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Output Feedback Stabilization of Switched Stochastic Nonlinear Systems Under Arbitrary Switchings

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Abstract: This paper is concerned with the problem of global output feedback stabilization in probability for a class of switched stochastic nonlinear systems under arbitrary switchings. The subsystems are assumed to be in output feedback form and driven by white noise. By introducing a common Lyapunov function, the common output feedback controller independent of switching signals is constructed based on the backstepping approach. It is proved that the zero solution of the closed-loop system is fourth-moment exponentially stable. An example is given to show the effectiveness of the proposed method.

Keywords: Stochastic nonlinear system, switched nonlinear system, output feedback, under arbitrary switchings, moment exponential stability.

1 Introduction

Switched control systems have drawn considerable attention for the past decades since they play an essential role in numerous applications such as mechanical systems, air traffic control systems, etc.[1−3]. Recently some attempts on switched systems have been investigated theoretically[4−6]. In the existing literatures, the study of switched nonlinear systems mainly focuses on stability and stabilization under arbitrary switchings[7−9] or under some designed switching law[10, 11]. The stability for arbitrary switching sequences is, in fact, a kind of robustness. On the other hand, stochastic nonlinear systems have been widely studied in theory and applied in practice. In the constructive methods for stabilization of stochastic nonlinear systems, the usage of Lyapunov functions can be mainly divided into two types: One is quadratic Lyapunov functions[12−14], and the other is the quartic Lyapunov function which can sometimes result in a more simple design algorithm than the former one[15, 16]. As an intersecting issue of the above two aspects, switched stochastic nonlinear systems have become an emerging hot research topic for the past decade. These researches are not only interesting and challenging, but also theoretically and practically significant[17−20].

As is known to all, the output feedback control is one of the most important problems of nonlinear systems since only the plant output can be measured in many cases. Generally speaking, the separation principle does not work for nonlinear systems. This makes the design of output feedback control complicated and difficult. Thus, it motivated many scholars to work on this interesting topic. Nale et al.[21] investigated the stability of switched systems with output feedback control methodology. Deng and Krstic[22] concentrated on the output feedback stabilization of stochastic nonlinear systems. Wu[23] further considered the backstepping control problem of stochastic nonlinear systems with Markovian switching, where the subsystems were assumed to be in the output feedback form. The nonlinear systems in the output feedback form are a kind of significant nonlinear systems. The feedback controllers of such a kind of nonlinear systems can be constructed systematically by the backstepping approach and its variations[24−28].

In this paper, we are interested in stability analysis and control synthesis of switched stochastic nonlinear systems under arbitrary switchings. Compared to full state feedback control problem, the design of the output feedback turns out to be much more difficult. By adopting the backstepping design used in [22], we extended the output feedback design problem of the stochastic systems in the strict-feedback form to a more general class of the switched stochastic nonlinear systems. In the switched systems, multiple Lyapunov functions are often employed to satisfy different performance requirements for individual subsystems. And a set of switching laws are then designed to guarantee stability of the overall switched closed-loop system. Although the existence of multiple Lyapunov functions is less conservative, a common Lyapunov function is necessary for asymptotic stability under arbitrary switching. It is uniform over the set of all switching signals for all the subsystems. In addition, the controller can not be dependent on switching signals because the nonlinear systems we studied here are under arbitrary switchings, and the switching signals are unknown a priori. So a common output feedback controller independent of switching signals is constructed in the paper such that the equilibrium at the origin of the closed-loop system is stable.

2 Problem formulation

Consider the following class of switched stochastic nonlinear systems

$$\begin{aligned}
\mathrm{d}x_i &= \left[x_{i+1} + f_{\sigma(t),i}\left(y\right)\right]\mathrm{d}t + g^{\mathrm{T}}_{\sigma(t),i}\left(y\right)\mathrm{d}\omega,\\
&\quad i = 1, 2, \cdots, n-1\\
\mathrm{d}x_n &= \left[u + f_{\sigma(t),n}\left(y\right)\right]\mathrm{d}t + g^{\mathrm{T}}_{\sigma(t),n}\left(y\right)\mathrm{d}\omega, \quad y = x_1 \qquad (1)
\end{aligned}$$

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where x_i, u and y are the state variable, the input and the outputs of the system, respectively. $\sigma(t) : [0, +\infty) \longrightarrow M = \{1, 2, \cdots, m\}$ is a piecewise constant switching signal, $\forall j \in M$, $\forall i \in \{1, 2, \cdots, n\}$, $f_{j,i}(\cdot)$ are C^1 functions with $f_{j,i}(0) = 0$, $g_{j,i}(\cdot)$ are r-vector-valued smooth functions with $g_{j,i}(0) = 0$, and ω is an independent r-dimensional standard Wiener process.

Remark 1. The switched stochastic nonlinear system (1) has the typical structure called strict-feedback form, which has been widely studied recently, such as [29] for the Markovian switching case, [27] for the non-stochastic and non-switched case, etc. Compared to [22], the class of systems we studied is a more general nonlinear system even in the non-switched case.

The goal of this paper is to design a controller for system (1) by backstepping approach such that the zero solution of the closed-loop system is globally asymptotically stable in probability under arbitrary switchings. Further, it will be proved that the zero equilibrium of the closed-loop system is of pth-moment globally exponential stability, i.e., there exist $k > 0$, $c > 0$, $p > 0$ such that $\mathrm{E}\left[\| x(t) \|_p^p\right] \leqslant k \| x(0) \|_p^p \, \mathrm{e}^{-ct}$, $\forall \| x(0) \| \in \mathbf{R}^n$.

