International Journal of Automation and Computing 12(2), April 2015, 125-133
DOI: 10.1007/s11633-015-0879-9

Decentralized Networked Control System Design Using Takagi-Sugeno (TS) Fuzzy Approach

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Abstract: This paper proposes a new method for control of continuous large-scale systems where the measures and control functions are distributed on calculating members which can be shared with other applications and connected to digital network communications. At first, the nonlinear large-scale system is described by a Takagi-Sugeno (TS) fuzzy model. After that, by using a fuzzy Lyapunov-Krasovskii functional, sufficient conditions of asymptotic stability of the behavior of the decentralized networked control system (DNCS), are developed in terms of linear matrix inequalities (LMIs). Finally, to illustrate the proposed approach, a numerical example and simulation results are presented.

Keywords: Continuous large-scale systems, decentralized static output feedback fuzzy control, networked control systems (NCS), Takagi-Sugeno (TS) fuzzy model, linear matrix inequalities (LMIs).

1 Introduction

Decentralized control of large-scale systems (also known as interconnected systems in some books) has been investigated as a branch of control theory and has received considerable attention over the past three decades due to its various applications such as power systems, aerospace systems, nuclear reactors, systems control process, etc.[1−3]

In fact, various techniques for distributed control using linear matrix inequalities (LMIs) were recently studied[4−9]. The systems consist of a large set of interconnected subsystems which can be far from each other. That′s why we introduce the notion of communication network to connect them, and thus it aims to ensure data transmission and coordinating manipulation among spatially distributed components. Compared with conventional point-to-point control systems, the advantages of networked control systems (NCS) are less wiring, lower installation cost as well as greater agility in diagnosis and maintenance. Because of these distinctive benefits, typical application of these systems ranges over various fields, such as automotive, mobile robotics, advanced aircraft, etc. It is well known that limited network resources, network-induced delays and data packets dropout through the network, may degrade the decentralized networked control system (DNCS) performance and lead to instability. It is mentioned that the communication delay, which has time-varying characteristics, is one of the important factors to be considered in NCS analysis and synthesis[10−19].

In this paper, the decentralized static output feedback control method for stabilization of nonlinear interconnected system, that takes into account problems of delay and data packets dropout in communication, is proposed. Based on Takagi-Sugeno (TS) fuzzy system, the static output feedback controller is designed. The sufficient condition is offered to guarantee the stability of the closed-loop system using Lyapunov Krasovskii functional. Its constructive conditions are presented in LMIs terms, taking effects of communication network into account.

The paper is organized as follows. Section 2 presents system description and preliminaries. Section 3 presents the main results, describing the control strategy for large-scale systems through a communication network. Section 4 shows simulation results. Finally, conclusions are given in Section 5.

Notations. $sym(W)$ stands for $W + W^{\mathrm{T}}$. The symbol $(*)$ within a matrix represents the symmetric entries.

2 Preliminaries and system description

Consider a large-scale system S composed of J interconnected subsystems $S_i, i = 1, 2, \cdots, J$. The i-th fuzzy subsystem S_i is described by the following TS fuzzy model:

$$S_i : \begin{cases} \text{If } \theta_{i1}(t) \text{ is } F_{i1}^l \text{ and } \theta_{ig}(t) \text{ is } F_{ig}^l \\ \quad \text{then } \dot{x}_i(t) = A_i^l x_i(t) + B_i^l u_i(t) + \displaystyle\sum_{j=1}^{J} f_{ij}(x_j(t)) \\ y_i(t) = C_{2i} x_i(t) \end{cases} \tag{1}$$

where $i = 1, 2, \cdots, J$, $l = 1, 2, \cdots, r_i$, $x_i(t)$ denotes the state vector, $y_i(t)$ denotes the measured output, $u_i(t)$ is the control input, A_i^l, B_i^l and C_{2i} are constant real matrices with appropriate dimensions and C_{2i} is full rank, $\theta_{i1}(t)$,

Regular Paper
Special Issue on Advances in Nonlinear Dynamics and Control
Manuscript received March 25, 2014; accepted September 25, 2014
Recommended by Associate Editor Fernando Tadeo

$\theta_{i2}(t), \cdots, \theta_{ig}(t)$ are some measurable premise variables for subsystems S_i, $F_{iq}^l (q = 1, 2, \cdots, g)$ represents the linguistic fuzzy sets of the rule, $f_{ij}(x_j(t))$ represents the interconnection of fuzzy rules in subsystem S_i and subsystem S_j, and r_i represents the number of fuzzy rules in subsystem S_i.

Using the central-average defuzzifier, the TS fuzzy system can be given as

$$\begin{cases} \dot{x}_i(t) = \sum\limits_{l=1}^{r_i} h_i^l(\theta_i(t))[A_i^l x_i(t) + B_i^l u_i(t) + \sum\limits_{j=1}^{J} f_{ij}(x_j(t))] \\ y_i(t) = C_{2i} x_i(t) \end{cases} \tag{2}$$

where

$$\begin{aligned} h_i^l(\theta_i(t)) &= \frac{\upsilon_i^l(\theta_i(t))}{\sum\limits_{l=1}^{r_i} \upsilon_i^l(\theta_i(t))} \\ \upsilon_i^l(\theta_i(t)) &= \prod_{q=1}^{g} F_{iq}^l(\theta_{iq}(t)) \end{aligned} \tag{3}$$

with $F_{iq}^l(\theta_{iq}(t))$ as the grade of membership of $\theta_{iq}(t)$ in the fuzzy set F_{iq}^l. $h_i^l(\theta_i(t))$ is the membership function for each fuzzy rule, which represents normalized grade of membership, and satisfies

$$0 \le h_i^l(\theta_i(t)) \le 1, \text{ for } l = 1, 2, \cdots, r_i, \ \sum_{l=1}^{r_i} h_i^l(\theta_i(t)) = 1. \tag{4}$$

We assume that system S will be controlled through network. Fig. 1 represents the structure of networked control sub system S_i with induced delays, where τ_{sci} is sensor-to-controller delay and τ_{cai} is the controller-to-actuator delay. It is assumed that the controller computational delay can be absorbed into either τ_{sci} or τ_{cai}.

