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Spreading speed of a food-limited population model with delay

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Abstract. This paper is concerned with the spreading speed of a food-limited population model with delay. First, the existence of the solution of Cauchy problem is proved. Then, the spreading speed of solutions with compactly supported initial data is investigated by using the general Harnack inequality. Finally, we present some numerical simulations and investigate the dynamical behavior of the solution.

§1 Introduction

In this paper, we consider the following food-limited population model with delay:

$$v_t(t,x) = \Delta v(t,x) + \mu v(t,x)\left(\frac{1-v(t-\tau,x)}{1+\beta v(t-\tau,x)}\right), \quad t \geq 0,\ x \in \mathbb{R}, \tag{1.1}$$

where the delay τ and the coefficients μ, β are positive. The archetype of this model can be found in the original paper of Smith [18]. We also refer the readers to the more detailed research about the food-limited model in [3,4,6-10,13,17,19]. In particular, there are some works [8,12,22,25,26,28] about the traveling wave solution of (1.1). It is worth noting that Trofimchuk et al. [22] presented a broad and explicit range of the parameters which permit the existence of monotone traveling waves. Furthermore, Hasik et al. [12] gave a more detailed analysis about the geometric diversity of wavefront of (1.1), which include the non-monotone and non-oscillating wavefronts.

However, there seem no results about the long-time behaviour of solutions of the Cauchy problem of (1.1). The long-time behaviors of solutions of reaction-diffusion equations with compactly supported initial data can well describe the propagation phenomenon in population invasion and epidemic spread. Usually, such a propagation phenomenon can be characterized

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by investigating the so-called spreading speed of solutions [1, §4]. For convenience, we first introduce the following definition:

Definition 1.1 (Spreading speed). *Assume that $v(t,x)$ is a nonnegative function for all $(t,x) \in (0,+\infty)\times\mathbb{R}$. Then, there exists a constant $\kappa > 0$, c^* is called the asymptotic speed of spreading of $v(t,x)$ if*

(i) $\lim\limits_{t\to+\infty,\ |x|\geq(c^*+\varepsilon)t} v(t,x) = 0$ *for any given* $\varepsilon > 0$*;*

(ii) $\liminf\limits_{t\to+\infty,\ |x|\leq(c^*-\varepsilon)t} v(t,x) > \kappa$ *for any given* $\varepsilon \in (0,c^*)$*.*

Up to now, there has been great progress on the study of the spreading speed of solutions with compactly supported initial data when the solution semiflow of the equations is monotone. Unfortunately, equation (1.1) is not quasi-monotone. Therefore, the whole point of this paper is to investigate the spreading speed of solutions of (1.1) with compactly supported initial data, namely, we consider the following initial valued problem:

$$\begin{cases} v_t(t,x) = \Delta v(t,x) + \mu v(t,x)\left(\dfrac{1-v(t-\tau,x)}{1+\beta v(t-\tau,x)}\right), \quad t\geq 0,\ x\in\mathbb{R}, \\ v(s,x) = v_0(s,x),\ s\in[-\tau,0],\ x\in\mathbb{R}. \end{cases} \tag{1.2}$$

Note that the delay in the reaction term makes the comparison principle unavailable, which results in that the abstract theory of the spreading speed for monotone semiflows is not applicable. Recently, replacing the delayed term $v(t-\tau,x)$ by a special convolution term

$$(g*v)(t,x) = \int_0^\tau k(s)\int_{\mathbb{R}} \frac{1}{\sqrt{4\pi s}} e^{-(x-y)^2/(4s)} v(t-s,y)dyds,$$

we [23] have determined the spreading speed by using the solution of the heat equation. Unfortunately, the method used in [23] is not applicable to equation (1.2). In particular, when $\beta = 0$, equation (1.2) is reduced to the following delayed Fisher-KPP equation:

$$\begin{cases} v_t(t,x) = \Delta v(t,x) + \mu v(t,x)\left(1-v(t-\tau,x)\right), \quad t\geq 0,\ x\in\mathbb{R}, \\ v(s,x) = v_0(s,x),\ s\in[-\tau,0],\ x\in\mathbb{R}. \end{cases} \tag{1.3}$$

It should be emphasized that determining of the spreading speed of solutions of (1.3) is still an open problem. Of course, if an instantaneous self-limitation term is introduced to the nonlinearity, the boundedness of the solutions could be obtained and hence, the spreading speed of solutions could be established [14,15].

