Research on application of ground penetrating radar array method based on plane beam signal in different geological models

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Abstract

The traditional common-offset ground penetrating radar method measures point by point along the survey line with a single transmitter and a single receiver. Due to the influence of the antenna radiation power and the low-pass filtering function of the earth medium, an intense amplitude gain cannot be obtained when the signal is intercepted. This article addresses a plane beam signal ground penetrating radar array observation method based on high radiation power gain. The transmitting antenna array simultaneously excites the pulse signal with the same centre frequency. All the transmitted signals interfere with each other at the near surface to form a plane beam signal, and the electromagnetic energy is superimposed mutually to increase the radiation power. We applied the plane beam signal ground penetrating radar array method to different geological models constructed by the finite difference time-domain (FDTD) algorithm for numerical simulation in this research. Since there are various offsets in the array ground penetrating radar observation method, we introduce a composite frequency shift-perfect matching layer (CFS-PML) based on the recursive convolution method as the absorbing boundary condition. It eliminates the problem of secondary reflection caused by the angle variation of the incident wave. The research result shows that the plane beam signal illuminates the target uniformly in space, can eliminate the discontinuity in profile data caused by the directivity of the antenna, improve the stability and quality of the echo signal, and enrich the target response parameters.

Keywords Plane beam signals · Ground penetrating radar array (GPR array) · Finite difference time domain (FDTD) · Recursive convolution · Complex frequency shift-perfect matching layer (CFS-PML)

Introduction

As an active source detection method that emits high-frequency pulse electromagnetic waves, GPR is widely used in various engineering inspections and surveys due to its advantages in high resolution and non-destructive detection of exploration targets (Bai and Sindield 2020). However,

with the continuous development of geophysical electromagnetic methods, breakthroughs, and the continuous improvement of engineering inspection quality requirements, the shortcomings of traditional GPR have become more prominent. For example, in single transmitting antenna and single receiving antenna mode, the antenna radiates pluses with a narrow time width, resulting in lower radiation power. And due to the scattering of signals by the underground medium, it makes the detection depth shallow. The target resolution is low, the antenna power gain effect is poor, the effective excitation signal and the follow-up echo signal are weak, and there are problems such as low signal anti-interference ability (Kundu 2020; Raza et al. 2020). To solve this series of problems, many scholars have done a lot of research on the improvement of GPR (Annan 2002). For example, Gazit (1988) improved the feed port of the Vivaldi antenna and used the tapered transition of the symmetrical double-sided slit line at the substrate feed end of the microstrip antenna to increase the working frequency band of the GPR antenna. Oyan et al. (2012) studied an ultra-wideband GPR

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and discussed the generation of frequencies from 500 MHz to 3 GHz and the processing of radar data and signals, which improved the dynamic range of the GPR system. Wang and Yuan (2014) and Arabi et al. (2020) had introduced multiple-input multiple-output (MIMO) technology into the GPR detection data to achieve signal diversity and suppress the rapid attenuation of electromagnetic waves. Unfortunately, system designers always hope that the transmitting antenna can transmit high-power, high-fidelity, and narrow-time pulse signals. However, because the GPR system design is restricted by developing the electronic device manufacturing level, it is difficult to achieve high-fidelity, ultra-narrow pulse emission.

Based on the signal diversity technology of the MIMO system, this research proposes a method of using signal aggregation. The spherical wave signal of the traditional GPR is integrated in space to form a plane beam signal. To study the wave field characteristics and propagation laws of plane beam GPR array signals in different media and geological models, numerical simulations are used to analyse different models. Numerical simulation is an effective method for studying the propagation of electromagnetic waves in shallow underground complex geological media. It is also an important basis for reverse time migration and comprehensive waveform inversion research (Liu et al. 2012). There are many numerical analysis methods for GPR, which are generally divided into two categories: the geometric ray method (ray tracing method) (Zeng et al. 1995; Huang et al. 2011) and the wave equation method. Wave equation methods include finite difference time domain (FDTD) (Feng and Dai 2011; Feng et al. 2016; An et al. 2019), finite element time domain (FETD) (Chilton and Lee 2007; Diamanti and Giannopoulos 2009; Howlader and Sattar 2015), pseudospectral time domain (PSTD) (Liu 1997) and Fourier method (Carcione et al. 1999). Compared with the geometric ray method, the electromagnetic wave field calculated by the wave equation method contains the kinematic information of electromagnetic and contains a wealth of dynamic information. This will provide more data and information for studying electromagnetic wave propagation mechanisms and the interpretation of complex structures. Moreover, the FDTD method as a full-waveform numerical calculation method (Knott 2003; Bai et al. 2013) was proposed by Yee (1966). This method has the advantages of simple principle and easy implementation (Lacanna and Ripepe 2020). Moreover, the FDTD algorithm, as a wide-band electromagnetic field numerical simulation technology, can calculate the whole multi-polarisation characteristics of broadband signals in one simulation to obtain the full-wave response characteristics of the signal at the same time (Chai et al. 2007). Therefore, the FDTD algorithm is used in the research process to simulate the GPR array based on the plane beam signal. CFS-PML is introduced as the