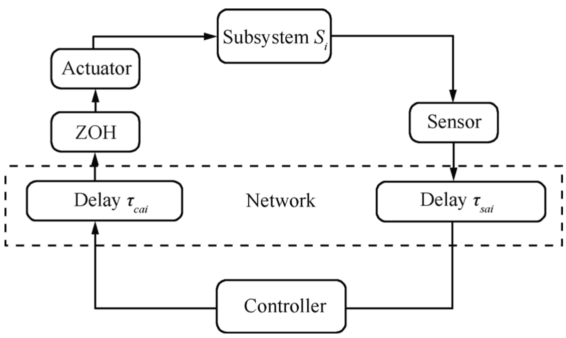

Fig. 1 Framework of networked control subsystem S_i

Assumption 1. All pairs (A_i^l, B_i^l) $(i = 1, 2, \cdots, J$ and $l = 1, 2, \cdots, r_i)$ are stabilizable.[20]

Assumption 2. The interconnection $f_{ij}(x_j(t))$ satisfies the following conditions: $f_{ij}(x_j(t)) = B_i^l f_{ij^l}(x_j(t))$ and $\|f_{ij}^l(x_j(t))\| \le \bar{f}_{ij}^l \|x_j(t)\|$, where $\bar{f}_{ii}^l = 0$, $\bar{f}_{ij}^l (i \ne j)$ is a positive constant and B_i^l is a constant real matrix with appropriate dimensions.[20]

Assumption 3. The sensors are clock driven, the controller and actuators are event driven.

Assumption 4. Data, either from measurement or for control, are transmitted in a single packet.

Assumption 5. The effect of signal quantization is not considered.

Assumption 6. The real input $u_i(t)$ for each subsystem, realized through a zero-order hold (ZOH), is a piecewise constant function.

It is worth mentioning that the sampling period of a sensor is pre-determined for control algorithm design, and thus the sensor can be assumed to be clock driven. However, an actuator does not change its output to the plant under control until an updated control signal is received, implying that the actuator is event driven.

To obtain our main results, the following lemmas are needed.

Lemma 1.[21] For each real vector ζ and ρ, it follows that

$$2\zeta^{\mathrm{T}}\rho \le \zeta^{\mathrm{T}} Z \zeta + \rho^{\mathrm{T}} Z^{-1} \rho \tag{5}$$

with $Z > 0$.

Lemma 2.[20] The following inequality is verified for each real vector $\nu_i \in \mathbf{R}^n$:

$$[\sum_{i=1}^{m} \nu_i]^{\mathrm{T}} [\sum_{i=1}^{m} \nu_i] \le m \sum_{i=1}^{m} \nu_i^{\mathrm{T}} \nu_i. \tag{6}$$

3 Main results

In this section, we are interested in the design of static output feedback controller in order to stabilize the system. Indeed, it is assumed that the states of the system (2) are not all available for measurement, that is why we achieve an output feedback control. The control scheme type parallel distributed compensation (PDC) will be considered for each subsystem S_i. The overall fuzzy PDC networked controller corresponding to S_i can be described as

$$u_i(t_k) = \sum_{l=1}^{r_i} h_i^l(\theta_i(t_k)) K_i^l y_i(t_k - \tau_{ki}).$$

From the ZOH, the input signal for each subsystem S_i for $t_k \le t \le t_{k+1}$ is given by

$$u_i(t) = \sum_{l=1}^{r_i} h_i^l(\theta_i(t_k)) K_i^l y_i(t_k - \tau_{ki}). \tag{7}$$

For network-induced delay (τ_{ki}), one major challenge for NCS design is the effect of network-induced delays in a control loop. It occurs when the system components exchange data across the network. It can degrade control performance significantly or even destabilize the system. The delays in NCS consist of a communication delay between sensors and controllers τ_{sci}, a communication delay between controller and actuators τ_{cai}, computational time in controller τ_c which can be generally included in the controller to actuator delay.

A natural assumption on τ_{ki} can be made as

$$0 < \tau_{mi} \le \tau_{ki} \le \tau_{Mi}. \tag{8}$$

Packet dropouts are network-induced effects which can be the consequence of a link failure. They can also be generated purposefully in order to avoid congestion or to guarantee the most recent data to be sent. Although most network protocols are equipped with transmission-retry mechanisms, they can only re-transmit for limited time. After this time has expired, the packets are dropped. Normally, feedback controllers can tolerate a certain amount of packet losses. But the consecutive packet losses have an adverse impact on the overall performance.

$$t_{k+1} - t_k = \bar{\sigma}_i T_e + \max_i\{\tau_{(k+1)i}\} - \min_i\{\tau_{ki}\} \tag{9}$$

where T_e denotes the sampling period, t_k denotes the sampling instant, and $\bar{\sigma}_i$ denotes the maximum number of packet dropouts in the updating periods. Using (2) and (7), the closed-loop networked control system can be written for $t_k \le t \le t_{k+1}$ as

$$\begin{cases} \dot{x}_i(t) = A(t)x_i(t) + H(t)x_i(t_k - \tau_{ki}) + f(x_i(t)) \\ y_i(t) = C_{2i}x_i(t) \end{cases} \tag{10}$$

with

$$\begin{aligned} A(t) &= \sum_{l=1}^{r_i} h_i^l A_i^l \\ B(t) &= \sum_{l=1}^{r_i} h_i^l B_i^l \\ H(t) &= B(t)\sum_{s=1}^{r_i} h_i^s K_i^s C_{2i} \\ f(x_i(t)) &= \sum_{l=1}^{r_i} h_i^l \sum_{j=1}^{J} f_{ij}(x_j). \end{aligned} \tag{11}$$

Defining $\eta_i(t) = t - t_k + \tau_{ki}, \quad t_k \le t \le t_{k+1}$, then

$$\tau_{ki} \le \eta_i(t) \le \bar{\sigma}_i T_e + \max_i\{\tau_{(k+1)i}\}.$$

Thus, we get from [22] that

$$\eta_{1i} \le \eta_i(t) \le \eta_{2i}, \quad \dot{\eta}_i(t) \le h_{di} \tag{12}$$

where

$$\eta_{1i} = \tau_{mi} \quad \text{and} \quad \eta_{2i} = \bar{\sigma}_i T_e + \max_i\{\tau_{Mi}\}.$$

As $\sum_{k=0}^{\infty}[t_k, t_{k+1}) = [0, \infty)$, we have

$$\begin{cases} \dot{x}_i(t) = A(t)x_i(t) + H(t)x_i(t - \eta_i(t)) + f(x_i(t)) \\ y_i(t) = C_{2i}x_i(t) \\ x_i(t) = \phi_i(t), t \in [t_0 - \eta_{2i}, t_0] \end{cases} \tag{13}$$

where $\phi_i(t)$ can be viewed as the initial condition of the closed-loop control system. Then based on (12), it is noted that the NCS (13) is equivalent to a system with an interval time-varying delay.