In this paper, we develop a new argument to establish the spreading speed of (1.2), which gets rid of the intricate investigations about the uniform boundedness of the solutions of (1.2). Precisely speaking, we will use the general Harnack inequality [5] and the comparison principle to study the spreading speed of solutions of (1.2). Before stating the main results, we give some notations. Let $\mathcal{Y} = BUC(\mathbb{R},\mathbb{R})$ be the Banach space of all bounded and uniformly continuous functions from $\mathbb{R}$ to $\mathbb{R}$ with the supremum norm $\|\cdot\|_{\mathcal{Y}}$. Let $\mathcal{Y}^+ := \{v\in\mathcal{Y} : v(x)\geq 0, x\in\mathbb{R}\}$. Then $\mathcal{Y}$ is a Banach lattice under the partial ordering induced by $\mathcal{Y}^+$. It follows from [2,Theorem

1.5] that the $\mathcal{Y}$-realization $\Delta_{\mathcal{Y}}$ of the Laplacian Δ generates a strongly continuous analytic semigroup $\Psi(t)$ on $\mathcal{Y}$ and $\Psi(t)\mathcal{Y}^+ \subset \mathcal{Y}^+$ for $t \geq 0$. In addition, we have

$$(\Psi(t)w)(x) = \frac{1}{\sqrt{4\pi t}} \int_{\mathbb{R}} e^{-\frac{(x-y)^2}{4t}} w(y)dy, \quad t > 0,\ x \in \mathbb{R},\ w(\cdot) \in \mathcal{Y}.$$

Let $\mathcal{C} = C([-\tau, 0], \mathcal{Y})$ be the Banach space of continuous functions from $[-\tau, 0]$ into $\mathcal{Y}$ with the supremum norm $\|\cdot\|_{\mathcal{C}}$ and let $\mathcal{C}^+ = \{\varphi \in \mathcal{C} : \varphi(s) \in \mathcal{Y}^+, s \in [-\tau, 0]\}$. Then $\mathcal{C}^+$ is a positive cone of $\mathcal{C}$. Usually, we identify an element $\varphi \in \mathcal{C}$ as a function from $[-\tau, 0] \times \mathbb{R}$ into $\mathbb{R}$ defined by $\varphi(s, x) = \varphi(s)(x)$. For $T > 0$, we define $\mathcal{C}_T = C([-\tau, T], \mathcal{Y})$ be the Banach space of continuous functions from $[-\tau, T]$ into $\mathcal{Y}$ with the supremum norm $\|\cdot\|_{\mathcal{C}_T}$.

***Theorem* 1.2.** *Let $v(t,x)$ be the solution of the Cauchy problem* (1.2) *with the initial condition $v_0 \in \mathcal{C}^+$ such that $v_0(s,x) \not\equiv 0$. Then there exists γ which is a positive constant independent of $v_0 \in \mathcal{C}^+$, such that*

$$\liminf_{t\to+\infty} \left(\min_{|x|\leq ct} v(t,x) \right) > \gamma^{-1}, \quad \forall\ 0 \leq c < 2\sqrt{\mu}. \tag{1.4}$$

Furthermore, if $v_0(s,x)$ is compactly supported, then

$$\lim_{t\to+\infty} \left(\max_{|x|\geq ct} v(t,x) \right) = 0, \quad \forall\ c > 2\sqrt{\mu}. \tag{1.5}$$

This paper is organized as follows: in Section 2, we give the rigorous proof of Theorem 1.2. In Section 3, some numerical simulations are presented to investigate the dynamical behavior of the solution of (1.2).