absorbing boundary condition, which avoids the problem of false reflection recording caused by secondary reflection at the truncated boundary position due to the incident angle variation (Li and Huang 2014; Tian et al. 2013). Finally, compared with the traditional common-offset GPR, the advantages of the GPR array based on plane beam signals in different geological models are analysed in terms of detection performance and data quality.

Method and principle

Principle of three-dimensional FDTD

The grid division of the FDTD algorithm uses the classic Yee grid, and the six components of the electric field and magnetic field are placed on an arbitrary rectangular grid in space. The distribution of the electric field and magnetic field in the three-dimensional space and the corresponding relationship between the node positions are shown in Fig. 1. Each magnetic field component is surrounded by four electric field components. In turn, four magnetic field components also surround each electric field component. In the solution process, the electric field and the magnetic field are different in time by half a time step to perform alternate iterative calculations (Sun et al. 2019). The FDTD algorithm is an effective method to accurately calculate various electromagnetic interaction problems, and different geological models can be constructed flexibly. It can be used to solve various anisotropic and nonlinear electromagnetic problems. And because it uses the differential form to solve the electric field and magnetic field component values in each direction of Maxwell's differential equations, the calculation results can reflect the full-wave characteristics of the wave. Therefore, use FDTD algorithm to simulate the GPR array based on plane beam signals; then, obtain the wave propagation

Fig. 1 Schematic diagram of Yee differential grid electromagnetic field space

laws under different field components and the response characteristics to different geological models.

Then another question is that when using the FDTD method, the problem of absorbing boundary conditions must be addressed. The traditional absorbing boundary condition is designed to divide the electromagnetic field component at the boundary of the study area and assign a different loss value to each divided field component. The reflection coefficient is a function of incident angle and wave impedance. The reflection coefficient can be equal to zero only when the angle of incidence is equal to the angle of refraction, or the wave is incident normal to the interface. Using the FDTD algorithm to simulate a GPR array based on a plane beam, the incident angle will change with the offset. Therefore, the perfect matching layer of complex frequency shift whose reflection coefficient is independent of angle is selected as the absorbing boundary condition. According to the principle of CFS-PML, in the time domain, the coordinate transformation is a convolution relationship. The direct solution of the convolution is complicated, and the calculation is onerous. The convolution is directly replaced under the discrete staggered grid, and the time-domain coordinate recursion equation is obtained by inverse Fourier transform. It is effective to use the CFS-PML as the absorbing boundary condition; as shown in Fig. 2, the electromagnetic wave enters the perfect matching layer without reflection at the boundary and is attenuated perfectly. When the conventional perfectly matched layer is used as the absorption boundary condition, to realise the fact that the reflection coefficient is equal to zero independent of frequency and incident angle and meets the impedance matching condition, the incident angle needs to be equal to the refraction angle. In practice,

such a medium does not exist, so part of the electromagnetic wave energy will be reflected at the boundary and return to the study area, as shown in Fig. 3. For the specific principles of the three-dimensional FDTD algorithm and how to apply CFS-PML as an absorbing boundary condition at the boundary position, please refer to Appendix.