The controller design is based on the following preliminary result given by the Lemma 3.

Lemma 3. For given scalars $\eta_{1i} > 0$ and $\eta_{2i} > 0$, the closed-loop system (13) is asymptotically stable, if there exist positive matrices P_i, Q_{1i}, Q_{2i}, Q_{3i}, Z_{1i}, and matrices G_{1i}, G_{2i} and G_{3i}, with appropriate dimensions, such that the following conditions hold:

$$\Phi_{ij} = \begin{bmatrix} \Phi_{11ij} & \Phi_{12i} & Z_{1i} & 0 & \Phi_{15i} \\ * & \Phi_{22i} & 0 & 0 & \Phi_{25ij} \\ * & * & -Q_{2i} - Z_{1i} & 0 & 0 \\ * & * & * & -Q_{3i} & 0 \\ * & * & * & * & \Phi_{55i} \end{bmatrix} < 0 \tag{14}$$

$$\begin{aligned} &\Phi_{11ij} = Q_{1i} + Q_{2i} + Q_{3i} + sym(G_{1i}^{\mathrm{T}}A(t)) - Z_{1i} + G_{1i}^{\mathrm{T}}G_{1i} + \\ &\quad (3J\sum_{j=1}^{J}\hat{f}_{ji}^2\|\hat{B}_j\|^2)I \\ &\Phi_{12i} = A(t)^{\mathrm{T}}G_{2i} + G_{1i}^{\mathrm{T}}H(t) \\ &\Phi_{22i} = sym(G_{2i}^{\mathrm{T}}H(t)) - (1 - h_{di})Q_{1i} + G_{2i}^{\mathrm{T}}G_{2i} \\ &\Phi_{15i} = P_i - G_{1i}^{\mathrm{T}} + A(t)^{\mathrm{T}}G_{3i} \\ &\Phi_{25ij} = -G_{2i}^{\mathrm{T}} + H^{\mathrm{T}}(t)G_{3i} \\ &\Phi_{55i} = \eta_{1i}^2 Z_{1i} - sym(G_{3i}) + G_{3i}^{\mathrm{T}}G_{3i}. \end{aligned}$$

Proof. Let the Lyapunov-Krasovskii functional candidate be

$$V(t) = \sum_{i=1}^{J} v_i(t), \ i = 1, 2, \cdots, J \tag{15}$$

where $v_i(t)$ denotes the Lyapunov-Krasovskii functional corresponding to fuzzy subsystem S_i. Each $v_i(t)$ is defined as

$$\begin{aligned} v_i(t) = &\ x_i^{\mathrm{T}}(t)P_i x_i(t) + \int_{t-\eta_i(t)}^{t} x_i^{\mathrm{T}}(s)Q_{1i}x_i(s)\,\mathrm{d}s + \\ &\int_{t-\eta_{1i}}^{t} x_i^{\mathrm{T}}(s)Q_{2i}x_i(s)\,\mathrm{d}s + \\ &\int_{t-\eta_{2i}}^{t} x_i^{\mathrm{T}}(s)Q_{3i}x_i(s)\,\mathrm{d}s + \\ &\eta_{1i}\int_{-\eta_{1i}}^{0}\Big(\int_{t+s}^{t}\dot{x}_i^{\mathrm{T}}(\upsilon)Z_{1i}\dot{x}_i(\upsilon)\,\mathrm{d}\upsilon\Big)\mathrm{d}s. \end{aligned} \tag{16}$$

The corresponding time derivative of $v_i(t)$ is given by

$$\begin{aligned} \dot{v}_i(t) \le &\ 2\dot{x}_i^{\mathrm{T}}(t)P_i x_i(t) + x_i^{\mathrm{T}}(t)(Q_{1i} + Q_{2i} + Q_{3i})x_i(t) - \\ &(1 - h_{di})x_i^{\mathrm{T}}(t - \eta_i(t))Q_{1i}x_i(t - \eta_i(t)) - \\ &x_i^{\mathrm{T}}(t - \eta_{1i})Q_{2i}x_i(t - \eta_{1i}) - \\ &x_i^{\mathrm{T}}(t - \eta_{2i})Q_{3i}x_i(t - \eta_{2i}) + \\ &\dot{x}_i^{\mathrm{T}}(t)(\eta_{1i}^2 Z_{1i})\dot{x}_i(t) - \\ &\eta_{1i}\int_{t-\eta_{1i}}^{t}\dot{x}_i^{\mathrm{T}}(\upsilon)Z_{1i}\dot{x}_i(\upsilon)\,\mathrm{d}\upsilon. \end{aligned} \tag{17}$$

Denoting $\psi_{1i} = x_i(t) - x_i(t - \eta_{1i})$, by Jensen inequality, we can obtain

$$-\eta_{1i}\int_{t-\eta_{1i}}^{t}\dot{x}_i^{\mathrm{T}}(\upsilon)Z_{1i}\dot{x}_i(\upsilon)\,\mathrm{d}\upsilon \le -\psi_{1i}^{\mathrm{T}}Z_{1i}\psi_{1i}. \tag{18}$$

