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Second Order Sliding Mode Control for Discrete Decouplable Multivariable Systems via Input-output Models

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Abstract: The problem of the chattering phenomenon is still the main drawback of the classical sliding mode control. To resolve this problem, a discrete second order sliding mode control via input-output model is proposed in this paper. The proposed control law is synthesized for decouplable multivariable systems. A robustness analysis of the proposed discrete second order sliding mode control is carried out. Simulation results are presented to illustrate the effectiveness of the proposed strategy.

Keywords: Discrete second order sliding mode control, decouplable multi-input multi-output systems, input-output model, chattering phenomenon, robustness analysis.

1 Introduction

The variable structure systems (VSS) were introduced in 1960s[1, 2]. They are characterized by their robustness against uncertainties, modeling errors and external disturbances[1, 3−5]. Sliding mode control (SMC) is a particular case of the variable structure systems. The main idea behind SMC is to synthesize a discontinuous control input to force the states trajectories to reach a specific surface called the sliding surface ($s = 0$) in finite time and to stay on it. However, in spite of the robustness of the sliding mode control, the chattering phenomenon, caused by the discontinuous term of the control law, is still the main problem of the SMC which involves sudden and rapid variation of the control signal leading to undesirable results[3, 5−7].

Many researchers were interested in solving the problem of the chattering phenomenon. One of the solutions was the higher order sliding mode control which has been introduced in the 1980s in order to overcome the chattering problem. The second order sliding mode control is a particular case of the higher order sliding mode control. It involves forcing the system to reach the sliding surface characterized by $\dot{s} = s = 0$ and to remain on it[8, 9].

The elaboration of the sliding mode control via input output model was limited to the classical sliding mode control. There are some works, in literature, which used input-output model for single-input single-output systems (SISO)[10−15]. For SISO systems, we proposed a new discrete second order sliding mode control in order to reduce the chattering phenomenon[16].

However, it is well known that many control systems are multivariable and the control problems of this type of systems are very difficult[17−19]. Therefore, it is necessary to synthesize robust control laws for multivariable systems. The adaptive control and the sliding mode control were combined in order to synthesize a discrete robust adaptive sliding mode controller for multivariable systems[17]. Moreover, in the last few years, a discrete sliding mode control via input-output model was developed for decouplable and nondecouplable multivariable systems, respectively[18, 19].

This work proposes a discrete second order sliding mode control for decouplable multivariable systems (2-MDSMC) via input-output model[20] and studies the robustness of this control. The 2-MDSMC was designed to resolve the problems of the chattering phenomenon and the external disturbances. In order to obtain good performance in terms of reduction of the chattering phenomenon and rejection of the external disturbances, a condition for the choice of the discontinuous term amplitude was given.

This paper is organized as follows. Section 2 describes the classical sliding mode control for decouplable multivariable systems. In Section 3, we propose a new discrete second order sliding mode control for the decouplable multi-input multi-output systems. A robustness analysis of the proposed discrete second order sliding mode control is presented in Section 4. Simulation results are given in Section 5. Section 6 concludes the paper.

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2 Multivariable discrete classical sliding mode control (1-MDSMC)

Consider the multi-input multi-output (MIMO) discrete time system described by the following model:

$$A(q^{-1})Y(k) = q^{-1}B(q^{-1})U(k) + V(k) \tag{1}$$

where $Y(k)$, $U(k)$ and $V(k)$ are the output, the input and the disturbance vectors, respectively.

$$\begin{aligned} Y(k) &= [y_1(k) \ \cdots \ y_p(k)]^{\mathrm{T}} \\ U(k) &= [u_1(k) \ \cdots \ u_p(k)]^{\mathrm{T}} \\ V(k) &= [v_1(k) \ \cdots \ v_p(k)]^{\mathrm{T}}. \end{aligned}$$

$A(q^{-1})$ and $B(q^{-1})$ are two polynomial matrices defined as

$$\begin{cases} A\left(q^{-1}\right) = I_p + A_1 q^{-1} + \cdots + A_{n_A} q^{-n_A}, \\ \quad \dim\left(A_{\tau_1}\right) = (p,p), \tau_1 \in [1, n_A] \\ \\ B\left(q^{-1}\right) = B_0 + B_1 q^{-1} + \cdots + B_{n_B} q^{-n_B}, \\ \quad \dim\left(B_{\tau_2}\right) = (p,p), \tau_2 \in [1, n_B]. \end{cases}$$

The sliding function vector is given by

$$\begin{aligned} S(k) = C\left(q^{-1}\right)\left(Y(k) - Y_r(k)\right) &= \\ C(q^{-1})E(k) &= \\ [s_1(k) \ \cdots \ s_p(k)]^{\mathrm{T}} \end{aligned} \tag{2}$$

where $C\left(q^{-1}\right)$ is a polynomial matrix defined as

$$\begin{aligned} C(q^{-1}) = I_p + C_1 q^{-1} + \cdots + C_{n_C} q^{-n_C} \\ \dim(C_{\tau_3}) = (p,p), \ \tau_3 \in [1, \ n_C]. \end{aligned}$$

$Y_r(k)$ is the desired trajectory vector. $E(k)$ is the error vector.

