On the Behavior of Mechanical Stress Fields at Indentation of Materials with Residual Stresses

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It is an obvious fact that residual stresses can have a detrimental effect on the mechanical integrity of structures. Measuring such stresses can often be a tedious task and for that reason sharp indentation testing has been proposed as an alternative for this purpose. Correlation between global indentation properties and residual stresses has been studied quite frequently, and a solid foundation has been laid down concerning this issue. Empirical, or semi-empirical, relations have been proposed yielding results of quite good accuracy. Further progress and mechanical understanding regarding this matter will require a more in-depth understanding of the field variables at this particular indentation problem and this is the subject of the present study. In doing so, finite element simulations are performed of sharp indentation of materials with and without residual stresses. Classical Mises plasticity and conical indentation are considered. The main conclusion from this study is that the development of stresses in materials with high or medium-sized compressive residual stresses differs substantially from a situation with tensile residual stresses, both as regards the level of elastic deformation in the contact region and the sensitivity of such stresses. Any attempt to include such stress states in a general correlation effort of indentation quantities is therefore highly unlikely to be successful.

Keywords correlation, Mises plasticity, mechanical stress fields, residual stresses, sharp indentation

1. Introduction

Sharp indentation contact is from many aspects a formidable mechanical problem involving contact between two bodies, stress singularities and nonlinearities also at linear elastic material behavior. The latter feature is a result of the fact that the contact area is dependent on the contact force. Despite of this though, analytical solutions have been achieved at linear elastic constitutive behavior, for example by Sneddon (Ref 1) for a number of indenter profiles including conical ones.

In case of other constitutive behaviors, than linear elastic, analytical solutions cannot be expected save for very particular cases most often of little or no practical relevance. Therefore, empirical or semi-empirical methods are almost without exception used to determine relevant parameters at sharp contact. Classical studies by Tabor (Ref 2), Johnson (Ref 3, 4) and Atkins and Tabor (Ref 5), relating global indentation parameters to constitutive properties, should be mentioned in this context.

It goes almost without saying that if residual stresses are present in the indented material, the contact problem becomes even more involved. Despite of this though, this is a very important issue as of course residual stresses can be detrimental for the load-bearing capacity of structures and needs to be measured in order to be accounted for at mechanical design.

There are many existing experimental techniques for this purpose such as indentation crack techniques, fracture-surface analysis, neutron and x-ray tilt techniques, beam bending, hole drilling, and layer removal. Many of these methods can, however, be complicated or expensive (or both) and for that reason indentation testing has emerged as a convenient alternative. This is particularly so remembering the rapid increase in commercially available experimental devices for indentation testing with very low loads and nanometer sized indentation depths, perhaps starting with the so-called nanoindenter as presented by Pethica et al. (Ref 6). This enables for example indentation testing of very thin films where residual stresses (or deformation-/temperature-induced stresses) are notarius.

Due to its obvious practical usefulness, indentation as a tool to determine residual stresses has been studied experimentally/numerically/theoretically very frequently during the latest 25 years. Perhaps the starting point of this was the important contributions by Pharr and co-workers (Ref 7, 8) where indentation of stressed aluminum alloy 8009 was investigated using nanoindentation and finite element simulations. Qualitative results concerning the relation between residual (applied) stresses and global indentation properties were presented. After that a large number of investigations have been published trying to correlate these two features and thereby enhancing the use of indentation techniques for this purpose (Ref 9–22).

Even though the references mentioned above, (Ref 7–22), is certainly not a complete list of studies, concerning indentation as a tool to determine residual stresses, it covers quite well the different topics pertinent to this feature. It involves experimental, numerical, and theoretical studies as well as approaches based on optimization (inverse modeling). There are of course not altogether unanimous conclusions drawn from these studies, but there are some basic results that can serve as a foundation for future investigations. First of all, there is agreement upon the fact that the contact area at indentation is

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dependent of residual stresses and can be used for determining such stresses. Furthermore, most studies indicate that the material hardness is independent of residual stresses, at least when plastic deformations are dominating at and close to the region of contact. The latter feature also indicates that residual stresses influence indentation variables differently when contact deformations are both elastic and plastic of equal magnitude.