From (13), we construct for appropriately dimensioned matrices G_{1i}, G_{2i}, and G_{3i} as the following zero-value expression:

$$2\big[x_i^{\mathrm{T}}(t)G_{1i}^{\mathrm{T}}+x_i^{\mathrm{T}}(t-\eta_i(t))G_{2i}^{\mathrm{T}}+\dot{x}_i^{\mathrm{T}}(t)G_{3i}^{\mathrm{T}}\big]\times$$
$$\big[-\dot{x}_i(t)+A(t)x_i(t)+H(t)x(t-\eta_i(t))+f(x_i(t))\big]=0$$
$$\Psi_i^{\mathrm{T}}(t)=$$
$$\begin{bmatrix}x_i^{\mathrm{T}}(t) & x_i^{\mathrm{T}}(t-\eta_i(t)) & x_i^{\mathrm{T}}(t-\eta_{1i}) & x_i^{\mathrm{T}}(t-\eta_{2i}) & \dot{x}_i^{\mathrm{T}}(t)\end{bmatrix}. \tag{19}$$

According to Lemmas 1 and 2, we have

$$\begin{aligned}&2x_i^{\mathrm{T}}(t)G_{1i}^{\mathrm{T}}\sum_{j=1}^{J}f_{ij}(x_j)\le\\&\quad x_i^{\mathrm{T}}(t)G_{1i}^{\mathrm{T}}G_{1i}x_i(t)+\sum_{j=1}^{J}f_{ij}^{\mathrm{T}}(x_j)\sum_{j=1}^{J}f_{ij}(x_j)\le\\&\quad x_i^{\mathrm{T}}(t)G_{1i}^{\mathrm{T}}G_{1i}x_i(t)+J\sum_{j=1}^{J}f_{ij}^{\mathrm{T}}(x_j)f_{ij}(x_j).\end{aligned}\tag{20}$$

Based on Assumptions 1 and 2, and defining $\hat{f}_{ij}=\max_l \bar{f}_{ij}^l$, $\|\hat{B}_i\|=\max_l\|B_i^l\|$, we have

$$\begin{aligned}&2x_i^{\mathrm{T}}(t)G_{1i}^{\mathrm{T}}\sum_{j=1}^{J}f_{ij}(x_j)\le\\&\quad x_i^{\mathrm{T}}(t)G_{1i}^{\mathrm{T}}G_{1i}x_i(t)+J\sum_{j=1}^{J}f_{ij}^{\mathrm{T}}(x_j)f_{ij}(x_j)\le\\&\quad x_i^{\mathrm{T}}(t)(G_{1i}^{\mathrm{T}}G_{1i}+J\sum_{j=1}^{J}\hat{f}_{ji}^2\|\hat{B}_j\|^2)Ix_i(t)\end{aligned}\tag{21}$$

$$\begin{aligned}&2x_i^{\mathrm{T}}(t-\eta_i(t))G_{2i}^{\mathrm{T}}\sum_{j=1}^{J}f_{ij}(x_j)\le\\&\quad x_i^{\mathrm{T}}(t-\eta_i(t))G_{2i}^{\mathrm{T}}G_{2i}x_i(t-\eta_i(t))+\\&\quad x_i^{\mathrm{T}}(t)J\sum_{j=1}^{J}\hat{f}_{ji}^2\|\hat{B}_j\|^2Ix_i(t)\end{aligned}\tag{22}$$

$$\begin{aligned}&2\dot{x}_i^{\mathrm{T}}(t)G_{3i}^{\mathrm{T}}\sum_{j=1}^{J}f_{ij}(x_j)\le\\&\quad \dot{x}_i^{\mathrm{T}}(t))G_{3i}^{\mathrm{T}}G_{3i}\dot{x}_i(t)+x_i^{\mathrm{T}}(t)J\sum_{j=1}^{J}\hat{f}_{ji}^2\|\hat{B}_j\|^2)Ix_i(t).\end{aligned}\tag{23}$$

Considering (17)−(19) and (21)−(23), the derivative of (15) along the closed loop system (13) can be described as

$$\dot{V}(t)=\sum_{i=1}^{J}\dot{v}_i(t)\le\sum_{i=1}^{J}\sum_{j=1}^{J}\sum_{l=1}^{r_i}\sum_{s=1}^{r_i}h_i^lh_j^s\Psi_i^{\mathrm{T}}(t)\Phi_{ij}\Psi_i(t)\le 0. \tag{24}$$

According to Lemma 3, we have $\dot{V}(t)<0$. So system (13) is asymptotically stable. □

The objective now is to determine the gain matrices K_i^l such that the static output feedback closed-loop system is asymptotically stable.

Theorem 1. For given scalars $\eta_{1i}>0$, $\eta_{2i}>0$, μ_1, μ_2, and μ_3, the closed-loop system (13) is asymptotically stable, if there exist positive matrices $\bar{P}_i$, $\bar{Q}_{1i}$, $\bar{Q}_{2i}$, $\bar{Q}_{3i}$, $\bar{Z}_{1i}$, matrices $\hat{G}_{11i}>0$, $\hat{G}_{21i}>0$, $\hat{G}_{22i}>0$, and Y_i^s, with appropriate dimensions, such that the following conditions hold

$$\bar{\Phi}_{ij}^{ll}<0 \tag{25}$$
$$\bar{\Phi}_{ij}^{ls}+\bar{\Phi}_{ij}^{sl}<0,\quad j>i,\quad s>l \tag{26}$$

where

$$\bar{\Phi}_{ij}^{ls}=$$
$$\begin{bmatrix}\bar{\Phi}_{11il} & \bar{\Phi}_{12ils} & \bar{Z}_{1i} & 0 & \bar{\Phi}_{15il} & \bar{G}_i^{\mathrm{T}}\\ * & \bar{\Phi}_{22ils} & 0 & 0 & \bar{\Phi}_{25ils} & 0\\ * & * & -\bar{Q}_{2i}-\bar{Z}_{1i} & 0 & 0 & 0\\ * & * & * & -\bar{Q}_{3i} & 0 & 0\\ * & * & * & * & \bar{\Phi}_{55i} & 0\\ * & * & * & * & * & \bar{\Phi}_{66ij}\end{bmatrix}$$