§2 Main proof

The aim of this section is to provide a mathematically rigorous proof of Theorem 1.2. First of all, we give the global existence of the solution of (1.4) by the contraction mapping principle, as well as the classical argument.

***Proposition* 2.1** (*Existence*)**.** *For any $v_0 \in \mathcal{C}^+$, equation* (1.2) *admits a unique mild solution $v(t,x) = v(t)(x) \in C([-\tau, \infty), \mathcal{Y})$ satisfying*

$$0 \leq v(t,x) \leq e^{\mu t}\|v_0\|_{\mathcal{C}_0}, \quad \forall\ t > 0,\ x \in \mathbb{R}.$$

In particular, when $t > \tau$, the mild solution is classical.

Proof. We first prove that the solution of (1.2) exists locally. Let $v_0 \in C([-\tau, 0], \mathcal{Y}^+)$ be the given initial value. For $0 < T < \infty$, fix $K > 1$, then we define

$$\Gamma_T := \left\{ v(t,x) \in \mathcal{C}_T : \begin{array}{l} v(s,x) = v_0(s,x) \text{ for } (s,x) \in [-\tau, 0] \times \mathbb{R}; \\ 0 \leq v(t,x) \leq K||v_0||_{\mathcal{C}_0} \text{ for } (t,x) \in [0,T] \times \mathbb{R}. \end{array} \right\}$$

It is clear that Γ_T is a complete metric space with a distance induced by the norm $\|\cdot\|_{\mathcal{C}_T}$. Let $L > \frac{\mu}{\beta}$ be a constant. For any $v \in \Gamma_T$, define

$$G(v)(t,x) := v(t,x) \left(L + \mu \frac{(1 - v(t-\tau, x))}{1 + \beta v(t-\tau, x)} \right)$$

and

$$u(t,x) := e^{-Lt}(\Psi(t)v_0(\cdot))(x) + \int_0^t e^{-L(t-s)}(\Psi(t-s)G(v)(s,\cdot))(x)ds \tag{2.1}$$

for $t \in [0,T]$ and $x \in \mathbb{R}$. According to [2,16,24], the function $u \in C([0,T],\mathcal{Y})$ defined by **representation** (2.1) is a mild solution of the following Cauchy problem

$$\begin{cases} u_t(t,x) - \Delta u(t,x) + Lu(t,x) = G(v)(t,x), & \forall t \in (0,T],\ x \in \mathbb{R}, \\ u(0,x) = v_0(x), & \forall x \in \mathbb{R}. \end{cases} \tag{2.2}$$

Define a map $\mathcal{T} : \Gamma_T \to \mathcal{C}_T$ by $\hat{u}(t,x) := (\mathcal{T}v)(t,x)$, where $\hat{u}(s,x) := v_0(s,x)$ for $(s,x) \in [-\tau,0]\times\mathbb{R}$, $\hat{u}(t,x) := u(t,x)$ for any $(t,x) \in (0,T]\times\mathbb{R}$. Then we have the following two steps.

Step 1: *For sufficiently small $T > 0$, $\mathcal{T}(\Gamma_T) \subset \Gamma_T$.*

Obviously, $\hat{u} \in C([-\tau,T],\mathcal{Y}^+)$. For any $(t,x) \in (0,T]\times\mathbb{R}$, it follows from (2.1) and $G(v)(t,x) \le (L+\mu)v(t,x)$ that

$$\begin{aligned} \hat{u}(t,x) &\le \|v_0(0,\cdot)\|_X e^{-Lt} + e^{-Lt}\frac{1}{L}(e^{Lt}-1)(L+\mu)K\|v_0\|_{\mathcal{C}_0} \\ &\le \left(1 + \frac{1}{L}(L+\mu)K\left(e^{LT}-1\right)\right)\|v_0\|_{\mathcal{C}_0} \\ &\le K\|v_0\|_{\mathcal{C}_0}, \end{aligned}$$

provided that $T > 0$ is sufficiently small such that $\left(1 + \frac{1}{L}(L+\mu)K\left(e^{LT}-1\right)\right) \le K$. Therefore, $\mathcal{T}v \in \Gamma_T$.

