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Delay-dependent Non-fragile H_∞ Control for Linear Systems with Interval Time-varying Delay

Jun-Jun Hui[1,2] He-Xin Zhang[1] Xiang-Yu Kong[1]
[1]The Xi′an Research Institute of High Technology, Xi′an, Shaanxi 710025, China
[2]Mailbox 150 extension 11, Baoji, Shaanxi 721013, China

Abstract: This paper considers the problem of delay-dependent non-fragile H_∞ control for a class of linear systems with interval time-varying delay. Based on the direct Lyapunov method, an appropriate Lyapunov-Krasovskii functional (LKF) with triple-integral terms and augment terms is introduced. Then, by using the integral inequalities and convex combination technique, an improved H_∞ performance analysis criterion and non-fragile H_∞ controller are formulated in terms of linear matrix inequalities (LMIs), which can be easily solved by using standard numerical packages. At last, two numerical examples are provided to demonstrate the effectiveness of the obtained results.

Keywords: Lyapunov-Krasovskii functional (LKF), non-fragile, H_∞ control, linear matrix inequality (LMI), interval time-varying delay.

1 Introduction

Time-delay is frequently a source of instability and poor performance, which is often encountered in various physical and engineering systems such as chemical engineering systems, biological systems, economic systems and networked control systems and so on. Hence, the subject of the stability analysis of systems with time-varying delay has received considerable attention in the past few years (see e.g. [1–19], and the references therein).

It is well known that the problem of H_∞ control has long been an important and challenging research topic in the control community. Therefore, in the past years, much attention has been paid to the H_∞ control problem for time-delay systems. Depending on whether the existence conditions of H_∞ controller include the information of delay or not, the existing results on H_∞ control for time-delay systems can be classified into two types: delay-independent ones and delay-dependent ones. Since delay-dependent ones are generally less conservative than delay-independent ones especially for system with small size delays. Hence, considerable attention has been paid to the delay-dependent stability[12−19]. Usually, the H_∞ performance index and the upper bound of the delay are two performance indices, which are used to evaluate the conservativeness of the stability conditions. For a prescribed upper bound of the delay, the smaller the value of performance index is the better the stability conditions are. For a prescribed performance index, the larger the value of upper bound is, the less conservative the stability conditions are.

Brief paper
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In practice, however, owing to the A/D conversion, D/A conversion, finite word length and round-off errors in numerical computations, there are often some perturbations appearing in the feedback controller gain, which may result from either the actuator degradations or the requirements for readjustment of controller gains during the controller implementation stage. So, it is necessary and reasonable that any controller should be able to tolerate some level of controller gain variations. Following this idea, in recent years, the non-fragile control problem has attracted the interest of many researchers. For example, the problems of robust non-fragile H_∞ control for stochastic systems with time-varying delay had been investigated in [20–22]. The problems of non-fragile guaranteed cost control for stochastic systems with time-varying delay had been studied in [23–25]. The H_∞ non-fragile observer-based control for uncertain time-delay systems had been presented in [26–28]. However, all these techniques assume that the delay-range varies from zero to an upper bound. Nevertheless, in certain time-delay systems, like networked control systems, the delay-range may have a non-zero lower bound. In this case, the criteria in the previous work are conservative since they do not take into account information of the lower bound of delay.

Recently, the triple integral forms of Lyapunov-Krasovskii functional (LKF) for stability of time-varying delays were proposed[4, 5], and showed its improvement of maximum delay bounds. Inspired by the works of [4, 5], in this paper, we contribute to the improvement of the H_∞ performance and non-fragile H_∞ control for a class of linear systems with interval time-varying delay. Based on the direct Lyapunov method, a new LKF with triple integral terms involving lower and upper bounds of interval time-varying delays have been introduced, and combining it with the newly established integral inequality, an improved

H_∞ performance analysis criterion and non-fragile H_∞ controller are formulated in terms of linear matrix inequalities. The improved results are mainly attributed to the appropriate LKF and tighter bounding technique for dealing with the cross-terms that emerge from the time derivative of the LKF. Numerical examples are given to illustrate the effectiveness of the proposed method.

The remainder of the paper is organized as follows. Section 2 states the problem formulation. Section 3 and Section 4 provides the improved results for H_∞ performance and non-fragile H_∞ controller. Two examples are illustrated in Section 5 to show the effectiveness of the proposed approaches, and the paper is concluded in Section 6.

2 Problem formulation

Consider a class of linear systems with interval time-varying delay described by

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-h(t)) + Bu(t) + B_\omega \omega(t) \\ z(t) = Cx(t) + C_d x(t-h(t)) + D_\omega \omega(t) + Du(t) \\ x(t) = \varphi(t), t \in [-h_2, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}^m$ is the control input vector, $\omega(t) \in \mathbf{R}^p$ is the disturbance input belonging to $L_2[0,\infty)$, $z(t) \in \mathbf{R}^l$ is the controlled output; A, A_d, B, B_ω, C, C_d, D_ω and D are known real constant matrices of appropriate dimensions; $\varphi(t)$ is continuous-time initial function defined on $[-h_2, 0]$; $h(t)$ is the time-varying delay of the system and is assumed to satisfy:

$$h_1 \le h(t) \le h_2, \quad \dot{h}(t) \le \mu \tag{2}$$

or

$$h_1 \le h(t) \le h_2 \tag{3}$$

where h_1, h_2 and μ are known constants.

