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Analysis of the selective traveling salesman problem with time-dependent profits

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Abstract

We consider a generalization of the selective traveling salesman problem (STSP) in which the benefit of visiting a location changes over time. This new problem, called the selective travelling salesman problem with time-dependent profits (STSP-TDP), is defined on a graph with time-dependent profits associated with the vertices, and consists of determining a circuit of maximal total profit. In the STSP-TDP the tour length must not exceed a maximum value, and its starting and ending times must both lie within a prespecified planning horizon. This problem arises in planning tourist itineraries, mailbox collection, military surveillance, and water sampling, where the traveler accumulates different profits upon visiting the locations throughout the day. We focus on analyzing several variants of the problem depending on the shape of the time-dependent profit function. If this function is not monotonic, it may be worth visiting a site more than once. We propose formulations for the single-visit case and for when multiple visits are allowed, in which case the problem reduces to an STSP, which is adapted to be solved as a longest path problem. These formulations are then solved for piecewise-linear profit functions using a general-purpose solver, and tested on several artificially created instances and on four TSPLib instances involving up to 535 vertices. A detailed analysis of the problem and the solution is performed.

Keywords Selective traveling salesman problem · Time-dependent profits · Multiple visits

Mathematics Subject Classifications 90B06 · 90C10 · 90C05 · 90C27

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1 Introduction

In several contexts, a traveler holds a list of potential sites of interest, but it may not be possible nor desirable to visit all of them. Some sites will be selected and then visited by means of a single tour. With each solution is associated a profit derived from visiting the sites. One example of this setting occurs in the planning of tourist trips. An interesting application cited in Vansteenwegen et al. (2011) is the Mobile Tourist Guide of Souffriau et al. (2008). Similar tourist trip problems are mentioned by Wang et al. (2008); Schilde et al. (2009) , and da Silva et al. (2018) where the profit is measured in terms of the interest associated with visiting the sites. Another example arises in the collection of letters from mailboxes. Here the benefit can be measured by the mail volume accumulated in the mailbox at each visit. Other applications are encountered in the surveillance activities of submarines or unmanned aircraft (Wang et al. 2008), and in water sampling Zhang et al. (2020). A recent application arises in the context of a parking warden distributing fines for irregularly parked cars (Bruglieri (2020)), where the profit associated with an inspection depends on the time elapsed since the previous inspection.

This problem is known as the selective traveling salesman problem (STSP) Laporte and Martello (1990), sometimes called orienteering problem (Golden et al. 1987; Vansteenwegen et al. 2011), if the profits are time-independent. However, there also exist several settings where the profits are time-dependent. For example, the pleasure associated with a tourist attraction is often related to crowdiness and, therefore, to the time of the day. In postal applications it is clear that mail accumulation is time-dependent. In military applications the profit associated with surveillance activities may vary over time. Hence, we study an extension of the STSP with time-dependent profits (STSP-TDP).

We assume that the problem is defined over a given time horizon and that the sites are to be visited by a single uncapacitated vehicle during a work shift whose length does not exceed that of the planning horizon. Waiting before starting the tour is allowed, i.e., the shift does not necessarily start at the beginning of the planning horizon. Each site has a time window and a service time, and it may be beneficial for the vehicle to wait along its route to let the profit increase at some sites, as is the case of the mail collection application. As we will describe, in some versions of the problem it may be beneficial to visit the same site more than once.

For reviews of the STSP the interested reader is referred to Feillet et al. (2005), Vansteenwegen et al. (2011), Archetti et al. (2014) and Gunawan et al. (2016). Of interest in the context of the STSP-TDP is the survey of Gavalas et al. (2014) which presents a detailed description of the tourist trip design problem. These authors observe that this problem can be viewed as a bicriteria generalization of the TSP, i.e., maximizing the collected profit and minimizing the travel cost. Yu et al. (2019b) investigate the team orienteering problem with time windows and time-dependent scores. In this work, each vertex is associated with a basic score and several recommendation factors that vary depending on the time of visit. The authors propose a mathematical formulation and a hybrid artificial bee colony algorithm to compute optimal solutions for small instances and near-optimal solutions for larger instances.

There also exists a rich literature on several variants of the STSP-TDP studied in this paper, which is summarized in Table 1 but, to the best of our knowledge, our variant is unique and we are the first to provide a formulation solved by means of an exact branch-and-cut algorithm capable of solving instances much larger than was previously achievable. In our version of the STSP-TDP, the starting time of the route is also optimized within the planning horizon and waiting at the sites is allowed in order to arrive at a site when the profit to be collected is optimal. Here we introduce, model, and solve this problem exactly. We perform tests on artificial instances and on TSPLib instances, and we derive insights based on the problem characteristics and on the computational results.

The remainder of this paper is organized as follows. In Sect. 2, we formally describe the problem and a number of its variants, and we propose corresponding mathematical programming models. An analysis of some properties of the problem depending on the shape of the profit functions is presented in Sect. 3. This is followed by computational results in Sect. 4 and by conclusions in Sect. 5.

2 Formal definition and mathematical models

The STSP-TDP is defined on a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V}$ is the vertex set, $0 \in \mathcal{V}$ is the depot and $\mathcal{V}' = \{1, \dots, n\}$ is the set of sites. The arc set is defined as $\mathcal{A} = \{(i,j) : i,j \in \mathcal{V}, i \neq j\}$. A travel time τ_{ij} is associated with each arc (i, j). The problem is defined over a planning horizon of length T, and time can be discretized into a set of moments $t \in \mathcal{T} = \{1, \dots, T\}$. Each site i has an associated profit $p_i(t)$ that depends on t. A solution is a closed circuit not exceeding L time units ($L \leq T$), leaving the depot at time t_0^d, returning to the depot at time $t_0^a \leq T$ and visiting a subset of the sites, where $t_0^a - t_0^d \leq L$. The goal of the problem is to find a solution of maximal total profit for the selective traveling salesman problem with time-dependent profits (STSP-TDP). We assume that there are no vehicle capacity constraints and that one of the following two time-dependent profits is considered:

1. Profit is cumulative, i.e., the profit function is monotonically increasing. The later a vertex is visited, the higher is the collected profit. It follows that if a vertex is visited several times, one can achieve the total profit by performing only the last visit. An example of this setting is letter collection in mailboxes.
2. Profit depends just on the time the vertex is visited. The profit function is not necessarily monotonically increasing. Then it may be beneficial to visit a vertex more than once. Examples include parking tickets, publicity delivery, or fund collection by humanitarian organizations.

It follows from the second profit function type that a site may be visited more than once, and in both cases imposing a waiting time between successive visits to the same site may be appropriate since the collected profit is the one observed upon arrival.

Table 1 Variants of the STSP-TDP

References	Length limit on tours	Time windows	Profit function	Multiple visits each visit	Collect all profit	Number of vehicles	Exact algorithm (largest size)	Heuristic
Erkut and Zhang (1996)	Yes	No	Decreasing	No	No	1	Branch-and-bound (20)	Greedy
Erdoğan and Laporte (2013)	Yes	No	Service time dependent	Yes	Yes	1	Branch-and-cut (200)	–
Afsar and Labadie (2013)	Yes	No	Decreasing	No	No	≥ 1	Column generation (100)	Evolutionary local search
Yu et al. (2019b)	Yes	Yes	Arbitrary	No	No	≥ 1	–	Bees
Yu et al. (2019a)	Yes	No	Service time dependent	No	No	1	–	Matheuristic
Yu et al. (2021)	Yes	Deadlines	Decreasing	No	No	≥ 1	Branch-and-cut (200)	Modified coordinate search
This paper	Yes	Yes	Increasing or arbitrary	Yes	Yes	1	Branch-and-cut (535)	–

2.1 At most one visit per site allowed

We first model the case in which at most one visit to each site is allowed. The formulation we propose requires that the profit functions be linear or piecewise linear, with time as a dependent variable. In this case, a time discretization is not required.

We define variables x_{ij} equal to one if and only if arc $(i,j) \in \mathcal{A}$ is used in the solution, and variables t_i equal to the time at which vertex $i \in \mathcal{V}'$ is visited. In what follows, M is a large positive number. The problem for the continuous single-visit case (SF) is then formulated as follows:

$$(SF) \quad \text{maximize} \sum_{i \in \mathcal{V}'} p_i(t_i) \tag{1}$$

subject to

$$\sum_{j \in \mathcal{V}, j \neq i} x_{ij} = \sum_{j \in \mathcal{V}, j \neq i} x_{ji} \leq 1 \quad i \in \mathcal{V}' \tag{2}$$

$$\sum_{j \in \mathcal{V}'} x_{0j} = \sum_{i \in \mathcal{V}'} x_{i0} = 1 \tag{3}$$

$$t_0^a - t_0^d \leq L \tag{4}$$

$$t_j \geq t_i + \tau_{ij} - M(1 - x_{ij}) \quad i,j \in \mathcal{V}', i \neq j \tag{5}$$

$$t_j \geq t_0^d + \tau_{0j} - M(1 - x_{0j}) \quad j \in \mathcal{V}' \tag{6}$$

$$t_0^a \geq t_j + \tau_{j0} - M(1 - x_{j0}) \quad j \in \mathcal{V}' \tag{7}$$

$$t_i \in [0, T] \quad i \in \mathcal{V}' \tag{8}$$

$$x_{ij} \in \{0, 1\} \quad (i,j) \in \mathcal{A}. \tag{9}$$

The objective function (1) represents the total collected profit to be maximized. Constraints (2) are the flow conservation conditions and ensure that there is at most one incoming and one outgoing arc for each site. Constraint (3) ensures that there is one incoming and one outgoing arc for the depot. Constraint (4) imposes a maximal route duration of L (time units). Constraints (5)–(7) ensure that travel times are respected and eliminate subtours. Finally, constraints (8) and (9) define the domains of the variables.

Note that the case of "exactly one visit" is a particular case arising in the traveling salesman problem with time-dependent profits (TSP-TDP), and hence, its formulation is obtained by just changing "≤ 1" with "$= 1$" in constraints (2).