$$\begin{aligned}&\bar{\Phi}_{11il}=\bar{Q}_{1i}+\bar{Q}_{2i}+\bar{Q}_{3i}+\mu_1 sym(A_i^l\bar{G}_i)-\bar{Z}_{1i}+\mu_1^2I\\&\bar{\Phi}_{12ils}=\mu_2\bar{G}_i^{\mathrm{T}}(A_i^l)^{\mathrm{T}}+\mu_1B_i^lY_i^sC_{2i}\\&\bar{\Phi}_{22ils}=\mu_2 sym(B_i^lY_i^sC_{2i})-(1-h_{di})\bar{Q}_{1i}+\mu_2^2I\\&\bar{\Phi}_{15il}=\bar{P}_i-\mu_1\bar{G}_i+\mu_3\bar{G}_i^{\mathrm{T}}(A_i^l)^{\mathrm{T}}\\&\bar{\Phi}_{25ils}=-\mu_2\bar{G}_i+\mu_3C_{2i}^{\mathrm{T}}(Y_i^s)^{\mathrm{T}}(B_i^l)^{\mathrm{T}}\\&\bar{\Phi}_{55i}=\eta_{1i}^2\bar{Z}_{1i}-\mu_3 sym(\bar{G}_i)+\mu_3^2I\\&\bar{\Phi}_{66ij}=-(3J\sum_{j=1}^{J}\hat{f}_{ji}^2\|\hat{B}_j\|^2)^{-1}I\\&\bar{G}_i=V_i\begin{bmatrix}\hat{G}_{11i} & 0\\ \hat{G}_{21i} & \hat{G}_{22i}\end{bmatrix}V_i^{\mathrm{T}}.\end{aligned}\tag{27}$$

Then, the desired controller gains are given by $K_i^s=Y_i^sW_iS_i\hat{G}_{11i}^{-1}S_i^{-1}W_i^{\mathrm{T}}$, where W_i, S_i and V_i are derived from singular value decomposition (SVD) of C_{2i}.

Proof. Under the conditions of the Theorem 1, a feasible solution satisfies the condition $\bar{\Phi}_{55i}<0$, which implies that $\bar{G}_i$ is nonsingular. Define $G_i=\bar{G}_i^{-1}$, $\bar{P}_i=\bar{G}_i^{\mathrm{T}}P_i\bar{G}_i$, $\bar{Q}_{1i}=\bar{G}_i^{\mathrm{T}}Q_{1i}\bar{G}_i$, $\bar{Q}_{2i}=\bar{G}_i^{\mathrm{T}}Q_{2i}\bar{G}_i$, $\bar{Q}_{3i}=\bar{G}_i^{\mathrm{T}}Q_{3i}\bar{G}_i$ and $\bar{Z}_{1i}=\bar{G}_i^{\mathrm{T}}Z_{1i}\bar{G}_i$.

Assume that C_{2i} is full rank, then the SVD decomposition exists such that $W_i^{\mathrm{T}}C_{2i}V_i=\begin{bmatrix}S_i & 0\end{bmatrix}$ and $\bar{G}_i=V_i\begin{bmatrix}\hat{G}_{11i} & 0\\ \hat{G}_{21i} & \hat{G}_{22i}\end{bmatrix}V_i^{\mathrm{T}}$. It is obtained that

$$\begin{aligned}&C_{2i}\bar{G}_i=W_i[S_i\quad 0]V_i^{\mathrm{T}}V_i\begin{bmatrix}\hat{G}_{11i} & 0\\ \hat{G}_{21i} & \hat{G}_{22i}\end{bmatrix}V_i^{\mathrm{T}}=\\&\quad W_i[S_i\hat{G}_{11i}\quad 0]V_i^{\mathrm{T}}=\\&\quad W_iS_i\hat{G}_{11i}S_i^{-1}W_i^{\mathrm{T}}W_i[S_i\quad 0]V_i^{\mathrm{T}}=\hat{G}_iC_{2i}.\end{aligned}$$

By letting $Y_i^l=K_i^l\hat{G}=K_i^lW_iS_i\hat{G}_{11i}S_i^{-1}W_i^{\mathrm{T}}$, using Schur complement and applying a congruence transformation to (25) and (26) by $\mathrm{diag}\{G_i,G_i,G_i,G_i,G_i\}$, we find that the condition (14) holds considering (4) and (11). Thus, there exists a fuzzy controller (7) such that the closed-loop system (13) is asymptotically stable. □

4 Simulation results

Example 1. To show the effectiveness of the proposed approach, we consider the numerical example given in [4], which is composed of two subsystems S_1 and S_2 described respectively by

$$\begin{cases} \dot{x}_1(t) = \sum_{l=1}^{2} h_1^l(\theta_1(t))[A_1^l x_1(t) + B_1^l u_1(t) + \sum_{j=1}^{J} f_{1j}(x_j(t))] \\ y_1(t) = C_{21}x_1(t) \end{cases} \tag{28}$$

with

$$A_1^1 = \begin{bmatrix} -6 & 6 & 0 \\ 0.5 & -3 & 1 \\ 0 & 0.2 & -1 \end{bmatrix}, \quad A_1^2 = \begin{bmatrix} -1 & 0.1 & 0 \\ -0.2 & -2 & 0 \\ 0.3 & 0 & -1 \end{bmatrix}$$

$$B_1^1 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad B_1^2 = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$C_{21} = \begin{bmatrix} 1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}, \quad f_{11} = 0$$

$$f_{12} = \begin{bmatrix} 0.02 & 0.01 \\ 0.01 & 0.4 \\ 0.01 & 0.1 \end{bmatrix} ||x_2||$$

$$h_1^1(x_1(t)) = \sin^2(x_{11}(t)), \quad h_1^2(x_1(t)) = \cos^2(x_{11}(t)).$$

For subsystem S_2,

$$\begin{cases} \dot{x}_2(t) = \sum_{l=1}^{2} h_2^l(\theta_2(t))[A_2^l x_2(t) + B_2^l u_2(t) + \sum_{j=1}^{J} f_{2j}(x_j(t))] \\ y_2(t) = C_{22}x_2(t) \end{cases} \tag{29}$$

with

$$A_2^1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad A_2^2 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B_2^1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad B_2^2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$C_{22} = \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix}, \quad f_{21} = \begin{bmatrix} 0.01 & 0.01 & 0.01 \\ 0.02 & 0.01 & 0.1 \end{bmatrix} ||x_1||$$

$$f_{22} = 0, \quad h_2^1(x_2(t)) = \sin^2(x_{21}(t))$$
$$h_2^2(x_2(t)) = \cos^2(x_{21}(t)).$$