Consider $\bar{F}\left(q^{-1}\right)$ and $\bar{G}\left(q^{-1}\right)$ as the two polynomial matrices solutions of the diophantine polynomial matrix equation:

$$C\left(q^{-1}\right) = \bar{F}\left(q^{-1}\right) A\left(q^{-1}\right) \Delta\left(q^{-1}\right) + q^{-1}\bar{G}\left(q^{-1}\right) \tag{3}$$

where

$$\begin{cases} \bar{F}\left(q^{-1}\right) = I_p \\ \bar{G}\left(q^{-1}\right) = \bar{G}_0 + \bar{G}_1 q^{-1} + \cdots + \bar{G}_{n_{\bar{G}}} q^{-n_{\bar{G}}} \\ \dim\left(\bar{G}_{\tau_4}\right) = (p,p), \ \tau_4 \in [0, n_{\bar{G}}] \\ \quad n_{\bar{G}} = \sup\left(n_C - 1, n_A\right) \end{cases}$$

and $\Delta\left(q^{-1}\right) = \left(1 - q^{-1}\right) I_p$ is a differential operator.

The equivalent control law was obtained when the following condition was satisfied:

$$S(k+1) = S(k) = 0.$$

We have

$$S(k+1) = C\left(q^{-1}\right)\left(Y(k+1) - Y_r(k+1)\right).$$

If we replace the expression of $Y(k+1)$, $S(k+1)$ becomes

$$\begin{aligned} &S(k+1) = \\ &\quad C\left(q^{-1}\right)\left[A\left(q^{-1}\right)\right]^{-1}\left[B\left(q^{-1}\right)U(k) + V(k+1)\right] - \\ &\quad C\left(q^{-1}\right)Y_r(k+1) = \\ &\quad C\left(q^{-1}\right)\left[A\left(q^{-1}\right)\right]^{-1}B\left(q^{-1}\right)U(k) - \\ &\quad C\left(q^{-1}\right)Y_r(k+1) + C\left(q^{-1}\right)\left[A\left(q^{-1}\right)\right]^{-1}V(k+1). \end{aligned}$$

Replacing $C\left(q^{-1}\right)$ by its expression defined in (3), we obtained

$$\begin{aligned} &S(k+1) = \\ &\quad \bar{F}\left(q^{-1}\right)B\left(q^{-1}\right)\Delta\left(q^{-1}\right)U(k) + \\ &\quad q^{-1}\bar{G}\left(q^{-1}\right)\left[A\left(q^{-1}\right)\right]^{-1}B\left(q^{-1}\right)U(k) + \\ &\quad C\left(q^{-1}\right)\left[A\left(q^{-1}\right)\right]^{-1}V(k+1) - C\left(q^{-1}\right)Y_r(k+1). \end{aligned}$$

By using (1), the sliding function $S(k+1)$ can be written as

$$\begin{aligned} S(k+1) = B\left(q^{-1}\right)\Delta\left(q^{-1}\right)U(k) + G\left(q^{-1}\right)Y(k) + \\ \Delta\left(q^{-1}\right)V(k+1) - C\left(q^{-1}\right)Y_r(k+1). \end{aligned}$$

In order to calculate the equivalent control law $U_{eq}(k)$, we assumed that the disturbances vector was null, the sliding function $S(k+1)$ became

$$\begin{aligned} S(k+1) = B\left(q^{-1}\right)\Delta\left(q^{-1}\right)U(k) + \bar{G}\left(q^{-1}\right)Y(k) - \\ C\left(q^{-1}\right)Y_r(k+1). \end{aligned}$$

Then, the equivalent control law was given by

$$\begin{aligned} U_{eq}(k) = \left[B\left(q^{-1}\right)\Delta\left(q^{-1}\right)\right]^{-1}C\left(q^{-1}\right)Y_r(k+1) - \\ \left[B\left(q^{-1}\right)\Delta\left(q^{-1}\right)\right]^{-1}\bar{G}\left(q^{-1}\right)Y(k). \end{aligned} \tag{4}$$

To ensure the robustness of the sliding mode control law, we added the discontinuous control term, such as

$$U_{dis}(k) = -\begin{bmatrix} m_1\mathrm{sgn}\left(s_1(k)\right) \\ m_2\mathrm{sgn}\left(s_2(k)\right) \\ \cdots \\ m_p\mathrm{sgn}\left(s_p(k)\right) \end{bmatrix} \tag{5}$$

where sgn is the signum function defined as

$$\mathrm{sgn}\left(s_i(k)\right) = \begin{cases} -1, & \text{if} \quad s_i(k) < 0 \\ 1, & \text{if} \quad s_i(k) > 0. \end{cases}$$

Then, the global sliding mode control law can be expressed as

$$\begin{aligned} U(k) = U_{eq}(k) + U_{dis}(k) &= \\ \left[B\left(q^{-1}\right)\Delta\left(q^{-1}\right)\right]^{-1} &\times \\ \left[-\bar{G}\left(q^{-1}\right)Y(k) + C\left(q^{-1}\right)Y_r(k+1)\right] &- \\ \begin{bmatrix} m_1\mathrm{sgn}\left(s_1(k)\right) \\ m_2\mathrm{sgn}\left(s_2(k)\right) \\ \cdots \\ m_p\mathrm{sgn}\left(s_p(k)\right) \end{bmatrix}&. \end{aligned} \tag{6}$$

3 Multivariable discrete second order sliding mode control (2-MDSMC)

In spite of the robustness of the sliding mode control, the chattering phenomenon is the main drawback of SMC. To overcome this problem, we propose using a discrete second order sliding mode control for the decouplable multivariable systems via input-output model.