We can impose the following bounds on the timing variables, which indicate that one cannot arrive at vertex i earlier than τ_{0i}:

$$t_i \geq \tau_{0i} \quad i \in \mathcal{V}'. \tag{10}$$

A similar reasoning can be applied at the end of the planning horizon: one can compute the latest arrival time at vertex i such that one still arrives back at the depot at time T. The following upper bound on t_i can be imposed:

$$t_i \leq T - \tau_{i0} \quad i \in \mathcal{V}'. \tag{11}$$

2.2 Multiple visits to sites allowed

There are two main ways to model problems in which multiple visits to the same site are allowed. Some models do not replicate the sites and allow multiple visits by controlling the number of times a site is visited (see, e.g., Bruck and Iori 2017; Salazar-González and Santos-Hernández 2015; Erdoğan and Laporte 2013, and Munari and Savelsbergh 2022). The other approach is to make several copies of the same site (see, e.g., Gribkovskaia et al. 2007 and Hoff et al. 2009). Here we adopt the second modeling technique. More specifically, we make use of an auxiliary two-dimensional directed graph $\overline{\mathcal{G}} = (\overline{\mathcal{V}}, \overline{\mathcal{A}})$ (see Fig. 1) whose x-axis represents the periods of the planning horizon and whose y-axis represents the sites and the depot. The graph $\overline{\mathcal{G}}$ extends the graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, so that for every vertex $v \in \mathcal{V}$ and every period $t \in \mathcal{T}$, there is one vertex $\overline{v} \in \overline{\mathcal{V}}$ identified with the pair (t, v), which is labelled as $\overline{v} = t \cdot |\mathcal{V}| + v$. It follows then that, for each given vertex $\overline{v} \in \overline{\mathcal{V}}$ we can obtain the corresponding period t and site v in graph $\mathcal{G}$ as follows: $t(\overline{v}) = \lfloor \overline{v}/|\mathcal{V}| \rfloor$ and $v(\overline{v}) = \overline{v}/|\mathcal{V}| - \lfloor \overline{v}/|\mathcal{V}| \rfloor$. We denote by $t(i)$ and $v(i)$ the time $t \in \mathcal{T}$ and the vertex $v \in \mathcal{V}$ corresponding to the vertex $i \in \overline{\mathcal{V}}$,

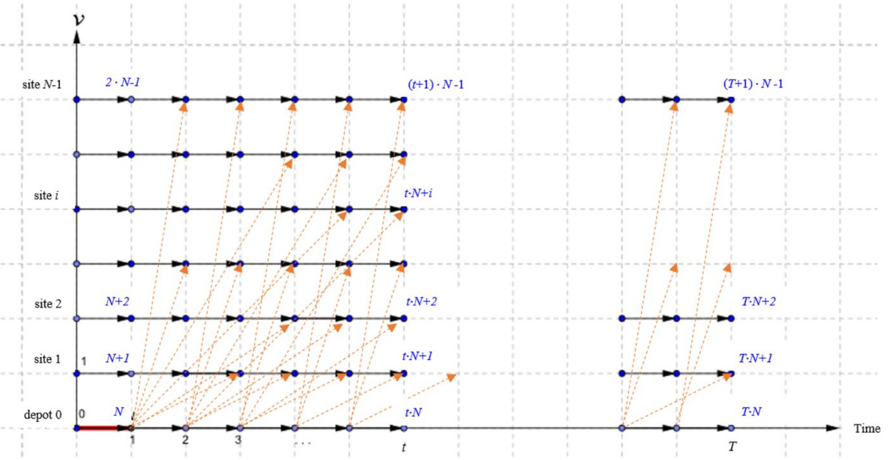


Fig. 1 Auxiliary graph $\overline{\mathcal{G}} = (\overline{\mathcal{V}}, \overline{\mathcal{A}})$, where $|\mathcal{V}| = N$

respectively. Therefore, the profit function at vertex $\overline{v}$ is $\overline{p}_{\overline{v}} = p_v(t)$. Note that not all arcs are allowed in $\overline{\mathcal{G}}$ and that the starting and ending vertices must correspond to the depot. After predefining the set $\overline{\mathcal{A}}$, we need to solve a selective TSP with profit on $\overline{\mathcal{G}}$, with a maximum route length L no larger than the duration T of the planning horizon. We have then no predefined starting and ending vertices, but these must be selected within a subset $\{t \cdot |\mathcal{V}| \, : \, t = 0, 1, ..., T\} \subset \overline{\mathcal{V}}$ representing the set of depot vertices (T copies of the depot). The set of site vertices is $\overline{\mathcal{V}}' = \overline{\mathcal{V}} \setminus \{t \cdot |\mathcal{V}| \, : \, t = 0, 1, ..., T\}$.

The set of available arcs $\overline{\mathcal{A}}$ is composed of the set $\overline{\mathcal{A}_1}$ of arcs linking vertices from the same site at consecutive periods (for waiting times), and the set $\overline{\mathcal{A}_2}$ of arcs linking vertices from different sites and whose difference between their periods equals the travel time between these sites. Therefore,

$$\overline{\mathcal{A}_1} = \{(i,j) \mid \big((v(i) = v(j)) \wedge (t(j) = t(i) + 1)\big)\},$$

$$\overline{\mathcal{A}_2} = \{(i,j) \mid \big((v(i) \neq v(j)) \wedge (t(j) - t(i) = \tau_{ij})\big)\},$$

and $\overline{\mathcal{A}} = \overline{\mathcal{A}_1} \cup \overline{\mathcal{A}_2} = \{(i,j) \mid \big((v(i) = v(j)) \wedge (t(j) = t(i) + 1)\big) \vee \big((v(i) \neq v(j)) \wedge (t(j) - t(i) = \tau_{ij})\big)\}$.

Note that Fig. 1 shows a representation of the horizontal arcs from the set $\overline{\mathcal{A}_1}$ and a partial representation of the set $\overline{\mathcal{A}_2}$ (dashed arcs). For the sake of clarity, from the latter we only represent arcs departing from the depot.

Note that if $L = T$, the problem over $\overline{\mathcal{G}}$ is very similar to the STSP with profits (see Laporte and Martello 1990 and Vansteenwegen et al. 2011). In the extended graph $\overline{\mathcal{G}} = (\overline{\mathcal{V}}, \overline{\mathcal{A}})$, the problem is similar to the STSP, where we have a set of $|\overline{\mathcal{V}}| = N \cdot (T + 1) - 1$ vertices, each with an associated profit.

The difference between our problem in the auxiliary graph and the STSP is that, in our case, the starting and end vertices are not fixed, but belong to the set of depot vertices with zero profit. In order to obtain the STSP, we add two artificial vertices B_1 and B_2, which will be the starting and ending vertices of the Hamiltonian path, respectively. Accordingly, a set of zero length arcs joining B_1 and the depot vertices and the latter with B_2 is also added. So, from now on, the problem is defined on $\hat{G} = (\hat{V}, \hat{\mathcal{A}})$, where $\hat{V} = \overline{\mathcal{V}} \cup \{B_1, B_2\}$ and $\hat{\mathcal{A}} = \overline{\mathcal{A}} \cup \{(i, B_2) | i \in \overline{\mathcal{V}} \setminus \overline{\mathcal{V}}'\} \cup \{(B_1, i) | i \in \overline{\mathcal{V}} \setminus \overline{\mathcal{V}}'\}$.

We use directed continuous variables y_{ij} representing the flow over arc (i, j) and binary variables γ_i equal one if the shift starts at vertex $i \in \{0, ..., T - L\}$. Note that the graph $\hat{G}$ is a time-ordered graph, which enables us (for fixed depot departure and arrival times t_0^d and t_0^a, being $t_0^a - t_0^d = L$) to model the problem as a longest path problem. In doing so, we transfer the profit associated with a vertex j to its incoming arcs in $\overline{\mathcal{A}_2}$. That is, each arc $(i,j) \in \overline{\mathcal{A}_2}$ has an associated profit $\overline{p}_{ij} = \overline{p}_j$. The profit associated with the remaining arcs in $\hat{\mathcal{A}}$ equals zero. The problem for the discretized multiple-visit case (MF) over $\hat{G}$ is formulated as follows:

$$(MF) \quad \text{maximize} \sum_{(i,j) \in \hat{\mathcal{A}}} \overline{p}_{ij} \cdot y_{ij} \tag{12}$$

subject to

$$\sum_{j\in\delta^+(i)} y_{ij} - \sum_{j\in\delta^-(i)} y_{ji} = \begin{cases} 1 & \text{if } i = B_1 \\ 0 & \text{if } i \neq B_1, B_2 \\ -1 & \text{if } i = B_2 \end{cases} \tag{13}$$

$$y_{Ai} \leq \gamma_{t(i)}, \quad i : v(i) = 0 \text{ and } t(i) \in \{0, ..., T - L\} \tag{14}$$

$$y_{i+L,B} \leq \gamma_{t(i)}, \quad i : v(i + L) = 0 \text{ and } t(i) \in \{0, ..., T - L\} \tag{15}$$

$$\sum_{i\in\{0,...,T-L\}} \gamma_i = 1 \tag{16}$$

$$\gamma_i \in \{0, 1\} \quad i \in \{0, ..., T - L\} \tag{17}$$

$$y_{ij} \geq 0 \quad (i,j) \in \hat{\mathcal{A}}. \tag{18}$$

The objective function (12) is the total collected profit. Recall that the profit is collected when the vehicle arrives at the vertex. Constraints (13), where $\delta^+(i)$ and $\delta^-(i)$ are the sets of successors and predecessors of vertex i in graph $\hat{G}$, respectively, ensure that exactly one arc departs from the artificial vertex A to the set of depot vertices, that one unit of flow arrives at B from the same set, and enforce the flow conservation conditions for the remaining vertices, which are the site vertices. Constraints (14) and (15) impose a maximal route duration by means of the γ variables. There are as many γ_{y_L} variables as $T - L$ time periods. By selecting only one of the γ_{y_L} variables, constraints (16) ensure that the route starts at one time instant and ends after exactly L periods. Finally, constraints (17) and (18) impose the binary condition of the γ variables and the non-negativity condition of the y variables, respectively.