Step 2: *For sufficiently small $T > 0$, $\mathcal{T} : \Gamma_T \to \Gamma_T$ is a contraction map.*

For any $v_1, v_2 \in \Gamma_T$, define $\hat{u}_1 = \mathcal{T}v_1$ and $\hat{u}_2 = \mathcal{T}v_2$. Then for any $(t,x) \in (0,T]\times\mathbb{R}$, by (2.1), we get

$$\begin{aligned} |\hat{u}_1(t,x) - \hat{u}_2(t,x)| &\le \int_0^t e^{-L(t-s)}(\Psi(t-s)\,|G(v_1)(s,\cdot) - G(v_2)(s,\cdot)|)(x)ds \\ &\le \int_0^t e^{-L(t-s)}\left(L+\mu+(1+\beta)(K\|v_0\|_{\mathcal{C}_0}+1)+\frac{\mu}{\beta}\right)\|v_1-v_2\|_{\mathcal{C}_T}ds \\ &\le \frac{1}{L}\left(e^{LT}-1\right)\left(L+\mu+(1+\beta)(K\|v_0\|_{\mathcal{C}_0}+1)+\frac{\mu}{\beta}\right)\|v_1-v_2\|_{\mathcal{C}_T}. \end{aligned}$$

Apparently, if T is sufficiently small, then there exists $0 < \rho < 1$ such that

$$\|\hat{u}_1 - \hat{u}_2\|_{\mathcal{C}_T} = \|\mathcal{T}v_1 - \mathcal{T}v_2\|_{\mathcal{C}_T} \le \rho\|v_1 - v_2\|_{\mathcal{C}_T}, \tag{2.3}$$

which indicates that $\mathcal{T}$ is a contraction map on Γ_T.

By the Banach's fixed point theorem, there exists a unique function $v \in \Gamma_T$, which is the fixed point of $\mathcal{T}$. Especially, $v \in \Gamma_T$ is the unique mild solution of (1.2) on $t \in (0,T]$ and satisfies

$$v(t,x) := e^{-Lt}(\Psi(t)v_0(\cdot))(x) + \int_0^t e^{-L(t-s)}(\Psi(t-s)G(v)(s,\cdot))(x)ds \tag{2.4}$$

for $(t,x) \in (0,T]\times\mathbb{R}$. This confirms the local existence of mild solution $v(t,x)$ of (1.2). To establish the global existence of mild solution $v(t,x)$ of (1.2), we can assume that $v(t,x)$ is the unique non-continuable mild solution of (1.2) on the maximal existence interval $[0,\sigma_{v_0})$ for

$\sigma_{v_0} > 0$. Apparently, $v \in C([-\tau, \sigma_{v_0}), \mathcal{Y}^+)$. Since $v(t,x)$ satisfies (2.4) for any $t > 0$, it follows that

$$v(t,x) \le e^{-Lt}(\Psi(t)v_0(\cdot))(x) + \int_0^t e^{-L(t-s)}(\Psi(t-s)(L+\mu)v(s,\cdot))(x)ds$$

for all $(t,x) \in (0, \sigma_{v_0}) \times \mathbb{R}$. Consider the function $v^+(t) := e^{\mu t}\|v_0\|_{\mathcal{C}_0}$, where $t \ge 0$. It is obvious that $v^+(t)$ satisfies equation $u_t = \Delta u(t,x) + \mu u(t,x)$ with the initial data $u(0,x) = \|v_0\|_{\mathcal{C}_0}$ for any $(t,x) \in (0,+\infty) \times \mathbb{R}$. By the comparison principle (see [16, Proposition 3]), we get that $v(t,x) \le v^+(t)$ for any $(t,x) \in [0, \sigma_{v_0}) \times \mathbb{R}$. Thus, $\sigma_{v_0} = +\infty$ and

$$v(t,x) \le e^{\mu t}\|v_0\|_{\mathcal{C}_0} \tag{2.5}$$

for any $(t,x) \in [0,+\infty) \times \mathbb{R}$. Furthermore, it is known that the mild solution $v(t,x)$ of (1.2) is a classical solution for $t > \tau$ [16, 24].