The network-related parameters for each subsystem S_i are assumed as $T_e = 3\,\text{ms}$, the minimum delay $\eta_{1i} = 4\,\text{ms}$, the maximum delay $\eta_{2i} = 20\,\text{ms}$ and the maximum number of packet dropouts is $\bar{\sigma}_i = 3$. The time varying delays between the sensors and controller as well as between controller and actuator are generated randomly such as $\min(\tau_{sci}+\tau_{cai}) \geq \eta_{1i}$, and $\max(\tau_{sci}+\tau_{cai}+(\bar{\sigma}_i+1)T_e) \leq \eta_{2i}$, and packet dropouts are also generated randomly such as $\max(N_e) \leq 3$, where N_e is the number of packet dropouts, $h_{di} = 0.1$, $\mu_1 = 1$, $\mu_2 = 0.3$ and $\mu_3 = 0.5$.

By Theorem 1, we find a feasible solution as $K_1^1 = \begin{bmatrix} 1.2494 & -3.3911 \\ -1.3490 & 3.2305 \end{bmatrix}$, $K_1^2 = \begin{bmatrix} 0.0596 & 0.5105 \\ -0.1445 & -0.5467 \end{bmatrix}$ for subsystem S_1, and $K_2^1 = \begin{bmatrix} -0.0559 & -0.0221 \end{bmatrix}$, $K_2^2 = \begin{bmatrix} -0.0311 & -0.0564 \end{bmatrix}$ for subsystem S_2.

For simulation, initial conditions are $x_1(0) = \begin{bmatrix} 1 & 0.5 & -1 \end{bmatrix}^{\mathrm{T}}$ and $x_2(0) = \begin{bmatrix} 2 & -2 \end{bmatrix}^{\mathrm{T}}$.

The state variables evolution of NCSs and control inputs are shown in Figs. 2−4 from which, we can note that all states converge to zero. Figs. 5 and 6 show the delays introduced by the network and packet loss data which are randomly generated. Therefore, according to Theorem 1, the closed-loop overall fuzzy large-scale system composed of two subsystems S_1 and S_2 is asymptotically stable. The simulation results are consistent with the analysis and support the effectiveness of the developed design strategy.

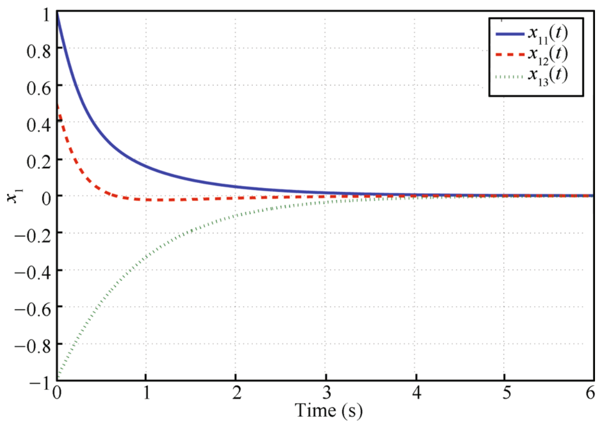

Fig. 2 Response of state x in the S_1

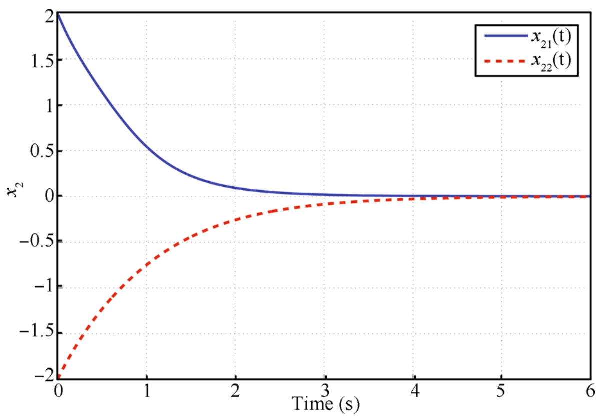

Fig. 3 Response of state x in the S_2

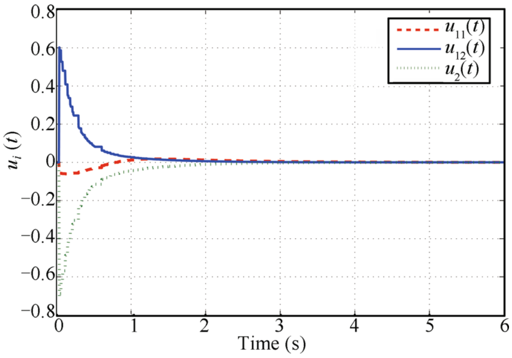

Fig. 4 Evolution of control input signals $u_i(t)$

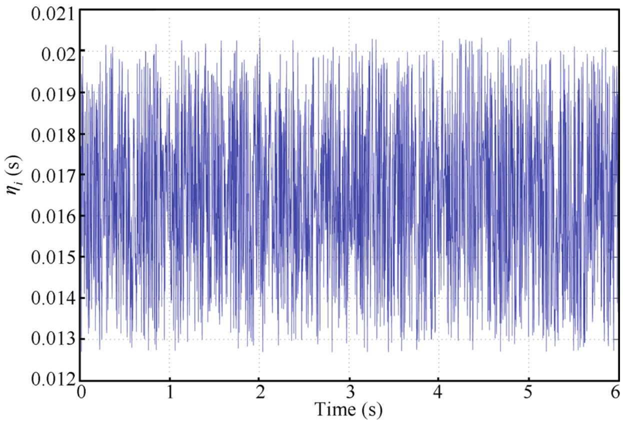

Fig. 5 Delay induced by communication networks

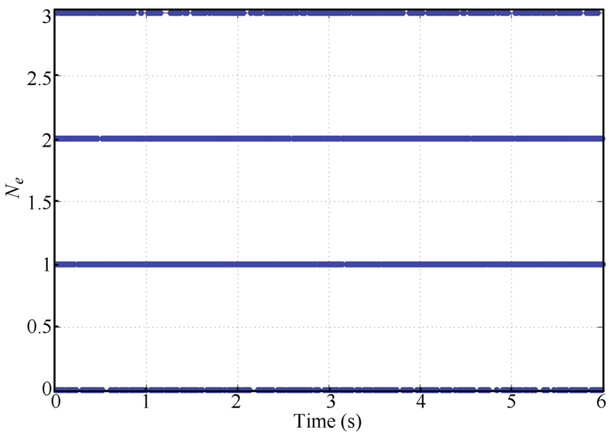

Fig. 6 Data packets dropout

Example 2. We consider the same large-scale system S composed of three fuzzy subsystems S_i, $i = 1, 2, 3$, as that in [20].