3 Analysis of the problem and shapes of the profit function

As explained in Sect. 2, depending on the shape of the profit function, the problem possesses certain characteristics. These will not only be influenced by the monotonicity condition of the profit functions, but also by their specific shape.

Observation 1. Let $p_i : \mathcal{T} \rightarrow \mathbb{R}$ represent the accumulated profit at vertex $i \in \mathcal{V}'$ arising from a positive non-cumulative profit function $p'_i : \mathcal{T} \rightarrow \mathbb{R}^+$, that is, the accumulated profit at vertex i during the time interval $[0, t]$ is calculated as $p_i(t) = \int_0^t p'_i(t)dt$. Then, p_i is a monotonically increasing function and the total profit collected visiting vertices i at time t_1 and $t_2 \in \mathcal{T}$ ($t_1 < t_2$) equals the profit collected by visiting vertices i only once at time t_2.

Proof If vertex $i \in \mathcal{V}'$ is visited at time units t_1 and $t_2 \in \mathcal{T}$, the total collected profit equals $\int_0^{t_1} p'_i(t)dt + \int_{t_1}^{t_2} p'_i(t)dt = \int_0^{t_2} p'_i(t)dt$.

If vertex $\mathcal{V}'$ is visited only once at time unit t_2, the total collected profit equals $p_i(t_2) = \int_0^{t_2} p_i'(t)dt$. □

Observation 2. If $p_i(t) = \alpha_i$ for all $i \in \mathcal{V}'$, $\alpha_i \geq 0$, then the STSP-TDP when at most one visit per site is allowed obviously becomes the STSP. In this case, it does not matter whether $L = T$ or $L < T$ since the collected profit is not time dependent.

Observation 3. Let us assume that L is large enough to visit all vertices in any order and that each site can be visited at most once (following formulation (SF) presented in Sect. 2.1). If all profit functions are linear, i.e., $p_i(t) = \alpha_i t$ for all $i \in \mathcal{V}'$, $\alpha_i \geq 0$, $\alpha_i \leq \alpha_{i+1}$, and $\tau_{0,i} \geq \tau_{0,i+1}$ for all $i \in \mathcal{V}' \setminus \{N\}$, then $x_{i,i+1} = 1$ for all $i \in \mathcal{V}' \setminus \{N, N-1\}$ and $x_{N0} = 1$. $x_{i,j} = 0$ for all $j \neq i+1$, $i \in \mathcal{V}' \setminus \{N, N-1\}$. That is, under the described conditions, if the vertices are labeled in increasing order with respect to the slope ($\alpha_i \leq \alpha_{i+1}$), the visiting order coincides with the labeling order.

Proof Given two vertices i and $i+1 \in \mathcal{V}'$ satisfying the above mentioned conditions, it is always beneficial to first visit the one with the lower slope (in this case i since $\alpha_i \leq \alpha_{i+1}$).

Let $p(i,\ j,\ t)$ be the profit collected at vertices i and j if vertex i is visited at time t and vertex j at time $t + \tau_{ij}$. Then, starting from vertex 0, the collected profit obtained by visiting first vertex i and then vertex $i+1$ is $p(i, i+1, \tau_{0,i}) = \alpha_i \tau_{0,i} + \alpha_{i+1}(\tau_{0,i} + \tau_{i,i+1})$. The collected profit if vertex $i+1$ is visited before vertex i is $p(i+1, i, \tau_{0,i+1}) = \alpha_{i+1}\tau_{0,i+1} + \alpha_i(\tau_{0,i+1} + \tau_{i+1,i})$.

Let us demonstrate that the collected profit is larger if vertex i is visited before vertex $i+1$, that is, $p(i, i+1, \tau_{0,i}) \geq p(i+1, i, \tau_{0,i+1})$:

$$p(i, i+1, \tau_{0,i}) = \alpha_i \tau_{0,i} + \alpha_{i+1}(\tau_{0,i} + \tau_{i,i+1}) = \alpha_i \tau_{0,i} + \alpha_{i+1}\tau_{0,i} + \alpha_{i+1}\tau_{i,i+1},$$
$$p(i+1, i, \tau_{0,i+1}) = \alpha_{i+1}\tau_{0,i+1} + \alpha_i(\tau_{0,i+1} + \tau_{i+1,i}) = \alpha_{i+1}\tau_{0,i+1} + \alpha_i \tau_{0,i+1} + \alpha_i \tau_{i+1,i}.$$

By *reductio ad absurdum*, if $p(i, i+1, \tau_{0,i}) < p(i+1, i, \tau_{0,i+1})$, then

$$\alpha_i \tau_{0,i} + \alpha_{i+1}\tau_{0,i} + \alpha_{i+1}\tau_{i,i+1} < \alpha_{i+1}\tau_{0,i+1} + \alpha_i \tau_{0,i+1} + \alpha_i \tau_{i+1,i}.$$

Since $\tau_{i+1,i} = \tau_{i,i+1}$, the inequality can be written as follows:

$$\tau_{i,i+1}(\alpha_{i+1} - \alpha_i) < (\alpha_{i+1} + \alpha_i)(\tau_{0,i+1} - \tau_{0,i}),$$

which is not respected given that $\tau_{i,i+1}$, $\alpha_{i+1} - \alpha_i$, and $\alpha_{i+1} + \alpha_i$ are positive values and $\tau_{0,i+1} - \tau_{0,i} \leq 0$. Therefore, the left-hand side of the inequality is positive and the right-hand side is negative. □

Observation 4. The STSP-TDP can be viewed as a generalization of the STSP with time windows by setting the profit function as a stepwise function equal to a negative number outside the time window, and equal to a positive constant during the time window interval. That is, if

$$p_i(t) = \begin{cases} -\beta & \text{if } t < a_i \\ \alpha_i & \text{if } a_i \leq t \leq b_i \\ -\beta & \text{if } b_i < t \end{cases}$$

for all $i \in \mathcal{V}'$, $\alpha_i \geq 0$, $\beta \geq 0$, then the STSP-TDP becomes the STSP with time windows $[a_i, b_i]$ for all $i \in \mathcal{V}'$. Vertex i would not be visited outside the time window since this yields a negative profit and a corresponding decrease of the objective function (total collected profit).

Observation 5. A new problem called the STSP-TDP with time windows, $[a_i, b_i]$ for all $i \in \mathcal{V}'$, can be considered if the profit function has the following shape:

$$p_i(t) = \begin{cases} -\beta & \text{if } t < a_i \\ f_i(t) & \text{if } a_i \leq t \leq b_i \\ -\beta & \text{if } b_i < t \end{cases}$$

for all $i \in \mathcal{V}'$, $\beta \geq \alpha_i \geq 0$, with $f_i(t) \geq 0$ being the profit function associated with each vertex $i \in \mathcal{V}'$ during the time interval $[a_i, b_i]$ in which the vertex may be visited. Since the objective function is to maximize the total collected profit, negative values of this profit outside its time window $[a_i, b_i]$ ensure that vertex $i \in \mathcal{V}'$ will not be visited outside the time window.

Note that Observations 4 and 5 may apply for both the single- and multiple-visit cases.

4 Computational results

When the profit functions are linear or piecewise linear functions of time, the formulation (*SF*) presented in Sect. 2.1 can be directly applied and there is no need to discretize the time. In the following, we perform two main types of experiments to provide insights into the models: first for artificially created instances defined on a mesh network, and second for benchmark instances from the TSPLib, assigning time-dependent profits to each vertex. These experiments were performed on a machine equipped with an Intel i7-8700 3.20GHz CPU. The model of Sect. 2.1 was implemented in Python 3.8 and solved with Gurobi version 9.1.

4.1 Mesh instances

The first set of experiments is designed to illustrate the behavior of the models when changing the maximum length of routes L and the duration of the planning horizon T, as well as to assess the solver's performance for different instance sizes. We have considered three instances sizes of 15, 30, and 50 vertices. In all the experiments, the network is constructed following a concentric clockwise labelling procedure, starting from vertex number 1, which acts as the depot and increasing the vertex number consecutively (see Fig. 2 for an illustration of the 15-vertex network). The horizontal

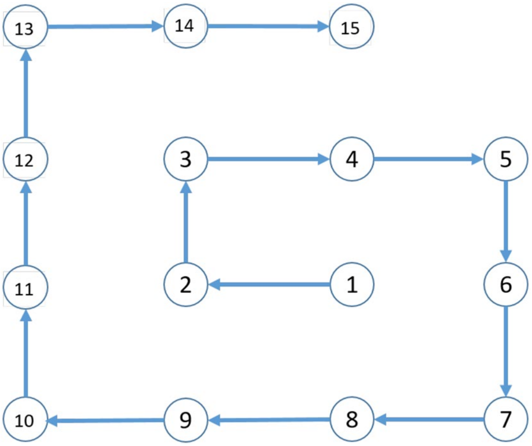


Fig. 2 Mesh network construction

and vertical distances between vertices are set to 1 and the arcs travel times have been obtained considering a travel speed equal to a tenth of the total distance needed to visit in order all the vertices, plus the Euclidean distance from the last vertex to the depot, ensuring there is no time to visit all vertices in any of the considered instances.

With respect to the piecewise linear profit functions, for each vertex we consider a non-cumulative one-peak profit function with the following shape (see Fig. 3):

$$p_i(t) = \begin{cases} a_i \cdot t & \text{if } t \leq t_i^* \\ b_i \cdot t + t_i^* \cdot (a_i - b_i) & \text{if } t \geq t_i^* \end{cases} \tag{19}$$

where $a_i \geq 0$, $b_i \leq 0$, and t_i^* are parameters associated to each vertex $i \in \mathcal{V}'$. Regarding the shape of the profit function, we have divided the first set of experiments into two subsets. The first one (experiments 1 to 15) considers values of t_i^* following an increasing vertex label order, i.e. the peak of the profit function of each vertex is proportional to the vertex number. Specifically, $t_i^* = 480/n \times (i-1)$, where n represents the number of vertices of each instance. Moreover, the slopes of the profit function (a_i, b_i) are also proportional to the vertex index: specifically $a_i = 3 \times (i-1)$ and $b_i = -3 \times (i-1)$. The second subset of experiments (experiments 16 to 30) considers values of t_i^* following a decreasing order with the index of the vertex, i.e., $t_i^* = 480/n \times (n - (i-1))$.