3 Main results

3.1 Controller design

Since states $x_2, \cdots, x_n$ are not measured, the observer is designed as

$$\begin{cases} \dot{\hat{x}}_i = \hat{x}_{i+1} + k_i (y - \hat{x}_1), \quad i = 1, 2, \cdots, n-1 \\ \dot{\hat{x}}_n = u + k_n (y - \hat{x}_1) \end{cases} \tag{2}$$

where $k_i > 0$ are the design parameters. Define the estimate error as

$$\tilde{x} = x - \hat{x} \tag{3}$$

where $\tilde{x} = [\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n]^{\mathrm{T}}$, $x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}}$ and $\hat{x} = [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_n]^{\mathrm{T}}$. Then one has

$$\mathrm{d}\tilde{x} = \left[A\tilde{x} + F_{\sigma(t)}(y)\right] \mathrm{d}t + G_{\sigma(t)}(y)\, \mathrm{d}\omega \tag{4}$$

where

$$A = \begin{bmatrix} -k_1 & & & \\ -k_2 & & & \\ \vdots & & I_{n-1} & \\ -k_n & 0 & \cdots & 0 \end{bmatrix}$$

$$G_{\sigma(t)}(y) = \begin{bmatrix} g^{\mathrm{T}}_{\sigma(t),1}(y) \\ \vdots \\ g^{\mathrm{T}}_{\sigma(t),n}(y) \end{bmatrix}$$

and

$$F_{\sigma(t)}(y) = \left[f_{\sigma(t),1}(y), f_{\sigma(t),2}(y), \cdots, f_{\sigma(t),n}(y)\right]^{\mathrm{T}}.$$

Here, the design parameters $k_i > 0$ are chosen such that A is a Hurwitz matrix. Now, the entire system can be described by

$$\begin{cases} \mathrm{d}\tilde{x} = \left[A\tilde{x} + F_{\sigma(t)}(y)\right] \mathrm{d}t + G_{\sigma(t)}(y)\, \mathrm{d}\omega \\ \mathrm{d}y = \left[\hat{x}_2 + \tilde{x}_2 + f_{\sigma(t),1}(y)\right] \mathrm{d}t + g^{\mathrm{T}}_{\sigma(t),1}(y)\, \mathrm{d}\omega \\ \mathrm{d}\hat{x}_i = \left[\hat{x}_{i+1} + k_i (y - \hat{x}_1)\right] \mathrm{d}t, \quad i = 2, \cdots, n-1 \\ \mathrm{d}\hat{x}_n = \left[u + k_n (y - \hat{x}_1)\right] \mathrm{d}t. \end{cases} \tag{5}$$

Define the following coordinate transformation

$$\begin{cases} z_1 = y \\ z_2 = \hat{x}_2 - \alpha_1(y) \\ z_i = \hat{x}_i - \alpha_{i-1}\left(\bar{\hat{x}}_{i-1}, y\right), \quad i = 3, \cdots, n \end{cases} \tag{6}$$

where $\alpha_{i-1}(\cdot)$ are the virtual stabilizing functions to be determined later and $\bar{\hat{x}}_i = [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_i]^{\mathrm{T}}$. Then according to Itô's differentiation rule, system (5) can be transformed into the following equalities

$$\begin{cases} \mathrm{d}\tilde{x} = \left[A\tilde{x} + F_{\sigma(t)}(y)\right] \mathrm{d}t + G_{\sigma(t)}(y)\, \mathrm{d}\omega \\ \mathrm{d}y = \left[\hat{x}_2 + \tilde{x}_2 + f_{\sigma(t),1}(y)\right] \mathrm{d}t + g^{\mathrm{T}}_{\sigma(t),1}(y)\, \mathrm{d}\omega \\ \mathrm{d}z_2 = \left\{\hat{x}_3 + k_2 (y - \hat{x}_1) - \dfrac{\partial \alpha_1}{\partial y}\left[\hat{x}_2 + \tilde{x}_2 + f_{\sigma(t),1}(y)\right] - \right. \\ \quad \left. \dfrac{\partial^2 \alpha_1}{2\partial y^2} g^{\mathrm{T}}_{\sigma(t),1} g_{\sigma(t),1}\right\} \mathrm{d}t - \dfrac{\partial \alpha_1}{\partial y} g^{\mathrm{T}}_{\sigma(t),1} \mathrm{d}\omega \\ \mathrm{d}z_i = \left\{\hat{x}_{i+1} - \displaystyle\sum_{l=2}^{i-1} \dfrac{\partial \alpha_{i-1}}{\partial \hat{x}_l}\left[\hat{x}_{l+1} + k_l (y - \hat{x}_1)\right] + \right. \\ \quad k_i (y - \hat{x}_1) - \dfrac{\partial^2 \alpha_{i-1}}{2\partial y^2} g^{\mathrm{T}}_{\sigma(t),1} g_{\sigma(t),1} - \\ \quad \left. \dfrac{\partial \alpha_{i-1}}{\partial y}\left[\hat{x}_2 + \tilde{x}_2 + f_{\sigma(t),1}(y)\right]\right\} \mathrm{d}t - \\ \quad \dfrac{\partial \alpha_{i-1}}{\partial y} g^{\mathrm{T}}_{\sigma(t),1} \mathrm{d}\omega \\ \mathrm{d}z_n = \left\{u - \displaystyle\sum_{l=2}^{n-1} \dfrac{\partial \alpha_{n-1}}{\partial \hat{x}_l}\left[\hat{x}_{l+1} + k_l (y - \hat{x}_1)\right] + \right. \\ \quad k_n (y - \hat{x}_1) - \dfrac{\partial^2 \alpha_{n-1}}{2\partial y^2} g^{\mathrm{T}}_{\sigma(t),1} g_{\sigma(t),1} - \\ \quad \left. \dfrac{\partial \alpha_{n-1}}{\partial y}\left[\hat{x}_2 + \tilde{x}_2 + f_{\sigma(t),1}(y)\right]\right\} \mathrm{d}t - \\ \quad \dfrac{\partial \alpha_{n-1}}{\partial y} g^{\mathrm{T}}_{\sigma(t),1} \mathrm{d}\omega. \end{cases} \tag{7}$$

Naturally, the goal is changed to design a controller for system (7) by backstepping approach such that the equilibrium at the origin of the closed-loop system is globally asymptotically stable in probability under arbitrary switchings. Similar to [22], instead of constructing the stabilizing functions α_i and u in a step-by-step fashion, we derive them simultaneously. In this paper, the stabilizing functions α_i and u are constructed as

$$\begin{cases} \alpha_1(y) = z_1 s_1(y) \\ \alpha_i\left(\bar{\hat{x}}_i, y\right) = z_i s_i\left(\bar{\hat{x}}_i, y\right), \quad i = 2, \cdots, n-1 \\ u = z_n s_n\left(\bar{\hat{x}}_n, y\right) \end{cases} \tag{8}$$

where $s_i(\cdot)$ to be determined later are all smooth functions defined on $\mathbf{R}^i$.