This completes the proof. □

Proof of Theorem 1.2:

Step 1: ***Proof of*** **(1.5).**

Suppose that $v_0(s,x) = 0$ for $|x| \ge l$, $s \in [-\tau, 0]$, where l is a positive constant. Since $v(t,x) \ge 0$ for all $t \ge -\tau$ and $x \in \mathbb{R}$, one has

$$\mu v(t,x)\left(\frac{1-v(t-\tau,x)}{1+\beta v(t-\tau,x)}\right) \le \frac{\mu v(t,x)}{1+\beta v(t-\tau,x)} \le \mu v(t,x), \quad (t,x) \in [0,+\infty) \times \mathbb{R}.$$

Suppose $w(t,x)$ with $w(0,x) = v_0(0,x)$ solve the equation $w_t(t,x) = w_{xx}(t,x) + \mu w(t,x)$, $t > 0$, $x \in \mathbb{R}$. It follows from the comparison principle that

$$0 \le v(t,x) \le w(t,x) := \frac{e^{\mu t}}{\sqrt{4\pi t}} \int_{-l}^{l} e^{-(x-y)^2/(4t)} v_0(0,y)dy, \quad \forall t \ge 0, \ x \in \mathbb{R}. \tag{2.6}$$

Let $c > 2\sqrt{\mu}$, we then obtain

$$0 \le v(t,x) \le \frac{e^{\mu t}\|v_0\|_{\mathcal{C}_0}}{\sqrt{4\pi t}} \int_{-l}^{l} e^{-(ct-L)^2/(4t)} dy, \ \forall \ t \ge \frac{l}{c}, \ |x| \ge ct,$$

which further yields (1.5). It is noteworthy that, because $v(t,x)$ is continuous, non-negative and converges to 0 at $\pm\infty$ (see (2.6)), the maximum of $v(t,x)$ on $(-\infty, -ct] \cup [ct, +\infty)$ can be reached.

Step 2: *The proof of* (1.4).

For any $(t,x) \in [2\tau, \infty) \times \mathbb{R}$, we have

$$\begin{aligned} v_t(t,x) &= v_{xx}(t,x) + \mu v(t,x)\left(\frac{1-v(t-\tau,x)}{1+\beta v(t-\tau,x)}\right) \\ &= v_{xx}(t,x) + \mu h(t,x)v(t,x). \end{aligned}$$

Since $h(t,x) := \frac{1-v(t-\tau,x)}{1+\beta v(t-\tau,x)}$ is continuous in $(t,x) \in [2\tau, \infty) \times \mathbb{R}$, $\mu|h(t,x)| \le \frac{\mu(1+\beta)}{\beta}$ for any $(t,x) \in [2\tau, \infty) \times \mathbb{R}$ and $v(t,x) > 0$ for any $(t,x) \in [2\tau, \infty) \times \mathbb{R}$. In order to apply the Harnack inequalities in [5, Lemma 3.8], we first take $t^* \ge 3\tau$, $x^* \in \mathbb{R}$, then choose $\theta = \frac{\tau}{3}$. Let $\Omega = (x^*-3, x^*+3)$, $U = (x^*-2, x^*+2)$ and $D = (x^*-1, x^*+1)$, in addition, to avoid the confusion, we denote τ in [5, Lemma 3.8] as $\hat{\tau}$, and let $\hat{\tau} = t^* - \frac{4}{3}\tau$, then there exists a constant

$\gamma > 0$ which is independent of t^* and x^*, such that

$$0 < \sup_{[t^*-\tau, t^*-\frac{2\tau}{3}]\times[x^*-1,x^*+1]} v(s,y) \le \gamma \inf_{[t^*-\frac{\tau}{3}, t^*]\times[x^*-1,x^*+1]} v(s,y).$$