In this section, we consider the same system defined by (1). In the case of second order sliding mode control, the sliding function is expressed in terms of $S(k+1)$ and $S(k)$. The new sliding function vector $\sigma(k)$ was selected as[8, 9]

$$\sigma(k) = S(k) + \beta S(k-1) \tag{7}$$

where $S(k)$ is the sliding function defined in (2), and $0 < \beta < 1$.

The equivalent control law was deduced from the following equation:

$$\sigma(k+1) = \sigma(k) = 0. \tag{8}$$

We had

$$S(k+1) = B(q^{-1})\Delta(q^{-1})U(k) + \bar{G}(q^{-1})Y(k) - C(q^{-1})Y_r(k+1).$$

Then, the sliding function vector $\sigma(k+1)$ can be written as

$$\sigma(k+1) = S(k+1) + \beta S(k) = B(q^{-1})\Delta(q^{-1})U(k) + \bar{G}(q^{-1})Y(k) - C(q^{-1})Y_r(k+1) + \beta S(k).$$

Using the last relation and (8), we obtained the expression of the equivalent control law as

$$U_{eq_2}(k) = -\left[B(q^{-1})\Delta(q^{-1})\right]^{-1}\beta S(k) - \left[B(q^{-1})\Delta(q^{-1})\right]^{-1}\bar{G}(q^{-1})Y(k) + \left[B(q^{-1})\Delta(q^{-1})\right]^{-1}C(q^{-1})Y_r(k+1) \tag{9}$$

with $\bar{G}(q^{-1})$ as the polynomial matrix solution of the diophantine polynomial matrix defined in (3).

In the case of the discrete second order sliding mode control, the discontinuous control law $U_{dis_2}(k)$ is given by[9, 16]:

$$U_{dis_2}(k) = U_{dis_2}(k-1) - \left[B(q^{-1})\Delta(q^{-1})\right]^{-1}T_e \begin{bmatrix} m'_1\mathrm{sgn}(\sigma_1(k)) \\ m'_2\mathrm{sgn}(\sigma_2(k)) \\ \vdots \\ m'_p\mathrm{sgn}(\sigma_p(k)) \end{bmatrix} \tag{10}$$

with T_e as the sampling rate.

Then, the global control law is written as

$$U(k) = U_{eq_2}(k) + U_{dis_2}(k) = \left[B(q^{-1})\Delta(q^{-1})\right]^{-1}\begin{bmatrix} -\beta S(k) - \bar{G}(q^{-1})Y(k) + \\ C(q^{-1})Y_r(k+1) \end{bmatrix} + U_{dis_2}(k-1) - \left[B(q^{-1})\Delta(q^{-1})\right]^{-1}T_e \begin{bmatrix} m'_1\mathrm{sgn}(\sigma_1(k)) \\ m'_2\mathrm{sgn}(\sigma_2(k)) \\ \cdots \\ m'_p\mathrm{sgn}(\sigma_p(k)) \end{bmatrix}. \tag{11}$$

4 Robustness analysis

In order to obtain good results in terms of reduction of the chattering phenomenon and rejection of external disturbances, we must choose an optimal value of the discontinuous term amplitude. Therefore, in this section, we determined a condition for the choice of the discontinuous term amplitude.

By applying the control law defined in (11) to the system (1), the sliding function vector at instant $k+1$ was written as

$$S(k+1) = B(q^{-1})\Delta(q^{-1})U(k) + \bar{G}(q^{-1})Y(k) + \Delta(q^{-1})V(k+1) - C(q^{-1})Y_r(k+1). \tag{12}$$

Replacing $U(k)$ by its expression (11), we had

$$S(k+1) = B(q^{-1})\Delta(q^{-1})(U_{eq_2}(k) + U_{dis_2}(k)) + \bar{G}(q^{-1})Y(k) + \Delta(q^{-1})V(k+1) - C(q^{-1})Y_r(k+1).$$

Replace $U_{eq_2}(k)$ by its expression, the sliding function vector became

$$S(k+1) = -\beta S(k) + B(q^{-1})\Delta(q^{-1})U_{dis_2}(k) + \Delta(q^{-1})V(k+1).$$

The difference between $S(k+1)$ and $S(k)$ gave

$$S(k+1) - S(k) = -\beta S(k) + B(q^{-1})\Delta(q^{-1})U_{dis_2}(k) + \Delta(q^{-1})V(k+1) + \beta S(k-1) - B(q^{-1})\Delta(q^{-1})U_{dis_2}(k-1) - \Delta(q^{-1})V(k).$$

The new sliding function vector $\sigma(k+1)$ can be rewritten as

$$\sigma(k+1) = \sigma(k) + \Delta(q^{-1})(V(k+1) - V(k)) - \begin{bmatrix} m''_1\,\mathrm{sgn}(\sigma_1(k)) \\ \cdots \\ m''_p\,\mathrm{sgn}(\sigma_p(k)) \end{bmatrix}$$

with $m''_i = T_e\, m'_i,\quad i \in [1 \ \cdots \ p]$.

Let

$$\tilde{V}(k) = \Delta(q^{-1})(V(k+1) - V(k)) = [\tilde{v}_1(k), \cdots, \tilde{v}_p(k)]^{\mathrm{T}}.$$

A convergent quasi-sliding mode control exists, if the following condition is satisfied:

$$|\sigma_i(k+1)| < |\sigma_i(k)|, \quad i \in [1, \cdots, p]. \tag{13}$$

This last equation is equivalent to

$$\begin{cases} (\sigma_i(k+1) - \sigma_i(k)), & \text{if } (\sigma_i(k)) < 0 \\ (\sigma_i(k+1) + \sigma_i(k)), & \text{if } (\sigma_i(k)) > 0. \end{cases} \tag{14}$$

We had

$$\sigma_i(k+1) = \sigma_i(k) - m''_i \operatorname{sgn}(\sigma_i(k)) + \tilde{v}_i(k).$$

Case 1. $\sigma_i(k) > 0$.