3.2 Stability analysis

Theorem 1. For system(8), there exists a control law taking the form of (7) such that the equilibrium at the origin of the closed-loop system is fourth-moment exponentially stable.

Proof. Since $\forall j \in M$, $f_{j,i}(y)$ and $g_{j,i}(y)$ are smooth functions with $f_{j,i}(0) = 0$ and $g_{j,i}(0) = 0$, by the mean value theorem, $f_{j,i}(y)$ and $g_{j,i}(y)$ can be expressed as

$$f_{j,i}(y)=y\varphi_{j,i}(y) \tag{9}$$

$$g_{j,i}(y)=y\psi_{j,i}(y) \tag{10}$$

where $\varphi_{j,i}(y)$ and $\psi_{j,i}(y)$ are both smooth functions.

Because of the term $\frac{1}{2}\mathrm{tr}\{g^{\mathrm{T}}(\frac{\partial^2 V}{\partial x^2})g\}$ caused by stochastic case, we employ quartic Lyapunov functions. Define a Lyapunov function of the form

$$V=\frac{1}{4}\sum_{i=1}^{n}z_i^4+\frac{1}{2}\left(\tilde{x}^{\mathrm{T}}P\tilde{x}\right)^2 \tag{11}$$

where P is a positive defined matrix which satisfies $A^{\mathrm{T}}P+PA=-I$. Then according to the Itô's differentiation rule, we have

$$\begin{aligned}\mathrm{d}V=&\mathcal{L}V\mathrm{d}t+\Bigg(z_1^3g_{\sigma(t),1}^{\mathrm{T}}-\sum_{i=2}^{n}z_i^3\frac{\partial\alpha_{i-1}}{\partial y}g_{\sigma(t),1}^{\mathrm{T}}+\\&2\tilde{x}^{\mathrm{T}}P\tilde{x}\tilde{x}^{\mathrm{T}}PG_{\sigma(t)}(y)\Bigg)\mathrm{d}\omega\end{aligned} \tag{12}$$

where

$$\begin{aligned}\mathcal{L}V=&y^3\left[\alpha_1+z_2+\tilde{x}_2+y\varphi_{\sigma(t),1}(y)\right]+\frac{3}{2}y^2g_{\sigma(t),1}^{\mathrm{T}}g_{\sigma(t),1}+\\&\tilde{x}^{\mathrm{T}}P\tilde{x}\left(2\tilde{x}^{\mathrm{T}}PF_{\sigma(t)}(y)-\|\tilde{x}\|^2\right)+z_2^3\Bigg\{\alpha_2+z_3+k_2\tilde{x}_1-\\&\frac{\partial\alpha_1}{\partial y}\left[\hat{x}_2+\tilde{x}_2+y\varphi_{\sigma(t),1}(y)\right]-\frac{\partial^2\alpha_1}{2\partial y^2}g_{\sigma(t),1}^{\mathrm{T}}g_{\sigma(t),1}\Bigg\}+\\&\sum_{i=3}^{n-1}z_i^3\Bigg\{\alpha_i+z_{i+1}+k_i\tilde{x}_1-\sum_{l=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_l}\left(\hat{x}_{l+1}+k_l\tilde{x}_1\right)-\\&\frac{\partial\alpha_{i-1}}{\partial y}\left[\hat{x}_2+\tilde{x}_2+y\varphi_{\sigma(t),1}(y)\right]-\\&\frac{\partial^2\alpha_{i-1}}{2\partial y^2}g_{\sigma(t),1}^{\mathrm{T}}g_{\sigma(t),1}\Bigg\}+\\&z_n^3\Bigg\{u+k_n\tilde{x}_1-\sum_{l=2}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_l}\left(\hat{x}_{l+1}+k_l\tilde{x}_1\right)-\\&\frac{\partial\alpha_{n-1}}{\partial y}\left[\hat{x}_2+\tilde{x}_2+y\varphi_{\sigma(t),1}(y)\right]-\\&\frac{\partial^2\alpha_{n-1}}{2\partial y^2}g_{\sigma(t),1}^{\mathrm{T}}g_{\sigma(t),1}\Bigg\}+\\&\frac{3}{2}\sum_{i=2}^{n}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2g_{\sigma(t),1}^{\mathrm{T}}g_{\sigma(t),1}-\tilde{x}^{\mathrm{T}}P\tilde{x}\tilde{x}^{\mathrm{T}}\tilde{x}+\\&\mathrm{Tr}\left[G_{\sigma(t)}^{\mathrm{T}}(y)\left(2P\tilde{x}\tilde{x}^{\mathrm{T}}P+\tilde{x}^{\mathrm{T}}P\tilde{x}P\right)G_{\sigma(t)}(y)\right].\end{aligned}$$

Next, the terms $\sum_{i=2}^{n-1}z_i^3z_{i+1}, y^3\tilde{x}_2, -z_i^3\sum_{l=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_l}k_l\tilde{x}_1$, y^3z_2, $2\tilde{x}^{\mathrm{T}}P\tilde{x}\tilde{x}^{\mathrm{T}}PF_{\sigma(t)}(y)$, $-\frac{1}{2}\sum_{i=2}^{n}z_i^3\frac{\partial^2\alpha_{i-1}}{\partial y^2}g_{\sigma(t),1}^{\mathrm{T}}g_{\sigma(t),1}$, $-z_i^3\frac{\partial\alpha_{i-1}}{\partial y}\hat{x}_2$, $-z_i^3\frac{\partial\alpha_{i-1}}{\partial y}y\varphi_{\sigma(t),1}(y)$, $-\sum_{i=2}^{n}z_i^3\frac{\partial\alpha_{i-1}}{\partial y}\tilde{x}_2$, $-z_i^3\sum_{l=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_l}\hat{x}_{l+1}$, $\frac{3}{2}\sum_{i=2}^{n}z_i^2\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2g_{\sigma(t),1}^{\mathrm{T}}g_{\sigma(t),1}$, $\mathrm{tr}\left[G_{\sigma(t)}^{\mathrm{T}}(y)\left(2P\tilde{x}\tilde{x}^{\mathrm{T}}P+\tilde{x}^{\mathrm{T}}P\tilde{x}P\right)G_{\sigma(t)}(y)\right]$ and $z_i^3k_i\tilde{x}_1$ are further handled one by one such that their effects on the negativity of $\mathcal{L}V$ can be canceled by s_i selected properly. To handle these terms, a special case of the well-known Young's inequality[25] plays a very important role, which states that the inequality