For a prescribed scalar $\gamma > 0$, we define the performance index as

$$J(\omega) = \int_0^\infty [z(t)^{\mathrm{T}} z(t) - \gamma^2 \omega^{\mathrm{T}}(t)\omega(t)]\mathrm{d}t. \tag{4}$$

This paper is concerned with the problems of non-fragile H_∞ control for linear systems with interval time-varying delay. Our attention is paid to the design of a memoryless non-fragile state feedback controller:

$$u(t) = K(t)x(t) \tag{5}$$

where $K(t) = K + \Delta K(t)$ and K is the controller gain, ΔK is a perturbed matrix, which is assumed to be

$$\Delta K(t) = D_c F_c(t) E_c, \quad F_c^{\mathrm{T}}(t) F_c(t) \le I \tag{6}$$

where D_c and E_c are known real constant matrices with appropriate dimensions, the time-varying uncertain matrix $F(t)$ satisfies $F^{\mathrm{T}}(t)F(t) \le I$, $\forall t$.

The purpose of this paper is to develop a delay-dependent H_∞ conditions such that, for any $h(t)$ satisfying (2) or (3):

1) The closed-loop system is asymptotically stable for $\omega(t) = 0$.

2) The closed-loop system guarantees under zero initial condition $||z(t)||_2 < \gamma||\omega(t)||_2$ for all nonzero $\omega(t) \in L_2[0,\infty)$ and a prescribed scalar $\gamma > 0$.

To end this section, we introduce the following lemmas, which are important for deriving the main results.

Lemma 1[2]. For any constant matrix $W \in \mathbf{R}^{n\times n}$, $W = W^{\mathrm{T}} > 0$, a scalar function $h := h(t) > 0$, and a vector-valued function $\dot{x} : [-h, 0] \to \mathbf{R}^n$, such that the following integrations are well defined, then:

$$-h\int_{t-h}^{t} \dot{x}^{\mathrm{T}}(s)W\dot{x}(s)\mathrm{d}s \le \zeta_1^{\mathrm{T}}(t)\begin{bmatrix} -W & W \\ W & -W \end{bmatrix}\zeta_1(t)$$

$$-\frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)W\dot{x}(s)\mathrm{d}s \le \zeta_2^{\mathrm{T}}(t)\begin{bmatrix} -W & W \\ W & -W \end{bmatrix}\zeta_2(t)$$

where

$$\zeta_1^{\mathrm{T}}(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-h) \end{bmatrix}$$
$$\zeta_2^{\mathrm{T}}(t) = \begin{bmatrix} hx^{\mathrm{T}}(t) & \int_{t-h}^{t} x^{\mathrm{T}}(s)\mathrm{d}s \end{bmatrix}.$$

Lemma 2[7]. Suppose $r_1 \le r(t) \le r_2$, where $r(t) : \mathbf{R}_+ \to \mathbf{R}_+$, then, for any $R = R^{\mathrm{T}} > 0$, the following integral inequality holds:

$$-\int_{t-r_2}^{t-r_1} \dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s \le \delta^{\mathrm{T}}(t)\Big\{(r_2 - r(t))TR^{-1}M^{\mathrm{T}} + (r(t) - r_1)YR^{-1}N^{\mathrm{T}} + [Y \quad -Y+T \quad -T] + [Y \quad -Y+T \quad -T]^{\mathrm{T}}\Big\}\delta(t)$$

where

$$\delta^{\mathrm{T}}(t) = \begin{bmatrix} x^{\mathrm{T}}(t-r_1) & x^{\mathrm{T}}(t-r(t)) & x^{\mathrm{T}}(t-r_2) \end{bmatrix}$$
$$T = \begin{bmatrix} T_1^{\mathrm{T}} & T_2^{\mathrm{T}} & T_3^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, Y = \begin{bmatrix} Y_1^{\mathrm{T}} & Y_2^{\mathrm{T}} & Y_3^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$

where T and Y are free matrices of appropriate dimensions.

Lemma 3[9]. Suppose $\gamma_1 \le \gamma(t) \le \gamma_2$, where $\gamma(\cdot)$: $\mathbf{R}_+ \to \mathbf{R}_+$. Then, for any constant matrices Ξ_1, Ξ_2 and Ω with proper dimensions, the following matrix inequality

$$\Omega + (\gamma(t) - \gamma_1)\Xi_1 + (\gamma_2 - \gamma(t))\Xi_2 < 0$$

holds, if and only if

$$\Omega + (\gamma_2 - \gamma_1)\Xi_1 < 0$$
$$\Omega + (\gamma_2 - \gamma_1)\Xi_2 < 0.$$

Lemma 4[3]. Given matrices $Q = Q^{\mathrm{T}}$, H, E, and $R = R^{\mathrm{T}}$ with appropriate dimensions, the inequality

$$Q + HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} < 0$$

holds for all F satisfying $F^{\mathrm{T}}F \le R$, if and only if there exists some scalar $\varepsilon > 0$, such that

$$Q + \varepsilon HH^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}RE < 0.$$

3 H_∞ performance analysis

In this section, we will establish a less conservative delay dependent stability criterion for the following unforced system:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-h(t)) + B_\omega \omega(t) \\ z(t) = Cx(t) + C_d x(t-h(t)) + D_\omega \omega(t) \\ x(t) = \varphi(t), \forall t \in [-h_2, 0]. \end{cases} \tag{7}$$