In this case, the conditions of existence of a quasi-sliding mode are

$$\begin{cases} \sigma_i(k+1) - \sigma_i(k) < 0 \\ \sigma_i(k+1) + \sigma_i(k) > 0. \end{cases}$$

1) $\sigma_i(k+1) - \sigma_i(k) = -m''_i + \tilde{v}_i(k) < 0$
$\Rightarrow m''_i > \tilde{v}_i(k)$

2) $\sigma_i(k+1) + \sigma_i(k) = 2\sigma_i(k) - m''_i + \tilde{v}_i(k) > 0$
$\Rightarrow m''_i < 2\sigma_i(k) + \tilde{v}_i(k)$.

Case 2. $\sigma_i(k) < 0$.

The conditions of existence of a quasi-sliding mode become

$$\begin{cases} \sigma_i(k+1) - \sigma_i(k) > 0 \\ \sigma_i(k+1) + \sigma_i(k) < 0. \end{cases}$$

1) $\sigma_i(k+1) - \sigma_i(k) = m''_i + \tilde{v}_i(k) > 0$
$\Rightarrow m''_i > -\tilde{v}_i(k)$

2) $\sigma_i(k+1) + \sigma_i(k) = 2\sigma_i(k) + m''_i + \tilde{v}_i(k) < 0$
$\Rightarrow m''_i < -2\sigma_i(k) - \tilde{v}_i(k)$.

Theorem 1. The discrete second order sliding mode control defined in (11) allows the reduction of the chattering phenomenon if and only if the gains m''_i satisfy

$$\tilde{v}_i(k) \operatorname{sgn}(\sigma_i(k)) < m''_i < 2|\sigma_i(k)| + \tilde{v}_i(k) \operatorname{sgn}(\sigma_i(k)), \quad i \in [1, \cdots, p]. \tag{15}$$

If the external disturbances $v_i(k)$ are constant, then the last relation becomes

$$0 < m''_i < 2|\sigma_i(k)| \tag{16}$$

as the sliding function $\sigma_i(k)$ tends to zero. Therefore, m_i'' must be very small.

5 Simulation example

Consider the multi-input multi-output system described by the following expression[19]:

$$A(q^{-1})Y(k) = q^{-1}B(q^{-1})U(k) + V(k)$$

with

$$\begin{cases} A(q^{-1}) = I_2 + A_1 q^{-1} + A_2 q^{-2} \\ B(q^{-1}) = B_0 + B_1 q^{-1} \end{cases}$$

where

$$A_1 = \begin{pmatrix} -1.856 & 0 \\ 0 & -1.925\,2 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0.860\,6 & 0 \\ 0 & 0.926\,5 \end{pmatrix}$$

$$B_0 = \begin{pmatrix} 0.095\,16 & 0.048\,77 \\ 0.048\,77 & 0.259\,7 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} -0.090\,5 & -0.044\,1 \\ -0.047\,5 & -0.247 \end{pmatrix}$$

$$Y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix}$$

$$U(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix}.$$

The polynomial matrix $C(q^{-1})$ was chosen as

$$C(q^{-1}) = I_2 - 0.02 I_2 q^{-1} + 0.000\,1 I_2 q^{-2}.$$

The desired trajectory vector was defined as

$$Y_r(k) = \begin{bmatrix} y_{r_1}(k) \\ y_{r_2}(k) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

The disturbances vector was chosen as

$$\begin{cases} V(k) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, & \text{if } 0 < k < 100 \\ V(k) = \begin{bmatrix} 0.6 \\ 0 \end{bmatrix}, & \text{if } k > 100 \\ V(k) = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}, & \text{if } k > 150. \end{cases}$$

In order to reduce the chattering phenomenon, the gains m''_i must satisfy the condition defined in (15). The external disturbances vector was chosen as constant disturbances. Therefore, the gains m''_i must satisfy the condition defined in (16).

We had

$$0 < m''_1 < 2|\sigma_1(k)| \quad \text{and} \quad 0 < m''_2 < 2|\sigma_2(k)|.$$

Firstly, we proposed that the gains m''_i do not satisfy the condition defined in (15). Then, we chose m''_i which satisfied the condition.

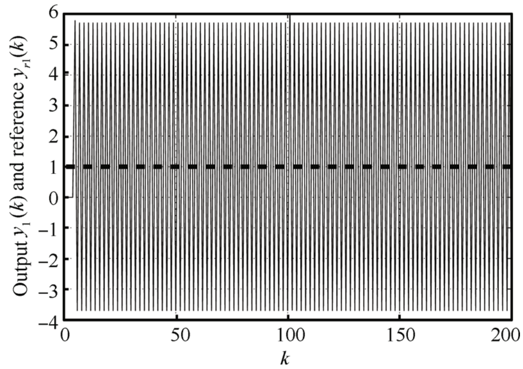
5.1 Condition not satisfied

In this case, we chose m''_1 and m''_2 such as condition (16) was not satisfied.

