Correction

Correction to: Relation-theoretic metrical fixed-point results via w -distance with applications

Tanusri Senapati and Lakshmi Kanta Dey

Correction to: J. Fixed Point Theory Appl. 19(2017) 2945–2961 <https://doi.org/10.1007/s11784-017-0462-9>

The Example 2.3 in [3] contains an inaccuracy as the mapping T considered here is not a self-mapping. Therefore, we can not employ our main result, [3, Theorem 2.1], to find the fixed points of the mapping. In this erratum, we amend the aforementioned error by presenting the following example instead of [3, Example 2.3]. However, the main results of [3] remain untouched by this correction.

Example 0.1. Let (X, d) be a metric space where $X = [0, 1]$ and d is the usual metric defined on X . We consider the following binary relation

$$\mathcal{R} = \left\{ (0, 0), \left(\frac{1}{2}, 1 \right), \left(\frac{1}{2}, 0 \right), \left(0, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right) \right\}$$

on X . Let T be a self-map on X defined by

$$T(x) = \begin{cases} 0, & x \in [0, \frac{1}{2}]; \\ \frac{1}{2}, & x > \frac{1}{2}. \end{cases}$$

Our primary goal is to check the hypotheses of [2, Theorem 3.1] due to Alam and Imdad. It is very easy to check that for all $(x, y) \in \mathcal{R} \Rightarrow (Tx, Ty) \in \mathcal{R}$, i.e., T is $\mathcal{R}$ -closed. Let (x_n) be an $\mathcal{R}$ -preserving sequence converging to x as

$n \rightarrow \infty$. Then we must have

$$(x_n, x_{n+1}) \in \left\{ (0, 0), \left(\frac{1}{2}, 0\right), \left(0, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right) \right\} \\ \Rightarrow x_n \in \left\{ 0, \frac{1}{2} \right\}.$$

This implies that either $x_n \rightarrow 0$ or $x_n \rightarrow \frac{1}{2}$ as $n \rightarrow \infty$ and clearly, we have $[x_n, x] \in \mathcal{R}$ for all $n \in \mathbb{N}$, where $x = 0$ and $\frac{1}{2}$. This shows that $\mathcal{R}$ is d -self-closed. Also, $T(X) = \{0, \frac{1}{2}\}$ is a closed set. Next, we check the contraction condition given in Alam and Imdad [2].

Let us consider $x = \frac{1}{2}$, $y = 1$. Then $d(Tx, Ty) = d(0, \frac{1}{2}) = \frac{1}{2}$ and $d(x, y) = \frac{1}{2}$. So we cannot find any $k \in [0, 1)$ such that

$$d(Tx, Ty) \leq kd(x, y)$$

holds. But if we choose a w -distance function p as $p(x, y) = x + y$, then for all $x, y \in X$, we have

$$p(Tx, Ty) \leq \lambda p(x, y)$$

with $(x, y) \in \mathcal{R}$ and for some $\lambda \in [0, 1)$.

Hence all the hypotheses of [3, Theorem 2.1] are satisfied and note that $x = 0$ is a fixed point of T and also, it is the unique fixed point.

Note: It is worth noting that the results of Ahmadullah et al. [1] are more generalized and improved than that of Alam and Imdad [2] but still in that example, we cannot employ the main result [1, Theorem 2.1] due to Ahmadullah et al. For $x = \frac{1}{2}, y = 1$, we obtain

$$M_T\left(\frac{1}{2}, 1\right) = \max \left\{ d\left(\frac{1}{2}, 1\right), d\left(\frac{1}{2}, 0\right), d\left(1, \frac{1}{2}\right), \frac{d(\frac{1}{2}, \frac{1}{2}) + d(1, 0)}{2} \right\} = \frac{1}{2}.$$

As ψ is a function with $\psi(t) < t$, $t > 0$, we cannot find any function ψ such that

$$d(Tx, Ty) \leq \psi(M_T(x, y))$$

holds. Hence, we cannot employ the results of Ahmadullah et al. [1] in this example.

References

- [1] Ahmadullah, M., Imdad, M., Gubran, R.: Relation-theoretic metrical fixed point theorems under nonlinear contractions. *Fixed Point Theory (to appear)*
- [2] Alam, A., Imdad, M.: Relation-theoretic contractive principle. *J. Fixed Point Theory Appl.* **17**(4), 693–702 (2015)
- [3] Senapati, T., Dey, L.K.: Relation-theoretic metrical fixed point results via w -distance with applications. *J. Fixed Point Theory Appl.* **19**(4), 2945–2961 (2017)

Tanusri Senapati and Lakshmi Kanta Dey

Department of Mathematics

National Institute of Technology Durgapur

Durgapur West Bengal

India

e-mail: senapati.tanusri@gmail.com;

lakshmikdey@yahoo.co.in