All the results mentioned above are essentially empirical and are not directly derived from in-depth analyses of the mechanical field variables at indentation. There are exceptions, cf., e.g., (Ref 9) and (Ref 18), but these studies only touch upon the issue and for example the difference between elastic-plastic and plastic indentation is not elaborated upon. In order to gain further insight into the problem, it therefore seems necessary to include also a proper analysis of the mechanical stress fields at different indentation situations.

Accordingly, this is the aim of the present investigation. In doing so, finite element simulations are performed for a variety of residual stress combinations. In particular, the development of the principal stresses is studied in relation to the Mises flow surface. For simplicity and clarity, but not out of necessity, the analysis is restricted to conical indentation of materials with equi-biaxial residual stress states.

2. Theoretical Background: Statement of the problem

In this section, the theoretical background (“state of the art”) of this particular indentation problem is outlined. In doing so, the analysis by Rydin and Larsson (Ref 18) is relied upon to some extent. However, important findings from other studies are also discussed and the need for the present analysis is explained.

There is quite a general agreement in the literature that the relative contact area at indentation is sensitive to the presence of residual stresses, while the material hardness is not, at least when metallic materials are at issue. The relative contact area at indentation is here denoted c^2 and defined as

$$c^2 = A/A_0 \quad (\text{Eq 1})$$

where A is the real contact area between indenter and material and A_0 is the nominal contact area that would have resulted if the material did not sink-in or pile-up at the contact boundary, see Fig. 1 for the case of conical indentation. The material hardness is here denoted H and defined as

$$H = F/A \quad (\text{Eq 2})$$

where F is the normal indentation load, see Fig. 1.

Rydin and Larsson (Ref 18) showed that the effect of residual stresses could be well-correlated based on the relative contact area c^2 by using the so-called Johnson (Ref 3, 4) parameter

$$\lambda = E \tan \beta / (\sigma_y (1 - \nu^2)) \quad (\text{Eq 3})$$

where E is Young’s modulus, ν is Poisson’s ratio, σ_y is the material yield stress and $\beta = 22^\circ$ according to Fig. 1. This parameter was introduced by Johnson (Ref 3, 4) in order to correlate indentation experiments focusing on the nature of the deformation in and around the contact region, i.e., the relation between plasticity and elasticity. A high value on λ , $\lambda > 30$

Fig. 1 Schematic of the geometry of cone indentation where $A = \pi a^2$ represents the real contact area with a being the contact radius. In the present investigation, $\beta = 22^\circ$ being representative of a standard conical indenter. The nominal contact area $A_0 = \pi h^2 / (\tan \beta)^2$ where h is the indentation depth

say, indicates dominating plasticity while for decreasing values elastic effects increase and indeed, if $\lambda < 3$ say, indentation is essentially purely elastic. The latter situation is in practice only of importance at indentation on the atomic scale (Ref 23). The way the Johnson (Ref 3, 4) parameter is presented in Eq 3, it is valid for an elastic-ideally plastic material and this limitation is commented upon below.

Rydin and Larsson (Ref 18) correlated residual stresses with the change of the relative contact area c^2 by noting that such stresses altered the effective (apparent) yield stress of the material, at indentation. Accordingly, good agreement with numerical and experimental results were found from the derived relations

$$\begin{aligned} c^2 &= c^2(0) - 0.35 \ln(1 + (0.52 \sigma_{\text{res}} / \sigma_y)), \sigma_{\text{res}} < 0 \\ c^2 &= c^2(0) - 0.35 \ln(1 + (1.77 \sigma_{\text{res}} / \sigma_y)), \sigma_{\text{res}} > 0 \end{aligned} \quad (\text{Eq 4})$$

where σ_{res} is a homogeneous equi-biaxial residual stress state and $c^2(0)$ is the relative contact area at indentation of a virgin material (a material with no residual stresses). As already mentioned, Eq 4 rests on the fact that the Johnson (Ref 3, 4) parameter in (3) is changed by residual stresses as such stresses alters the effective yield stress of the material at indentation, and that the relative contact area c^2 depends linearly on $\ln \lambda$ for a wide range of λ -values. In a strict manner, Eq 4 is only valid for elastic-ideally plastic materials and cone indentation. However, as suggested by Johnson (Ref 3, 4) pertinent to initially stress-free materials, strain-hardening can be accounted for by using the yield stress at a representative value on the effective plastic strain. Furthermore, results for cone indentation and pyramid indentation, i.e., Vickers and Berkovich indentation, are generally close, at least so in the present case with $\beta = 22^\circ$.