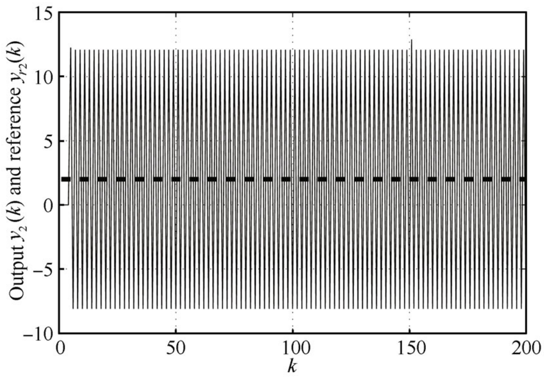
$$m''_1 = 0.5$$
$$m''_2 = 0.5.$$

5.1.1 Case 1: Multivariable discrete classical sliding mode control (1-MDSMC)

The simulation results of classical sliding mode control are shown in Figs. 1 and 2. Fig. 1 presents the evolution of the output vector and the desired reference trajectory vector. The evolution of the sliding functions $s_1(k)$ and $s_2(k)$ is given in Fig. 2.



(a) Evolution of the output $y_1(k)$ (——) and the desired trajectory $y_{r1}(k)$ (– – –)



(b) Evolution of the output $y_2(k)$ (——) and the desired trajectory $y_{r2}(k)$ (- - -)

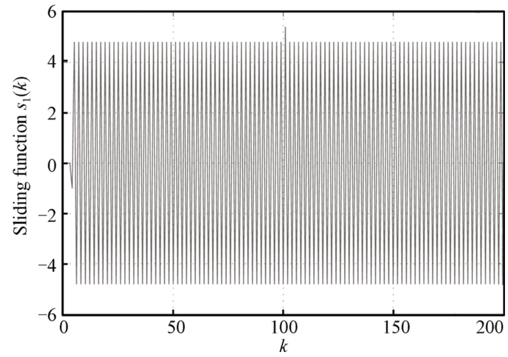
Fig. 1 Evolution of the output vector $Y(k)$ and the desired trajectory vector $Y_r(k)$

Figs. 1 and 2 show that the chattering phenomenon appears in the output vector $Y(k)$ and in the sliding function vector $S(k)$.

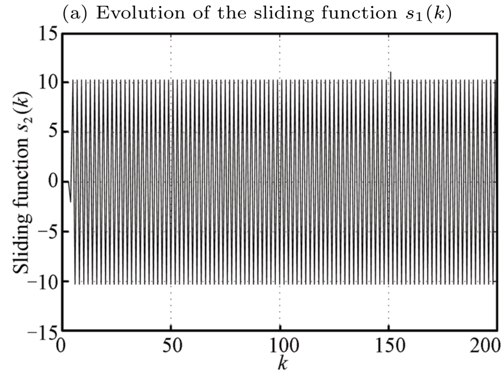
5.1.2 Case 2: Discrete second order sliding mode control (2-MDSMC)

Figs. 3 and 4 illustrate the evolution of the outputs $y_1(k)$ and $y_2(k)$ and the sliding functions $\sigma_1(k)$ and $\sigma_2(k)$, respectively. The parameter β was chosen as $\beta = 0.1$.

We observe from Figs. 3 and 4 the presence of the chattering phenomenon when condition (16) was not satisfied.

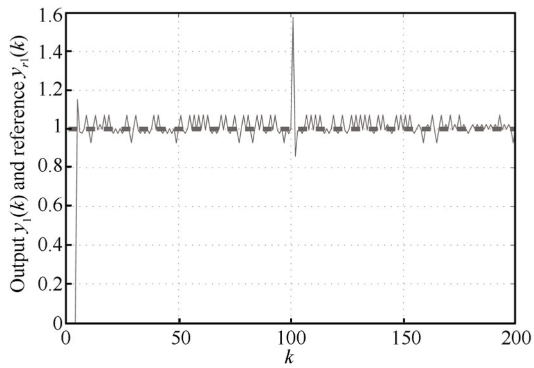


(a) Evolution of the sliding function $s_1(k)$

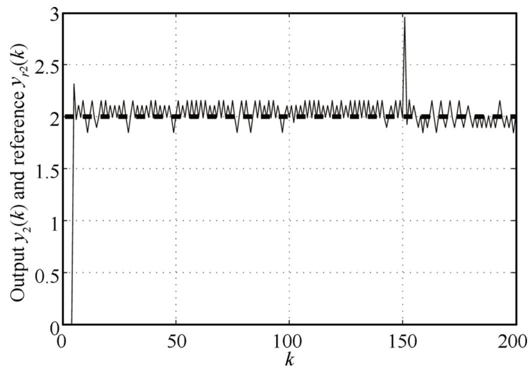


(b) Evolution of the sliding function $s_2(k)$

Fig. 2 Evolution of the sliding function vector $S(k)$

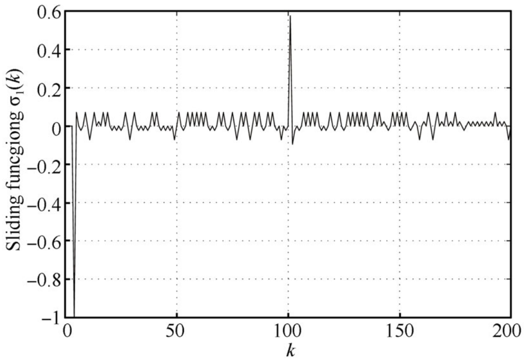


(a) Evolution of the output $y_1(k)$ (——) and the desired trajectory $y_{r1}(k)$ (- - -)