For subsystem S_1:

Rule 1 :

If $x_{11}(t)$ is small and $x_{12}(t)$ is big

$$\text{then } \dot{x}_1(t) = A_1^1 x_1(t) + B_1^1 u_1 + \sum_{j=1}^{3} f_{1j}(x_j(t))$$

$$y_1(t) = C_{11} x_1(t).$$

Rule 2 :

If $x_{11}(t)$ is small and $x_{12}(t)$ is small

$$\text{then } \dot{x}_1(t) = A_1^2 x_1(t) + B_1^2 u_1 + \sum_{j=1}^{3} f_{1j}(x_j(t))$$

$$y_1(t) = C_{11} x_1(t).$$

Rule 3 :

If $x_{11}(t)$ is big and $x_{12}(t)$ is small

$$\text{then } \dot{x}_1(t) = A_1^3 x_1(t) + B_1^3 u_1 + \sum_{j=1}^{3} f_{1j}(x_j(t))$$

$$y_1(t) = C_{11} x_1(t).$$

For these rules,

$$A_1^1 = \begin{bmatrix} -2 & 3 \\ 1.5 & -2.2 \end{bmatrix}, \quad A_1^2 = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$$

$$A_1^3 = \begin{bmatrix} -2 & 3 \\ -6 & -11 \end{bmatrix}, \quad B_1^1 = \begin{bmatrix} 0.15 \\ 0.1 \end{bmatrix}$$

$$B_1^2 = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}, \quad B_1^3 = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}$$

$$C_{11} = \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix}, \quad f_{11} = 0$$

$$f_{12} = \begin{bmatrix} 0.08 \\ 0.05 \end{bmatrix} ||x_2||, \quad f_{13} = \begin{bmatrix} 0.09 \\ 0.06 \end{bmatrix} ||x_3||.$$

For subsystem S_2:

Rule 1 :

If $x_{21}(t)$ is small and $x_{22}(t)$ is small

$$\text{then } \dot{x}_2(t) = A_2^1 x_2(t) + B_2^1 u_2 + \sum_{j=1}^{3} f_{2j}(x_j(t))$$

$$y_2(t) = C_{11} x_2(t).$$

Rule 2 :

If $x_{21}(t)$ is big and $x_{22}(t)$ is small

$$\text{then } \dot{x}_2(t) = A_2^2 x_2(t) + B_2^2 u_2 + \sum_{j=1}^{3} f_{2j}(x_j(t))$$

$$y_2(t) = C_{11} x_2(t).$$

For these rules,

$$A_2^1 = \begin{bmatrix} -3 & 1 \\ 5 & -3 \end{bmatrix}, A_2^2 = \begin{bmatrix} -2 & 1 \\ 3 & -0.3 \end{bmatrix}$$

$$B_2^1 = \begin{bmatrix} 0.1 \\ 0.6 \end{bmatrix}, B_2^2 = \begin{bmatrix} 0.2 \\ 1.2 \end{bmatrix}$$

$$f_{21} = \begin{bmatrix} 0.02 \\ 0.12 \end{bmatrix} ||x_1||, \quad f_{22} = 0$$

$$f_{23} = \begin{bmatrix} 0.06 \\ 0.36 \end{bmatrix} ||x_3||.$$

For subsystem S_3:

Rule 1 :

If $x_{31}(t)$ is big and $x_{32}(t)$ is big

$$\text{then } \dot{x}_3(t) = A_3^1 x_3(t) + B_3^1 u_3 + \sum_{j=1}^{3} f_{3j}(x_j(t))$$

$$y_3(t) = C_{11} x_3(t).$$

Rule 2 :

If $x_{31}(t)$ is small and $x_{32}(t)$ is big

$$\text{then } \dot{x}_3(t) = A_3^2 x_3(t) + B_3^2 u_3 + \sum_{j=1}^{3} f_{3j}(x_j(t))$$

$$y_3(t) = C_{11} x_3(t).$$

For these rules,

$$A_3^1 = \begin{bmatrix} -3 & 1 \\ 4 & -2 \end{bmatrix}, A_3^2 = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$$

$$B_3^1 = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}, B_3^2 = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}$$

$$f_{31} = \begin{bmatrix} 0.48 \\ 0.64 \end{bmatrix} ||x_1||, \quad f_{32} = \begin{bmatrix} 0.24 \\ 0.32 \end{bmatrix} ||x_2||$$

$$f_{33} = 0.$$

It is seen that all f_{ij} satisfy the matching condition (2) with $\hat{f}_{11}^1 = \hat{f}_{11}^2 = \hat{f}_{11}^3 = 0, \hat{f}_{12}^1 = 0.5, \hat{f}_{12}^2 = 0.125, \hat{f}_{12}^3 = 0.25, \hat{f}_{13}^1 = 0.6, \hat{f}_{13}^2 = 0.15$ and $\hat{f}_{13}^3 = 0.3$ for subsystem S_1. All f_{ij} satisfy (2) with $\hat{f}_{21}^1 = 0.2, \hat{f}_{21}^2 = 0.1, \hat{f}_{22}^1 = \hat{f}_{22}^2 = 0, \hat{f}_{23}^1 = 0.6$ and $\hat{f}_{23}^2 = 0.3$ for subsystem S_2, All f_{ij} satisfy (2) with $\hat{f}_{31}^1 = 0.8, \hat{f}_{31}^2 = 1.6, \hat{f}_{32}^1 = 0.4, \hat{f}_{32}^2 = 0.8$ and $\hat{f}_{33}^1 = \hat{f}_{33}^2 = 0$ for subsystem S_3.

