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A New Adaptive Tracking Control Approach for Uncertain Flexible Joint Robot System

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Abstract: The adaptive tracking problem for uncertain flexible joint robot system is studied in this paper. By utilizing the adaptive backstepping method, an adaptive controller is constructed at the beginning. By utilizing the modified adaptive dynamic surface control technique, a new adaptive controller is presented afterwards to avoid the overparametrization problem and the explosion of complexity problem existing in the adaptive backstepping method. All the signals of the closed-loop system are rendered globally/semi-globally uniformly ultimately bounded, and the tracking error can be made arbitrarily small by tuning the designed parameters. A simulation example is given to show the validity of the control algorithm.

Keywords: Flexible joint robots, adaptive control, backstepping method, dynamic surface control, position tracking.

1 Introduction

The research of flexible joint robots (FJR) has received considerable attention in the past two decades[1]. To obtain good control performance, no matter in modeling or in control design, the joint flexibility, which is usually caused by harmonic drives, shaft windup, and bearing deformation, cannot be ignored. In the literature, there have been many methods proposed, for instance, the singular perturbation approach[2], the passivity approach[3], the sliding mode approach[4], and the neural network approach[5, 6].

It is known that the backstepping technique is also an important method. A robust controller was proposed to guarantee the tracking of any given reference trajectory with arbitrary accuracy[7]. An adaptive output-feedback controller was designed for the single link robotic manipulator[8] and the adaptive backstepping method for rigid-link flexible-joint robots was studied. In addition to these methods[9, 10], the dynamic surface control method is a recently proposed control algorithm. With this method, Zhang et al.[11, 12] considered adaptive dynamic surface control for nonlinear systems with uncertainties, Hou and Duan[13] studied how to design controller for integrated missile guidance and autopilot. Moreover, some other approaches such as model reference adaptive control[14, 15], robust adaptive control[16] and adaptive iterative learning control[17] are also useful to regulate such nonlinear systems.

The adaptive tracking problem for FJR has been studied before. However, there exist some drawbacks in the existing methods. The first one may be the "overparametrization" problem. The reason is that the control design procedure depends on the linear property of unknown system parameters. Another may be the "explosion of complexity" problem, i.e., the designed controller is usually made quite complicated because of the repeated differentiations of virtual controllers in control design procedure. In this paper, we will investigate how to design an adaptive tracking controller for FJR to avoid the above problems. Mainly motivated by the continuous control ideas[18, 19] and flexibly using algebraic techniques, we present a new control design method for FJR. Then, we construct an appropriate Lyapunov function and show that the designed controllers can guarantee all the signals of the resulting closed-loop system globally/semi-globally uniformly ultimately bounded, and the tracking errors can be rendered arbitrarily small.

The main contributions of the paper are characterized by the following specific features: 1) The "explosion of complexity" problem and the "overparametrization" problem of the existing control methods are avoided. 2) It is not easy to find an appropriate Lyapunov function which is well-behaved in stability analysis. In this paper, by using flexible algebraic techniques, two new Lyapunov functions are recursively constructed in the control design procedure.

2 Preliminaries and problem statement

Consider the dynamic equations of the flexible joint robots given as[20, 21]

$$M(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + h(q_1) + \tau_c + f_1(q_1, \dot{q}_1) = 0 \quad (1)$$

$$B\ddot{q}_2 - \tau_c + f_2(q_2, \dot{q}_2) = u \quad (2)$$

$$\tau_c = K(q_1 - q_2) \quad (3)$$

where $q_1 \in \mathbf{R}^n$ is the joint angular position, $q_2 \in \mathbf{R}^n$ is the motor angular position, $\dot{q}_1, \dot{q}_2 \in \mathbf{R}^n$ are the respective veloc-

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ities, $M(q_1)\in\mathbf{R}^{n\times n}$, $B\in\mathbf{R}^{n\times n}$ are positive and symmetric inertia matrices, $K\in\mathbf{R}^{n\times n}$ is diagonal matrix whose entries are elastic constants k_i of the joints, $C(q_1,\dot q_1)\dot q_1\in\mathbf{R}^n$ is the centrifugal and coriolis force, $h(q_1)\in\mathbf{R}^n$ is the gravity force vector, $f_1(q_1,\dot q_1)$, $f_2(q_2,\dot q_2)$ are the frictional terms, and $u\in\mathbf{R}^n$ is the input torque. The angular positions q_1, q_2, and the respective velocities $\dot q_1$, $\dot q_2$ are assumed to be measurable.

The purpose of the paper is to design an adaptive state feedback controller for the system (1)–(3). We specified the control problems as: 1) Given the desired reference trajectory q_d, design a controller if possible, such that the link position tracking error $z_1=q_1-q_d$ converges to 0 as much as possible. 2) Meanwhile, all the closed-loop signals are rendered bounded.

We need Assumptions 1–3.

Assumption 1. The desired trajectory vectors are continuous and available, and $[q_d^{\rm T},\dot q_d^{\rm T},\ddot q_d^{\rm T}]^{\rm T}\in\Omega_d$ with known compact set $\Omega_d=\left\{[q_d^{\rm T},\dot q_d^{\rm T},\ddot q_d^{\rm T}]^{\rm T}\in\mathbf{R}^{3n}\big||q_d|^2+|\dot q_d|^2+|\ddot q_d|^2\le A\right\}$.

Assumption 2. There exist positive constants b_i, m_i, k_i, $i=1,2$, such that $m_1\le\lambda_{\min}(M)\le\|M\|_2\le\lambda_{\max}(M)\le m_2$, $b_1\le\lambda_{\min}(B)\le\|B\|_2\le\lambda_{\max}(B)\le b_2$, $k_1\le\lambda_{\min}(K)\le\|K\|_2\le\lambda_{\max}(K)\le k_2$, where only parameter b_2 is known.

Assumption 3. There exist unknown positive constant θ_i, and known smooth functions $\phi_i(\cdot)$, $i=0,1,2$, such that

$$|C(q_1,\dot q_1)\dot q_1+h(q_1)+Kq_1|^2\le\theta_0\phi_0(q_1,\dot q_1)\tag{4}$$
$$|f_1(q_1,\dot q_1)|^2\le\theta_1\phi_1(q_1,\dot q_1)\tag{5}$$
$$|f_2(q_2,\dot q_2)|^2\le\theta_2\phi_2(q_2,\dot q_2).\tag{6}$$

Remark 1. These assumptions are reasonable and not stronger than the existing ones. Assumption 1 gives basic conditions using dynamic surface control method. Assumption 2 provides that only one constant b_2 is known in this paper, while in [8, 10], all the parameters b_i, m_i, k_i, $i=1,2$ were assumed to be known. Assumption 3 is similar to Assumption 2 in [19] for the rigid joint case.