Theorem 1. Given scalars $\gamma > 0$, $0 \le h_1 \le h_2$, and μ, the system (7) with conditions (2) and (3) is asymptotically stable and satisfying $||z(t)||_2 \le \gamma ||\omega(t)||_2$ for any nonzero $\omega(t) \in L_2[0,\infty)$, under the zero initial condition if there exist real symmetric positive definite matrices $P = \left[\begin{smallmatrix} P_{11} & P_{12} & P_{13} \\ * & P_{22} & P_{23} \\ * & * & P_{33} \end{smallmatrix}\right]$, Q_i $(i = 1,2,3)$, R_j, Z_j $(j = 1,2)$; free matrices S_1, S_2, Y_a, T_a $(a = 1,2,3)$ of appropriate dimensions such that the following linear matrix inequalities (LMIs) hold:

$$\begin{bmatrix} \Xi & \sqrt{h_\delta} Y & \Upsilon \\ * & -R_2 & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{8}$$

$$\begin{bmatrix} \Xi & \sqrt{h_\delta} T & \Upsilon \\ * & -R_2 & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{9}$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & S_1A_d & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & S_1B_\omega \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & P_{23}^{\mathrm{T}} & P_{33}^{\mathrm{T}} & 0 \\ * & * & \Xi_{33} & \Xi_{34} & A_d^{\mathrm{T}} S_2^{\mathrm{T}} & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & 0 & \Xi_{46} & \Xi_{47} & 0 \\ * & * & * & * & \Xi_{55} & P_{12} & P_{13} & S_2B_\omega \\ * & * & * & * & * & -Z_1 & 0 & 0 \\ * & * & * & * & * & * & -Z_2 & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Xi_{11} = P_{12} + P_{12}^{\mathrm{T}} + Q_1 - R_1 - h_2^2 Z_1 - h_\delta^2 Z_2 + S_1 A + A^{\mathrm{T}} S_1^{\mathrm{T}}$$

$$\Xi_{12} = R_1 + P_{13},\ \Xi_{14} = -P_{12} - P_{13}, \Xi_{15} = P_{11} - S_1 + A^{\mathrm{T}} S_2^{\mathrm{T}}$$

$$\Xi_{16} = P_{22}^{\mathrm{T}} + h_2 Z_1,\ \Xi_{17} = P_{23} + h_\delta Z_2$$

$$\Xi_{22} = -Q_1 + Q_2 + Q_3 - R_1 + Y_1 + Y_1^{\mathrm{T}}$$

$$\Xi_{23} = -Y_1 + T_1 + Y_2^{\mathrm{T}},\ \Xi_{24} = -T_1 + Y_3^{\mathrm{T}}$$

$$\Xi_{33} = -(1-\mu)Q_2 - Y_2 - Y_2^{\mathrm{T}} + T_2 + T_2^{\mathrm{T}}$$

$$\Xi_{34} = -T_2 - Y_3^{\mathrm{T}} + T_3^{\mathrm{T}},\ \Xi_{44} = -Q_3 - T_3 - T_3^{\mathrm{T}}$$

$$\Xi_{46} = -P_{22}^{\mathrm{T}} - P_{23}^{\mathrm{T}},\ \Xi_{47} = -P_{32}^{\mathrm{T}} - P_{33}^{\mathrm{T}}$$

$$\Xi_{55} = H - S_2 - S_2^{\mathrm{T}},\ h_\delta = h_2 - h_1$$

$$H = h_1^2 R_1 + h_\delta R_2 + \frac{1}{4} h_2^4 Z_1 + \frac{1}{4}(h_2^2 - h_1^2)^2 Z_2$$

$$Y = \begin{bmatrix} 0 & Y_1^{\mathrm{T}} & Y_2^{\mathrm{T}} & Y_3^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$T = \begin{bmatrix} 0 & T_1^{\mathrm{T}} & T_2^{\mathrm{T}} & T_3^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$S = \begin{bmatrix} S_1^{\mathrm{T}} & 0 & 0 & 0 & S_2^{\mathrm{T}} & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$\Upsilon = \begin{bmatrix} C & 0 & C_d & 0 & 0 & 0 & 0 & D_\omega \end{bmatrix}^{\mathrm{T}}.$$

Proof. Construct an appropriate LKF as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{10}$$

where

$$\begin{aligned} V_1(t) =& \xi^{\mathrm{T}}(t) P \xi(t) \\ V_2(t) =& \int_{t-h_1}^{t} x^{\mathrm{T}}(s) Q_1 x(s) \mathrm{d}s + \int_{t-h(t)}^{t-h_1} x^{\mathrm{T}}(s) Q_2 x(s) \mathrm{d}s + \\ & \int_{t-h_2}^{t-h_1} x^{\mathrm{T}}(s) Q_3 x(s) \mathrm{d}s + \\ & h_1 \int_{-h_1}^{0} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) R_1 \dot{x}(s) \mathrm{d}s \mathrm{d}\theta + \\ & \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) R_2 \dot{x}(s) \mathrm{d}s \mathrm{d}\theta \\ V_3(t) =& \frac{h_2^2}{2} \int_{-h_2}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{x}^{\mathrm{T}}(s) Z_1 \dot{x}(s) \mathrm{d}s \mathrm{d}\lambda \mathrm{d}\theta + \\ & \frac{(h_2^2 - h_1^2)}{2} \int_{-h_2}^{-h_1} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{x}^{\mathrm{T}}(s) Z_2 \dot{x}(s) \mathrm{d}s \mathrm{d}\lambda \mathrm{d}\theta \\ \xi^{\mathrm{T}}(t) =& \begin{bmatrix} x^{\mathrm{T}}(t) & \int_{t-h_2}^{t} x^{\mathrm{T}}(s) \mathrm{d}s & \int_{t-h_2}^{t-h_1} x^{\mathrm{T}}(s) \mathrm{d}s \end{bmatrix}. \end{aligned}$$

