Discussion

Discussion of "A Commentary on Reaction Kinetics in Processes of Nucleation and Growth"*

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The reprinting of the classic article by Johnson and Mehl[1] is most welcome, in particular since the unpublished Appendices A through E are now included. The history of their equation in the United States is well described in the commentary, but a few additional details may be interesting. The Russian metallurgist, I.L. Mirkin, studied the kinetics of the isothermal formation of pearlite, and in 1935, he published an approximate solution of the problem of hard impingement between pearlite colonies.[2] Later he contacted the famous statistician, A.N. Kolmogorov, and introduced him to the problem. When he published his exact solution of the problem in 1937,[3,4] he extended his gratitude to I.L. Mirkin, who interested him in this problem and kindly provided him with all the necessary material. Kolmogorov considered "metal crystallization," in general, but Mirkin applied it to the formation of pearlite in1938.[5] In the Soviet Union, it then became known as the Kolmogorov–Mirkin equation.

In the Western countries, it was long known as the Johnson–Mehl equation and later Avrami's name was added. In 1961, Kolmogorov's priority was realized,[6] and gradually it became known as the KJMA equation. It seems that Mirkin's contribution should also be acknowledged, as it was in the Soviet Union. It is thus proposed that we should refer to the KMJMA equation or KM-JMA equation.

Through the publication of the Appendices from the article by Johnson and Mehl, one can now compare the two ways of deriving the famous equation. This illustrates the advantage of consulting a mathematician. A derivation for the simple case of N pearlite colonies per volume, nucleated at random positions but at the same time, would be as follows, according to Kolmogorov's method.

Instead of considering the volume fraction, f, transformed to pearlite, Kolmogorov started with the probability, S_i, that a randomly selected point in the volume V of the system would not fall inside a particular nodule of pearlite, $S_i = 1 - V_i/V$. The probability that it falls outside all the nodules is obtained as the product $S = \Pi S_i = \Pi(1 - V_i/V)$ independent of how they may have impinged on each other because the individual volumes, V_i, will be evaluated without considering impingement. This probability is identical to the volume fraction of the untransformed material. The logarithm of this equation yields

$$\ln(1 - f) = \Sigma \ln(1 - V_i/V) \cong \Sigma(-V_i/V) = -(1/V)\Sigma V_i$$

[1]

The approximation is very good if the total volume is much larger than the individual volumes, i.e., if there are many pearlite colonies. With a time-independent growth rate, G, i.e., for isothermal conditions, one obtains

$$\ln(1 - f) = -(1/V) \cdot NV \cdot (4\pi/3)(Gt)^3 = -(4\pi/3)NG^3t^3$$

[2]

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