Remark 2. It is easy to see that the FJR system is underactuated. Tracking control problems of underactuated systems are more difficult, since there are fewer inputs than degrees of freedom. In addition, when the system is suffered by external disturbances, only some semi-global results can be obtained, rather than achieving global results. See for instance, the FJR system[7, 9], the ships[22], the wheeled inverted pendulums[23], and more general underactuated system[24].

Before the control design procedure, we define $x_1=q_1$, $x_2=\dot q_1$, $x_3=q_2$, $x_4=\dot q_2$, and introduce the following transformations:

$$\begin{cases} z_1=x_1-q_d, & z_2=\dot z_1+(c_1+1)z_1\\ z_3=x_3-x_3^*, & z_4=x_4-x_4^*\end{cases}\tag{7}$$

where x_i^* is the virtual control to be determined later. Then, we can deduce that

$$\begin{cases}\dot z_1=z_2-(c_1+1)z_1\\ \dot z_2=M^{-1}(x_1)Kx_3-F_1\\ \dot z_3=x_4-\dfrac{\partial x_3^*}{\partial\hat\theta}\dot{\hat\theta}-F_2\\ \dot z_4=B^{-1}\left(u+K(x_1-x_3)-f_2\right)-\dfrac{\partial x_4^*}{\partial\hat\theta}\dot{\hat\theta}-F_3\end{cases}\tag{8}$$

where F_i are defined as $F_1=M^{-1}\left({\rm C}x_2+h+Kx_1+f_1\right)+\ddot q_d-(c_1+1)z_2+(c_1+1)^2z_1$, $F_2=\frac{\partial x_3^*}{\partial z_1^{\rm T}}\dot z_1+\frac{\partial x_3^*}{\partial z_2^{\rm T}}\dot z_2+\frac{\partial x_3^*}{\partial q_d^{\rm T}}\dot q_d+\frac{\partial x_3^*}{\partial\dot q_d^{\rm T}}\ddot q_d$, and $F_3=\frac{\partial x_4^*}{\partial z_1^{\rm T}}\dot z_1+\frac{\partial x_4^*}{\partial z_2^{\rm T}}\dot z_2+\frac{\partial x_4^*}{\partial z_3^{\rm T}}\dot z_3+\frac{\partial x_4^*}{\partial q_d^{\rm T}}\dot q_d+\frac{\partial x_4^*}{\partial\dot q_d^{\rm T}}\ddot q_d$. Define positive parameters $a=\frac{m_2}{k_1^2}$ and $\theta=\frac{k_2^2}{a^2m_1^2}\times\max\left\{\theta_0,\theta_1,m_1^2,k_2^2,a^2m_1^2,\frac{am_1^2}{k_2},\frac{\theta_2a^2m_1^2}{k_2^2}\right\}$. Parameter a will be used for constructing the Lyapunov function subsequently, and parameter θ is the only one to be estimated in the control design. The following lemma 1, which is the well known Young′s inequality[19], will play a key role in proving the main results of this paper.

Lemma 1. For vectors $x,y\in\mathbf{R}^n$, and scalar positive numbers $\epsilon>0$, $p>0$, there holds

$$x^{\rm T}y\ \le\ \frac{\epsilon^p}{p}|x|^p+\frac{1}{q\epsilon^p}|y|^q$$

where $q=\frac{p}{p-1}$.

Lemma 2. There exist smooth nonnegative functions ψ_i, $i=1,\cdots,4$, and positive constant ν such that

$$-z_3^{\rm T}F_2\le(z_3^{\rm T}z_3)(\psi_1\theta+\psi_2)+\nu\tag{9}$$
$$-z_4^{\rm T}F_3\le(z_4^{\rm T}z_4)(\psi_3\theta+\psi_4)+\nu.\tag{10}$$

Proof. See Appendix. □

3 Control of FJR system

In this subsection, we will construct an adaptive state-feedback controller for FJR system, which will be addressed in a step-by-step manner.

Step 1. Suppose $\hat\theta$ is the estimate of θ, and the corresponding error is defined as $\tilde\theta=\hat\theta-\theta$. Then, we introduce the transformations: $z_1=x_1-q_d$ and $z_2=\dot z_1+(c_1+1)z_1$ and choose the candidate Lyapunov function $V_1(z_1,z_2,\tilde\theta)=\frac12z_1^{\rm T}z_1+\frac{1}{2a}z_2^{\rm T}Kz_2+\frac{1}{2\gamma}\tilde\theta^2$, where $\gamma>0$ is a designed parameter. Taking the time derivative of V_1, we get

$$\begin{aligned}\dot V_1=&-(c_1+1)z_1^{\rm T}z_1+z_1^{\rm T}z_2+\frac1az_2^{\rm T}KM^{-1}(x_1)Kx_3^*+\\&\frac1az_2^{\rm T}KM^{-1}K(x_3-x_3^*)-\frac1az_2^{\rm T}KF_1+\frac1\gamma\tilde\theta\dot{\hat\theta}.\end{aligned}\tag{11}$$

By using Lemma 1, one can conclude that

$$z_1^{\rm T}z_2\le\frac12z_1^{\rm T}z_1+\frac12z_2^{\rm T}z_2.\tag{12}$$

From Assumption 2 and Lemma 1, it leads to

$$\begin{aligned}&-\frac{1}{a}z_2^{\mathrm{T}}KM^{-1}\left(Cx_2+h(x_1)+Kx_1+f_1\right)\leq\\&\frac{k_2}{am_1}|z_2||Cx_2+h+Kx_1+f_1|\leq\\&\frac{1}{\delta_1}\left(\frac{k_2}{am_1}\right)^2|z_2|^2\left(\theta_0\phi_0+\theta_1\phi_1\right)+\frac{\delta_1}{2}\leq\\&\frac{1}{\delta_1}\theta z_2^{\mathrm{T}}z_2\left(\phi_0+\phi_1\right)+\frac{\delta_1}{2}.\end{aligned}\tag{13}$$

According to Assumption 2 and Lemma 1, it yields

$$-\frac{1}{a}z_2^{\mathrm{T}}K\ddot{q}_d\leq\frac{\theta}{2\delta_2}z_2^{\mathrm{T}}z_2+\frac{\delta_2}{2}|\ddot{q}_d|^2.\tag{14}$$

Similarly, we have

$$\begin{aligned}&-\frac{1}{a}z_2^{\mathrm{T}}K\left(-(c_1+1)z_2+(c_1+1)^2z_1\right)\leq\\&\left(c_1+1+\frac{(c_1+1)^4}{2}\right)\max\{\frac{k_2}{a},\frac{k_2^2}{a^2}\}|z_2|^2+\frac{1}{2}|z_1|^2\leq\\&\left(c_1+1+\frac{(c_1+1)^4}{2}\right)\theta z_2^{\mathrm{T}}z_2+\frac{1}{2}z_1^{\mathrm{T}}z_1.\end{aligned}\tag{15}$$