Then, taking the time derivative of $V(t)$ with respect to t along the system (7) yields:

$$\begin{aligned} \dot{V}(t) \le & 2\xi^{\mathrm{T}}(t) P \dot{\xi}(t) + x^{\mathrm{T}}(t) Q_1 x(t) + \dot{x}^{\mathrm{T}}(t) H \dot{x}(t) - \\ & x^{\mathrm{T}}(t-h_1)(Q_1 - Q_2 - Q_3) x(t-h_1) - \\ & (1-\mu) x^{\mathrm{T}}(x-h(t)) Q_2 x(t-h(t)) - \\ & x^{\mathrm{T}}(t-h_2) Q_3 x(t-h_2) - \\ & h_1 \int_{t-h_1}^{t} \dot{x}^{\mathrm{T}}(s) R_1 \dot{x}(s) \mathrm{d}s - \int_{t-h_2}^{t-h_1} \dot{x}^{\mathrm{T}}(s) R_2 \dot{x}(s) \mathrm{d}s \\ & - \frac{h_2^2}{2} \int_{-h_2}^{0} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) Z_1 \dot{x}(s) \mathrm{d}s \mathrm{d}\theta - \\ & \frac{(h_2^2 - h_1^2)}{2} \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) Z_2 \dot{x}(s) \mathrm{d}s \mathrm{d}\theta \end{aligned} \tag{11}$$

where $H = h_1^2 R_1 + h_\delta R_2 + \frac{1}{4} h_2^4 Z_1 + \frac{1}{4}(h_2^2 - h_1^2)^2 Z_2$.

From Lemmas 1 and 2, respectively, we obtain the following inequality:

$$-h_1\int_{t-h_1}^{t}\dot{x}^{\rm T}(s)R_1\dot{x}(s){\rm d}s\le \begin{bmatrix} x(t)\\ x(t-h_1)\end{bmatrix}^{\rm T}\begin{bmatrix} -R_1 & R_1\\ * & -R_1\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-h_1)\end{bmatrix} \quad (12)$$

$$-\frac{h_2^2}{2}\int_{-h_2}^{0}\int_{t+\theta}^{t}\dot{x}^{\rm T}(s)Z_1\dot{x}(s){\rm d}s{\rm d}\theta\le \begin{bmatrix} h_2x(t)\\ \int_{t-h_2}^{t}x(s){\rm d}s\end{bmatrix}^{\rm T}\begin{bmatrix} -Z_1 & Z_1\\ Z_1 & -Z_1\end{bmatrix}\begin{bmatrix} h_2x(t)\\ \int_{t-h_2}^{t}x(s){\rm d}s\end{bmatrix} \quad (13)$$

$$-\frac{(h_2^2-h_1^2)}{2}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{x}^{\rm T}(s)Z_2\dot{x}(s){\rm d}s{\rm d}\theta\le \begin{bmatrix} (h_2-h_1)x(t)\\ \int_{t-h_2}^{t-h_1}x(s){\rm d}s\end{bmatrix}^{\rm T}\begin{bmatrix} -Z_2 & Z_2\\ Z_2 & -Z_2\end{bmatrix}\begin{bmatrix} (h_2-h_1)x(t)\\ \int_{t-h_2}^{t-h_1}x(s){\rm d}s\end{bmatrix} \quad (14)$$

and

$$-\int_{t-h_2}^{t-h_1}\dot{x}^{\rm T}(s)R_2\dot{x}(s){\rm d}s\le\delta^{\rm T}(t)\Big[(h_2-h(t))TR_2^{-1}T^{\rm T}+ (h(t)-h_1)YR_2^{-1}Y^{\rm T}+[Y\quad -Y+T\quad -T]+ [Y\quad -Y+T\quad -T]^{\rm T}\Big]\delta(t). \quad (15)$$

From the system (7), the following equation holds:

$$2[x^{\rm T}(t)S_1+\dot{x}^{\rm T}(t)S_2]\times [Ax(t)+A_dx(t-h(t))-\dot{x}(t)+B_\omega\omega(t)]=0 \quad (16)$$

where S_1, S_2 are free matrices with appropriate dimensions.

By substituting (12)–(16) in (11), and defining an augmented state vector $\zeta(t)$ as

$$\zeta^{\rm T}(t)=\Big[\ x^{\rm T}(t)\quad x^{\rm T}(t-h_1)\quad x^{\rm T}(t-h(t))\quad x^{\rm T}(t-h_2)\quad \dot{x}^{\rm T}(t)\quad \int_{t-h_2}^{t}x^{\rm T}(s){\rm d}s\quad \int_{t-h_2}^{t-h_1}x^{\rm T}(s){\rm d}s\quad \omega^{\rm T}(t)\ \Big].$$

Then $\dot{V}(t)$ can be expressed as

$$\dot{V}(t)\le\zeta^{\rm T}(t)[\Pi+(h_2-h(t)]TR_2^{-1}T^{\rm T}+ (h(t)-h_1)YR_2^{-1}Y^{\rm T})\zeta(t) \quad (17)$$

where $\Pi=$

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & S_1A_d & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & S_1B_\omega\\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & P_{23}^{\rm T} & P_{33}^{\rm T} & 0\\ * & * & \Xi_{33} & \Xi_{34} & A_d^{\rm T}S_2^{\rm T} & 0 & 0 & 0\\ * & * & * & \Xi_{44} & 0 & \Xi_{46} & \Xi_{47} & 0\\ * & * & * & * & \Xi_{55} & P_{12} & P_{13} & S_2B_\omega\\ * & * & * & * & * & -Z_1 & 0 & 0\\ * & * & * & * & * & * & -Z_2 & 0\\ * & * & * & * & * & * & * & 0 \end{bmatrix}.$$