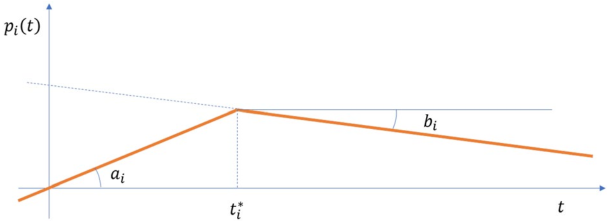


Fig. 3 Piecewise linear profit function

With these artificially created instances we can test the behavior and performance of the solver on the models. For the first subset of experiments, depending on the values of L and T, since the profit peaks increase with the vertex order it is expected that the optimal routes for the single-visit case will follow a clockwise direction. Alternatively, for the second subset, the optimal routes for the single-visit case should follow a counterclockwise direction. For each subset and for each size (15, 30 and 50 vertices), five experiments were carried out, considering $L = 240$, 480 and $T = 240$, 480 and 1440 (which produces the next five combinations $240 - 240$, $240 - 480$, $240 - 1440$, $480 - 480$, $480 - 1440$). The maximum computation time was fixed to $60 \times n$ seconds.

Tables 2 and 3 show the results for the mesh shape instances considering the single-visit (Formulation (*SF*)) and the multiple-visit (Formulation (*MF*)) cases, respectively . The table headings are defined as follows:

- Exp. #: experiment number.
- $|\mathcal{V}|$: the size of the instance.
- L: maximum length of the route.
- T: planning horizon.
- Profit: total profit obtained, which is the objective function value.
- t_0^d: departure time from the depot.
- Gap (%): optimality gap calculated as $\frac{UB-LB}{UB} \cdot 100$, where LB and $UB =$ (profit) are the best known lower and upper bounds.
- #Vertices visited: number of visited vertices in the solution.
- Time: computation time in seconds.
- Max. time: maximum allowed computation time in seconds.

All instances are solved using the standard truncated branch-and-cut algorithm implemented in Gurobi, truncated after a predefined CPU time limit "Max. time”. As expected, the collected profit is higher for the multiple-visit case and, for both the single- and multiple-visit cases, the collected profit increases with L and T. Moreover, for a given value of L the profit tends to increase with T. Note that all the instances are selective and that, for each instance and fixed values of T, the number of visited vertices in the single-visit case tends to increase with L.

Figures 4 and 5 depict the resulting selective routes for each of the 30 vertices instances of Table 2. Observe that in Fig. 4, since the profit peak vertices are disposed in increasing order, not all vertices can be visited and the visited ones are those with larger indices, i.e., larger profits. The visiting order follows a clockwise direction, except for some instances for which the optimal solution is not achieved within Max. time. The behavior in experiments 21 to 25, where profit peaks follow a decreasing order, is the opposite. In these experiments, the numbers over the vertices correspond to the profit collected and the visit time. As illustrated, regardless of the instance size, the higher the ratio T/L, the higher is the collected profit. When T is larger than L, the model can move the starting time of the route (column t_0^d in Table 2). Note that for the cases where $T = L$ (240–240, 480–480), the departure from the depot occurs at time $t = 0.00$, thus using the full route length to visit as

Table 2 Mesh shape instances. Formulation (*SF*)

Exp. #	$\lvert\mathcal{V}\rvert$	L	T	Profit	t_0^d	Gap (%)	#Vertices visited	Time	Max. time
Peak profit in increasing order of labels									
1	15	240	240	14434.50	0.00	0.00	6	2.6	900
2	15	240	480	31342.50	239.99	0.00	3	559	900
3	15	240	1440	34825.50	282.99	0.00	3	610	900
4	15	480	480	76612.50	0.00	0.00	11	1.5	900
5	15	480	1440	89025.00	59.50	0.00	10	0.3	900
6	30	240	240	66702.68	0.00	2.15	12	1800	1800
7	30	240	480	216056.94	239.99	0.81	9	1800	1800
8	30	240	1440	248829.87	297.55	0.63	9	1800	1800
9	30	480	480	299362.05	0.00	2.80	25	1800	1800
10	30	480	1440	379818.84	68.35	0.00	22	3.5	1800
11	50	240	240	187330.34	0.00	2.59	19	3000	3000
12	50	240	480	585414.32	240.00	0.95	17	3000	3000
13	50	240	1440	608244.64	273.98	0.91	16	3000	3000
14	50	480	480	944748.27	0.00	0.21	39	3000	3000
15	50	480	1440	974375.83	55.74	0.19	38	3000	3000
Peak profit in decreasing order of labels									
16	15	240	240	14512.50	0.00	0.00	6	6.1	900
17	15	240	480	22561.50	197.99	0.00	6	824.3	900
18	15	240	1440	22561.50	198.00	0.10	6	900	900
19	15	480	480	41082.00	0.00	0.00	12	34.5	900
20	15	480	1440	42826.50	22.03	0.00	11	4.6	900
21	30	240	240	71006.64	0.00	1.22	12	1800	1800
22	30	240	480	90106.62	89.96	1.32	12	1800	1800
23	30	240	1440	90106.62	89.96	1.32	12	1800	1800
24	30	480	480	169988.71	0.00	0.22	25	1800	1800
25	30	480	1440	181398.50	3.22	0.15	24	1800	1800
26	50	240	240	168029.25	0.00	1.84	19	3000	3000
27	50	240	480	183822.86	149.53	2.22	20	3000	3000
28	50	240	1440	244929.40	120.79	1.42	21	3000	3000
29	50	480	480	460253.75	0.00	0.29	40	3000	3000
30	50	480	1440	443507.33	8.86	0.34	38	3000	3000

many vertices as possible, arriving again to the depot at time $t = L$. In general, for a given size of the instance, the number of visited vertices tends to increase with T, but this is not always true. See, for instance, experiments 1, 2, and 3, with $L = 240$: although the collected profit increases, the number of visited vertices decreases. Since the model can freely move the route duration interval along the planning horizon, a higher profit can be collected visiting fewer but more profitable vertices. This behavior can also be observed for experiments 4–5, 9–10, and 11–13 in the case

Table 3 Mesh shape instances. Formulation (*MF*)

Exp. #	$\|\mathcal{V}\|$	L	T	Profit	t_0^d	Gap (%)	# Vertices visited	Time	Max. time
Peak profit in increasing order of labels									
31	15	240	240	15948.00	0.00	0.00	4	0.18	900
32	15	240	480	45018.00	240.00	0.00	3	4.18	900
33	15	240	1440	50664.00	292.00	0.00	4	172.75	900
34	15	480	480	92790.00	0.00	0.00	4	0.93	900
35	15	480	1440	127968.00	218.00	0.00	4	71.77	900
36	30	240	240	81918.00	0.00	0.00	10	143	1800
37	30	240	480	227259.00	240.00	0.00	7	19.65	1800
38	30	240	1440	260679.00	322.00	0.00	7	1131.83	1800
39	30	480	480	413841.00	0.00	0.00	6	7.59	1800
40	30	480	1440	617916.00	214.00	0.00	10	161.92	1800
41	50	240	240	268898.40	0.00	0.00	7	7.35	3000
42	50	240	480	769442.40	240.00	0.00	6	40.3	3000
43	50	240	1440	862028.40	342.00	0.00	7	1719.79	3000
44	50	480	480	1194255.00	0.00	0.00	7	26.63	3000
45	50	480	1440	1744662.00	210.00	0.00	6	621.96	3000
Peak profit in decreasing order of labels									
46	15	240	240	14265.00	0.00	0.00	4	0.24	900
47	15	240	480	24084.00	134.00	0.00	5	3.51	900
48	15	240	1440	24084.00	131.00	0.00	5	27.08	900
49	15	480	480	42912.00	0.00	0.00	10	1.30	900
50	15	480	1440	43896.00	47.00	0.00	8	37.31	900
51	30	240	240	71748.00	0.00	0.00	10	1.31	1800
52	30	240	480	96168.00	167.00	0.00	11	114.66	1800
53	30	240	1440	96168.00	168.00	0.00	11	261.68	1800
54	30	480	480	185502.00	0.00	0.00	19	7.12	1800
55	30	480	1440	189180.00	19.00	0.00	19	114.27	1800
56	50	240	240	186520.20	0.00	0.00	15	6.81	3000
57	50	240	480	277503.00	114.00	0.00	19	57.36	3000
58	50	240	1440	277503.00	114.00	0.00	19	520.56	3000
59	50	480	480	491557.79	0.00	0.00	31	26.72	3000
60	50	480	1440	505301.39	44.00	0.00	34	495.26	3000

of peak profits following an increasing order with time. Similarly, in the opposite order of profit peaks, experiments 19–20, 24–25, and 29–30 exhibit this decrease in the number of visited vertices. Graphical representations of the remaining instances (with 15 and 50 vertices) are provided in the Appendix.