Combining (13)–(15), it follows that

$$-\frac{1}{a}z_2^{\mathrm{T}}KF_1\leq\frac{1}{2}z_1^{\mathrm{T}}z_1+\theta(\omega_1-1)z_2^{\mathrm{T}}z_2+\frac{\delta_1}{2}+\frac{\delta_2}{2}|\ddot{q}_d|^2\tag{16}$$

where $\omega_1=\frac{1}{\delta_1}(\phi_0+\phi_1)+\frac{1}{2\delta_2}+c_1+\frac{(c_1+1)^4}{2}+2$. It follows from Assumption 2 and Lemma 1 that

$$\frac{1}{a}z_2^{\mathrm{T}}KM^{-1}K(x_3-x_3^*)\leq\frac{1}{2}z_3^{\mathrm{T}}z_3+\theta z_2^{\mathrm{T}}z_2.\tag{17}$$

Defining $d_1=\frac{\delta_1}{2}+\frac{\delta_2}{2}|\ddot{q}_d|^2$, and substituting (12)–(17) into (11), we obtain

$$\begin{aligned}\dot{V}_1\leq&-c_1z_1^{\mathrm{T}}z_1+\frac{1}{a}z_2^{\mathrm{T}}KM^{-1}(x_1)Kx_3^*+\left(\frac{1}{2}+\omega_1\theta\right)z_2^{\mathrm{T}}z_2+\\&\frac{1}{2}z_3^{\mathrm{T}}z_3+d_1+\frac{1}{\gamma}\tilde{\theta}\dot{\hat{\theta}}.\end{aligned}\tag{18}$$

We choose the first virtual control

$$x_3^*(z_1,z_2,q_d,\dot{q}_d,\hat{\theta})=-\left(c_2+\frac{1}{2}+\omega_1\hat{\theta}\right)z_2\tag{19}$$

where $c_2>0$ is a design parameter. The adaptive law is designed as

$$\dot{\hat{\theta}}=\Psi-\sigma\hat{\theta}\tag{20}$$

where $\Psi\geq0$ will be determined later. Actually, the adaptive law can guarantee $\hat{\theta}(t)>0$ for any positive initial value, i.e., if $\hat{\theta}(t_0)>0$. From (20), we have

$$\hat{\theta}(t)=\hat{\theta}(t_0)\mathrm{e}^{-\sigma(t-t_0)}+\int_{t_0}^{t}\mathrm{e}^{-\sigma(t-s)}\Psi(s)\mathrm{d}s>0.$$

In view of $a>0$ and $\hat{\theta}>0$, it follows that

$$\frac{1}{a}z_2^{\mathrm{T}}KM^{-1}(x_1)Kx_3^*\leq-(c_2+\frac{1}{2}+\omega_1\hat{\theta})z_2^{\mathrm{T}}z_2.\tag{21}$$

Substituting (21) into (18), we obtain

$$\dot{V}_1\leq-\sum_{i=1}^{2}c_iz_i^{\mathrm{T}}z_i+\frac{1}{2}z_3^{\mathrm{T}}z_3+\frac{\tilde{\theta}}{\gamma}(\dot{\hat{\theta}}-\Psi_1)+d_1\tag{22}$$

where $\Psi_1=\gamma\omega_1z_2^{\mathrm{T}}z_2$. This completes Step 1. It can be viewed as the initialization of the whole design procedure.

Step 2. Choosing the candidate Lyapunov function $V_2(z_1,z_2,z_3,\tilde{\theta})=V_1+\frac{1}{2}z_3^{\mathrm{T}}z_3$, and taking the time derivative, it leads to

$$\begin{aligned}\dot{V}_2\leq&-\sum_{i=1}^{2}c_iz_i^{\mathrm{T}}z_i+\frac{1}{2}z_3^{\mathrm{T}}z_3+z_3^{\mathrm{T}}x_4^*+\frac{\tilde{\theta}}{\gamma}(\dot{\hat{\theta}}-\Psi_1)+\\&d_1-z_3^{\mathrm{T}}\frac{\partial x_3^*}{\partial\hat{\theta}}\dot{\hat{\theta}}+z_3^{\mathrm{T}}(x_4-x_4^*)-z_3^{\mathrm{T}}F_2.\end{aligned}\tag{23}$$

By Lemma 1, we can deduce that

$$z_3^{\mathrm{T}}(x_4-x_4^*)\leq\frac{1}{2}z_4^{\mathrm{T}}z_4+z_3^{\mathrm{T}}z_3.\tag{24}$$

Using Lemma 2, we have

$$-z_3^{\mathrm{T}}F_2\leq(z_3^{\mathrm{T}}z_3)\left(\psi_1\theta+\psi_2\right)+\nu.\tag{25}$$

Defining $\Psi_2=\Psi_1+\gamma\psi_1z_3^{\mathrm{T}}z_3$ and choosing the virtual control, we have

$$x_4^*(\cdot)=-(c_3+1+\psi_2+\psi_1\hat{\theta})z_3-\frac{\partial x_3^*}{\partial\hat{\theta}}(\Psi_2-\sigma\hat{\theta})\tag{26}$$

where $c_3>0$ is a design parameter. Substituting (26) into (23), it yields that

$$\begin{aligned}\dot{V}_2\leq&-\sum_{i=1}^{3}c_iz_i^{\mathrm{T}}z_i+\frac{1}{2}z_4^{\mathrm{T}}z_4+(\frac{\tilde{\theta}}{\gamma}-z_3^{\mathrm{T}}\frac{\partial x_3^*}{\partial\hat{\theta}})\times\\&(\dot{\hat{\theta}}-\Psi_2+\sigma\hat{\theta})-\frac{\sigma}{\gamma}\tilde{\theta}\hat{\theta}+\mathrm{d}_2\end{aligned}\tag{27}$$

where $d_2=d_1+\nu$ is a positive constant.

Step 3. Choosing the candidate Lyapunov function as $V_3(z_1,z_2,z_3,z_4,\varrho_1,\varrho_2,\tilde{\theta})=V_2+\frac{1}{2}z_4^{\mathrm{T}}Bz_4$, and taking the time derivative of V_3 while noticing (7) and (8), we have

$$\begin{aligned}\dot{V}_3\leq&-\sum_{i=1}^{3}c_iz_i^{\mathrm{T}}z_i+\frac{1}{2}z_4^{\mathrm{T}}z_4+(\frac{\tilde{\theta}}{\gamma}-z_3^{\mathrm{T}}\frac{\partial x_3^*}{\partial\hat{\theta}})\times\\&(\dot{\hat{\theta}}-\Psi_2-\sigma\hat{\theta})+\frac{\sigma}{\gamma}\tilde{\theta}\hat{\theta}+z_4^{\mathrm{T}}u-z_4^{\mathrm{T}}\frac{\partial x_4^*}{\partial\hat{\theta}}\dot{\hat{\theta}}+\\&z_4^{\mathrm{T}}\left(K(x_1-x_3)-f_2\right)-z_4^{\mathrm{T}}F_3+d_2.\end{aligned}\tag{28}$$