The membership functions of each state are shown in Fig. 1 of [20].

The network-related parameters for each subsystem S_i are assumed as $T_e = 5$ ms, the minimum delay $\eta_{1i} = 6$ ms, the maximum delay $\eta_{2i} = 20$ ms and the maximum number of packet dropouts is $\bar{\sigma}_i = 2$. The time varying delays between the sensors and controller as well as between controller and actuator are generated randomly such as $\min(\tau_{sci}+\tau_{cai}) \geq \eta_{1i}$, and $\max(\tau_{sci}+\tau_{cai}+(\bar{\sigma}_i+1)T_e) \leq \eta_{2i}$ and packet dropouts are also generated randomly such as $\max(N_e) \leq 2$, $h_{di} = 0.1$, $\mu_1 = 1$, $\mu_2 = 0.5$ and $\mu_3 = 0.9$.

Applying Theorem 1, the solutions of LMIs can be obtained as $K_1^1 = \begin{bmatrix} -4.2341 & -4.6081 \end{bmatrix}$, $K_1^2 = \begin{bmatrix} -1.1609 & -1.0426 \end{bmatrix}$ and $K_1^3 = \begin{bmatrix} 0.6774 & 0.2271 \end{bmatrix}$ for subsystem S_1, $K_2^1 = \begin{bmatrix} -2.2344 & -2.9514 \end{bmatrix}$ and $K_2^2 = \begin{bmatrix} -1.8578 & -1.5973 \end{bmatrix}$ for subsystem S_2, and $K_3^1 = \begin{bmatrix} -0.8119 & -0.8278 \end{bmatrix}$ and $K_3^2 = \begin{bmatrix} -1.9309 & -1.6042 \end{bmatrix}$ for subsystem S_3.

For simulation, initial conditions are $x_1(0) = \begin{bmatrix} 1.5 & -1 \end{bmatrix}^{\mathrm{T}}$, $x_2(0) = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix}^{\mathrm{T}}$ and $x_3(0) = \begin{bmatrix} 0.7 & -0.3 \end{bmatrix}^{\mathrm{T}}$.

The state variable evolution of NCSs and control inputs are shown in Figs. 7−10 from which, we can note that all states converge to zero. Therefore, according to Theorem 1, the closed-loop overall fuzzy large-scale system composed of three subsystems S_1, S_2 and S_3 is asymptotically stable. Thus, we have shown that the proposed decentralized static output feedback controller makes the nonlinear interconnected system in network communication exhibit asymptotic stability.

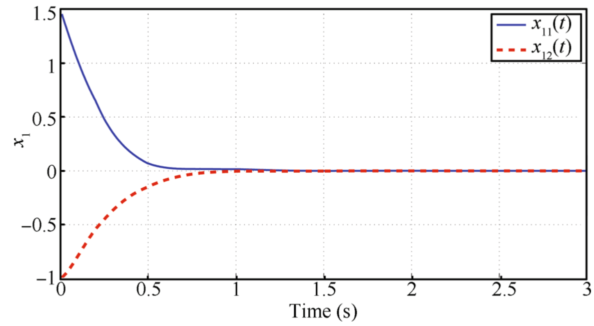

Fig. 7 State responses x in S_1

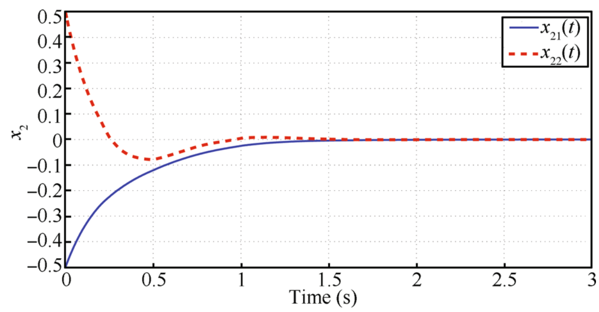

Fig. 8 State responses x in S_2

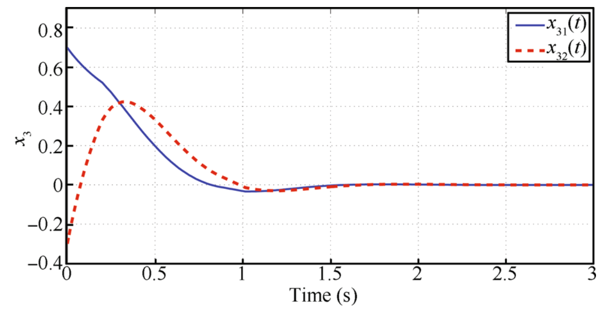

Fig. 9 State responses x in S_3

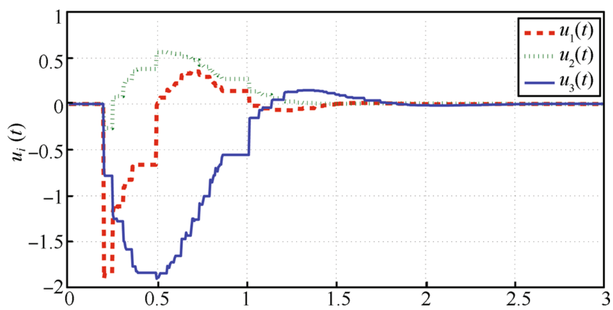

Fig. 10 Control signal trajectories $u_i(t)$

5 Conclusions

In this paper, based on Lyapunov-Krasovskii functional, new stabilization conditions have been established for networked controlled large-scale system. Furthermore, using these conditions in presence of the delay and data packets dropout in the network communication, networked fuzzy static output feedback controller gains have been obtained. The simulation results are shown to prove the advantages of the developed method.

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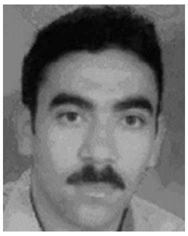

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