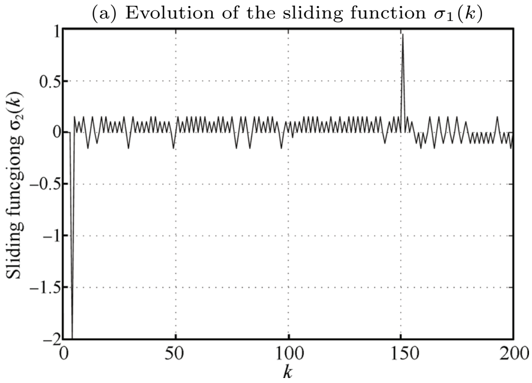


(b) Evolution of the output $y_2(k)$ (——) and the desired trajectory $y_{r2}(k)$ (– – –)

Fig. 3 Evolution of the output vector $Y(k)$ and the desired trajectory vector $Y_r(k)$



(a) Evolution of the sliding function $\sigma_1(k)$



(b) Evolution of the sliding function $\sigma_2(k)$

Fig. 4 Evolution of the sliding function vector $\sigma(k)$

A comparison between the results obtained by the discrete classical sliding mode control with those obtained by the proposed discrete second order sliding mode control, is given in Fig. 5.

The comparison of the two methods show that the results obtained by the proposed control law are better than those obtained by the classical discrete sliding mode control in terms of reduction of the chattering phenomenon even if the condition was not satisfied.

5.2 Condition satisfied

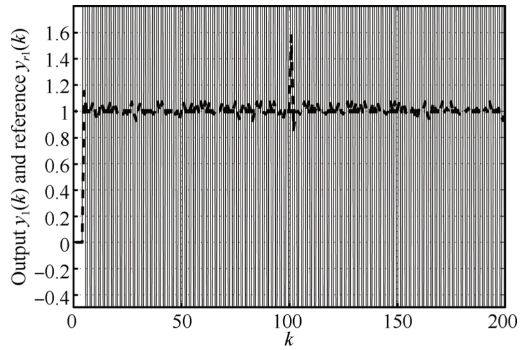
In this section, we assumed that condition (16) was satisfied.

We chose m''_1 and m''_2 as

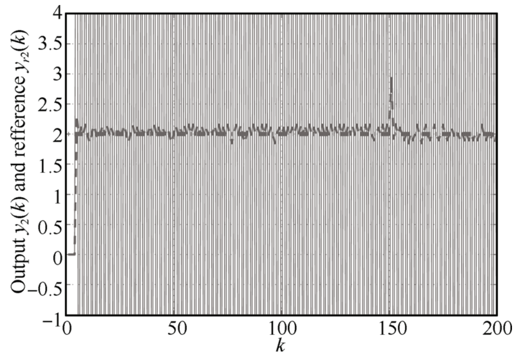
$$m''_1 = 0.001$$
$$m''_2 = 0.01.$$

5.2.1 Case 3: Multivariable discrete classical sliding mode control (1-MDSMC)

In this case, we assumed that condition (6) was satisfied. The simulation results of the system controlled by the controller defined in (6) are shown in Figs. 6−8. The evolution of the real system outputs $y_1(k)$ and $y_2(k)$ and the desired trajectories $y_{r1}(k)$ and $y_{r2}(k)$ are given in Fig. 6. Fig. 7 shows the evolution of the control signals $u_1(k)$ and $u_2(k)$. The evolution of the sliding functions $s_1(k)$ and $s_2(k)$ is presented in Fig. 8.



(a) Comparison between the output $y_1(k)$ of 1-MDSMC (——) and 2-MDSMC (− − −)



(b) Comparison between the output $y_2(k)$ of 1-MDSMC (——) and 2-MDSMC (− − −)

Fig. 5 Comparison between the output of 1-MDSMC and 2-MDSMC

It is clear that the chattering phenomenon cannot be reduced by the classical discrete sliding mode control even if condition (16) was satisfied.

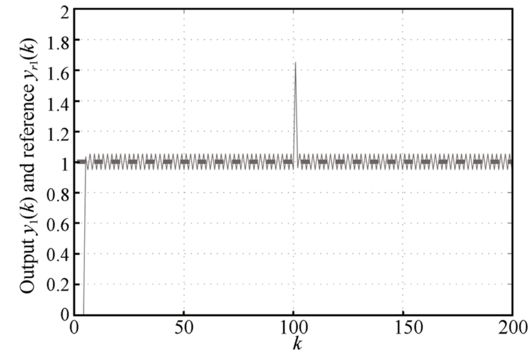
5.2.2 Case 4: Discrete second order sliding mode control (2-MDSMC)

The simulation results of the system controlled by the controller defined in (11) are shown in Figs. 9–11. The evolution of the real system outputs $y_1(k)$ and $y_2(k)$ and the desired trajectories $y_{r1}(k)$ and $y_{r2}(k)$ are given in Fig. 9. Fig. 10 shows the evolution of the control signals $u_1(k)$ and $u_2(k)$. The evolution of the sliding functions $\sigma_1(k)$ and $\sigma_2(k)$ is presented in Fig. 11.

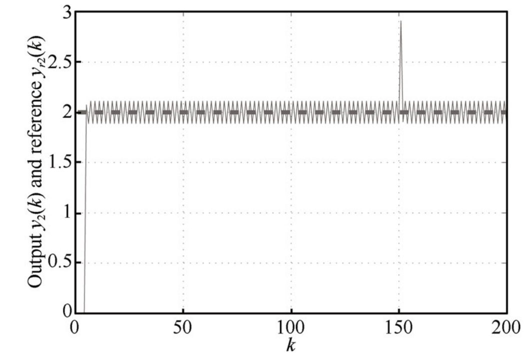
Figs. 9–11 prove that very satisfactory performance is recorded in the reduction of the chattering phenomenon and the rejection of external disturbances.