By using Lemma 1, it leads to

$$z_4^{\mathrm{T}}K(x_1-x_3)\leq\frac{\theta}{2\epsilon_1}|z_4|^2|x_1-x_3|^2+\frac{\epsilon_1}{2}.\tag{29}$$

Similarly, from Lemma 1, we can deduce that

$$-z_4^{\mathrm{T}}f_2\leq\frac{\theta_2}{2\epsilon_2}|z_4|^2\phi_2+\frac{\epsilon_2}{2}\leq\frac{\theta}{2\epsilon_2}|z_4|^2\phi_2+\frac{\epsilon_2}{2}.\tag{30}$$

By Lemma 2, it yields

$$-z_4^{\mathrm{T}}F_3\leq(z_4^{\mathrm{T}}z_4)\left(\psi_3\theta+\psi_4\right)+\nu.\tag{31}$$

Defining $\omega_2 = \frac{1}{2\epsilon_1}|x_1 - x_3|^2 + \frac{1}{2\epsilon_2}\phi_2 + \psi_3$, $\bar{\psi}_3 = \psi_3 - \gamma\psi_3 z_3^{\rm T}\frac{\partial x_3^*}{\partial\hat{\theta}}$, $\Psi = \Psi_2 + \gamma\omega_2 z_4^{\rm T} z_4$, $d_3 = d_2 + \nu + \frac{\epsilon_1}{2} + \frac{\epsilon_2}{2}$, and choosing the adaptive controller, we have

$$\begin{cases} u = -\left(c_4 + \dfrac{1}{2} + \bar{\psi}_3 + \omega_2\hat{\theta}\right) z_4 - \dfrac{\partial x_4^*}{\partial\hat{\theta}}(\Psi - \sigma\hat{\theta}) \\ \dot{\hat{\theta}} = \Psi - \sigma\hat{\theta}, \quad \hat{\theta}(0) > 0. \end{cases} \tag{32}$$

Substituting (24) and (31) into (28), we have

$$\dot{V}_3 \leq -\sum_{i=1}^{3} c_i z_i^{\rm T} z_i + \frac{1}{2} z_4^{\rm T} z_4 + \frac{\sigma}{\gamma}\tilde{\theta}\hat{\theta} + d_3. \tag{33}$$

It is not difficult to get

$$-\frac{1}{\gamma}\sigma\tilde{\theta}\hat{\theta} = -\frac{1}{\gamma}\sigma\tilde{\theta}^2 - \frac{1}{\gamma}\sigma\tilde{\theta}\theta \leq -\frac{1}{2\gamma}\sigma\tilde{\theta}^2 + \frac{1}{2\gamma}\sigma\theta^2. \tag{34}$$

Substituting (34) into (33), it follows that

$$\dot{V}_3 \leq -cV_3 + d \tag{35}$$

where $d = d_3 + \frac{1}{2\gamma}\sigma\theta^2$. Theorem 1 summarizes the main results of this section.

Theorem 1. Consider the flexible joint robots dynamic system (1)–(3) under Assumptions 1–3, we can design the adaptive controller (32), such that all the closed-loop signals are rendered globally uniformly ultimately bounded and the tracking error z_1 can be rendered arbitrarily small.

Proof. As can be seen, by (35), we can deduce that $\dot{V}_3 \leq 0$ on $V_3 = \rho$ when $c > \frac{d}{\rho}$. Hence, $V_3 \leq \rho$ is an invariant set, i.e., if $V_3(0) \leq \rho$, then $V_3(t) \leq \rho$ for all $t \geq 0$. Thus, all the closed-loop signals are globally uniformly ultimately bounded. Moreover, by adjusting parameters σ, γ, c_i, δ_j, ϵ_j, $i = 1, \cdots, 4$, $j = 1, 2, 3$, we can make the tracking error z_1 arbitrarily small. □

Remark 3. This section presents an adaptive backstepping based control algorithm for FJR system. The overparametrization problem is avoided. However, the designed controller is very complicated here. This problem can be solved by using a modified adaptive dynamic surface method, see the next section for detail.

4 Extensions

In this section, a modified adaptive dynamic surface method will be adopted to obtain a simple adaptive controller. We need to introduce the following transformations:

$$\begin{cases} \bar{z}_1 = x_1 - q_d, \quad \bar{z}_2 = \dot{\bar{z}}_1 + (\bar{c}_1 + 1)\bar{z}_1 \\ \bar{z}_3 = x_3 - \bar{x}_3, \quad \bar{z}_4 = x_4 - \bar{x}_4 \end{cases} \tag{36}$$

where $\bar{x}_i$, $i = 3, 4$ are the filtered virtual control achieved by the following first-order filter

$$\tau_i\dot{\bar{x}}_i + \bar{x}_i = x_i^*, \qquad \bar{x}_i(0) = x_i^*(0) \tag{37}$$

where $\tau_i > 0$ is a positive constant and x_i^* is the virtual control to be determined later. Define $\varrho_1 = \bar{x}_3 - x_3^*$, $\varrho_2 = \bar{x}_4 - x_4^*$, then from (36) and (37), we have

$$\begin{cases} \dot{\bar{z}}_1 = z_2 - (\bar{c}_1 + 1)z_1 \\ \dot{\bar{z}}_2 = M^{-1}(x_1)Kx_3 - F(x_1, x_2, q_d, \dot{q}_d, \ddot{q}_d) \\ \dot{\bar{z}}_3 = x_4 + \dfrac{1}{\tau_3}\varrho_1 \\ \dot{\bar{z}}_4 = B^{-1}\left(u + K(x_1 - x_3) - f_2(x_3, x_4)\right) + \dfrac{1}{\tau_4}\varrho_2 \end{cases} \tag{38}$$

where F is defined as

$$F = M^{-1}\left(Cx_2 + h + Kx_1 + f_1\right) + \ddot{q}_d - \\ (\bar{c}_1 + 1)\bar{z}_2 + (\bar{c}_1 + 1)^2\bar{z}_1.$$

Now, we give the design procedure in detail.