Concerning other studies, it should first be mentioned that the results presented in Eq 4 are close to the ones presented by Suresh and Giannakopoulos (Ref 9) even though these authors used a somewhat different approach. Also in this study, (Ref 9), the focus is on how residual stresses change the relative contact area, while in (Ref 13) and (Ref 14) the indentation load-indentation depth-curve was studied for this purpose.

A completely different approach was taken by, for example, Bocciarelli and G. Maier (Ref 15) using inverse analysis based on experimental/numerical results to determine the residual stress fields. Other aspects of residual stress determination using such a technique can be found in (Ref 17) and (Ref 20).

Spherical indentation in relation to residual stresses has also been studied in detail by for example Swadener et al (Ref 12) and Huber and Heerens (Ref 16). It was found, particularly emphasized in (Ref 12), that the global indentation parameters at spherical indentation is more sensitive to residual stresses than in the sharp contact case. It should be mentioned though that the results from a spherical test could be quite hard to interpret due to the fact that the global parameters then are dependent on the indentation depth. This is in contrast to the sharp contact case as then the problem is self-similar with no characteristic length. At least when large specimens of classical elastoplastic materials are indented.

Regarding further aspects of the problem, it is important to emphasize that the amount of elastic deformations at the contact region also plays an important role for the behavior at indentation of materials with residual stresses. It has been shown by Larsson (Ref 21, 22) that when significant elastic deformations are present in the contact region, for materials such as ceramics and polymers, also the hardness will be dependent on residual stresses as is always the case for the relative contact area. This is in contrast to a situation where plasticity dominates, pertinent to most metals and alloys, as then hardness is invariant of residual stresses. This will further complicate the problem but as emphasized by Larsson (Ref 21, 22), it also has the advantage that residual stresses can be correlated with an additional quantity.

Even though the analyses above include a number of theoretical considerations, it is undoubtedly so that the most important results are derived in an empirical and/or semi-empirical manner and not from an in-depth analysis based on the mechanical field variables at indentation. Optimization and inverse analysis, (Ref 15, 17) and (Ref 20), can also be included in such a categorization. It goes without saying that understanding the mechanics of the problem is essential for building a theoretical foundation and also for moving the research front forward (and also to understand the limitation of a proposed correlation scheme).

Therefore, the present analysis aims at determining and analyzing the development of the stress state close to the contact region at sharp indentation of materials with residual stresses. In particular, the stress state in relation to the Mises flow cylinder is studied in detail to determine how the principal stresses develop during the indentation process. The initial stress state is determined by the residual stresses. Different indentation situations are analyzed in order to capture the different features discussed above. To summarize these features:

- The difference between indentation of materials with compressive and tensile residual stress states.
- The effect from substantial elastic deformations in the contact region.
- The sensitivity (sometimes the insensitivity) of global indentation parameters to residual stresses at sharp indentation.

The analysis is performed using the finite element method at a variety of residual stress combinations. As already mentioned

above, the development of the principal stress state is studied in relation to the Mises flow surface. For simplicity and clarity, but not out of necessity, the analysis is restricted to conical indentation of materials with equi-biaxial residual stress states.

3. Finite Element Analysis

Finite element analysis of sharp indentation tests is nowadays, following classical initial contributions (Ref 24-27), quite a straightforward procedure. Residual stresses in finite element simulations of sharp indentation were perhaps first introduced in (Ref 8) and then followed a significant number of such studies. Many of these studies are included in (Ref 9-22).

Presently, the commercial finite element package ABAQUS (Ref 28) was used to solve the boundary value problem at hand where a rigid, sharp cone, with $\beta = 22^\circ$, is pressed normally and frictionless into a large specimen, see Fig. 1. It is assumed that the specimen is so large that outer boundary effects are negligible. The material is described by classical Mises elastoplasticity with isotropic hardening. Only ideal plasticity is considered so

$$\sigma_e = \sigma_y \quad (\text{Eq 5})$$

at plastic deformation while elastic loading and unloading is governed by a hypoelastic formulation of Hooke's law. Large deformation theory is relied upon.