Now, for a prescribed scalar γ, consider the performance index $J(\omega)$, by adding $z(t)^{\rm T}z(t)-\gamma^2\omega^{\rm T}(t)\omega(t)$ to the both sides of (17), we can rewrite (17) as

$$\dot{V}(t)+z(t)^{\rm T}z(t)-\gamma^2\omega^{\rm T}(t)\omega(t)\le \zeta^{\rm T}(t)(\bar{\Pi}+\Theta^{\rm T}\Theta+(h_2-h(t))TR_2^{-1}T^{\rm T}+ (h(t)-h_1)YR_2^{-1}Y^{\rm T})\zeta(t) \quad (18)$$

where $\bar{\Pi}=\Pi+{\rm diag}\left\{\ 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad -\gamma^2I\ \right\}$ and $\Theta=\left[\ C\quad 0\quad C_d\quad 0\quad 0\quad 0\quad 0\quad D_\omega\ \right]$.
If

$$\bar{\Pi}+\Theta^{\rm T}\Theta+(h_2-h(t))TR_2^{-1}T^{\rm T}+(h(t)-h_1)YR_2^{-1}Y^{\rm T}<0 \quad (19)$$

then

$$\dot{V}(t)+z(t)^{\rm T}z(t)-\gamma^2\omega^{\rm T}(t)\omega(t)\le 0. \quad (20)$$

Now, when $\omega(t)=0$, $\dot{V}(t)<0$, then, the system is asymptotically stable. For $\omega(t)\ne 0$, integrating both sides of (20) from 0 to t, and letting $t\to\infty$ with zero initial condition, we get

$$\int_0^\infty z^{\rm T}(s)z(s){\rm d}s\le\int_0^\infty\gamma^2\omega^{\rm T}(s)\omega(s){\rm d}s. \quad (21)$$

That is $||z(t)||_2<\gamma||\omega||_2$, so the closed-loop system (7) has an H_∞ disturbance attenuation level γ, under zero initial conditions.

Applying the Schur complement to (19), we deduce the LMIs stated in Theorem 1. □

4 Non-fragile robust H_∞ controller synthesis

In this section, based on the result stated in the Section 3, we now design a non-fragile state feedback controller (5) to make system (1) robustly asymptotically stable with the disturbance attenuation γ .

Theorem 2. Given scalars $\gamma>0$, $0\le h_1\le h_2$ and μ, system (1) with the non-fragile controller (5) is robustly asymptotically stable and satisfying $||z(t)||_2\le\gamma||\omega(t)||_2$ for any nonzero $\omega(t)\in L_2[0,\infty)$ under the zero initial condition, if there exist positive scalars $\tilde{\varepsilon}$, real symmetric positive definite matrices $\tilde{P}=\begin{bmatrix}\tilde{P}_{11} & \tilde{P}_{12} & \tilde{P}_{13}\\ * & \tilde{P}_{22} & \tilde{P}_{23}\\ * & * & \tilde{P}_{33}\end{bmatrix}$, $\tilde{Q}_i(i=1,2,3)$, $\tilde{R}_j$, $\tilde{Z}_j(j=1,2)$; free matrices $\tilde{Y}_a$, $\tilde{T}_a(a=1,2,3)$, X, Y of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} \tilde{\Lambda}_1 & \tilde{\Gamma}_a & \tilde{\Gamma}_E^{\rm T}\\ * & -\tilde{\varepsilon}I & 0\\ * & * & -\tilde{\varepsilon}I\end{bmatrix}<0 \quad (22)$$

$$\begin{bmatrix} \tilde{\Lambda}_2 & \tilde{\Gamma}_a & \tilde{\Gamma}_E^{\rm T}\\ * & -\tilde{\varepsilon}I & 0\\ * & * & -\tilde{\varepsilon}I\end{bmatrix}<0 \quad (23)$$

where

$$\tilde{\Lambda}_1 = \begin{bmatrix} \tilde{\Xi}' & \sqrt{h_\delta}\tilde{Y} & \tilde{\Upsilon}' \\ * & -\tilde{R}_2 & 0 \\ * & * & -I \end{bmatrix}, \quad \tilde{\Lambda}_2 = \begin{bmatrix} \tilde{\Xi}' & \sqrt{h_\delta}\tilde{T} & \tilde{\Upsilon}' \\ * & -\tilde{R}_2 & 0 \\ * & * & -I \end{bmatrix}$$

$$\tilde{\Xi}' = \begin{bmatrix} \tilde{\Xi}'_{11} & \tilde{\Xi}_{12} & A_d X & \tilde{\Xi}_{14} & \tilde{\Xi}'_{15} & \tilde{\Xi}_{16} & \tilde{\Xi}_{17} & B_\omega \\ * & \tilde{\Xi}_{22} & \tilde{\Xi}_{23} & \tilde{\Xi}_{24} & 0 & \tilde{P}_{23}^{\mathrm{T}} & \tilde{P}_{33}^{\mathrm{T}} & 0 \\ * & * & \tilde{\Xi}_{33} & \tilde{\Xi}_{34} & XA_d^{\mathrm{T}} & 0 & 0 & 0 \\ * & * & * & \tilde{\Xi}_{44} & 0 & \tilde{\Xi}_{46} & \tilde{\Xi}_{47} & 0 \\ * & * & * & * & \tilde{\Xi}_{55} & \tilde{P}_{12} & \tilde{P}_{13} & B_\omega \\ * & * & * & * & * & -\tilde{Z}_1 & 0 & 0 \\ * & * & * & * & * & * & -\tilde{Z}_2 & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\tilde{\Xi}'_{11} = \tilde{P}_{12} + \tilde{P}_{12}^{\mathrm{T}} + \tilde{Q}_1 - \tilde{R}_1 - h_2^2\tilde{Z}_1 - h_\delta^2\tilde{Z}_2 + AX + BY + XA^{\mathrm{T}} + Y^{\mathrm{T}}B^{\mathrm{T}}$$

$$\tilde{\Xi}_{12} = \tilde{R}_1 + \tilde{P}_{13},\ \tilde{\Xi}_{14} = -\tilde{P}_{12} - \tilde{P}_{13}$$

$$\tilde{\Xi}'_{15} = \tilde{P}_{11} - X + XA^{\mathrm{T}} + Y^{\mathrm{T}}B^{\mathrm{T}},\ \tilde{\Xi}_{16} = \tilde{P}_{22}^{\mathrm{T}} + h_2\tilde{Z}_1$$