Step 1. Choosing the candidate Lyapunov function $U_1(\bar{z}_1, \bar{z}_2, \tilde{\theta}) = \frac{1}{2}\bar{z}_1^{\rm T}\bar{z}_1 + \frac{1}{2a}\bar{z}_2^{\rm T}K\bar{z}_2 + \frac{1}{2\bar{\gamma}}\tilde{\theta}^2$, similar to Section 3, we have

$$-\frac{1}{a}\bar{z}_2^{\rm T}KF \leq \frac{1}{2}\bar{z}_1^{\rm T}\bar{z}_1 + \theta(\omega_1 - 1)\bar{z}_2^{\rm T}\bar{z}_2 + \frac{\bar{\delta}_1}{2} + \frac{\bar{\delta}_2}{2}|\ddot{q}_d|^2. \tag{39}$$

According to Assumption 2, Lemma 1 and noticing $x_3 - x_3^* = \bar{z}_3 + \varrho_1$, it follows that

$$\frac{1}{a}\bar{z}_2^{\rm T}KM^{-1}K(x_3 - x_3^*) \leq \frac{1}{2}\bar{z}_3^{\rm T}\bar{z}_3 + \frac{1}{2}\varrho_1^{\rm T}\varrho_1 + \theta\bar{z}_2^{\rm T}\bar{z}_2. \tag{40}$$

Choosing the virtual control

$$x_3^*(z_1, z_2, q_d, \dot{q}_d, \hat{\theta}) = -\left(\bar{c}_2 + \frac{1}{2} + \omega_1\hat{\theta}\right) z_2 \tag{41}$$

where $\bar{c}_2 > 0$ is a design parameter, and using (39) and (40), we have

$$\dot{U}_1 \leq -\sum_{i=1}^{2} \bar{c}_i\bar{z}_i^{\rm T}\bar{z}_i + \frac{1}{2}\bar{z}_3^{\rm T}\bar{z}_3 + \frac{1}{2}\varrho_1^{\rm T}\varrho_1 + \frac{\tilde{\theta}}{\bar{\gamma}}(\dot{\hat{\theta}} - \Lambda_1) + d_1 \tag{42}$$

where $\Lambda_1 = \bar{\gamma}\omega_1\bar{z}_2^{\rm T}\bar{z}_2$. Let x_3^* pass the first-order filter (37), then we get the filtered virtual control $\bar{x}_3$.

Step 2. Choosing the candidate Lyapunov function $U_2(z_1, z_2, \bar{z}_3, \tilde{\theta}) = U_1 + \frac{1}{2}\bar{z}_3^{\rm T}\bar{z}_3$ and taking the time derivative, we have

$$\dot{U}_2 \leq -\sum_{i=1}^{2} \bar{c}_i z_i^{\rm T} z_i + \frac{1}{2}\bar{z}_3^{\rm T}\bar{z}_3 + \frac{1}{2}\varrho_1^{\rm T}\varrho_1 + \frac{\tilde{\theta}}{\bar{\gamma}}(\dot{\hat{\theta}} - \Lambda_1) + \\ \bar{z}_3^{\rm T}(x_4 - x_4^*) + \bar{z}_3^{\rm T}(x_4^* + \frac{1}{\tau_3}\varrho_1) + d_1. \tag{43}$$

From Lemma 1, it yields that

$$\bar{z}_3^{\rm T}(x_4 - x_4^*) = \bar{z}_3^{\rm T}(\bar{z}_4 + \varrho_2) \leq \frac{1}{2}\bar{z}_4^{\rm T}\bar{z}_4 + \bar{z}_3^{\rm T}\bar{z}_3 + \frac{1}{2}\varrho_2^{\rm T}\varrho_2. \tag{44}$$

Choose the virtual control

$$x_4^*(\bar{z}_3, \varrho_1) = -\left(\frac{3}{2} + \bar{c}_3\right)\bar{z}_3 - \frac{1}{\tau_3}\varrho_1 \tag{45}$$

where $\bar{c}_3 > 0$ is a designed parameter. Defining $\bar{d}_1 = \frac{\bar{\delta}_1}{2} + \frac{\bar{\delta}_2}{2}|\ddot{q}_d|^2$, and substituting (45) into (43), it yields that

$$\dot{U}_2 \le -\sum_{i=1}^{3}\bar{c}_i\bar{z}_i^{\rm T}\bar{z}_i + \frac{1}{2}\bar{z}_4^{\rm T}\bar{z}_4 + \frac{1}{2}\varrho_1^{\rm T}\varrho_1 + \frac{1}{2}\varrho_2^{\rm T}\varrho_2 + \frac{\tilde{\theta}}{\gamma}(\dot{\hat{\theta}} - \Lambda_1) + \bar{d}_1. \tag{46}$$

Let the virtual control x_4^* pass the first-order filter (37), then we can get the filtered virtual control $\bar{x}_4$.

Step 3. From the definition of ϱ_1, ϱ_2, it follows that

$$\begin{cases}\dot{\varrho}_1 = -\frac{1}{\tau_3}\varrho_1 + \eta_1(\bar{z}_1,\cdots,\bar{z}_4,\varrho_1,q_d,\dot{q}_d,\ddot{q}_d,\hat{\theta})\\ \dot{\varrho}_2 = -\frac{1}{\tau_4}\varrho_2 + \eta_2(\bar{z}_1,\cdots,\bar{z}_4,\varrho_1,\varrho_2,q_d,\dot{q}_d,\ddot{q}_d,\hat{\theta})\end{cases} \tag{47}$$

where continuous functions η_1, η_2 are defined as

$$\eta_1 = \frac{\partial x_3^*}{\partial\bar{z}_1^{\rm T}}\dot{\bar{z}}_1 + \frac{\partial x_3^*}{\partial\bar{z}_2^{\rm T}}\dot{\bar{z}}_2 + \frac{\partial x_3^*}{\partial q_d^{\rm T}}\dot{q}_d + \frac{\partial x_3^*}{\partial\dot{q}_d^{\rm T}}\ddot{q}_d + \frac{\partial x_3^*}{\partial\hat{\theta}}\dot{\hat{\theta}} \tag{48}$$

$$\eta_2 = \frac{\partial x_4^*}{\partial\bar{z}_3^{\rm T}}\dot{\bar{z}}_3 + \frac{\partial x_4^*}{\partial\varrho_1^{\rm T}}\dot{\varrho}_1. \tag{49}$$

Choosing the candidate Lyapunov function $U_3(\bar{z}_1,\bar{z}_2,\bar{z}_3,\bar{z}_4,\varrho_1,\varrho_2,\tilde{\theta}) = U_2 + \frac{1}{2}\bar{z}_4^{\rm T}B\bar{z}_4 + \frac{1}{2}\varrho_1^{\rm T}\varrho_1 + \frac{1}{2}\varrho_2^{\rm T}\varrho_2$, and taking the time derivative of V_3 while noticing (36) and (38), we have

$$\dot{U}_3 \le -\sum_{i=1}^{3}\bar{c}_i\bar{z}_i^{\rm T}\bar{z}_i + \frac{1}{2}\bar{z}_4^{\rm T}\bar{z}_4 + \frac{1}{2}\varrho_1^{\rm T}\varrho_1 + \frac{1}{2}\varrho_2^{\rm T}\varrho_2 + \bar{d}_1 + \frac{\tilde{\theta}}{\bar{\gamma}}(\dot{\hat{\theta}} - \Lambda_1) + \bar{z}_4^{\rm T}u - \varrho_1^{\rm T}(\frac{1}{\tau_3}\varrho_1 - \eta_1) - \varrho_2^{\rm T}(\frac{1}{\tau_4}\varrho_2 - \eta_2) + \bar{z}_4^{\rm T}\big(K(x_1 - x_3) - f_2 + \frac{1}{\tau_4}B\varrho_2\big). \tag{50}$$