Residual (applied) stresses are enforced by prescribed boundary displacements prior to indentation. These displacements give rise to a homogeneous equi-biaxial stress state with principal stresses

$$\sigma_1 = \sigma_2 \quad (\text{Eq 6})$$

in the specimen. The indentation direction is parallel with the negative X_2 -axis as shown in Fig. 1. Initially then, the principal stress $\sigma_3 = \sigma_{X2} = 0$. Note that no distinction is made between residual and applied stresses in this analysis and that the applied prestresses are kept within the elastic limit.

The discretization of the indented specimen is shown in Fig. 2. The finite element mesh consists of approximately 4500 four-noded axisymmetric and hybrid elements and approximately 4800 nodes. The choice of hybrid elements is made in order to facilitate convergence at dominating plastic deformation.

4. Results and Discussion

In this section, the results from the finite element simulations are presented and discussed in relation to the three features listed above at the end of section 2. In particular, the development of the principal stress state is studied in relation to the Mises flow surface. The material at issue has the constitutive properties (E , ν , σ_y) so that λ is approximately equal to 55. This means that at indentation of the material without residual stresses: Plastic deformations dominate in and around the contact region.

Even though it was stated above that finite element simulation of sharp indentation problems nowadays is a standard numerical analysis, it still seems appropriate to discuss the numerical convergence of important indentation quantities.

Such a study also highlights some important features of the sharp indentation problem. Accordingly, in Fig. 3 the normalized hardness, $H = H/\sigma_y$, is depicted as function of the number of elements in contact. It can be noted that 8–9 elements in contact are sufficient in order to achieve steady-state conditions. Presently, the simulations were stopped when 25 elements were in contact. At such conditions

$$H = 2.54 \quad (\text{Eq 7})$$

which is exactly the same value as found by Atkins and Tabor (Ref 5) at cone indentation in a situation where plastic

Fig. 2 Details of the part of the finite element mesh close to the region of contact. The indentation direction is vertical downwards. The indenter tip is applied at the upper left corner

Fig. 3 Convergence rate for the numerically determined normalized hardness, H/σ_y , as function of the number of finite elements in contact. (dashed line) Atkins and Tabor (Ref 5). The finite element results are represented by filled circles, and the values presented are associated with one simulation, where data are extracted continuously with increasing indentation depth

deformations are dominating. It can also be seen that when steady-state conditions are achieved, the hardness becomes independent of the indentation depth h (the depth scales linearly with the number of elements in contact). This is due to the fact, as pointed out above, that the problem at issue is self-similar with no characteristic length.

The corresponding results for the relative contact area c^2 are shown in Fig. 4. Essentially the same conclusions as above from Fig. 3 can be drawn with steady-state conditions achieved when 8–9 elements are in contact. Naturally, based on the same reasoning as for the hardness, the relative contact area is also independent of indentation depth h .

It should be mentioned that the results in Fig. 3 and 4 are pertinent to materials without residual stresses. The inclusion of such stresses in the analysis does, however, not change the convergence rate. Therefore, with the accuracy of the numerical approach established, it seems appropriate to start discussing the main issue of the present study namely the behavior of the principal stresses and the Mises flow surface during sharp indentation of materials with residual stresses.

In doing so, the stress history of a material point on the surface just outside the final contact boundary is examined. When indentation is terminated, this material point is situated approximately 2 nodes outside the contact boundary (the final contact radius is, as mentioned above, approximately 25 elements in size). It is emphasized that this material point is fixed throughout the tracking of the stress history. The initial stress state of the stress history is defined by the equi-biaxial residual stresses. The final stress state is determined at a maximum indentation depth being the same in all simulations. Accordingly, the results will show the stress path at increasing indentation depths for different levels (and signs) of residual stress.