$$xy\leqslant\frac{\epsilon^p}{p}|x|^p+\frac{1}{\epsilon^q q}|y|^q,\ \forall(x,y)\in\mathbf{R}^2 \tag{13}$$

holds for any constants $p>1$ and $q>1$ satisfying $(p-1)(q-1)=1$.

By simple calculation and using Young's inequality, one has

$$\sum_{i=2}^{n-1}z_i^3z_{i+1}\leqslant\frac{3}{4}\sum_{i=2}^{n-1}\delta_i^{\frac{4}{3}}z_i^4+\frac{1}{4}\sum_{i=3}^{n}\frac{1}{\delta_{i-1}^4}z_i^4 \tag{14}$$

$$y^3\tilde{x}_2\leqslant\frac{3}{4}\epsilon_1^{\frac{4}{3}}y^4+\frac{1}{4\epsilon_1^4}\tilde{x}_2^4\leqslant\frac{3}{4}\epsilon_1^{\frac{4}{3}}y^4+\frac{1}{4\epsilon_1^4}\|\tilde{x}\|^4 \tag{15}$$

$$y^3z_2\leqslant\frac{3}{4}\delta_1^{\frac{4}{3}}y^4+\frac{1}{4\delta_1^4}z_2^4 \tag{16}$$

$$\begin{aligned}&-z_i^3\sum_{l=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_l}k_l\tilde{x}_1\leqslant\\&\frac{3}{4}\left(\kappa_i\sum_{l=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_l}k_l\right)^{\frac{4}{3}}z_i^4+\frac{1}{4\kappa_i^4}\tilde{x}_1^4\leqslant\\&\frac{3}{4}\left(\kappa_i\sum_{l=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_l}k_l\right)^{\frac{4}{3}}z_i^4+\frac{1}{4\kappa_i^4}\|\tilde{x}\|^4\end{aligned} \tag{17}$$

$$\begin{aligned}&\mathrm{tr}\left[G_{\sigma(t)}^{\mathrm{T}}(y)\left(2P\tilde{x}\tilde{x}^{\mathrm{T}}P+\tilde{x}^{\mathrm{T}}P\tilde{x}P\right)G_{\sigma(t)}(y)\right]\leqslant\\&n\left\|G_{\sigma(t)}^{\mathrm{T}}(y)\left(2P\tilde{x}\tilde{x}^{\mathrm{T}}P+\tilde{x}^{\mathrm{T}}P\tilde{x}P\right)G_{\sigma(t)}(y)\right\|_\infty\leqslant\\&n\sqrt{n}\left\|G_{\sigma(t)}^{\mathrm{T}}(y)\left(2P\tilde{x}\tilde{x}^{\mathrm{T}}P+\tilde{x}^{\mathrm{T}}P\tilde{x}P\right)G_{\sigma(t)}(y)\right\|\leqslant\\&3n\sqrt{n}y^2\left\|\psi_{\sigma(t)}(y)\right\|^2\|P\|^2\|\tilde{x}\|^2\leqslant\\&\frac{3n\sqrt{n}}{2\epsilon_2^2}y^4\left\|\psi_{\sigma(t)}(y)\right\|^4+\frac{3n\sqrt{n}\epsilon_2^2\|P\|^4\|\tilde{x}\|^4}{2}\leqslant\\&\frac{3n\sqrt{n}}{2\epsilon_2^2}y^4\sum_{i=1}^{n}\sum_{j=1}^{m}\|\psi_{j,i}(y)\|^4+\frac{3n\sqrt{n}\epsilon_2^2\|P\|^4\|\tilde{x}\|^4}{2}\end{aligned} \tag{18}$$

$$\begin{aligned}&-\frac{1}{2}\sum_{i=2}^{n}z_i^3\frac{\partial^2\alpha_{i-1}}{\partial y^2}g_{\sigma(t),1}^{\mathrm{T}}g_{\sigma(t),1}=\\&-\frac{1}{2}\sum_{i=2}^{n}z_i^3\frac{\partial^2\alpha_{i-1}}{\partial y^2}\psi_{\sigma(t),1}^{\mathrm{T}}\psi_{\sigma(t),1}y^2\leqslant\\&\frac{3}{8}\sum_{i=2}^{n}\left(\tau_i\frac{\partial^2\alpha_{i-1}}{\partial y^2}\sum_{j=1}^{m}\psi_{j,1}^{\mathrm{T}}\psi_{j,1}\right)^{\frac{4}{3}}z_i^4+\sum_{i=2}^{n}\frac{1}{8\tau_i^4}y^8\end{aligned} \tag{19}$$

$$\begin{aligned}&-z_i^3\frac{\partial\alpha_{i-1}}{\partial y}\hat{x}_2=\\&-z_i^3\frac{\partial\alpha_{i-1}}{\partial y}z_2-z_i^3\frac{\partial\alpha_{i-1}}{\partial y}ys_1\leqslant\\&\frac{3}{4}z_i^4\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^{\frac{4}{3}}+\frac{1}{4}z_2^4+\frac{1}{4}y^4+\frac{3}{4}z_i^4\left(s_1\frac{\partial\alpha_{i-1}}{\partial y}\right)^{\frac{4}{3}}\end{aligned} \tag{20}$$