Application of GPR array based on planar beam signal in different geological models

Three-dimensional metal ball model of double-layer medium

We set the size of the model area in the x -, y -, and z -directions to 0.5 m. The upper part is air with a thickness of 0.1 m, and the lower part is a double-layer uniform half-space. The first layer fill with concrete with a thickness of 0.15 m. The relative permittivity is 6.0, the conductivity is 0.001 S/m, the magnetic permeability is 0.1 H/m, and the magnetic loss is 0 Ω /m. Fill the second layer of soil medium with a relative permittivity of 9.0, the electrical conductivity of 0.001 S/m, magnetic permeability of 0 H/m, and magnetic loss of 0 Ω /m. The second layer of the soil medium buried a metal ball with radius of 0.05 m—the centre of the ball at (0.25, 0.25, 0.125). As shown in Fig. 4, an 8×8 GPR array is arranged on the surface of the concrete. The transmitting antenna and receiving antenna of the GPR array are arranged in a straight line along the y -axis. The spatial position of the first transmitting antenna is (0.035, 0.035, 0.40), the spatial position of the first receiving antenna is (0.085, 0.035, 0.40), and the distance between each transmitting antenna is 0.05 m, and the receiving antenna distance is

Fig. 2 CFS-PML absorption boundary condition

Fig. 3 PML absorption boundary condition

Fig. 4 Three-dimensional metal ball model structure diagram

also 0.05 m. The eight transmitting antennae use Hertzian dipoles polarised in the z -direction as the excitation source, which uses a Ricker wavelet waveform with a dominant frequency of 900 MHz and a maximum amplitude scaling of 1 A. We divide Yee grids between the x -, y -, and z -directions. The spatial size of each grid is 0.004 m. The transmitting antenna and the receiving antenna are stepped along the x -axis simultaneously in steps of 0.004 m (97 channels of data are scanned), and the time window is set to 120 ns. The complex frequency shift perfectly matched layer (CFS-PML) is used as the absorbing boundary condition, and the thickness of the fully matched layer on the six sides of the model is six spatial steps. The medium parameters of the single-shot single-receive model are the same as those of the array radar, and the spatial position relationship of the model is the same. The excitation point and the receiving point are located at (0.25, 0.25, 0.40) and (0.085, 0.250, 0.40), and the other parameters are consistent with the array radar model. The spatial position relationship of the model is shown in Fig. 4.

Signal integration and wave field reconstruction

In the conventional common-offset GPR observation mode, the target echo signal is reflected by the target body along the incident path of the transmitted signal and then transmitted back to the receiving antenna. The wavelet wave front forms a spherical envelope. The received signal is the product of the gain of the transmitting antenna and the receiving antenna different directions. The ball envelope irradiates the target body at a single point, so the time–distance curve of

Fig. 5 Schematic diagram of array radar wave field reconstruction

the received signal is a hyperbola. The GPR array based on plane beam signals adopts a signal integration. Each transmitting antenna excites a pulse signal of the same centre frequency, and each pulse signal interferes with each other near the surface. The wave field is reconstructed in space to form a plane beam signal, as shown in Fig. 5. Use MATLAB program to simulate the fusion process of the pulse signal. As shown in Fig. 6, eight excitation sources are located in the middle of the diagonal of the square simulation area, and each excitation source is 0.5 m apart. At the 150th time step, the excitation source generated eight pulse signals. With time stepping, the pulse signal spreads out to the surrounding space. At the 180th time step, the diffuse waves begin to integrate into a plane beam. When the wave field forms a plane beam, the antenna radiates uniformly along with the underground medium and illuminates the target uniformly in all directions in space. The energy carried by each pulse is also superimposed on each other, which enhances the power of the radiated signal and the strength of the echo signal.

Comparison and analysis of amplitude characteristics

The conventional common-offset GPR observation method is used to make point-by-point observations. The data of single-point observation are random, and the captured target space information is relatively vague, which reduces the accuracy and detection efficiency of detection data because the transmitting and receiving antenna adopts a small-distance observation method. In data processing and interpretation, it is generally considered that the signal received by the receiving antenna is perpendicular to the transmitted signal. After being reflected by the target, it returns to the ground along the vertical path and is received by the receiving antenna. This kind of data acquisition method, data processing, and interpretation method has insufficient target information acquisition, complicated data processing method, too many assumptions, excessive reliance on experience in the interpretation process, and insufficient target detection

Fig. 6 Signal fusion wave field at different time steps

resolution in practical applications. However, under the observation method based on the plane beam signal GPR array, the ray path of electromagnetic wave propagation is

shown in Fig. 7. The eight transmitting antennas contribute energy to each receiving antenna, and the electromagnetic waves radiated by each transmitting antenna can be received

Fig. 7 Electromagnetic wave propagation ray path diagram

by each receiving antenna after interference and superposition in space. When reflection occurs in the underground stratum, one scan can achieve multi-point coverage of the target body while capturing multi-point data covered multiple times.