The figures pertinent to these results will show, in ascending order, the following ratios of residual (applied) stress to yield stress; $[-0.9, -0.5, 0, 0.5, 0.9]$. The Mises yield surface is calculated according to

$$\sigma_y = \left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 \right) / 2 \quad (\text{Eq 8})$$

Fig. 4 Convergence rate for the numerically determined relative contact area c^2 as function of the number of finite elements in contact. (dashed line) Approximate steady-state value. The finite element results are represented by filled circles, and the values presented are associated with one simulation, where data are extracted continuously with increasing indentation depth

where σ_1 , σ_2 and σ_3 denote, as above, the principal stresses. The principal stress state will then, as already stated above, be discussed in relation to

- The difference between indentation of materials with compressive and tensile residual stress states.
- The effect from substantial elastic deformations in the contact region.
- The sensitivity (sometimes the insensitivity) of global indentation parameters to residual stresses at sharp indentation.

In doing so, it seems appropriate to first look at the development of principal stresses in relation to the flow surface at sharp indentation of virgin materials, i.e., materials with no residual stresses. The results are shown in Fig. 5 and as can be seen the stress state moves from the origin (no residual stresses present) to the Mises flow surface along a fairly straight line indicating close to a proportional stress state. Due to the applied contact pressure, it is natural that compressive stresses dominate at the final state. Note once again that the stress state is recorded at a fixed material point, as discussed above, situated just outside the contact region at maximum indentation depth.

Corresponding results for the case of residual stresses have been presented very rarely previously. Some results for the ratio of residual (applied) stress to yield stress being -0.5 and 0.5 were, however, discussed by Rydin and Larsson (Ref 18), and these are shown in Fig. 6. It can be seen that the results differ quite substantially between compressive (Fig. 6a) and tensile residual stresses (Fig. 6b). Essentially the compressive case is just a shorter version of the stress-free results, while at tensile stresses, the stress path is substantially different. It should be noted that the final state is approximately the same for all cases in Fig. 5 and 6 which indicates that the concept of an effective (apparent) yield stress is physically sound, at least for the residual stress states investigated in those figures. It is clear that much more elastic deformation is involved at indentation of a material with tensile residual stresses as then a long path is necessary before the flow cylinder is reached and also then a rearrangement of stresses occurs before the final state is achieved.

Fig. 5 Stress history in the principal stress space with the von Mises flow cylinder included. Initially, the material is stress-free, no residual stresses present, and the final stress state is recorded at the maximum indentation depth. (o), initial stress state. (x), intermediate stress states. (square), final stress state

With the results in Fig. 5 and 6 at hand, it seems obvious that an investigation of the extreme cases (of ratio of residual (applied) stress to yield stress) -0.9 and 0.9 will give further and very interesting information. Starting with the case -0.9 , the results are depicted in Fig. 7. It can be seen that this graph is similar to the one for -0.5 in Fig. 6(a). This indicates that indentation properties and fields, and in particular principal stresses, are rather insensitive to compressive residual stresses. This has been emphasized previously based on empirical studies, cf. (Ref 29), but can here be directly explained in a deterministic manner by the behavior of the stresses close to the contact region.

Corresponding results for high tensile stresses, ratio of residual (applied) stress to yield stress being 0.9 , are shown in Fig. 8. Clearly, the stress evolution is quite similar to the one for the ratio 0.5 , as shown in Fig. 6(b), but the stress path is longer including even more elastic deformation.

In order to further emphasize the matters discussed above, the results shown in Fig. 5-8 are collected in Fig. 9. It is then clearly visualized that the stress paths for compressed materials are very similar and also follow quite closely the path for an initially stress-free material. Furthermore, tensile residual stress paths differ substantially from the compressed ones with significant stress rearrangement and are much dependent on the level of residual stress.

In this context, it should also be mentioned that as an alternative; the results in Fig. 5-9 could be equally well shown as a two-dimensional presentation of the von Mises yield circle. This is due to the fact that the present analysis is based on classical elastoplasticity and accordingly, the hydrostatic pressure will not alter the effective stress in the plastic deformation regime. Presently though, it is judged that a three-dimensional representation (as in Fig. 5-9) is slightly more illustrative, but the discussion above about the role of the hydrostatic pressure is important to remember.

The results in Fig. 5-8 (and in Fig. 9) will now be summarized in relation to the features of interest listed above.

