Gumenyuk**

Published online: 6 July 2012


**Erratum to: Complex Anal. Oper. Theory (2010) 4:541–587
DOI 10.1007/s11785-010-0057-6**

The authors apologize for several mistakes and misprints found after the paper "Geometry behind chordal Loewner chains" was published. The following corrections are necessary in Lemmas 2.1, 2.2 and 3.6.

**I. Lemma 2.1 should be as follows:**

**Lemma 2.1** *Let* $\varphi$ *be a holomorphic univalent self-mapping of the unit disk* $\mathbb{D}$ *with* $\varphi(0) = 0$. *Then:*

(1) *for all* $z \in \mathbb{D}$,

$$|\varphi(z) - z| \leq |1 - \varphi'(0)| + \frac{|z|^2}{\sqrt{1 - |z|^2}} \sqrt{1 - |\varphi'(0)|^2};$$

The online version of the original article can be found under doi:10.1007/s11785-010-0057-6.

M. D. Contreras (✉) · S. Díaz-Madrigal
Camino de los Descubrimientos, s/n, Departamento de Matemática Aplicada II,
Escuela Técnica Superior de Ingenieros, Universidad de Sevilla, Sevilla 41092, Spain
e-mail: contreras@us.es

S. Díaz-Madrigal
e-mail: madrigal@us.es

P. Gumenyuk
Department of Mathematics, University of Bergen, Johannes Brunsgate 12, Bergen 5008, Norway
e-mail: Pavel.Gumenyuk@math.uib.no

(2) *if* $\varphi'(0) > 0$, *then for all* $z \in \mathbb{D}$,

$$|\varphi(z) - z| \le C_0\sqrt{1-\varphi'(0)},$$

*where* $C_0 := 1 + \dfrac{\sqrt{2}\,r^2}{\sqrt{1-r^2}}$ *and* $r := |z|$.

*Proof* Write $\varphi(z) = \sum_{n=1}^{\infty} a_n z^n$. Let us recall that

$$\sum_{n=1}^{\infty} n|a_n|^2 = m(\varphi(\mathbb{D})) \le m(\mathbb{D}) = 1,$$

where $m(\cdot)$ stands for the normalized Lebesgue measure in the unit disc. Let $z \in \mathbb{D}$ and $r := |z|$. Then

$$|\varphi(z) - z| \le \left|\sum_{n=2}^{\infty} a_n z^n\right| + |1-\varphi'(0)| \le \sqrt{\sum_{n=2}^{\infty} r^{2n}}\sqrt{\sum_{n=2}^{\infty} |a_n|^2} + |1-\varphi'(0)|$$
$$= \frac{r^2}{\sqrt{1-r^2}}\sqrt{\sum_{n=2}^{\infty} |a_n|^2} + |1-\varphi'(0)| \le \frac{r^2}{\sqrt{1-r^2}}\sqrt{\sum_{n=2}^{\infty} n|a_n|^2} + |1-\varphi'(0)|$$
$$\le \frac{r^2}{\sqrt{1-r^2}}\sqrt{1-|\varphi'(0)|^2} + |1-\varphi'(0)|.$$

This proves (1). Assertion (2) is now an immediate consequence of (1). □

**II. The last line in the proof of Lemma 2.2 has to be changed as follows:**

$$|f(z) - g(z)| \le g'(0)\frac{2C_0}{(1-r)^3}\sqrt{1-\varphi'(0)} = \frac{2C_0}{(1-r)^3}\sqrt{g'(0)(g'(0)-f'(0))}.$$

**III. The proof of Lemma 3.6 contains an error.** Arguing in a more direct way we get a better estimate under weaker conditions $|z| > b > 0$ and $|\arg z - \pi/2| \le \pi/3$:

$$\frac{1 + r_{\mathbb{H}}(z, ib)}{1 - r_{\mathbb{H}}(z, ib)} = \frac{|z+ib| + |z-ib|}{|z+ib| - |z-ib|} = \frac{\big(|z+ib| + |z-ib|\big)^2}{|z+ib|^2 - |z-ib|^2}$$
$$= \frac{\big(|z+ib| + |z-ib|\big)^2}{4b\,\mathsf{Im}\,z} \le \frac{\big(2|z| + 2b\big)^2}{4b\cdot\frac{1}{2}|z|} \le \frac{2}{b}\,|z|.$$

**IV. Misprints:**

1. Condition (B.2) in Theorem 4.8 should contain "$\Omega_t$" instead of "$\Omega_t(\mathbb{D})$".
2. The estimate of $|V(z,x)/z|$ in the proof of Proposition 5.9, see page 572, should contain "$|\mathsf{Re}\,z|$" instead of "$\mathsf{Re}\,z$".
3. The Eq. (7.5) on p. 585 should contain the formula $u(t) := H^{-1}(\lambda(t)) \neq 1$ rather than $u(t) := H(\lambda(t)) \neq 1$.