As an illustration of the problem with time windows described in Sect. 3, we consider the 15 vertices mesh instances described above but whose vertices have negative profit function values outside predefined time windows. We have

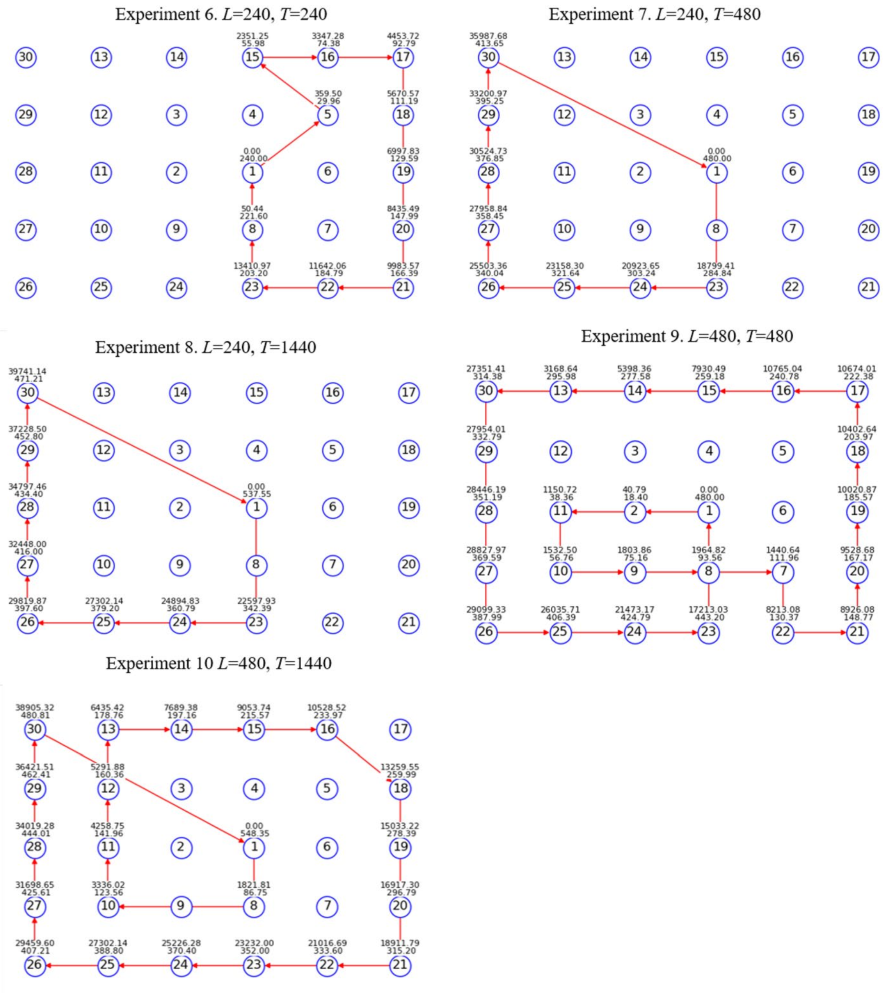
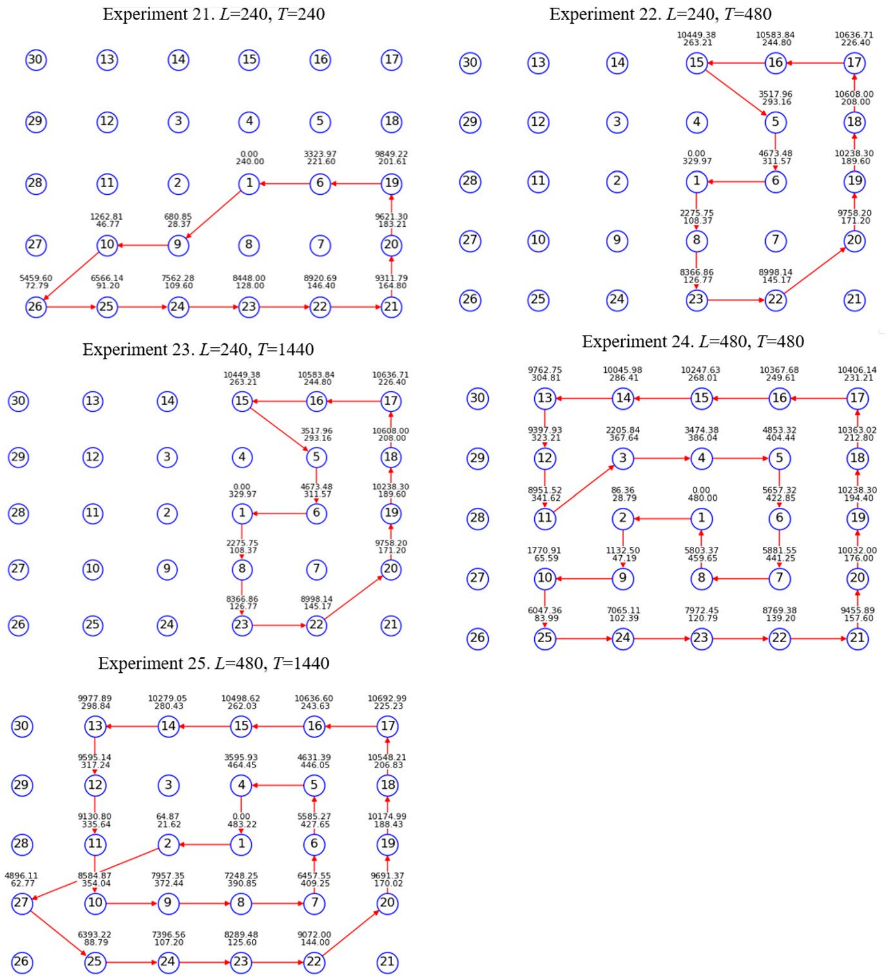


Fig. 4 Mesh instances. Experiments 6 to 10 with 30 vertices, clockwise order

considered piecewise linear profit functions similar to those illustrated in Fig. 3, but with time windows around the profit function peaks (see Fig. 6). Table 4 shows the results obtained for the multiple-visit case. It can be observed that, as commented in Observations 4 and 5, the vertices that are visited are indeed visited within their corresponding time window. Recall that, for these experiments, the time windows do not need to be added to the formulation, but they are naturally respected by assigning negative profit function values outside their bounds.


Experiment 21. L=240, T=240

Experiment 22. L=240, T=480

Experiment 23. L=240, T=1440

Experiment 24. L=480, T=480

Experiment 25. L=480, T=1440

Fig. 5 Mesh instances. Experiments 21 to 25 with 30 vertices, counterclockwise order

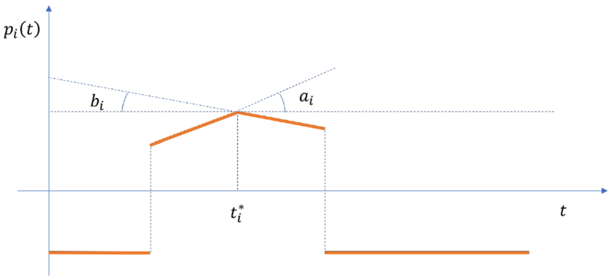


Fig. 6 Piecewise linear profit functions with negative values outside time windows

Table 4 Mesh shape instances with time windows. Formulation *(MF)*

Peak profit in increasing order of labels

$\lvert\mathcal{V}\rvert$	L	T	Profit	t_0^d	Gap (%)	Vertices visited	Time window	Visiting times	Time	Max. time
15	240	240	10515.00	0.00	0.00	4	[– 4, 196]	49	0.16	900
						5	[28, 228]	87		
						6	[60, 260]	125		
						7	[92, 292]	163		
						8	[124, 324]	201		
						1	–	240		
15	240	480	31242.00	240.00	0.00	14	[316,516]	366	12.21	900
						15	[348,548]	404		
						1	–	480		
15	240	1440	49884.00	310.00	0.00	14	[316, 516]	436	32.94	900
						15	[348, 548]	398, 474		
						1	–	550		
15	480	480	62328.00	0.00	0.00	4	[– 4, 196]	52	2.31	900
						5	[28, 228]	90		
						6	[60, 260]	128		
						8	[124, 324]	182		
						9	[156, 356]	220		
						11	[220, 420]	274		
						12	[252, 452]	312		
						14	[316, 516]	366		
						15	[348, 548]	404		
						1	–	480		
15	480	1440	103566.00	140.00	0.00	8	[124, 324]	178	15.85	900
						9	[156, 356]	216		
						11	[220, 420]	270		
						12	[252, 452]	308		
						13	[284, 484]	346		
						14	[316, 516]	384, 460		
						15	[348, 548]	422, 498		
						1	–	584		

Formulation (*MF*)

4.2 TSPLib instances

In this section, we consider different TSTLib instances adapted to the characteristics of our problem, that is, assigning one-peak time-dependent profit functions see Eq. (19) to vertices and considering different space measure scales.

Assuming that the TSPLib coordinates are given in km, we consider constant speed and different scales to study the problem resolution. In this way, e.g., a scale of 1/1000 means that the coordinates are taken in meters. Tables 5 and 6 illustrate the sensitivity of the single- and multiple-visit models, respectively, to different scales for instance *eil*51 and the following parameter values: $L = 480$, $T = 1440$, speed = 35.5 km/h. The CPU time limit has been set as Max. time=7200 seconds for these experiments. The table headings are defined as follows:

- Scale: different values are considered to rescale the instance *eil*51. Scale 1 indicates that distance is taken in kilometers and 1/1000 in meters.
- Profit: total profit obtained, which is the objective function value.
- Total distance: total traveled distance in the corresponding scale measure.
- Total distance (km): total traveled distance in km.
- Time: computation time in seconds.
- Gap (%): optimality gap calculated as $\frac{UB-LB}{UB} \cdot 100$.
- t_0^d: departure time from the depot.
- #Vertices visited: number of visited vertices in the solution.

Since we consider a constant speed, the smaller the scale, the smaller the distances and, therefore, the more time is available to visit the vertices. It can be observed that this effect influences the resulting solutions, mostly in terms of total collected profit and computation time. Observe that, for the single-visit case (Table 5), the higher the scale, the smaller the total profit obtained and the larger the computing times, being even not possible to obtain a feasible solution within the CPU time limit Max. time=7200 for scale 1/1. The starting time of the shift also changes with the scale, tending to start earlier when the scale is smaller. This is due to the fact that the considered speed is constant and, therefore, the smaller the scale, the larger the freedom to visit the vertices. Table 6, which shows the results obtained for the multiple-visit case, exhibits a similar behavior in terms of the objective function. Although, as expected, much higher values of the objective function are obtained for the multiple-visit case, these values also increase as the scale decreases. However, with respect to the computing times, for the multiple-visit case, the behavior of the computing times is not uniform. In this case, different scales lead to different travel times and, therefore, different arc set $\overline{\mathcal{A}}_2$. Since the model allows to wait at the same site, shorter travel times enable to arrive with more flexibility at a site, thus leading to equal or better profits but not necessarily better computing times.