$$-\sum_{i=2}^{n} z_i^3 \frac{\partial \alpha_{i-1}}{\partial y} \tilde{x}_2 \leqslant$$
$$\frac{3}{4}\sum_{i=2}^{n} \eta_i^{\frac{4}{3}} \left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^{\frac{4}{3}} z_i^4 + \frac{1}{4}\sum_{i=2}^{n} \frac{1}{\eta_i^4}\tilde{x}_2^4 \leqslant$$
$$\frac{3}{4}\sum_{i=2}^{n} \eta_i^{\frac{4}{3}} \left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^{\frac{4}{3}} z_i^4 + \frac{1}{4}\sum_{i=2}^{n} \frac{1}{\eta_i^4}\|\tilde{x}\|^4 \tag{21}$$

$$-z_i^3 \frac{\partial \alpha_{i-1}}{\partial y} y \varphi_{\sigma(t),1}(y) \leqslant$$
$$-\frac{3}{4}\left(\frac{\partial \alpha_{i-1}}{\partial y}\varphi_{\sigma(t),1}\right)^{\frac{4}{3}} z_i^4 - \frac{1}{4}y^4 \leqslant$$
$$-\frac{3}{4}\left(\frac{\partial \alpha_{i-1}}{\partial y}\sum_{j=1}^{m}\varphi_{j,1}\right)^{\frac{4}{3}} z_i^4 - \frac{1}{4}y^4 \tag{22}$$

$$\frac{3}{2}\sum_{i=2}^{n} z_i^2 \left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 g_{\sigma(t),1}^{\mathrm{T}} g_{\sigma(t),1} \leqslant$$
$$\frac{3}{4}\sum_{i=2}^{n} \frac{1}{\xi_i^2}\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^4 z_i^4 + \frac{3}{4}\sum_{i=2}^{n} \xi_i^2 \left(g_{\sigma(t),1}^{\mathrm{T}} g_{\sigma(t),1}\right)^2 \leqslant$$
$$\frac{3}{4}\sum_{i=2}^{n} \frac{1}{\xi_i^2}\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^4 z_i^4 + \frac{3}{4}\sum_{i=2}^{n}\sum_{j=1}^{m} \xi_i^2 \left(\psi_{j,1}^{\mathrm{T}} \psi_{j,1}\right)^2 y^4 \tag{23}$$

$$-z_i^3 \sum_{l=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_l}\hat{x}_{l+1} =$$
$$-z_i^3 \sum_{l=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_l}(z_{l+1} + z_l s_l) =$$
$$-z_i^4 \frac{\partial \alpha_{i-1}}{\partial \hat{x}_{i-1}} - z_i^3 \sum_{l=2}^{i-2} z_{l+1}\frac{\partial \alpha_{i-1}}{\partial \hat{x}_l} - z_i^3 \sum_{l=2}^{i-1} z_l \frac{\partial \alpha_{i-1}}{\partial \hat{x}_l} s_l \leqslant$$
$$-z_i^4 \frac{\partial \alpha_{i-1}}{\partial \hat{x}_{i-1}} + \frac{3}{4} z_i^4 \sum_{l=2}^{i-2}\left(\frac{\partial \alpha_{i-1}}{\partial \hat{x}_l}\right)^{\frac{4}{3}} + \frac{1}{2}\sum_{k=1}^{i-1} z_k^4 +$$
$$\frac{3}{4} z_i^4 \sum_{l=2}^{i-1}\left(\frac{\partial \alpha_{i-1}}{\partial \hat{x}_l}s_l\right)^{\frac{4}{3}} \tag{24}$$

$$2\tilde{x}^{\mathrm{T}} P \tilde{x} \tilde{x}^{\mathrm{T}} P F_{\sigma(t)}(y) \leqslant$$
$$\frac{3}{2}\|\tilde{x}\|^4 + \frac{1}{2}\lambda_{\max}^8(P) y^4 \sum_{i=1}^{n}\sum_{j=1}^{m}\varphi_{j,i}^4(y) \tag{25}$$

and

$$z_i^3 k_i \tilde{x}_1 \leqslant \frac{3}{4}\theta_i^{\frac{4}{3}} k_i^{\frac{4}{3}} z_i^4 + \frac{1}{4\theta_i^4}\tilde{x}_1^4 \leqslant \frac{3}{4}\theta_i^{\frac{4}{3}} k_i^{\frac{4}{3}} z_i^4 + \frac{1}{4\theta_i^4}\|\tilde{x}\|^4. \tag{26}$$

Based on the above analysis, we have

$$\mathcal{L}V \leqslant -\|\tilde{x}\|^4 p + y^4(s_1 + \mathscr{A}) + z_2^4(s_2 + \mathscr{B}) +$$
$$\sum_{i=3}^{n-1} z_i^4 (s_i + \mathscr{C}) + z_n^4(s_n + \mathscr{D}) \tag{27}$$

where $\forall j \in M$,

$$\mathscr{A} = \frac{-n^2 + 5n - 8}{8} + \frac{3}{2}\sum_{j=1}^{m}\psi_{j,1}(y)^{\mathrm{T}}\psi_{j,1}(y) + \frac{3}{4}\delta_1^{\frac{4}{3}} +$$
$$\sum_{i=2}^{n}\frac{1}{8\tau_i^4}y^4 + \frac{3}{4}\sum_{i=2}^{n}\sum_{j=1}^{m}\xi_i^2\left(\psi_{j,1}(y)^{\mathrm{T}}\psi_{j,1}(y)\right)^2 +$$
$$\frac{3n\sqrt{n}}{2\epsilon_2^2}\sum_{i=1}^{n}\sum_{j=1}^{m}\|\psi_{j,i}(y)\|^4 + \frac{3}{4}\epsilon_1^{\frac{4}{3}} +$$
$$\frac{1}{2\epsilon_3^4}\lambda_{\max}^8(P)\sum_{i=1}^{n}\sum_{j=1}^{m}\varphi_{j,i}^4(y) + \sum_{j=1}^{m}\varphi_{j,1}(y) \tag{28}$$