There exist positive constants $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$ such that

$$\bar{z}_4^{\rm T}\big(K(x_1 - x_3) - f_2\big) \le \theta\omega_2\bar{z}_4^{\rm T}\bar{z}_4 + \frac{1}{2}\varrho_2^{\rm T}\varrho_2 + \frac{\bar{\epsilon}_1}{2} + \frac{\bar{\epsilon}_2}{2} \tag{51}$$

where ω_2 is defined as $\omega_2 = \frac{1}{2\bar{\epsilon}_1}|x_1 - x_3|^2 + \frac{1}{2\bar{\epsilon}_2}\phi_2$. By using Lemma 1, we can deduce

$$\bar{z}_4^{\rm T}\times\frac{1}{\tau_4}B\varrho_2 \le \frac{1}{\bar{\epsilon}_3\tau_4^2}|\bar{z}_4|^2 + \frac{\bar{\epsilon}_3 b_2^2}{4}\varrho_2^2. \tag{52}$$

Defining $\bar{d}_2 = \bar{d}_1 + \frac{\bar{\epsilon}_1}{2} + \frac{\bar{\epsilon}_2}{2}$, and substituting (52) and (51) into (50), we have

$$\dot{U}_3 \le -\sum_{i=1}^{3}\bar{c}_i\bar{z}_i^{\rm T}\bar{z}_i + \frac{\tilde{\theta}}{\bar{\gamma}}(\dot{\hat{\theta}} - \Lambda) + \bar{z}_4^{\rm T}u + \varrho_1^{\rm T}\eta_1 + \varrho_2^{\rm T}\eta_2 + (\frac{1}{2} + \frac{1}{\bar{\epsilon}_3\tau_4^2} + \omega_2\hat{\theta})\bar{z}_4^{\rm T}\bar{z}_4 + \big(-\frac{1}{\tau_3} + \frac{1}{2}\big)\varrho_1^{\rm T}\varrho_1 + \big(-\frac{1}{\tau_4} + 1 + \frac{\bar{\epsilon}_3 b_2^2}{4}\big)\varrho_2^{\rm T}\varrho_2 + \bar{d}_2 \tag{53}$$

where $\Lambda = \Lambda_1 + \bar{\gamma}\omega_2\bar{z}_4^{\rm T}\bar{z}_4$. Choosing the adaptive controller

$$\begin{cases}u = -\left(\frac{1}{2} + \frac{1}{\bar{\epsilon}_3\tau_4^2} + \bar{c}_4 + \omega_2\hat{\theta}\right)\bar{z}_4\\ \dot{\hat{\theta}} = \Lambda - \bar{\sigma}\hat{\theta},\quad \hat{\theta}(0) > 0\end{cases} \tag{54}$$

where $\bar{\sigma}$ is a positive constant, and substituting (54) into (53), it leads to

$$\dot{U}_3 \le -\sum_{i=1}^{4}\bar{c}_i\bar{z}_i^{\rm T}\bar{z}_i - \frac{1}{\bar{\gamma}}\bar{\sigma}\tilde{\theta}\hat{\theta} + \big(-\frac{1}{\tau_3} + \frac{1}{2}\big)\varrho_1^{\rm T}\varrho_1 + \big(-\frac{1}{\tau_4} + 1 + \frac{\epsilon_3 b_2^2}{4}\big)\varrho_2^{\rm T}\varrho_2 + \varrho_1^{\rm T}\eta_1 + \varrho_2^{\rm T}\eta_2 + d_2. \tag{55}$$

Now, we have theorem 2, which summarizes the main results of this section.

Theorem 2. Consider the flexible joint robots dynamic system (1) – (3) under Assumptions 1–3, one can design the adaptive state-feedback controller (54), such that:

1) For any initial conditions satisfying $U_3(0) \le \rho$, $\rho > 0$, there exist σ, $\bar{\gamma}$, $\bar{c}_i$, $\bar{\delta}_j$, $\bar{\epsilon}_j$, τ_k, $i = 1,\cdots,4$, $j = 1,2,3$, $k = 3,4$, guaranteeing that the tracking error z_1 can be made arbitrarily small by adjusting these designed parameters.

2) All the closed-loop signals are rendered semi-globally uniformly ultimately bounded.

Proof. Choosing the Lyapunov function V_3, and defining the set $\Omega = \big\{[\bar{z}_1^{\rm T},\bar{z}_2^{\rm T},\bar{z}_3^{\rm T},\bar{z}_4^{\rm T},\varrho_1^{\rm T},\varrho_2^{\rm T},\tilde{\theta}]^{\rm T}\in\mathbf{R}^{6n+1}\big|\bar{z}_1^{\rm T}\bar{z}_1 + \frac{1}{a}\bar{z}_2^{\rm T}K\bar{z}_2 + \bar{z}_3^{\rm T}\bar{z}_3 + \bar{z}_4^{\rm T}B\bar{z}_4 + \varrho_1^{\rm T}\varrho_1 + \varrho_2^{\rm T}\varrho_2 + \frac{1}{\bar{\gamma}}\tilde{\theta}^2 \le 2\rho\big\}$, we see that Ω_d in Assumption 1 and Ω are compact sets. Then, from the definition of $\eta_1(\cdot)$ and $\eta_2(\cdot)$, it follows that there exist positive constants $\bar{\eta}_1$ and $\bar{\eta}_2$, such that on the compact set $\Omega_d\times\Omega$, $|\eta_1(\cdot)| \le \bar{\eta}_1$ and $|\eta_2(\cdot)| \le \bar{\eta}_2$. By using Lemma 1, we have

$$\varrho_1^{\rm T}\eta_1 \le \frac{\bar{\eta}_1^2}{2\bar{\delta}_3}\varrho_1^{\rm T}\varrho_1 + \frac{\bar{\delta}_3}{2},\quad \varrho_2^{\rm T}\eta_2 \le \frac{\bar{\eta}_2^2}{2\bar{\delta}_4}\varrho_2^{\rm T}\varrho_2 + \frac{\bar{\delta}_4}{2}. \tag{56}$$