Table 7 presents the results obtained for the single- and multiple-visit cases for TSPLib instances *eil*51, *st*70, *a*280, and *ali*535. For these experiments, the maximum length of the route $L = 480$, the planning horizon $T = 1440$, the threshold on the computing time is Max. time = 7200 s and the scale is 1/1000, which is

Table 5 Variations to scale for instance *eil*51

Scale	Profit	Total distance	Total distance (km)	Time	Gap (%)	t_0^d	#Vertices visited
1/1	–	–	–	–	–	–	–
1/5	3034136.16	246.34	1231.70	7200	0.40	475.75	51
1/10	3047909.70	131.35	1313.50	9.0	0.00	472.94	51
1/20	3048000.00	65.67	1313.40	0.9	0.00	471.76	51
1/50	3048000.00	26.27	1313.50	0.9	0.00	471.06	51
1/100	3048000.00	13.14	1314.00	0.9	0.00	470.82	51

Single-visit case (Formulation (*SF*))

Table 6 Variations to scale for instance *eil*51

Scale	Profit	Total distance	Total distance (km)	Time	Gap (%)	t_0^d	#Vertices visited
1/1	–	–	–	–	–	–	
1/5	54331764.71	215.75	1078.74	611.6	0.00	688	3
1/10	54726074.12	108.77	1087.68	281.08	0.00	688	3
1/20	55918907.67	226.14	4522.74	688.92	0.00	692	4
1/50	58765875.17	250.77	12538.24	259.88	0.00	706	3
1/100	58765875.17	125.38	12538.24	431.39	0.00	706	3

Multiple-visit case (Formulation (*MF*))

equivalent to considering the original coordinates in meters instead of in kilometers. The table headings are defined as follows:

- TSPLib instance: indicates the name of the benchmark instances.
- Profit: the total profit obtained, which is the objective function value.
- Gap (%): optimality gap calculated as $\frac{UB-LB}{UB} \cdot 100$.
- #Vertices visited: number of visited vertices in the solution.
- Time: computation time in seconds.
- Total distance: total traveled distance in meters (corresponding to scale 1/1000).
- t_0^d: departure time from the depot.

As expected, for both cases, the larger the number of vertices, the larger the total collected profit and the computation time. For the multiple-visit case, only instances *eil*51 and *st*70 could be solved to optimality within Max. time. For these instances, although the number of visited nodes in the multiple-visit case is considerably lower than for the single-visit case, these nodes are visited several times obtaining more than ten times higher values of the objective function (collected profit). Figure 7 illustrates the resulting routes with Formulation (*SF*) for the instances presented in Table 7. Since the problem aims at maximizing the total collected profit, the resulting routes differ significantly from those obtained with the usual TSP, where the objective is to minimize the total distance traveled.

As expected, when the objective function is to maximize the total collected profit, the total distance increases with respect to that obtained from a TSP solution. A trade-off analysis between the collected profit and the total distance may shed some light on these differences. To this end, it becomes appropriate to use a multi-objective approach.

4.3 Trade-off between the collected profit and the total distance. Multi-objective approach

Starting from formulation (*SF*), we consider the ϵ-constraints method proposed by Marglin (1967). That is, we include constraint (20) limiting the total distance of the route to a maximum of ϵ and the route is then obtained by maximizing the total collected profit and varying parametrically the maximum total distance ϵ:

$$\sum_{\substack{i,j \in \mathcal{V} \\ i \neq j}} \tau_{ij} x_{ij} \leq \epsilon. \tag{20}$$

Table 8 represents the results obtained by the ϵ-constraints method for the TSPLib instance *eil*51 and the following parameter values: $L = 480$, $T = 1440$, speed = 35.5km/h, scale = $1/10$. The table headings are defined as follows:

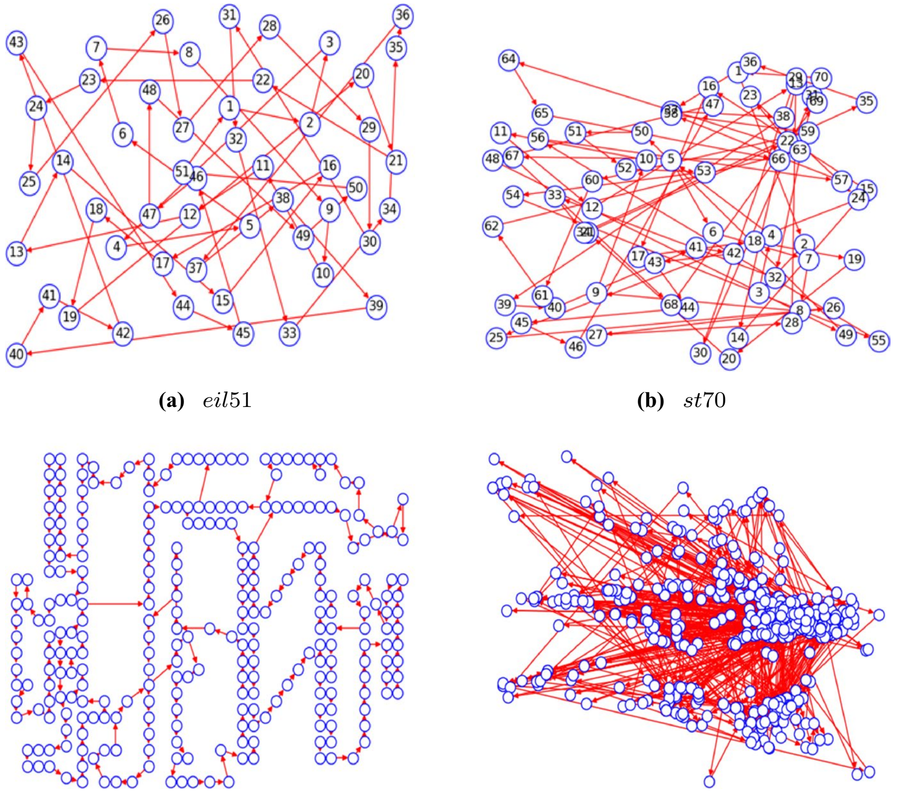
- Threshold ϵ: threshold on the route total distance.
- Profit: total profit obtained, which is the objective function value.
- Total distance: total traveled distance in hectometers (corresponding to scale 1/10).
- Time: computation time in seconds.
- Gap (%): optimality gap calculated as $\frac{UB-LB}{UB} \cdot 100$.

Table 7 Results for TSPLib instances for scale 1/1000, L = 480, T = 1440

TSPLib instance	Formulation	Profit	Gap (%)	#Vertices visited	Time	Total distance	t_0^d
*eil*51	(*SF*)	3048000.00	0.00	51	1.2	1.31	470.61
*eil*51	(*MF*)	58765875.17	0.00	3	261.65	12.5	706
*st*70	(*SF*)	5779440.00	0.00	70	5.5	3.41	473.18
*st*70	(*MF*)	81698999.99	0.00	3	709.09	3.89	710
*a*280	(*SF*)	93677040.00	0.00	280	185.4	2.82	478.32
*a*280	(*MF*)	–	–	–	–	–	–
*ali*535	(*SF*)	342699840.00	0.00	535	499.1	36.99	479.25
*ali*535	(*MF*)	–	–	–	–	–	–

Observe that, except for the first four rows, the value of the total distance tends to be similar to that of the upper threshold ϵ. This follows from the fact that the larger the total distance, the less constrained is the problem and, therefore, the larger is the total collected profit, which is the objective function. The first four rows stand for values of ϵ larger than 131.35, which is the total collected profit when no constraints on the total distance are imposed (see Table 5, third row). This explains the constant values for the first four rows of the third column.

The first and second columns of Table 8 are represented in Figure 8 in the form of a Pareto curve for a trade-off between collected profit and maximum total distance. It can be observed that, as expected, the total collected profit increases as the threshold on the total distance increases. However, the profit stabilizes when the threshold ϵ reaches 130. In fact, if no threshold is imposed and only the total collected profit is maximized, then the resulting total collected distance equals 131.35. Therefore, for ϵ values larger than 131.35, the total collected profit remains constant.



(a) $eil51$

(b) $st70$

(c) $a280$

(d) $ali535$

Fig. 7 Representation of the resulting routes for instances of Table 7 with scale 1/1000

The routes associated with some of the solutions of Table 8 are depicted in Figure 9. As ϵ decreases, the route length decreases and tends to look like a usual TSP solution, in which the total distance is minimized and profit is not considered.