$$\mathscr{B} = \frac{3n}{4} - \frac{5}{4} + \frac{3}{4}\delta_2^{\frac{4}{3}} + \frac{1}{4\delta_1^4} + \frac{3}{4}\theta_2^{\frac{4}{3}}k_2^{\frac{4}{3}} + \frac{3}{4}\left(\frac{\partial \alpha_1}{\partial y}\right)^{\frac{4}{3}} +$$
$$\frac{3}{4}\left(s_1\frac{\partial \alpha_1}{\partial y}\right)^{\frac{4}{3}} + \frac{3}{8}\left(\tau_2\frac{\partial^2 \alpha_1}{\partial y^2}\sum_{j=1}^{m}\psi_{j,1}^{\mathrm{T}}\psi_{j,1}\right)^{\frac{4}{3}} +$$
$$\frac{3}{4}\eta_2^{\frac{4}{3}}\left(\frac{\partial \alpha_1}{\partial y}\right)^{\frac{4}{3}} + \frac{3}{4}\frac{1}{\xi_2^2}\left(\frac{\partial \alpha_1}{\partial y}\right)^4 -$$
$$\frac{3}{4}\left(\frac{\partial \alpha_1}{\partial y}\sum_{j=1}^{m}\varphi_{j,1}\right)^{\frac{4}{3}} \tag{29}$$

$$\mathscr{C} = \frac{n-i}{2} + \frac{3}{4}\sum_{l=2}^{i-1}\left(\frac{\partial \alpha_{i-1}}{\partial \hat{x}_l}s_l\right)^{\frac{4}{3}} + \frac{3}{4}\sum_{l=2}^{i-2}\left(\frac{\partial \alpha_{i-1}}{\partial \hat{x}_l}\right)^{\frac{4}{3}} -$$
$$\frac{\partial \alpha_{i-1}}{\partial \hat{x}_{i-1}} + \frac{3}{4}\delta_i^{\frac{4}{3}} + \frac{1}{4\delta_{i-1}^4} + \frac{3}{4}\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^{\frac{4}{3}} + \frac{3}{4}\theta_i^{\frac{4}{3}}k_i^{\frac{4}{3}} +$$
$$\frac{3}{4}\left(s_1\frac{\partial \alpha_{i-1}}{\partial y}\right)^{\frac{4}{3}} + \frac{3}{8}\left(\tau_i\frac{\partial^2 \alpha_{i-1}}{\partial y^2}\sum_{j=1}^{m}\psi_{j,1}^{\mathrm{T}}\psi_{j,1}\right)^{\frac{4}{3}} +$$
$$\frac{3}{4}\eta_i^{\frac{4}{3}}\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^{\frac{4}{3}} + \frac{3}{4}\left(\kappa_i\sum_{l=2}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \hat{x}_l}k_l\right)^{\frac{4}{3}} +$$
$$\frac{3}{4}\frac{1}{\xi_i^2}\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^4 - \frac{3}{4}\left(\frac{\partial \alpha_{i-1}}{\partial y}\sum_{j=1}^{m}\varphi_{j,1}\right)^{\frac{4}{3}},$$
$$i = 3, \cdots, n-1 \tag{30}$$

$$\mathscr{D} = \frac{3}{4}\sum_{l=2}^{n-1}\left(\frac{\partial \alpha_{n-1}}{\partial \hat{x}_l}s_l\right)^{\frac{4}{3}} + \frac{3}{4}\sum_{l=2}^{n-2}\left(\frac{\partial \alpha_{n-1}}{\partial \hat{x}_l}\right)^{\frac{4}{3}} -$$
$$\frac{\partial \alpha_{n-1}}{\partial \hat{x}_{n-1}} + \frac{1}{4\delta_{n-1}^4} + \frac{3}{4}\theta_n^{\frac{4}{3}}k_n^{\frac{4}{3}} + \frac{3}{4}\left(\frac{\partial \alpha_{n-1}}{\partial y}\right)^{\frac{4}{3}} +$$
$$\frac{3}{4}\left(s_1\frac{\partial \alpha_{n-1}}{\partial y}\right)^{\frac{4}{3}} + \frac{3}{8}\left(\tau_n\frac{\partial^2 \alpha_{n-1}}{\partial y^2}\sum_{j=1}^{m}\psi_{j,1}^{\mathrm{T}}\psi_{j,1}\right)^{\frac{4}{3}} +$$
$$\frac{3}{4}\eta_n^{\frac{4}{3}}\left(\frac{\partial \alpha_{n-1}}{\partial y}\right)^{\frac{4}{3}} + \frac{3}{4}\left(\kappa_n\sum_{l=2}^{n-1}\frac{\partial \alpha_{n-1}}{\partial \hat{x}_l}k_l\right)^{\frac{4}{3}} +$$
$$\frac{3}{4}\frac{1}{\xi_n^2}\left(\frac{\partial \alpha_{n-1}}{\partial y}\right)^4 - \frac{3}{4}\left(\frac{\partial \alpha_{n-1}}{\partial y}\sum_{j=1}^{m}\varphi_{j,1}\right)^{\frac{4}{3}} \tag{31}$$

and the following equality is used.

$$\frac{1}{2}\sum_{i=3}^{n}\sum_{k=1}^{i-1} z_k^4 = \frac{1}{2}\sum_{i=3}^{n-1} z_i^4 (n-i) + \frac{1}{2} z_1^4 (n-2) + \frac{1}{2} z_2^4 (n-2). \quad (32)$$

In (27), ϵ, δ, η, θ, κ and ξ are positive constants to be chosen. $\lambda > 0$ is the smallest eigenvalue of P. We will choose ϵ_1, ϵ_2, ϵ_3, θ_i, κ_i and η_i to satisfy

$$p = 2\lambda - \frac{3}{2}\epsilon_3^{\frac{4}{3}} - \frac{3n\sqrt{n}\epsilon_2^2 \|P\|^4}{2} - \frac{1}{4\epsilon_1^4} - \frac{1}{4}\sum_{i=2}^{n}\frac{1}{\eta_i^4} - \frac{1}{4}\sum_{i=2}^{n}\frac{1}{\theta_i^4} - \frac{1}{4}\sum_{i=3}^{n}\frac{1}{\kappa_i^4} > 0. \quad (33)$$