$$\tilde{\Xi}_{17} = \tilde{P}_{23} + h_\delta\tilde{Z}_2,\ \tilde{\Xi}_{22} = -\tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 - \tilde{R}_1 + \tilde{Y}_1 + \tilde{Y}_1^{\mathrm{T}}$$

$$\tilde{\Xi}_{23} = -\tilde{Y}_1 + \tilde{T}_1 + \tilde{Y}_2^{\mathrm{T}},\ \tilde{\Xi}_{24} = -\tilde{T}_1 + \tilde{Y}_3^{\mathrm{T}}$$

$$\tilde{\Xi}_{33} = -(1-\mu)\tilde{Q}_2 - \tilde{Y}_2 - \tilde{Y}_2^{\mathrm{T}} + \tilde{T}_2 + \tilde{T}_2^{\mathrm{T}}$$

$$\tilde{\Xi}_{34} = -\tilde{T}_2 - \tilde{Y}_3^{\mathrm{T}} + \tilde{T}_3^{\mathrm{T}}, \tilde{\Xi}_{44} = -\tilde{Q}_3 - \tilde{T}_3 - \tilde{T}_3^{\mathrm{T}}$$

$$\tilde{\Xi}_{46} = -\tilde{P}_{22}^{\mathrm{T}} - \tilde{P}_{23}^{\mathrm{T}},\ \tilde{\Xi}_{47} = -\tilde{P}_{32}^{\mathrm{T}} - \tilde{P}_{33}^{\mathrm{T}}$$

$$\tilde{\Xi}_{55} = \tilde{H} - X - X^{\mathrm{T}},\ h_\delta = h_2 - h_1$$

$$\tilde{H} = h_1^2\tilde{R}_1 + h_\delta\tilde{R}_2 + \frac{1}{4}h_2^4\tilde{Z}_1 + \frac{1}{4}(h_2^2 - h_1^2)^2\tilde{Z}_2$$

$$\tilde{\Gamma}_a = \begin{bmatrix} \tilde{\varepsilon}B^{\mathrm{T}} & 0 & 0 & 0 & \tilde{\varepsilon}B^{\mathrm{T}} & 0 & 0 & 0 & D_u^{\mathrm{T}} & 0 \end{bmatrix}^{\mathrm{T}} D_c$$

$$\tilde{\Gamma}_E = \begin{bmatrix} E_c X^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Upsilon}' = \begin{bmatrix} CX^{\mathrm{T}} + DY & 0 & C_d X^{\mathrm{T}} & 0 & 0 & 0 & 0 & D_\omega \end{bmatrix}^{\mathrm{T}}$$

$$\tilde{Y} = \begin{bmatrix} 0 & \tilde{Y}_1^{\mathrm{T}} & \tilde{Y}_2^{\mathrm{T}} & \tilde{Y}_3^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$\tilde{T} = \begin{bmatrix} 0 & \tilde{T}_1^{\mathrm{T}} & \tilde{T}_2^{\mathrm{T}} & \tilde{T}_3^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}.$$

In this case, an appropriate non-fragile state feedback controller can be chosen by $u(t) = K(t)x(t)$, $K = YX^{-\mathrm{T}}$.

Proof. Firstly, replacing A, C by $A + B(K + \Delta K)$, $C + D_u(K + \Delta K)$ in (8), (9) and separating the uncertain ΔK, we have

$$\Lambda_1 + \Gamma_a F_c(t)\Gamma_E + \Gamma_E^{\mathrm{T}} F_c^{\mathrm{T}}(t)\Gamma_a < 0 \qquad (24)$$

$$\Lambda_2 + \Gamma_a F_c(t)\Gamma_E + \Gamma_E^{\mathrm{T}} F_c^{\mathrm{T}}(t)\Gamma_a < 0 \qquad (25)$$

where

$$\Lambda_1 = \begin{bmatrix} \Xi' & \sqrt{h_\delta}Y & \Upsilon' \\ * & -\tilde{R}_2 & 0 \\ * & * & -I \end{bmatrix}, \Lambda_2 = \begin{bmatrix} \Xi' & \sqrt{h_\delta}T & \Upsilon' \\ * & -\tilde{R}_2 & 0 \\ * & * & -I \end{bmatrix}$$

$$\Xi' = \begin{bmatrix} \Xi'_{11} & \Xi_{12} & S_1 A_d & \Xi_{14} & \Xi'_{15} & \Xi_{16} & \Xi_{17} & S_1 B_\omega \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & P_{23}^{\mathrm{T}} & P_{33}^{\mathrm{T}} & 0 \\ * & * & \Xi_{33} & \Xi_{34} & A_d^{\mathrm{T}} S_2^{\mathrm{T}} & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & 0 & \Xi_{46} & \Xi_{47} & 0 \\ * & * & * & * & \Xi_{55} & P_{12} & P_{13} & S_2 B_\omega \\ * & * & * & * & * & -Z_1 & 0 & 0 \\ * & * & * & * & * & * & -Z_2 & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Xi'_{11} = P_{12} + P_{12}^{\mathrm{T}} + Q_1 - R_1 - h_2^2 Z_1 - h_\delta^2 Z_2 + S_1(A + BK) + (A + BK)^{\mathrm{T}} S_1^{\mathrm{T}}$$

$$\Xi'_{15} = P_{11} - S_1 + (A + BK)^{\mathrm{T}} S_2^{\mathrm{T}}$$

$$\Upsilon' = \begin{bmatrix} (C + D_u K) & 0 & C_d & 0 & 0 & 0 & 0 & D_\omega \end{bmatrix}^{\mathrm{T}}.$$