- *The difference between indentation of materials with compressive and tensile residual stress states:* At tensile residual stresses, the stress path, from initial to final state, in the principal stress space is long and includes substantial stress rearrangements with elastic deformations. This is not so for compressive residual stresses as then the stress path is straight and short similar to the situation at indentation of stress-free materials. Accordingly, indentation properties and fields, and in particular principal stresses, are rather insensitive to compressive residual stresses. These aspects will be further discussed in detail in the points directly below.
- *The effect from substantial elastic deformations in the contact region:* Due to the long stress path in case of tensile residual stresses, substantial elastic deformations are present at the contact region. It has been shown previously, (Ref 21, 22) that high tensile stresses make the indentation response more elastic, in particular for metals and other materials (for example polymers and ceramics) with a relatively low value on the Johnson (Ref 3, 4) parameter λ and can also lead to a loss of hardness invariance. Compressive residual stresses on the other hand will increase the plastic deformation in the contact region. This is evident in Fig. 6(a) and 7, showing that the yield surface is reached almost immediately after the indentation

Fig. 6 Stress history in the principal stress space with the von Mises flow cylinder included. The residual stress-state is equi-biaxial according to $\sigma_1 = \sigma_2 = \sigma_{res}$, and the final stress state is recorded at the maximum indentation depth. (o), initial stress state. (x), intermediate stress states. (square), final stress state. The results are taken from (Ref 18). (a) $(\sigma_{res}/\sigma_y) = -0.5$. (b) $(\sigma_{res}/\sigma_y) = 0.5$

Fig. 7 Stress history in the principal stress space with the von Mises flow cylinder included. The residual stress-state is equi-biaxial according to $\sigma_1 = \sigma_2 = \sigma_{res}$, and the final stress state is recorded at the maximum indentation depth. (o), initial stress state. (x), intermediate stress states. (square), final stress state. $(\sigma_{res}/\sigma_y) = -0.9$

Fig. 8 Stress history in the principal stress space with the von Mises flow cylinder included. The residual stress-state is equi-biaxial according to $\sigma_1 = \sigma_2 = \sigma_{res}$, and the final stress state is recorded at the maximum indentation depth. (o), initial stress state. (x), intermediate stress states. (square), final stress state. $(\sigma_{res}/\sigma_y) = 0.9$

process is started.

- *The sensitivity (sometimes the insensitivity) of global indentation parameters to residual stresses at sharp indentation:* The stress paths shown in Fig. 6(a) and 7 are almost identical and very similar to the stress-free case shown in Fig. 5. Essentially no rearrangement of stresses occurs after initial plasticity. This indicates that sensitivity of global indentation variables and fields is an issue when compressive residual stresses are to be determined using sharp indentation. This issue has been discussed previously by investigators of insight, cf. (Ref 12), but here a solid theoretical understanding is given for this result. Indeed, spherical indentation aiming at residual stress determination is described frequently in the literature, cf. (Ref 12, 16, 30-32) and the advantage of such an approach is of course as already discussed, the sensitivity to residual stresses, while the disadvantage concerns the fact that at spherical indentation global properties are dependent of the indentation depth. Due to the latter feature, the interpretation of the results from a spherical indentation test

Fig. 9 Stress history in the principal stress space with the von Mises flow cylinder included. Summary of the results in Fig. 5-8

becomes substantially more intricate. The sensitivity to tensile residual stresses is much higher due to the long stress path during indentation.

5. Conclusions

Sharp indentation of initially stressed classical elastoplastic materials has been investigated using the finite element method. The aim of the investigation was to create a solid theoretical understanding of earlier results derived from empirical considerations.

In the study, the development of the principal stress state was analyzed in relation to the von Mises flow surface. In doing so, the stresses in a material point were followed from the initial state prior to indentation until the point enters the highly stressed part of the contact region. Such an approach has not been presented previously in a complete manner.

Based on the results, the mechanical behavior at indentation of pre-stressed materials is now understood from a deterministic approach. In particular, the analysis focuses on knowledge about (1) the difference between indentation of materials with compressive and tensile residual stress states, (2) the effect from substantial elastic deformations in the contact region, (3) the sensitivity (sometimes the insensitivity) of global indentation parameters to residual stresses at sharp indentation.

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