Table 8 Trade-off between collected profit and total distance for instance *eil*51 at scale 1/10

Threshold ϵ	Profit	Total distance	Time	Gap (%)
200	3047909.70	131.35	20.8	0.00
180	3047909.70	131.35	19.8	0.00
160	3047909.70	131.35	19.9	0.00
140	3047909.70	131.35	16.6	0.00
130	3047793.97	129.54	24.5	0.00
120	3044886.66	119.54	1800	0.00
100	3030961.07	99.75	1800	0.01
80	3004438.23	79.93	1800	0.01
60	2922777.87	59.95	1800	0.04
50	2849024.68	49.85	1800	0.07
45	2797871.66	44.92	1800	0.06
44	2785162.11	44.00	1800	0.03
43	2760263.52	42.95	139.5	0.00

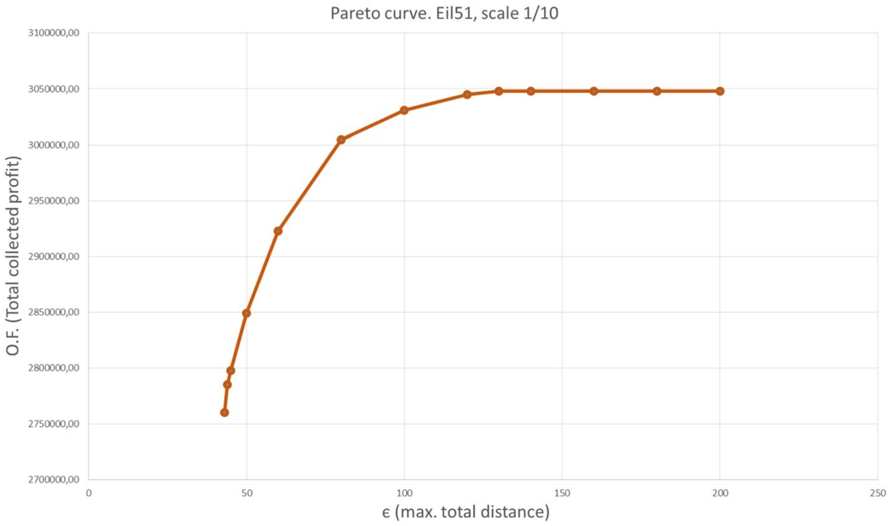
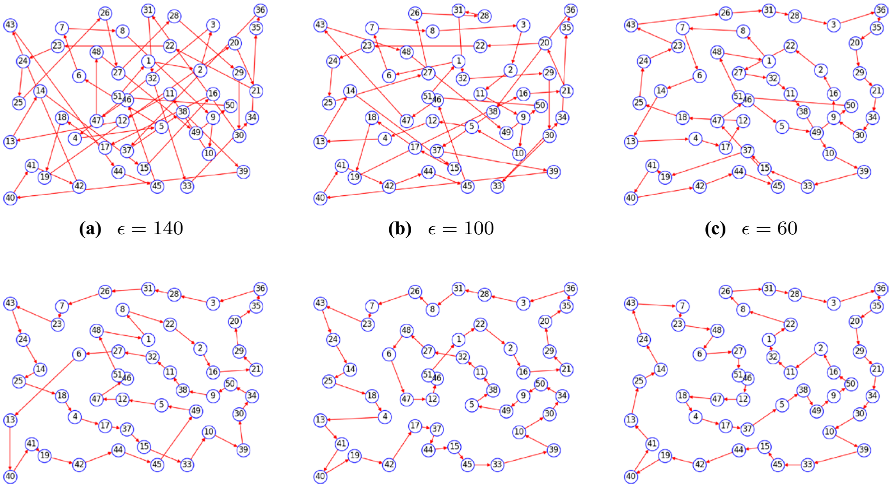


Fig. 8 Trade-off between collected profit and total distance for instance *eil*51 at scale 1/10



(a) $\epsilon = 140$ **(b)** $\epsilon = 100$ **(c)** $\epsilon = 60$

(d) $\epsilon = 50$ **(e)** $\epsilon = 45$ **(f)** $\epsilon = 43$

Fig. 9 Representation of the resulting routes for different values of ϵ

5 Conclusions

We have introduced, modeled, and solved the STSP-TDT. The STSP-TDT is a generalization of other well-known variants of the traveling salesman problems such as the TSP itself, the STSP, as well as the TSP and the STSP with time windows. Some interesting properties arise in the context of time-dependent profits. For example, it sometimes makes sense to visit the same vertex multiple times and to optimize the starting time of the tour. We have proposed and tested a formulation of the single-visit problem that can be used if the profit functions are linear or piecewise linear.

We have also proposed and tested an extended formulation for the case when multiple visits to the same vertex are allowed and desirable. In this case, we formulate the problem by means of an extended graph so that this variant reduces to an STSP, adapted to be solved as a longest path problem.

We have analyzed the problem by studying the shape of the time-dependent profit functions. We have also performed several experiments on artificially created instances and on some TSPLib instances considering the single- and multiple-visit cases. Our results on artificially created instances validate the model performance and highlight the importance of allowing a variable starting time of the route. We have considered several scale measures and optimally solved instances containing up to 535 vertices for the single-visit case. We have also compared the results obtained for the single- and multiple-visit cases and analyzed the results of our single-visit case experiments by means of a Pareto frontier representing the trade-off between the collected profit and the total distance traveled. We observe that these objectives are naturally opposed to one another.

Appendix. Results of 15 and 50 vertices mesh instances

See Figs. 10, 11, 12 and 13.

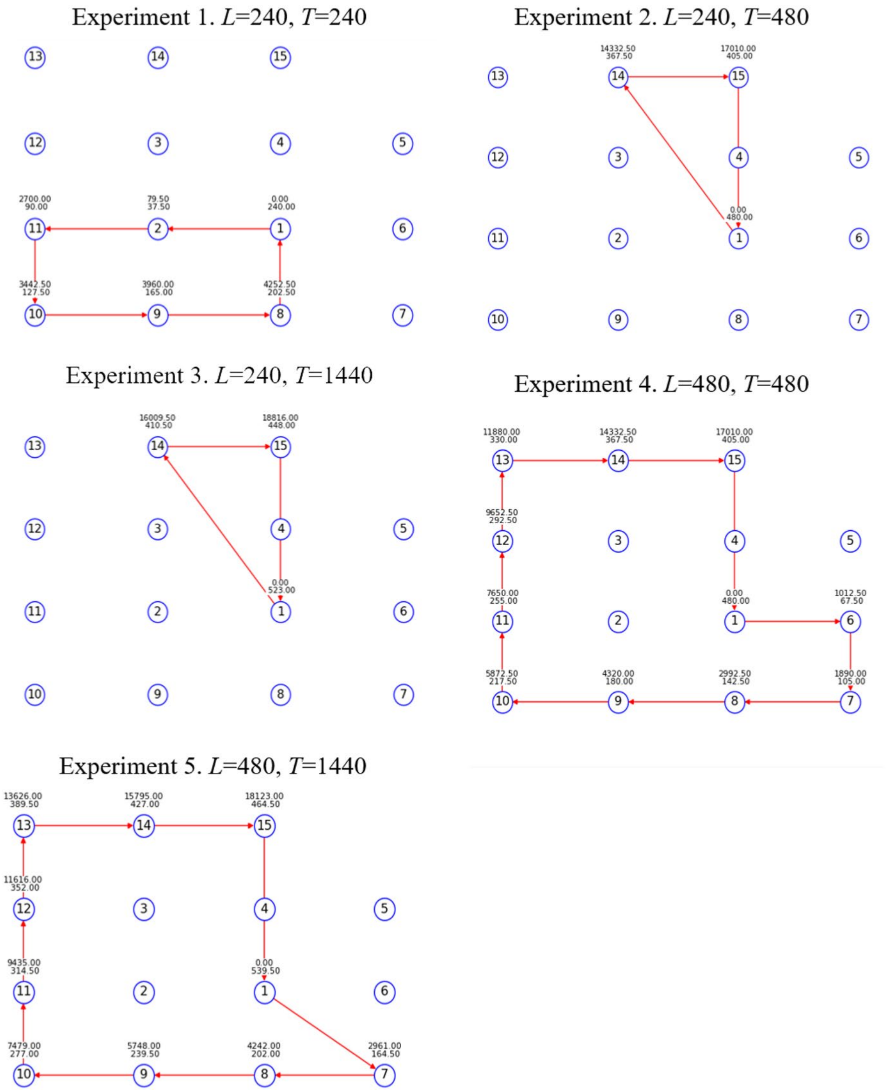
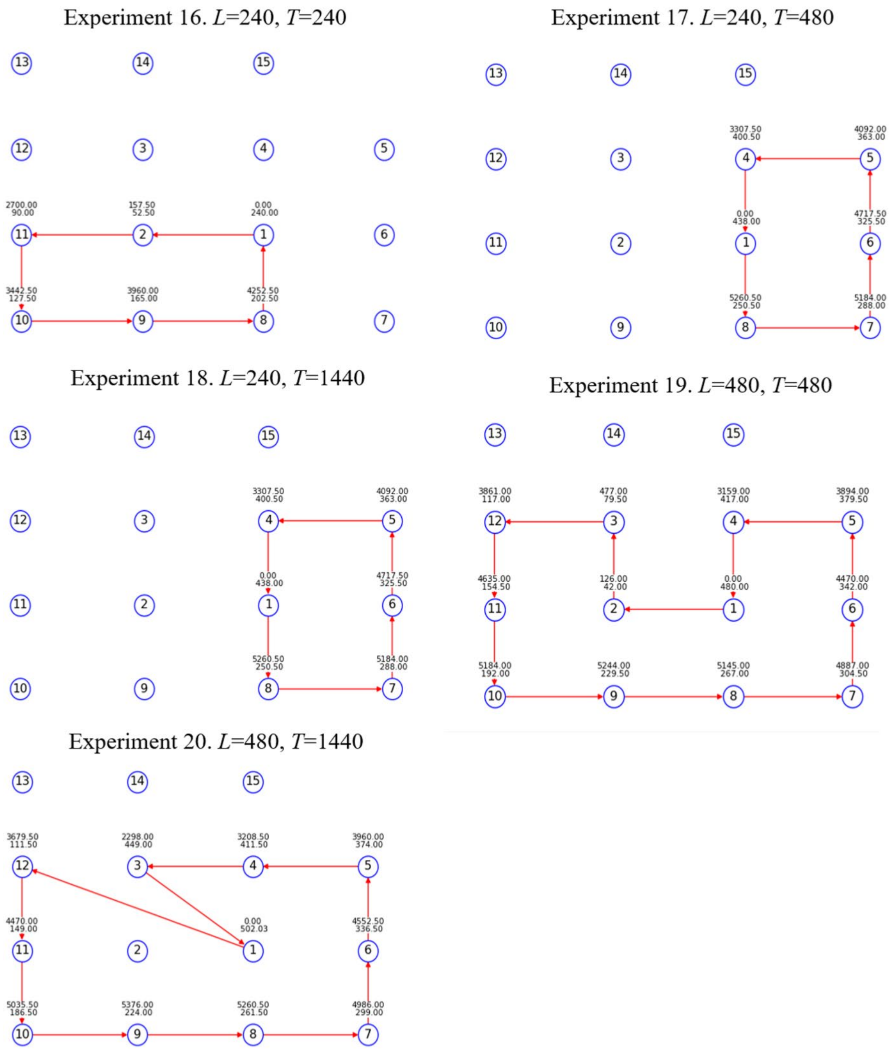