From Lemma 4, we have

$$\Lambda_1 + \varepsilon^{-1}\Gamma_a\Gamma_a^{\mathrm{T}} + \varepsilon\Gamma_E^{\mathrm{T}}\Gamma_E < 0 \qquad (26)$$

$$\Lambda_2 + \varepsilon^{-1}\Gamma_a\Gamma_a^{\mathrm{T}} + \varepsilon\Gamma_E^{\mathrm{T}}\Gamma_E < 0 \qquad (27)$$

where

$$\Gamma_a = \begin{bmatrix} (S_1 B)^{\mathrm{T}} & 0 & 0 & 0 & (S_2 B)^{\mathrm{T}} & 0 & 0 & 0 & 0 & D_u^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} D_c$$

$$\Gamma_E = \begin{bmatrix} E_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Applying the Schur complement to (26) and (27), we have

$$\begin{bmatrix} \Lambda_1 & \Gamma_a & \varepsilon\Gamma_E^{\mathrm{T}} \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0 \qquad (28)$$

$$\begin{bmatrix} \Lambda_2 & \Gamma_a & \varepsilon\Gamma_E^{\mathrm{T}} \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0. \qquad (29)$$

Now, in order to obtain an LMI based result, we set $S_1 = S_2 = X^{-1}$, where X is a nonsingular matrix. Then pre- and post-multiplying (28), (29) by $\mathrm{diag}\left\{\underbrace{X \cdots X}_{7}\ I\ I\ X\ \varepsilon^{-1}I\ \varepsilon^{-1}I\right\}$, and its transpose, defining $\tilde{P}_{3\times 3} = XP_{3\times 3}X^{\mathrm{T}}$, $\tilde{Q}_j = XQ_jX^{\mathrm{T}}$, $\tilde{Y}_j = XY_jX^{\mathrm{T}}$, $\tilde{T}_j = XT_jX^{\mathrm{T}}$, $j = 1, 2, 3$; $R_i = XR_iX^{\mathrm{T}}$, $Z_i = XZ_iX^{\mathrm{T}}$, $i = 1, 2$, $\tilde{\varepsilon} = \varepsilon^{-1}$, $Y = KX^{\mathrm{T}}$, we deduce the LMIs stated in Theorem 2. □

5 Numerical examples

In this section, two examples are given to demonstrate the improved H_∞ performance analysis criterion as well as non-fragile H_∞ control results obtained in this paper.

Example 1[14]. Consider time-delay system (1) with the following parameters

$$A=\begin{bmatrix}-0.6238 & -1.0132\\ 2.0116 & -0.2106\end{bmatrix}, D=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$A_d=\begin{bmatrix}-0.5011 & -0.7871\\ -0.3002 & 0.5231\end{bmatrix}, C=\begin{bmatrix}0.2134 & -0.0191\\ 0.1119 & -0.1665\end{bmatrix}$$
$$B_\omega=\begin{bmatrix}-0.4326 & 0.1253\\ -1.6656 & 0.2877\end{bmatrix}, C_d=\begin{bmatrix}0.0816 & 0.1290\\ 0.0712 & 0.0669\end{bmatrix}. \tag{30}$$

In this example, we will provide two performance indexes, that is the H_∞ performance index γ and the upper bound h_2. For comparing our results with those in [11,12,14,19], we assume $\mu=0$ and $h_1=0$. Table 1 gives the maximum allowable delay bound h_2 for a prescribed γ. Table 2 gives the minimum allowed γ for a given h_2 .

Table 1 The maximum allowable delay bound h_2 for a given γ

γ	2.0	2.5	3.0	3.5	4.0
[11]	0.4057	0.4660	0.5047	0.5316	0.5515
[12]	0.4057	0.4660	0.5046	0.5316	0.5515
[14]	0.4203	0.4779	0.5146	0.5401	0.5589
[19]	0.4734	0.5237	0.5545	0.5754	0.5904
Theorem 1	0.6620	0.7040	0.7300	0.7470	0.7595

Table 2 The minimum allowable γ for a delay bound h_2

h_2	0.1	0.2	0.3	0.4	0.5
[11]	1.0714	1.2426	1.5067	1.9634	2.2981
[12]	1.0714	1.2425	1.5067	1.9634	2.2981
[14]	1.0577	1.2112	1.4515	1.8733	2.7757
Theorem 1	0.9331	0.9525	1.0216	1.1204	1.2843

Through the comparison in Tables 1 and 2, it can be found that for a prescribed performance index γ, Theorem 1 can allow larger delay bound; on the other hand, for a prescribed upper bound h_2 of the delay, we can obtain smaller γ. Hence the stability criterion in Theorem 1 is less conservative than those in [11, 12, 14, 19].

Example 2[29]. Let us look into a practical example of a linearized model of aircraft control system, vertical take-off and landing (VTOL) control problem of helicopters described by

$$\begin{cases}\dot{x}(t)=Ax(t)+A_dx(t-h(t))+Bu(t)+B_\omega\omega(t)\\ z(t)=Cx(t)+Du(t)+D_\omega\omega(t)\\ x(t)=\varphi(t),\forall t\in[-h_2,0]\end{cases} \tag{31}$$

where

$$A=\begin{bmatrix}-0.0366 & 0.0271 & 0.0188 & -0.4555\\ 0.0482 & -1.0100 & 0.0024 & -4.0208\\ 0.1002 & 0.3681 & -0.7070 & 1.4200\\ 0 & 0 & 1.0000 & 0\end{bmatrix}$$

$$A_d=0.3A,\ B=\begin{bmatrix}0.4422 & 0.1761\\ 3.5446 & -7.5922\\ -5.5200 & 4.4900\\ 0 & 0\end{bmatrix}, B_\omega=\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$
$$C=\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}, D=\begin{bmatrix}0.1 & 0.1\end{bmatrix}\ D_\omega=0.$$

Note that the unforced system is unstable for the delay-free case, the state $x(t)$ of the open-loop system is shown in Fig. 1.