Defining $\bar{d} = \bar{d}_2 + \frac{\bar{\delta}_3}{2} + \frac{\bar{\delta}_4}{2} + \frac{1}{2\bar{\gamma}}\bar{\sigma}\theta^2$, and substituting (34) and (56) into (55) yields

$$\dot{U}_3 \le -\sum_{i=1}^{4}\bar{c}_i\bar{z}_i^{\rm T}\bar{z}_i - \frac{1}{2\bar{\gamma}}\bar{\sigma}\tilde{\theta}^2 - \big(\frac{1}{\tau_3} - \frac{1}{2} - \frac{\bar{\eta}_1^2}{2\bar{\delta}_3}\big)\varrho_1^{\rm T}\varrho_1 - \big(\frac{1}{\tau_4} - 1 - \frac{\bar{\epsilon}_3 b_2^2}{4} - \frac{\bar{\eta}_2^2}{2\bar{\delta}_4}\big)\varrho_2^{\rm T}\varrho_2 + \bar{d}. \tag{57}$$

Choosing parameters such that $\bar{c}_i > 0$, $i = 1,\cdots,4$, $\bar{\sigma} > 0$, $\frac{1}{\tau_3} \ge \tau_3^* + \frac{1}{2} + \frac{\bar{\eta}_1^2}{2\bar{\delta}_3}$, $\frac{1}{\tau_4} \ge \tau_4^* + 1 + \frac{\bar{\epsilon}_3 b_2^2}{4} + \frac{\bar{\eta}_2^2}{2\bar{\delta}_4}$, where $\tau_3^* > 0$, $\tau_4^* > 0$, we have

$$\dot{U}_3 \le -cU_3 + \bar{d} \tag{58}$$

where c satisfies $0 < \bar{c} \le 2\cdot\min\{\bar{c}_1, \frac{a\bar{c}_2}{k_2}, \bar{c}_3, \frac{\bar{c}_4}{b_2}, \frac{1}{2}\bar{\sigma}, \tau_3^*, \tau_4^*\}$. It follows from (58) that $\dot{U}_3 \le 0$ on the surface $U_3 = \rho$ when $\bar{c} > \frac{d}{\rho}$. Hence, $V_3 \le \rho$ is an invariant set, i.e., if $U_3(0) \le \rho$, then $U_3(t) \le \rho$ for all $t \ge 0$. Consequently, all the closed-loop signals are semi-globally uniformly ultimately bounded. Moreover, by adjusting parameters $\bar{\sigma}$, $\bar{\gamma}$, $\bar{c}_i$, $\bar{\delta}_j$, $\bar{\epsilon}_j$, τ_k, $i = 1,\cdots,4$, $j = 1,2,3$, $k = 3,4$, we can make the tracking error $\bar{z}_1$ arbitrarily small. □

Remark 4. To achieve the tracking errors arbitrarily small, one can adjust the design parameters accordingly. To be specific, increasing $\bar{c}_i$, $\bar{\sigma}$, and decreasing τ_k will help to increase $\bar{c}$. Decreasing $\bar{\delta}_j$, $\bar{\epsilon}_j$ and increasing $\bar{\gamma}$ will help to decrease $\bar{d}$. From (58), it leads to $U_3 \le U_3(0){\rm e}^{-\bar{c}t} + \frac{\bar{d}}{\bar{c}}(1 - {\rm e}^{-\bar{c}t})$. With increasing value of $\bar{c}$, and decreasing value of $\frac{\bar{d}}{\bar{c}}$, the tracking error is made smaller.

5 Simulation

Consider the single-link flexible joint robot[7, 9], whose dynamic equations are as follows:

$$\begin{aligned}&I\ddot{q}_1 + Mgl\sin(q_1) + K(q_1 - q_2) + f_1(q_1, \dot{q}_1) = 0\\ &J\ddot{q}_2 + B\dot{q}_2 + K(q_2 - q_1) + f_2(q_2, \dot{q}_2) = u.\end{aligned}$$

The desired trajectory for this robot model is given as $q_d = 0.5\sin(0.5t)$ rad. When the external disturbances are 0, we choose the parameters as $I = 1$ kg·m^2, $Mgl = 3$ N·m, $K = 5$ N·m/rad, $J = 1$ kg·m^2, $B = 4$ N·m·s/rad. It is easy to verify that Assumption 1 and Assumption 2 hold. Assumption 3 holds with $\phi_0 = \sin^2 q_1 + q_1^2$, $\phi_1 = 0$, $\phi_2 = \dot{q}_2^2$. In Figs. 1–3, the responses of the resulting closed-loop system for this case are characterized by the different curves (the position of the link is q_{1a}, the parameter estimation is θ_a, and the control input is u_a). When the external disturbances are $f_1 = -0.5\cos(q_1)$N and $f_2 = -0.5\cos(q_2)$N, we choose another group of the parameters as $I = 0.5$ kg·m^2, $Mgl = 1.5$ N·m, $K = 2.5$ N·m/rad, $J = 0.5$ kg·m^2, and $B = 2$ N·m·s/rad, the responses of the resulting closed-loop system for this case are characterized by the different curves (the position of the link is q_{1b}, the parameter estimation is θ_b, and the control input is u_b).

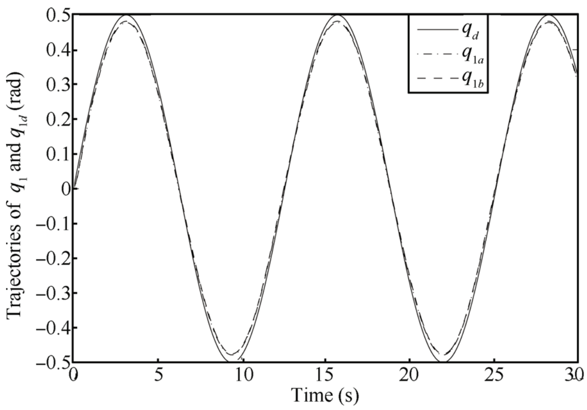

Fig. 1 The position of q_d and q_1

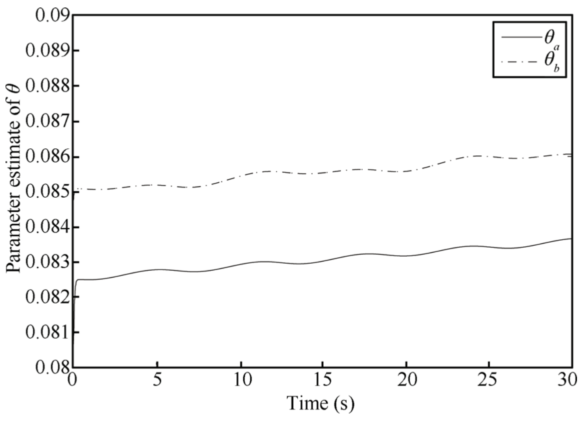

Fig. 2 The trajectories of $\hat{\theta}$

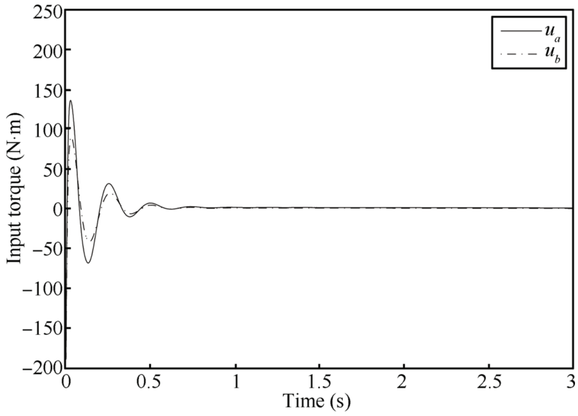

Fig. 3 The trajectories of input u

Since the responses of the presented controllers are similar, in this example, we only provide the simulation for the control method shown in Section 4. By (7), we have