According to the principle (Xu et al. 2011) of interference and superposition of electromagnetic waves, multiple transmitting antennae simultaneously excite electromagnetic waves of the same centre frequency. The source current on the antenna will generate radiation in space. If multiple current cells satisfy the coherence relationship in space, the radiated electromagnetic fields of the multiple currents will be superimposed in space, forming an interference phenomenon. It results in the in-phase superposition of the regional spatial fields, and the field is strengthened. The radiated electromagnetic field generated by the current distribution J in a uniform medium can be expressed as:

$$\mathbf{E} = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla\cdot\mathbf{A}) \quad (1)$$

$$\mathbf{H} = \frac{1}{\mu}\nabla\times\mathbf{A}. \quad (2)$$

In Eq. (2), $\mathbf{A}$ is known as the magnetic vector potential, which satisfies the following Helmholtz equation:

$$\nabla^2\mathbf{A} + k^2\mathbf{A} = -\mu\mathbf{J}. \quad (3)$$

When the current density J is distributed along the z -direction line as $zJz(z')$ and the length of the distribution line is L , then at some far-field observation point $P(x, y, z)$, the solution of the Helmholtz equation is:

$$\mathbf{A} = z\frac{\mu}{4\pi}\int_L \frac{Jz(z')e^{-jkR}}{R}dz'. \quad (4)$$

When multiple dipole antennae are used for radiation, the current can be discretised into the sum of N small parts in total:

$$L = \sum_{n=1}^N dln. \quad (5)$$

Use zn' to represent the centre coordinates of a small part dln of each dipole strip, if dln is small enough, the current density of each dipole antenna is uniform and constant in each small part, and it can be expressed as $J(z1'), J(z2'), J(z3'), \dots, kJ(zN')$. Then Eq. (4) can be expressed as:

$$\mathbf{A} = \sum_{n=1}^N \mathbf{A}n = z\sum_{n=1}^N \mathbf{A}zn = z\frac{\mu}{4\pi}\sum_{n=1}^N \frac{J(zn')dln e^{-jkRn}}{Rn}, \quad (6)$$

where $J(zn')dln$ represents the total current on a dipole, which is:

$$In = J(zn')dln \quad n = 1, 2, 3, \dots, N. \quad (7)$$

Substituting Eq. (7) into Eq. (6) gives:

$$\mathbf{A} = \sum_{n=1}^N \mathbf{A}n = z\frac{\mu}{4\pi}\sum_{n=1}^N \frac{Ine^{-j\omega Rn}}{Rn}. \quad (8)$$

It can be seen from Eq. (8) that the total radiation field vector potential in space can be expressed as the superposition of N radiation field magnetic vector potentials; therefore, the $\mathbf{E}$ and $\mathbf{H}$ of the total radiation electromagnetic field are also formed by the superposition of N parts, namely:

$$\mathbf{E} = \sum_{n=1}^N \mathbf{E}n \quad (9)$$

$$\mathbf{H} = \sum_{n=1}^N \mathbf{H}n. \quad (10)$$

The many small current sources of the array antenna replace the continuous current distribution of a single antenna. The current distribution on the conductor or medium of the antenna depends on the boundary conditions around the area in which it is located. Once the material, shape, structure, installation position, and method of excitation of the antenna are determined, the current distribution on the antenna is determined, and the resulting radiation field and radiation characteristics are also determined. However, when a single wire antenna or surface antenna is discretised into an array antenna composed of small radiating elements, each small radiating element's feed amplitude and phase can be adjusted, combined with the artificial control of the number and position of the small radiating elements. We obtain the electromagnetic field distribution of almost any target. The electromagnetic fields generated by the excitation of the discrete current elements are superimposed to form a total radiation field, forming a high-power radiated electromagnetic wave in space. Figure 8 shows the 97-channel E_y

Fig. 8 Waveform profiles of 97 channels and their E_y component data of common-offset GPR

Fig. 9 Waveform profiles of 97 channels and their E_y component data of plane beam GPR array

component radar data waveform obtained by scanning the double-layer metal ball model with a conventional common-offset GPR, and the time window size is 120 ns. The electromagnetic wave energy decays quickly in the first layer of the medium, resulting in weak energy of the second layer of the medium, so there is no obvious fluctuation in the position of the metal ball. The metal ball model was scanned with a GPR array based on plane beam signal, and 97 channels of E_y component data were obtained. According to the principle of electromagnetic interference superposition, when there are multiple transmitting antennas, the radiated energy will increase, thereby enhancing the echo signal of the target body. As shown in Fig. 9, the direct wave energy increases linearly, with a maximum value of 30 V/m, which increases the radiation energy by nearly ten times compared to the traditional single-transmit and single-receive GPR. It suppresses the attenuation of electromagnetic waves to a certain extent. Obvious fluctuations occurred at the position of the metal ball.