Obviously, it is a simple task to select s_i, $\forall i \in \{1, 2, \cdots, n\}$ such that

$$s_1 \leqslant -c_1 - \mathscr{A} \quad (34)$$

$$s_2 \leqslant -c_2 - \mathscr{B} \quad (35)$$

$$s_i \leqslant -c_i - \mathscr{C}, \quad i = 3, \cdots, n-1 \quad (36)$$

$$s_n \leqslant -c_n - \mathscr{D} \quad (37)$$

where $c_i > 0$. Then we get

$$\mathcal{L}V \leqslant -\sum_{i=1}^{n} c_i z_i^4 - p\|\tilde{x}\|^4 \leqslant -cV \quad (38)$$

where $c = \min\left\{4c_i, \frac{2p}{\lambda_{\max}^2}\right\}$, $i = 1, 2, \cdots, n$, $\lambda_{\max} > 0$ is the maximal eigenvalue of P.

Observe that

$$\begin{aligned} \mathrm{d}\left(\mathrm{e}^{ct}V\right) =& \mathrm{e}^{ct} cV \mathrm{d}t + \mathrm{e}^{ct}\mathrm{d}V = \\ & \mathrm{e}^{ct}\left(cV + \mathcal{L}V\right)\mathrm{d}t + \mathrm{e}^{ct}\Big(z_1^3 g_{\sigma(t),1}^{\mathrm{T}} - \\ & \sum_{i=1}^{n-1} z_i^3 \frac{\partial \alpha_{i-1}}{\partial y} g_{\sigma(t),1}^{\mathrm{T}} + \\ & 2\tilde{x}^{\mathrm{T}} P\tilde{x}\tilde{x}^{\mathrm{T}} P G_{\sigma(t)}(y)\Big)\mathrm{d}\omega \leqslant \\ & \mathrm{e}^{ct}\Bigg(z_1^3 g_{\sigma(t),1}^{\mathrm{T}} - \sum_{i=1}^{n-1} z_i^3 \frac{\partial \alpha_{i-1}}{\partial y} g_{\sigma(t),1}^{\mathrm{T}} + \\ & 2\tilde{x}^{\mathrm{T}} P\tilde{x}\tilde{x}^{\mathrm{T}} P G_{\sigma(t)}(y)\Bigg)\mathrm{d}\omega. \end{aligned} \quad (39)$$

For a given time t, suppose that switchings occur at the time instants $t_1, \cdots, t_K$, thus the time interval can be divided as $[0, t_1) \cup \cdots \cup [t_{K-1}, t_K) \cup [t_K, t]$. Integrating both sides of the above inequality on each subinterval and taking expectations yield

$$\mathrm{E}\left[\mathrm{e}^{ct}V(x(t))\right] - \mathrm{E}\left[\mathrm{e}^{ct_K}V(x(t_K))\right] \leqslant 0 \quad (40)$$

and

$$\mathrm{E}\left[\mathrm{e}^{ct_k^-}V\left(x\left(t_k^-\right)\right)\right] - \mathrm{E}\left[\mathrm{e}^{ct_{k-1}}V(x(t_{k-1}))\right] \leqslant 0 \quad (41)$$

where $k = 1, \cdots, K-1$ and $t_0 = 0$. Since the system state does not jump at the switching instants and V is a common Lyapunov function for all subsystems

$$\mathrm{E}\left[\mathrm{e}^{ct_k^-}V\left(x\left(t_k^-\right)\right)\right] = \mathrm{E}\left[\mathrm{e}^{ct_k}V(x(t_k))\right]. \quad (42)$$

Then it follows from (40) and (41) that

$$\mathrm{E}\left[V(x(t))\right] \leqslant \mathrm{E}\left[V(0)\right]\mathrm{e}^{-ct} = V(0)\,\mathrm{e}^{-ct}. \quad (43)$$

As we stated previously, the Lyapunov function we employ satisfies

$$\begin{aligned} V =& \frac{1}{4}\sum_{i=1}^{n} z_i^4 + \frac{1}{2}\left(\tilde{x}^{\mathrm{T}} P\tilde{x}\right)^2 \leqslant \\ & \frac{1}{4}\sum_{i=1}^{n} z_i^4 + \frac{\lambda_{\max}^2}{2}\|x\|^4 \leqslant \\ & \frac{1 + 2\lambda_{\max}^2}{4}\left(\sum_{i=1}^{n} z_i^4 + \|x\|^4\right) \leqslant \\ & \frac{n\left(1 + 2\lambda_{\max}^2\right)}{4}\|Y\|_4^4 \end{aligned} \quad (44)$$

where $Y = [z_1, z_2, \cdots, z_n, x_1, x_2, \cdots, x_n]^{\mathrm{T}}$. Hence,

$$\mathrm{E}\left[V(x(t))\right] \leqslant \frac{n\left(1 + 2\lambda_{\max}^2\right)}{4}\|Y(0)\|_4^4\,\mathrm{e}^{-ct}. \quad (45)$$

□

4 An illustrative example

In the following, an example shown in [30] is presented to illustrate the effectiveness of our result. The continuously stirred tank reactor with two-mode feeding stream is molded as the following system

$$\begin{cases} \dot{x}_1 = x_2 + f_{\sigma(t),1}(y) \\ \dot{x}_2 = u \end{cases} \quad (46)$$

with $f_{1,1}(x_1) = \frac{1}{2}x_1$ and $f_{2,1}(x_1) = 2x_1$. It is assumed that there exists white noise due to the term $f_{\sigma(t),1}$ in the above system. As a result, we have the following stochastic nonlinear system

$$\begin{cases} \mathrm{d}x_1 = \left[x_2 + f_{\sigma(t),1}(y)\right]\mathrm{d}t + f_{\sigma(t),1}(y)\,\mathrm{d}\omega \\ \mathrm{d}x_2 = u\mathrm{d}t \\ y = x_1. \end{cases} \quad (47)$$