$$\begin{cases}\dot{z}_1 = z_2 - (c_1+1)z_1\\ \dot{z}_2 = I^{-1}Kx_3 - F(x_1, x_2, q_d, \dot{q}_d, \ddot{q}_d)\\ \dot{z}_3 = x_4 + \dfrac{1}{\tau_3}\varrho_1\\ \dot{z}_4 = J^{-1}\left(u + Bx_4 + K(x_1 - x_3) - f_2\right) + \dfrac{1}{\tau_4}\varrho_2\end{cases} \tag{59}$$

where $F = I^{-1}\left(Mgl\sin x_1 + Kx_1 + f_1\right) + \ddot{q}_d - (c_1+1)z_2 + (c_1+1)^2 z_1$. We choose the first virtual control

$$x_3^*(z_1, z_2, q_d, \dot{q}_d, \hat{\theta}) = -\left(\frac{1}{2} + c_2 + \omega_1\hat{\theta}\right)z_2 \tag{60}$$

where $\omega_1 = \frac{1}{\delta_1}(\sin^2 x_1 + x_1^2) + \frac{1}{2\delta_2} + c_1 + \frac{(c_1+1)^4}{2} + 2$. Let the virtual control x_3^* pass the first-order filter (37), then we can get the filtered virtual control $\bar{x}_3$. Next, we choose the second virtual control

$$x_4^*(z_1, z_2, z_3, q_d, \dot{q}_d, \hat{\theta}) = -\left(\frac{3}{2} + c_3\right)z_3 - \frac{1}{\tau_3}\varrho_1. \tag{61}$$

Let the virtual control x_4^* pass the first-order filter (37), then we can get the filtered virtual control $\bar{x}_4$. At last, we choose the actual control and the adaptive law as

$$\begin{cases}u = -\left(\dfrac{1}{2} + \dfrac{1}{\epsilon_3\tau_4^2} + c_4 + \omega_2\hat{\theta}\right)z_4\\ \dot{\hat{\theta}} = \gamma(\omega_1 z_2^{\rm T}z_2 + \omega_2 z_4^{\rm T}z_4) - \sigma\hat{\theta},\quad \hat{\theta}(0) = 0.1\end{cases} \tag{62}$$

where $\omega_2 = \frac{1}{2\epsilon_1}|x_1 - x_3|^2 + \frac{1}{2\epsilon_2}x_4^2$.

In the simulation, we choose the parameters as $c_1 = 1.5$, $c_2 = 3$, $c_3 = 2.5$, $c_4 = 3$, $\delta_1 = 0.005$, $\delta_2 = 0.04$, $\tau_3 = 0.001$, $\tau_4 = 0.03$, $\epsilon_1 = 0.1$, $\epsilon_2 = 0.01$, $\epsilon_3 = 8.5$, $\sigma = 0.000\,4$ and $\gamma = 0.000\,55$. The initial values are $\bar{x}_3(0) = 1.5$ and $\bar{x}_3(0) = 3.5$. The simulation demonstrates that the tracking objective of flexible joint robots can be achieved with satisfactory responses by the designed controller (62).

6 Concluding remarks

Tracking problems of the FJR system which is underactuated, are more difficult than the fully actuated system.

This paper gives a new adaptive tracking control method for uncertain flexible joint robots. The designed controllers can make the position tracking error arbitrarily small, while keeping all the closed signals globally/semi-globally uniformly ultimately bounded. In this direction, there are still remaining problems to be investigated. For example, an interesting research problem is how to design an adaptive tracking controller for the flexible joint robots in random vibration environment.

Appendix

Proof of Lemma 2. From Assumptions 1 and 2, we have

$$|M_1^{-1}(x_1)\,(Cx_2+h(x_1)+Kx_1+f_1)\,| \leq \psi_{11}(x_1,x_2)\max\{\frac{\theta_0^{\frac{1}{2}}}{m_1},\frac{\theta_1^{\frac{1}{2}}}{m_1}\} \quad (A1)$$

where $\psi_{11}(x_1,x_2)=\phi_0^{\frac{1}{2}}+\phi_1^{\frac{1}{2}}$. Then, we can deduce that

$$|F_1|\leq \psi_{11}\bar{\theta}+\psi_{12}+\ddot{q}_d. \quad (A2)$$

From (8) and the definition of F_2 and F_3, it is not difficult to get that there exist smooth functions such that

$$|F_2|\leq \psi_{21}\bar{\theta}+\psi_{22}+\psi_{23}\ddot{q}_d \quad (A3)$$
$$|F_3|\leq \psi_{31}\bar{\theta}+\psi_{32}+\psi_{33}\ddot{q}_d. \quad (A4)$$

Thus, we can deduce that

$$\begin{aligned}-z_3^{\mathrm{T}}F_2 &\leq z_3^{\mathrm{T}}\left(\psi_{21}\bar{\theta}+\psi_{22}+\psi_{23}\ddot{q}_d\right)\leq\\ &(z_3^{\mathrm{T}}z_3)\left(\psi_{21}^2\theta+\psi_{22}^2+\psi_{23}^2\right)+\frac{\theta}{4}+\frac{1}{4}+\frac{\ddot{q}_d}{4}=\\ &(z_3^{\mathrm{T}}z_3)\left(\psi_1\theta+\psi_2\right)+\nu\end{aligned} \quad (A5)$$

where $\psi_1=\psi_{21}^2$, $\psi_2=\psi_{22}^2+\psi_{23}^2$ and $\nu=\frac{\theta}{4}+\frac{1}{4}+\frac{\ddot{q}_d}{4}$. Similarly, we can deduce that

$$\begin{aligned}-z_4^{\mathrm{T}}F_3 &\leq z_4^{\mathrm{T}}\left(\psi_{31}\bar{\theta}+\psi_{32}+\psi_{33}\ddot{q}_d\right)\leq\\ &(z_4^{\mathrm{T}}z_4)\left(\psi_{31}^2\theta+\psi_{32}^2+\psi_{33}^2\right)+\frac{\theta}{4}+\frac{1}{4}+\frac{\ddot{q}_d}{4}=\\ &(z_4^{\mathrm{T}}z_4)\left(\psi_3\theta+\psi_4\right)+\nu\end{aligned} \quad (A6)$$

where $\psi_3=\psi_{31}^2$ and $\psi_4=\psi_{32}^2+\psi_{33}^2$. □

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