Due to the arbitrariness of t^* and x^*, it holds

$$0 < v(t-\tau, x) \le \gamma v(t,x), \ \forall\, t > 3\tau, \ x \in \mathbb{R}.$$

Let $g(\varpi) = \frac{1-\varpi}{1+\beta\varpi}$. Since $g(\varpi)$ is decreasing with respect to $\varpi \in [0,\infty)$, we could obtain

$$\begin{aligned} v_t(t,x) &= v_{xx}(t,x) + \mu v(t,x)\left(\frac{1-v(t-\tau,x)}{1+\beta v(t-\tau,x)}\right) \\ &\ge v_{xx}(t,x) + \mu v(t,x)\left(\frac{1-\gamma v(t,x)}{1+\beta\gamma v(t,x)}\right), \ \forall\, t > 3\tau, \ x \in \mathbb{R}. \end{aligned}$$

Let $z(t,x)$ with $z(3\tau,x) = v(3\tau,x)$ solve

$$z_t(t,x) = z_{xx}(t,x) + \mu z(t,x)\left(\frac{1-\gamma z(t,x)}{1+\beta\gamma z(t,x)}\right), \ \forall\, t > 3\tau, \ x \in \mathbb{R}. \tag{2.7}$$

Clearly, $v(t,x)$ is a upper solution for equation (2.7). Besides, equation (2.7) admits two equilibria $z = 0$ and $z = \gamma^{-1}$. It follows from the classical theory of asymptotic spreading in [1, §4] that the solution of (2.7) satisfies

$$\liminf_{t\to\infty} \min_{|x|\le ct} z(t,x) \to \gamma^{-1} \quad \text{for all } 0 \le c < 2\sqrt{\mu}.$$

Then by the comparison principle, we get

$$\liminf_{t\to\infty} \min_{|x|\le ct} v(t,x) \ge \limsup_{t\to\infty} \max_{|x|\le ct} z(t,x) \ge \liminf_{t\to\infty} \min_{|x|\le ct} z(t,x) = \gamma^{-1}$$

for any $0 \le c < 2\sqrt{\mu}$.

This completes the proof. □

§3 Numerical simulations

In this section, the numerical simulations are demonstrated by the finite difference scheme. Precisely, we consider the case of a finite spatial domain with homogeneous Neumann boundary condition, which models a closed environment with reflecting boundaries, i.e., the individual cannot leave the domain. For equation (1.2), we choose $\mu = 1$, $x \in [-100, 100] := \Omega$, $t \in [0, 250]$, that is,

$$\begin{cases} v_t(t,x) = v_{xx}(t,x) + v(t,x)\left(\dfrac{1-v(t-\tau,x)}{1+\beta v(t-\tau,x)}\right), \\ v_x(-100,t) = v_x(100,t) = 0, \\ v(s,x) = v_0(s,x), \quad s \in [-\tau, 0]. \end{cases} \tag{3.1}$$

The initial condition is

$$v_0(s,x) = \begin{cases} 1, & x \in [-\alpha,\alpha], \\ 0, & x \in \Omega\backslash[-\alpha,\alpha], \end{cases}$$

where $\alpha > 0$. We then use the finite difference method to discretize (3.1). Define the uniform partition of the domain $\Omega = [-100, 100]$ by:

$$-100 = x_1 < x_2 < x_3 < \cdots < x_{n-1} < x_n < x_{n+1} = 100,$$

where $h = 200/n = 0.2$ and $x_i = x_1 + (i-1) \cdot h$, $i = 1, 2, \cdots, n+1$. The time domain $[0, 250]$ is treated as $k = 250/m = 0.02$, $t_j = t_1 + (j-1) \cdot k$, $j = 1, 2, \cdots, m+1$. The following are numerical simulations:

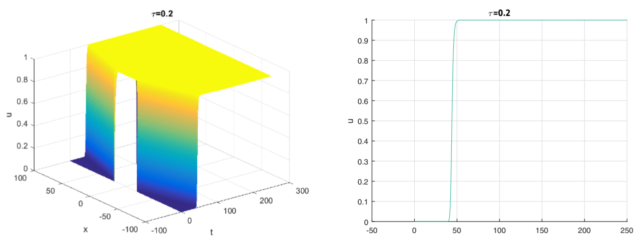

Figure 1: Choose $\alpha = 20$, $\beta = 0.5$ and $\tau = 0.2$. The left is the 3D-graph of $v(t, x)$ and the right is the graph of $v(t, 100)$. The solution approaches positive steady states.

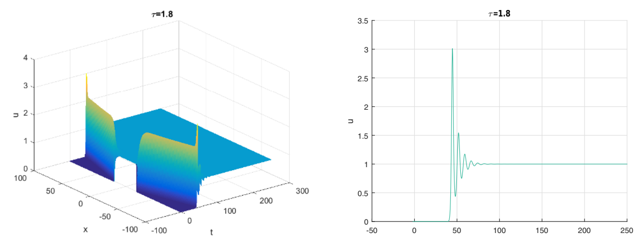

Figure 2: Choose $\alpha = 20$, $\beta = 0.5$ and $\tau = 1.8$. The left is the 3D-graph of $v(t, x)$ and the right is the graph of $v(t, 100)$. The solution approaches positive steady states but with oscillations.

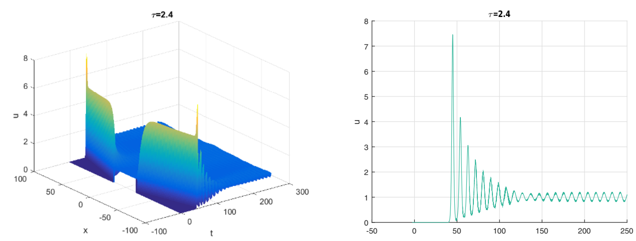

Figure 3: Choose $\alpha = 20$, $\beta = 0.5$ and $\tau = 2.4$. The left is the 3D-graph of $v(t, x)$ and the right is the graph of $v(t, 100)$. The solution is oscillatory and eventually converges to a time-periodic solution.

According to the figures given above, as time increases, for small τ, the nonnegative steady sates for (3.1) are $v \equiv 0$ and $v \equiv 1$; for sufficiently large τ, the steady $v \equiv 1$ becomes unstable, the non-constant bounded steady sates may exist. In particular, if we choose $\tau = 0.2$ and $\tau = 1.8$, the solution finally connects the equilibria 0 to 1. When choosing $\tau = 2.4$, as time increases, there exists an oscillating solution, but the solution still has upper and lower bounds and it seems to be a spatially uniform temporally periodic solution. Note that, for the rigorous proof of the existence of the time periodic solution of the delayed food-limited model, readers can refer to [20,21,27]. Especially, Su et al. [20,21] investigated the spatial structure of system (3.1) with Dirichlet boundary conditions, and showed that the solution tends to a stable spatial nonhomogeneous time-periodic solution when delay τ is large enough.

Moreover, the simulations support our theoretical results, i.e., Theorem 1.2. As we can see from figure 1 and 2, there hold

(i) $\lim_{t\to+\infty,\ |x|\geq ct} v(t,x) = 0,\ \forall\ c > 2;$

(ii) $\liminf_{t\to+\infty,\ |x|\leq ct} v(t,x) = 1,\ \forall\ 0 < c < 2.$

In figure 3, there exists a constant $\kappa > 0$ such that

(i) $\lim_{t\to+\infty,\ |x|\geq ct} v(t,x) = 0,\ \forall\ c > 2;$

(ii) $\liminf_{t\to+\infty,\ |x|\leq ct} v(t,x) > \kappa,\ \forall\ 0 < c < 2.$

It follows from these simulations that the spreading speed c^* of solution of (3.1) is equal to 2, which is the same as the conclusion of Theorem 1.2.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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