The signal received by the plane beam-based array radar observation method is the combined response of the transmitted waves incident at different angles and as scattered by the target. The radar scatters section is the integral of the incidence coefficient and the reflection coefficient. The result of the integration is the statistical parameter of the radar scatter section in the angular range, reflecting the target's comprehensive reflection ability. Decompose the incident coefficient and reflection coefficient terms of the target radar in the cross section:

$$rc(\theta t, \theta r) = \xi(\theta t)\xi(\theta r), \quad (11)$$

where θt is the incident direction angle and θr is the reflection direction angle, due to the single-shot single-receive radar having a small offset, therefore, $\theta t \approx \theta r, \xi(\theta t) \approx \xi(\theta r)$. The reflection coefficient of the receiving antenna observing the target in the θr direction is:

$$T_{\text{single}}(\theta r) = \int_V \xi\sigma(\theta r)\xi\sigma(\theta t)d\sigma = \int_V \xi\sigma^2(\theta r)d\sigma. \quad (12)$$

T_{single} is the reflection coefficient under the single-shot single-receive radar observation mode, V is the target body volume, and x is a certain point on the target body. In FDTD calculation, the space is discretised, and the target volume V is discretised into multiple scattering cells $d\sigma$. According to Eq. (12), in the common-offset GPR observation mode, the reflection coefficient $T_{\text{single}}(\theta r)$ is the integral of the volume with respect to the quadratic function of the reflection angle; therefore, when this changes very little with the scattering angle, the received signal can show larger flicker. When using GPR to detect target, the target body is not smooth. Especially when performing FDTD numerical simulations, the space is discretised into an infinite number of cube units so that the target appears to have a rough surface. Therefore, the target has different reflection or scattering characteristics due to electromagnetic waves being incident in various directions, which are generally a function of frequency and observation angle, and it is represented by radar scattering cross section. The higher the frequency, the more the target reflection coefficient changes with the scattering angle. The variation of the radar scattering cross section with the angle caused by the rough surface of the target will generate discontinuities in the observed data in the horizontal direction. As shown in Fig. 10, after eliminating the direct wave, obtain the conventional common-offset GPR profile data. The data are discontinuous in the horizontal direction, and multiple reflection stripes appear in the vertical direction. It reduced the signal-to-noise ratio of the data.

In the GPR array system, the receiving antenna observes the target in the direction θr and the approximate reflection coefficient of the target is:

Fig. 10 Common-offset GPR profile data

$$T_{\text{multit}}(\theta r) = \int_V \xi\sigma(\theta r) \sum_{m=1}^n \xi\sigma(\theta t) d\sigma. \quad (13)$$

When the transmitting antenna emits the same centre frequency and the same transmit waveform, the transmit array signals are superimposed in space to form a plane beam. Then the reflection coefficient of the receiving antenna observing the target in the θr direction can be expressed by the following continuous integral expression:

$$T_{\text{multit}}(\theta r) = \int_V \left[\xi\sigma(\theta r) \int_{\theta_1}^{\theta_2} \xi\sigma(\theta) d\theta \right] d\sigma. \quad (14)$$

The ratio of the reflection coefficient of the conventional common-offset GPR and the plane beam GPR array observation is:

$$\begin{aligned} \eta &= \frac{T_{\text{multit}}(\theta r)}{T_{\text{single}}(\theta)} = \frac{\int_V [\xi\sigma(\theta r) \int_{\theta_1}^{\theta_2} \xi\sigma(\theta) d\theta] d\sigma}{\int_V \xi\sigma^2(\theta r) d\sigma} \\ &\approx T_{\text{single}}(\theta r) \cdot \int_V \left[\int_{\theta_1}^{\theta_2} \xi\sigma(\theta) d\theta \right] d\sigma. \end{aligned} \quad (15)$$

T_{multit} is the reflection coefficient under the GPR array observation method based on plane beam, and n is the total number of transmitting antennae. The size of the reflection coefficient has a linear relationship with the number of transmitting antennae. The GPR array observation method based on the plane beam can obtain most of the information about the target scattering cross section, reflecting the statistics pertinent to the scattering points of the target in all directions and the integrity of the target reflection coefficient to a certain extent. When the targets is on or near a surface, the radiation aperture is about $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Its statistical

Fig. 11 GPR array profile data

characteristics suggest that the relationship between the directionality of radar transmission and reception and the observations of the target are weak. As shown in Fig. 11, making the profile data stable in the horizontal direction, thereby effectively suppressing noise signal, weakens the flickering seen in the target signal.