Fig. 12 gives the comparison between the output vector of the discrete classical sliding mode control and the proposed discrete second order sliding mode control.

It can be observed that, in the case of classical discrete sliding mode control, the chattering phenomenon was always present in the output vector, in the sliding function vector and also in the control input vector, even though the condition was satisfied. While, the proposed discrete second order sliding mode control gave good results in terms of reducing the chattering phenomenon and rejecting external disturbances when condition (16) was satisfied.

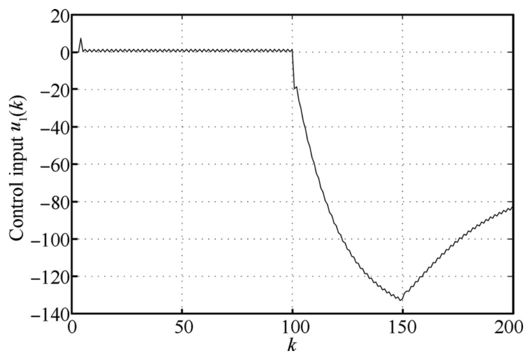


(a) Evolution of the output $y_1(k)$ (——) and the desired trajectory $y_{r1}(k)$ (− − −)

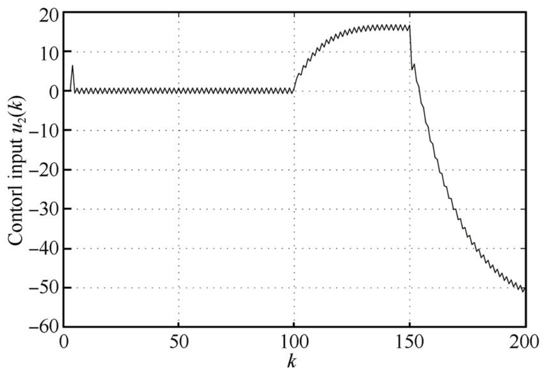


(b) Evolution of the output $y_2(k)$ (——) and the desired trajectory $y_{r2}(k)$ (− − −)

Fig. 6 Evolution of the output vector $Y(k)$ and the desired trajectory vector $Y_r(k)$

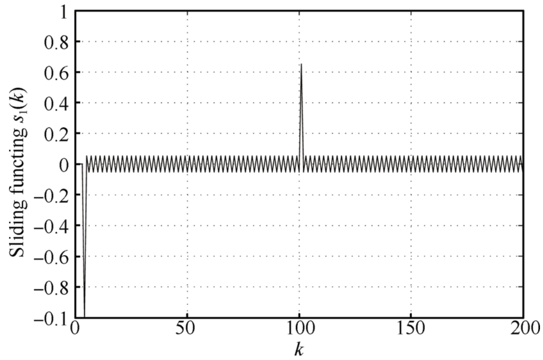


(a) Evolution of the control input $u_1(k)$

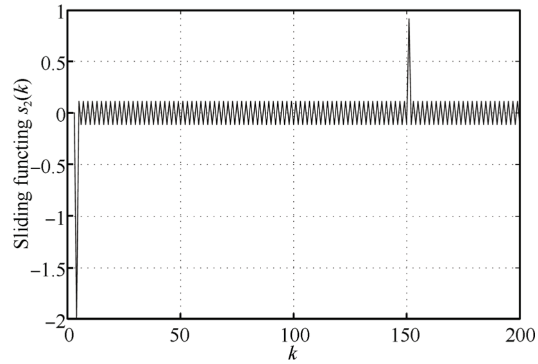


(b) Evolution of the control input $u_2(k)$

Fig. 7 Evolution of the control input vector $U(k)$

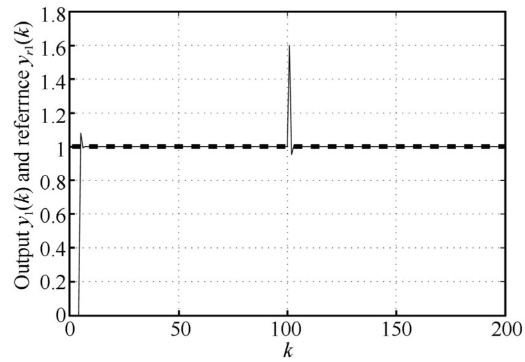


(a) Evolution of the sliding function $s_1(k)$

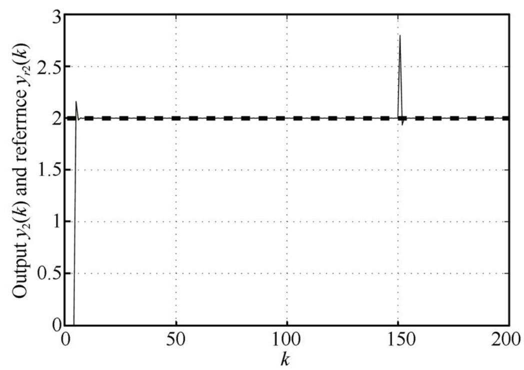


(b) Evolution of the sliding function $s_2(k)$

Fig. 8 Evolution of the sliding function vector $S(k)$

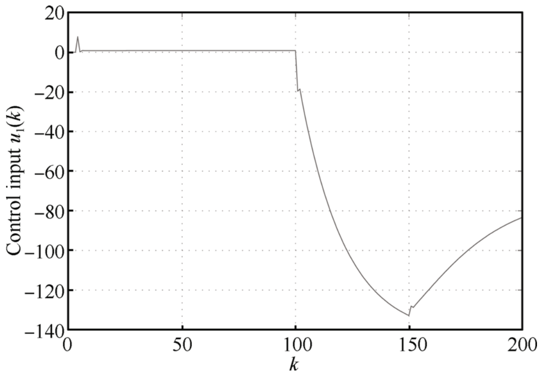


(a) Evolution of the output $y_1(k)$ (——) and the desired trajectory $y_{r1}(k)$ (− − −)