Fig. 10 Mesh instances. Experiments 1 to 5. 15 vertices. Clockwise order



Experiment 16. *L*=240, *T*=240

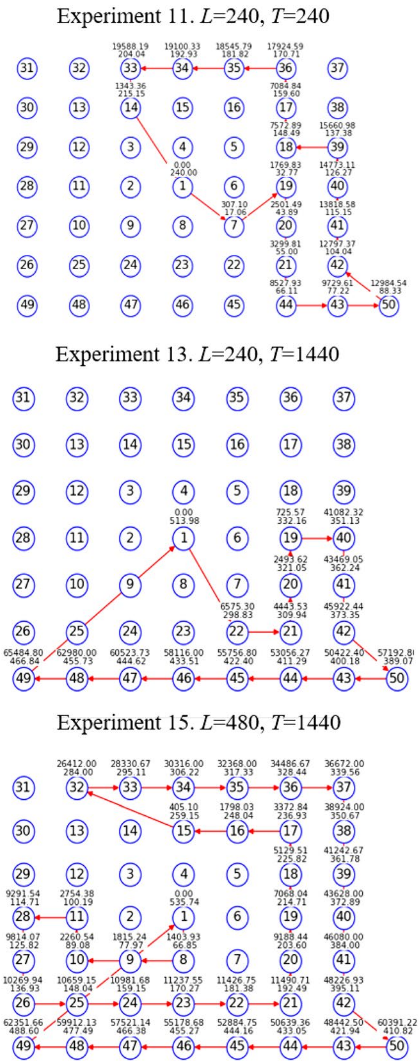
Experiment 17. *L*=240, *T*=480

Experiment 18. *L*=240, *T*=1440

Experiment 19. *L*=480, *T*=480

Experiment 20. *L*=480, *T*=1440

Fig. 11 Mesh instances. Experiments 16 to 20. 15 vertices. Counterclockwise order


Experiment 11. *L*=240, *T*=240

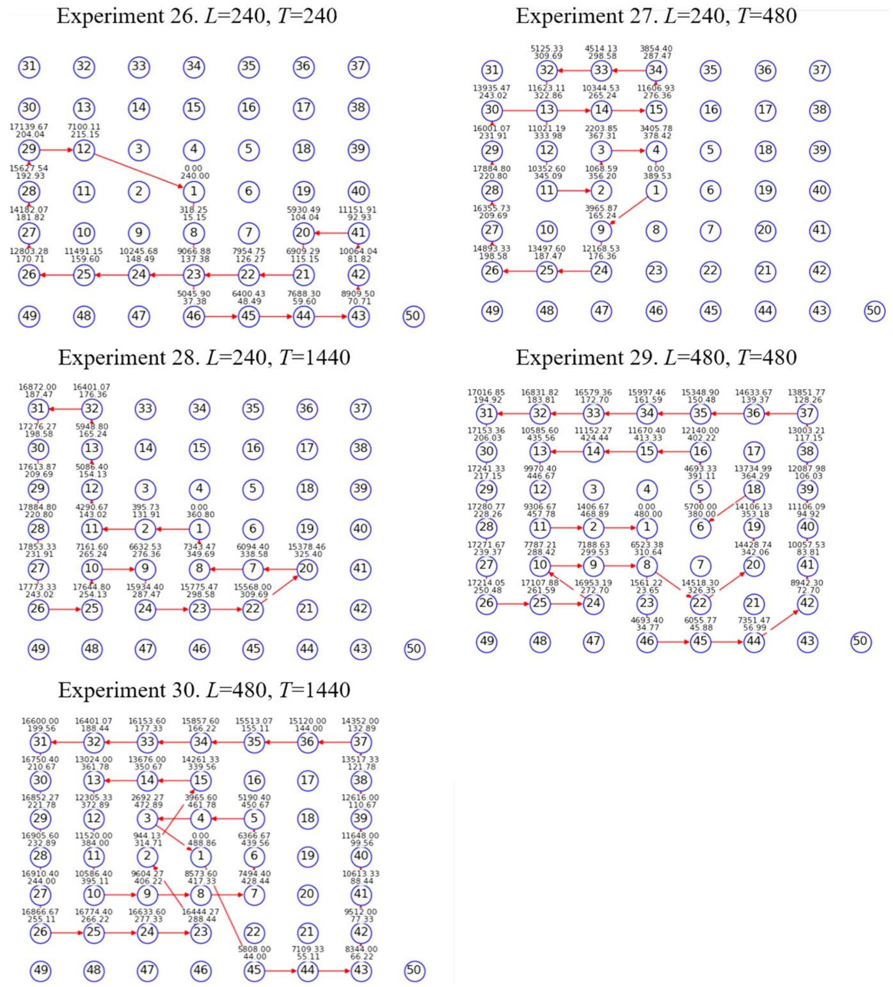
Experiment 12. *L*=240, *T*=480

Experiment 13. *L*=240, *T*=1440

Experiment 14. *L*=480, *T*=480

Experiment 15. *L*=480, *T*=1440

Fig. 12 Mesh instances. Experiments 11 to 15. 50 vertices. Clockwise order



Experiment 26. *L*=240, *T*=240

Experiment 27. *L*=240, *T*=480

Experiment 28. *L*=240, *T*=1440

Experiment 29. *L*=480, *T*=480

Experiment 30. *L*=480, *T*=1440

Fig. 13 Mesh instances. Experiments 26 to 30. 50 vertices. Counterclockwise order

References

Afsar H, Labadie N (2013) Team orienteering problem with decreasing profits. Electronic Notes in Discrete Mathematics 41:285–293

Archetti C, Speranza MG, Vigo D (2014) Vehicle routing problems with profits. In: Toth P, Vigo D (eds) Vehicle routing: problems, methods, and applications, monographs on discrete mathematics and applications, vol 18. MOS-SIAM Series on Optimization, Philadelphia, pp 87–116

Bruck BP, Iori M (2017) Non-elementary formulations for single vehicle routing problems with pickups and deliveries. Operat Res 65(6):1597–614

Bruglieri M (2020) The parking warden tour problem. Networks 76:539–554

da Silva AA, Morabito R, Pureza V (2018) Optimization approaches to support the planning and analysis of travel itineraries. Expert Syst Appl 112:321–330

Erdoğan G, Laporte G (2013) The orienteering problem with variable profits. Networks 61(2):104–116

Erkut E, Zhang J (1996) The maximum collection problem with time-dependent rewards. Naval Res Logist 43(5):749–763

Feillet D, Dejax P, Gendreau M (2005) Traveling salesman problems with profits. Transp Sci 39(2):188–205

Gavalas D, Konstantopoulos C, Mastakas K, Pantziou G (2014) A survey on algorithmic approaches for solving tourist trip design problems. J Heuristics 20(3):291–328

Golden BL, Levy L, Vohra R (1987) The orienteering problem. Naval Res Logist 34:307–318

Gribkovskaia I, Halskau Ø, Laporte G, Vlček M (2007) General solutions to the single vehicle routing problem with pickups and deliveries. Eur J Operat Res 180(2):568–584

Gunawan A, Lau HC, Vansteenwegen P (2016) Orienteering problem: a survey of recent variants, solution approaches and applications. Eur J Operat Res 255:315–332

Hoff A, Gribkovskaia I, Laporte G, Løkketangen A (2009) Lasso solution strategies for the vehicle routing problem with pickups and deliveries. Eur J Operat Res 192(3):755–766

Laporte G, Martello S (1990) The selective travelling salesman problem. Discrete Appl Math 26(2–3):193–207

Marglin SA (1967) Public investment criteria. Studies in the Economic Development of India 4, George Allen & Unwin, London

Munari P, Savelsbergh MWP (2022) Compact formulations for split delivery routing problems. Transp Sci. https://doi.org/10.1287/trsc.2021.1106

Salazar-González JJ, Santos-Hernández B (2015) The split-demand one-commodity pickup-and-delivery travelling salesman problem. Transp Res Part B: Method 75:58–73

Schilde M, Doerner KF, Hartl RF, Kiechle G (2009) Metaheuristics for the bi-objective orienteering problem. Swarm Intell 3(3):179–201

Souffriau W, Vansteenwegen P, Vertommen J, Vanden Berghe G, Van Oudheusden D (2008) A personalized tourist trip design algorithm for mobile tourist guides. Appl Artif Intell 22(10):964–985

Vansteenwegen P, Souffriau W, Van Oudheusden D (2011) The orienteering problem: a survey. Eur J Operat Res 209(1):1–10

Wang X, Golden BL, Wasil EA (2008) Using a genetic algorithm to solve the generalized orienteering problem. In: Golden BL, Raghavan S, Wasil EA (eds) The vehicle routing problem: latest advances and new challenges. Springer, New York, pp 263–274

Yu Q, Fang K, Zhu N, Ma S (2019) A matheuristic approach to the orienteering problem with service time dependent profits. Eur J Operat Res 273(2):488–503

Yu Q, Adulyasak Y, Rousseau LM, Zhu N, Ma S (2021) Team orienteering with time-varying profit. INFORMS J Comput. https://doi.org/10.1287/ijoc.2020.1026

Yu VF, Jewpanya P, Lin S, Redi A (2019) Team orienteering problem with time windows and time-dependent scores. Comput Industr Eng 127:213–224

Zhang W, Wang K, Wang S, Laporte G (2020) Clustered coverage orienteering problem of unmanned surface vehicles for water sampling. Naval Res Logist 67(5):353–367

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