In general, the observation mode of the array antenna based on the plane beam signal can reduce the influences of the antenna's directional instability on the profile observation. Whether it is a simple ideal electric dipole point source or an antenna entity with a certain structure, the transmitted and received signals of the antenna are related to the antenna position, near field and far field, polarisation direction, and structure parameters. In a single antenna unit, the radiation direction is either high or low gain in a specific direction. Therefore, receiving antennae at different positions, due to the different receiving direction angles of the target and the receiving antenna, the intensities of the incident signal and the target signal obtained are also different, and this makes the profile data discontinuous and the target response more complicated. Under low-frequency and near-field conditions, the multi-transmit and multi-receive transmit wave field is a plane waveform. Suppose a uniform plane wave illuminates the target object, the incident wave field is independent of the radiation direction characteristics of a single antenna element, so the receiving antenna is only affected by the radiation from the target object and the receiving direction.

Low-lying modeling of complex geological structure

Under complex geological structure conditions, the process of electromagnetic wave propagation will produce reflected waves and diffracted waves. Diffracted waves will propagate at different angles. To verify the detection effect of the GPR array based on plane beam signals in complex geological structures, and in order not to lose the geological structure

information and reduce the number of grid divisions in the numerical simulation process, improve the calculation efficiency. Construct a complex low-lying ideal geological model as shown in Fig. 12. Simulate two normal faults with different tendencies in the real geological structure. The two inclination angles are along with the northeast and northwest directions, and the inclination angles are 45° and 135° , respectively. Suppose the grid size of the model area to 300×400 , and the physical parameters of the simulation area are the uppermost medium is the air layer, the relative permittivity $\epsilon_0 = 1$, the conductivity $\sigma_0 = 0$, the relative permittivity of the second medium is subgrade (mainly filled with small gravel with air gaps), the relative permittivity $\epsilon_1 = 5$, its conductivity $\sigma_1 = 0.001 \text{ S/m}$, the relative permittivity of the third medium is soil layer, the relative permittivity $\epsilon_2 = 8$, and its conductivity $\sigma_2 = 0.003 \text{ S/m}$. All excitation sources use Ricker wavelet pulses with a dominant frequency of 900 MHz, with a space step of 0.006 m and a time step of 0.015 ns. The positions of the 26 excitation sources are distributed along the x -axis, and the distance between each excitation source is 5 spatial steps.

Figure 13 illustrates the wave field snapshots of the wave front envelope of the plane electromagnetic wave at time steps 100, 200, 400, and 500, respectively. The plane beam spreads downwards uniformly and reflects and diffracts at the fault in the medium. The echo signal is obvious, and the energy is balanced. The radar data profile shown in Fig. 14a is formed. Compared with traditional common-offset radars (Fig. 14b), plane beam radars with multiple excitation sources have stronger electromagnetic energy at the same depth, forming a clearer radar data profile with better quality data.

Comparative analysis of polarisation characteristics

When the excitation source uses a Hertzian dipole polarised in the z -direction, a uniform plane wave propagating along the z -axis will produce linear polarisation (Bianchi et al.

2020). The electric field is in a plane parallel to the xOy plane, and the electric field can always be decomposed into two components, E_x and E_y :

$$\mathbf{E} = \mathbf{a}_x E_x + \mathbf{a}_y E_y, \quad (16)$$

where $\mathbf{a}_x$ and $\mathbf{a}_y$ are the direction vectors in the x - and y -directions, respectively, and $\mathbf{E}$ represents the sum of the vector field linearly polarised along the x - and y -directions.