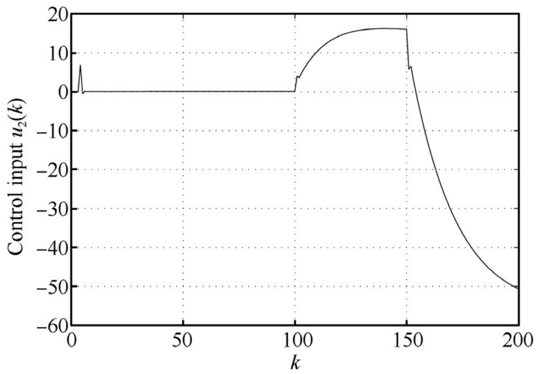


(b) Evolution of the output $y_2(k)$ (——) and the desired trajectory $y_{r2}(k)$ (− − −)

Fig. 9 Evolution of the output vector $Y(k)$ and the desired trajectory vector $Y_r(k)$

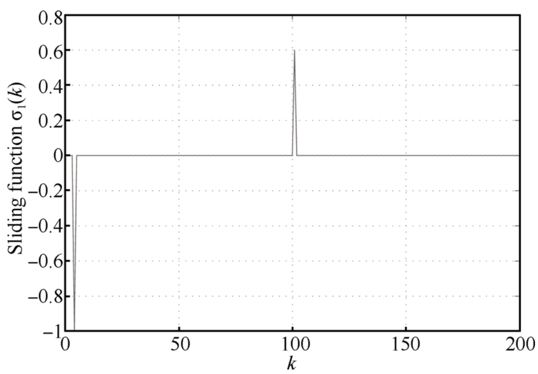


(a) Evolution of the control input $u_1(k)$

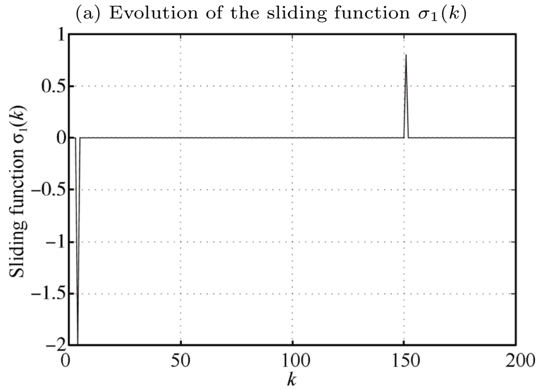


(b) Evolution of the control input $u_2(k)$

Fig. 10 Evolution of the control input vector $U(k)$

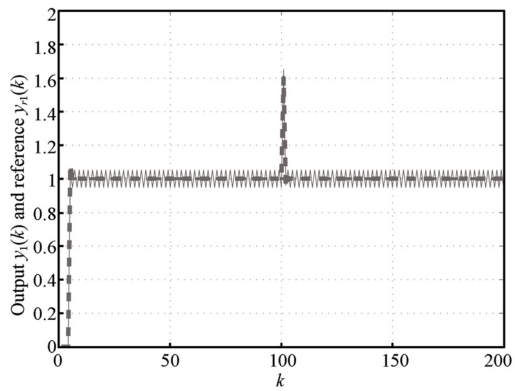


(a) Evolution of the sliding function $\sigma_1(k)$

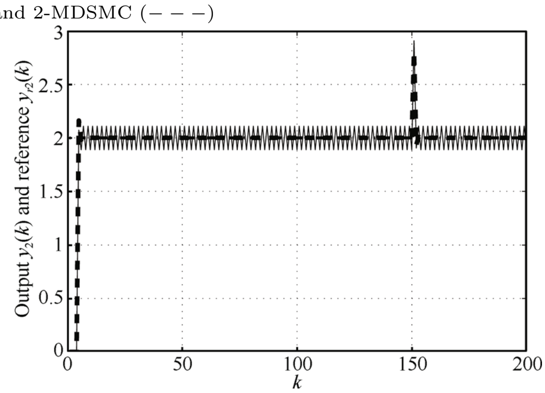


(b) Evolution of the sliding function $\sigma_2(k)$

Fig. 11 Evolution of the sliding function vector $\sigma(k)$



(a) Comparison between the output $y_1(k)$ of 1-MDSMC (——) and 2-MDSMC (− − −)



(b) Comparison between the output $y_2(k)$ of 1-MDSMC (——) and 2-MDSMC (− − −)

Fig. 12 Comparison between the output of 1-MDSMC and 2-MDSMC

6 Conclusions

In this paper, a discrete second order sliding mode control via input-output model for decouplable multivariable systems was proposed. Then, a condition for the choice of the discontinuous term amplitude was elaborated. Finally, a numerical example showed good performance in terms of reduction of the chattering phenomenon and rejection of external disturbances when the condition of the choice of the discontinuous term amplitude was satisfied.

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