For this system, the estimator is

$$\dot{\hat{x}}_1 = \hat{x}_2 + k_1(y - \hat{x}_1)$$

$$\dot{\hat{x}}_2 = u + k_2(y - \hat{x}_1).$$

It is easy to obtain that $\varphi_{1,1} = \frac{1}{2}$, $\varphi_{2,1} = 2$, $\psi_{1,1} = \frac{1}{2}$, $\psi_{2,1} = 2$ and $\varphi_{j,2} = \psi_{j,2} = 0$ for $j = 1, 2$. Based on (30), a smooth function s_1 should be chosen such that

$$\begin{aligned}s_1 =& -c_1 - \Bigg[\frac{1}{4} + \frac{3}{2}\sum_{j=1}^{2}\psi_{j,1}(y)^{\mathrm{T}}\psi_{j,1}(y) + \frac{3}{4}\delta_1^{\frac{4}{3}} + \\ & \frac{1}{8\tau_2^4}y^4 + \frac{3}{4}\sum_{j=1}^{2}\xi_2^2\left(\psi_{j,1}(y)^{\mathrm{T}}\psi_{j,1}(y)\right)^2 + \\ & \frac{3\sqrt{2}}{\epsilon_2^2}\sum_{i=1}^{2}\sum_{j=1}^{2}\|\psi_{j,i}(y)\|^4 + \frac{3}{4}\epsilon_1^{\frac{4}{3}} + \\ & \frac{1}{2\epsilon_3^4}\lambda_{\max}^8(P)\sum_{i=1}^{2}\sum_{j=1}^{2}\varphi_{j,i}^4(y) + \sum_{j=1}^{2}\varphi_{j,1}(y)\Bigg]. \end{aligned} \quad (46)$$

According to the first equation of (8), the virtual control is expressed as

$$\alpha_1 = s_1 y. \quad (47)$$

Then, based on (37), a smooth function s_2 should be chosen such that

$$\begin{aligned}s_2 =& -c_2 - \Bigg[\frac{1}{4\delta_1^4} + \frac{3}{4}\theta_2^{\frac{4}{3}}k_2^{\frac{4}{3}} + \frac{3}{4}\left(\frac{\partial\alpha_1}{\partial y}\right)^{\frac{4}{3}} + \\ & \frac{3}{4}\left(s_1\frac{\partial\alpha_1}{\partial y}\right)^{\frac{4}{3}} + \frac{3}{8}\left(\frac{17\tau_2}{64}\frac{\partial^2\alpha_1}{\partial y^2}\right)^{\frac{4}{3}} + \\ & \frac{3}{4}\eta_2^{\frac{4}{3}}\left(\frac{\partial\alpha_1}{\partial y}\right)^{\frac{4}{3}} + \frac{3}{4}\frac{1}{\xi_2^2}\left(\frac{\partial\alpha_1}{\partial y}\right)^4 - \frac{3}{4}\left(\frac{5\partial\alpha_1}{2\partial y}\right)^{\frac{4}{3}}\Bigg]\end{aligned}$$

which leads to the following control law by the third equation of (8)

$$u = [\hat{x}_2 - \alpha_1(y)]\, s_2. \quad (48)$$

In the numerical simulation, we chose $k_1 = 3$, $k_2 = 4.5$, $c_1 = -0.01$, $c_2 = -0.1$, $\delta_1 = 1$, $\xi_2 = 1$, $\tau_2 = 1$, $\epsilon_1 = 3$, $\epsilon_2 = 0.125$, $\epsilon_3 = 1$,$\eta_2 = 3$, and $\theta_2 = 3$ and set the initial condition to be $x_1(0) = 0.1$, $x_2(0) = 0$, $\hat{x}_1(0) = 0$ and $\hat{x}_2(0) = 0$. The switching signal generated randomly is shown in Fig. 1. The trajectories of the states and the estimates are shown in Figs. 2 and 3, respectively. From Fig. 2, one can see that the outputs of the closed-loop system converges to zero asymptotically. The simulation results show the effectiveness of the proposed control method.

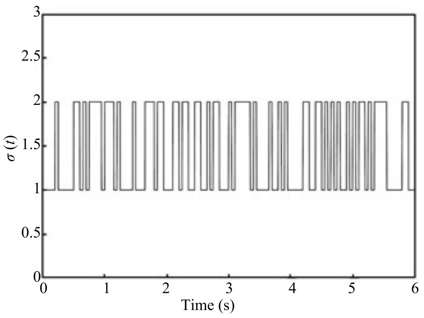

Fig. 1 The switching signal generated randomly

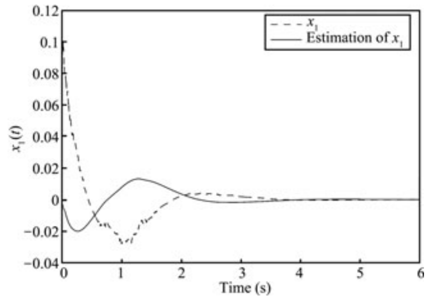

Fig. 2 The curves of the state one and its estimation

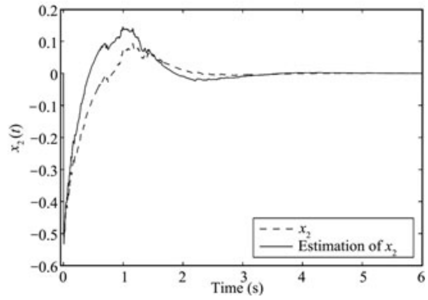

Fig. 3 The curves of the state two and its estimation

5 Conclusion

This paper has investigated the problem of global stabilization of switched stochastic output feedback nonlinear systems under arbitrary switchings by backstepping approach. A common output feedback controller independent of switching signals is constructed to guarantee that the equilibrium at the origin of the closed-loop system is fourth-moment exponentially stable. And an example is employed to show the effectiveness of the proposed method.

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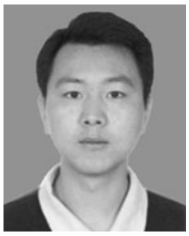

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