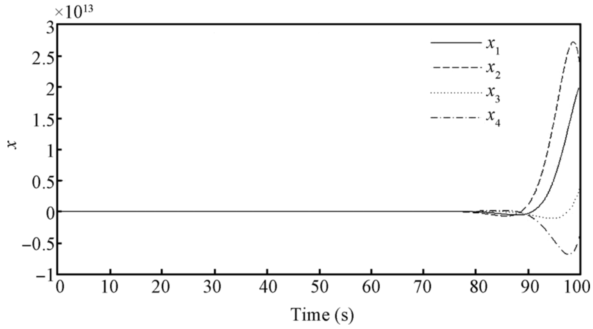

Fig. 1 State response of the open-loop system

To show the effectiveness and robustness of non-fragile H_∞ control results obtained in this paper, suppose we know that the time-varying delay $h(t)$ satisfies $0\le h(t)\le 5$.

Case 1. When $\Delta K=0$, then the non-fragile controller became the normal robust H_∞ controller. In this case, for given $\mu=0.3$ and H_∞ performance index $\gamma=0.6716$, the corresponding H_∞ state feedback controller is computed as

$$K_1=\begin{bmatrix}-0.0690 & 0.3498 & 2.2545 & 2.5134\\ -0.0223 & 0.4086 & 1.5923 & 0.2799\end{bmatrix}.$$

Case 2. When $\Delta K\neq 0$, and the uncertain parameter is satisfying (6) with

$$D_c=\begin{bmatrix}0.2 & 0\\ 0 & 0.1\end{bmatrix}, E_c=\begin{bmatrix}0.1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.2\end{bmatrix}.$$

For the same value of μ and γ, the corresponding non-fragile H_∞ controller is computed as

$$K_2=\begin{bmatrix}-0.2282 & 0.3588 & 1.9494 & 2.0676\\ -0.0949 & 0.4434 & 1.3840 & 0.0649\end{bmatrix}.$$

The state trajectories of the system under the non-fragile controller are shown in Fig. 2. Clearly, system (30) with the non-fragile state-feedback controller (5) is asymptotically stable.

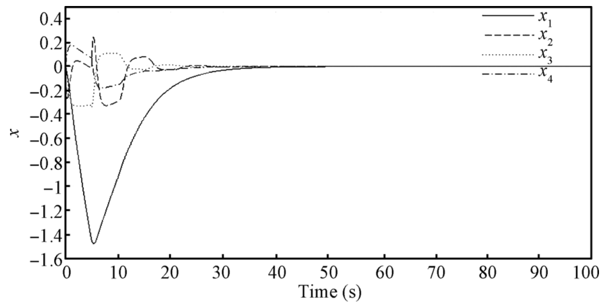

Fig. 2 State response of the close-loop system

Next, for comparing the robust controller with the non-fragile controller, we take the state $x_1(t)$ as research object. Under the same condition, the following Fig. 3 is showing the state trajectories of $x_1(t)$ under the controller K_1 and K_2 .

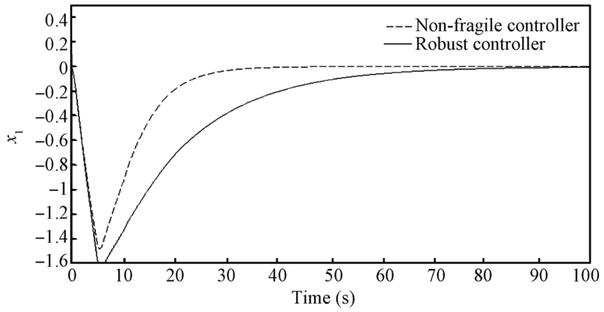

Fig. 3 The state $x_1(t)$ of the close-loop system

From the above figure, we can see that when there exist some perturbations in the feedback controller gain, the non-fragile controller can tolerate some level of controller gain variations and has better performance than the normal robust H_∞ controller.

6 Conclusions

In this paper, we have studied the problems of non-fragile H_∞ control for a class of linear systems with interval time-varying delay. The key features of the approach include an appropriate LKF with triple-integral terms, augment terms and a tighter integral inequality for bounding the cross-terms without neglecting any useful terms, thus more information on the time delay can be employed, and hence it yields less conservative delay-range bounds. Numerical examples have illustrated the effectiveness of the proposed method.

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Jun-Jun Hui received his B. Sc. and M. Sc. degrees in control science and engineering from Xi′an Research Institute of High Technology, China in 2001 and 2005, respectively. He is currently pursuing his Ph. D. degree in control science and engineering at the 301 Section, Xi′an Research Institute of High Technology, China.

His research interests include time-delay systems and networked control systems.

E-mail: ep22stone@163.com (Corresponding author)

ORCID iD: 0000-0002-6220-8857

He-Xin Zhang received his M. Sc. degree in control science and engineering from Northwestern Polytechnical University, China in 1994, and his Ph. D. degree in control science and engineering from Xi′an Research Institute of High Technology, China in 2001. Now he works as a professor in the 301 Section, Xi′an Research Institute of High Technology, China.

His research interests include nonlinear system identification and control, measurement and control, and adaptive control.

E-mail: epzhx301@163.com

Xiang-Yu Kong received his B. Sc. degree in control science and engineering from the Beijing Institute of Technology, China in 1990, his M. Sc. degree in control science and engineering from Xi′an Research Institute of High Technology, China in 2000, and Ph. D. degree in control science and engineering from Xi′an Jiaotong University in 2005. Now he works as a professor in the Xi′an Research Institute of High Technology, China.

His research interests include adaptive signal processing, nonlinear system modeling and its applications, and fault diagnosis.

E-mail: xiangyukong01@163.com