Suppose the amplitude of E_x is E_{x0} , the amplitude of E_y is E_{y0} , and the phase lag is ϕ , and Eq. (47) is expressed as:

$$\mathbf{E}(z) = \mathbf{a}_x E_{x0} e^{-jkz} + \mathbf{a}_y E_{y0} e^{-jkz} e^{-j\phi}. \quad (17)$$

According to Eq. (17), the transient expression of the electric field at time t in the z -plane is obtained:

$$\begin{aligned} \mathbf{E}(z, t) &= \text{Re}\{[\mathbf{a}_x E_{x0} + \mathbf{a}_y E_{y0} e^{-j\phi}] e^{-jkz} e^{-j\omega t}\} \\ &= \mathbf{a}_x E_{x0} \cos(\omega t - kz) + \mathbf{a}_y E_{y0} \cos(\omega t - kz - \phi), \end{aligned} \quad (18)$$

where k is the wave number, ω is the angular frequency, and j is the complex unit. It can be seen from Eq. (18) that the polarisation of the electric field on the z -plane at any time t is related to the electric field amplitude, and the transient polarisation component along the y -direction lags ϕ phases in the x -direction. Due to the vector nature of electromagnetic waves, polarisation characteristics can deal with target characteristic signals and capture rich target information (Løseth and Ursin 2007; Bakunov and Zhukov 2012). At $t = 45 \text{ ns}$, analyse the characteristic polarisation field of the double-layer metal ball model at the longitudinal position $z = 0.35 \text{ m}$ and the upper dielectric $z = 0.125 \text{ m}$. The electric field plane wave polarisation of the array radar based on the plane wave observation method is shown in Fig. 15. When multiple transmitting antennae are separated by less than half a wavelength and simultaneously transmit pulses of the same centre frequency, the signals are superimposed in phase and the amplitude is enhanced. According to Eq. (18), when the amplitude of the electric field increases, the amplitude of the electric field component along the x - and y -directions on the z -plane at any time t will be increased accordingly. The polarisation characteristics will be more obvious. It can be seen in Fig. 15 that the array radar observation method generates a very good polarisation effect in the xOy plane. After the metal ball scatters the electromagnetic wave, obvious polarisation occurs in the x - and y -directions. The electric field is a uniform plane wave when it propagates in the xOy plane. The size and shape of the target can be judged according to the polarisation component. When using the conventional common-offset GPR observation method, the electromagnetic wave spreads in a circle from a centre point on the x and y -planes. As shown in Fig. 16, no obvious directional polarisation arises in the plane containing the metal sphere.

Fig. 12 Schematic diagram of a low-lying model structure

Fig. 13 GPR array wave field based on plane beam signal at different time steps

Fig. 14 Low-lying model radar data record. **a** GPR array data profile. **b** Traditional common-offset radar data profile

Fig. 15 Polarisation characteristic field of array GPR at 45 ns

The calculation of electromagnetic polarisation fields in media is often practical when simplifying a problem for interpretational purposes. Planar electromagnetic waves can always be decomposed into two orthogonal linearly polarised waves. When a linearly polarised electromagnetic wave is emitted, the electromagnetic wave has a depolarisation effect after being scattered by a natural target, so there is not only a homopolarisation effect in the echo signal but also the polarisation has a cross-polarisation component. When the polarisation states of the receiving antenna and the echo signal are the same, the received echo energy reaches the maximum value; the GPR array will have a

full polarisation effect in space. So it can better capture the morphological characteristics of the target as it increases the energy borne by the echo. In order to further illustrate the advantages of the polarisation characteristics of the GPR array observation method based on the plane beam signal, established a metal pipeline model, and a metal pipeline was placed in the second layer of soil medium under the same conditions as the other model parameters and the metal ball model. FDTD simulation is used to obtain the E_z composite polarised wave field, as shown in Fig. 17. The electromagnetic wave is linearly polarised along the direction of the metal pipe. The polarised wave

Fig. 16 Polarisation characteristic field of common-offset GPR at 45 ns

field describes the outline of the metal pipe and the corresponding plane position relationship.

Wave field of different polarisation receiving direction

When the transmitting antenna and the receiving antenna are polarised in different directions, the target response signals obtained are different. When the transmit polarisation direction is the same as the receive polarisation direction, it is known as co-directional polarisation. When the transmit polarisation direction is perpendicular to the received direction of polarisation, it is known as orthogonal polarisation. The xOy plane is horizontal. Electromagnetic waves polarised along the x or y -axes are called horizontally polarised emission waves, and electromagnetic waves polarised along the z -axis are called vertically polarised waves. Figure 18 shows a snapshot of the electric field wave field received along with different polarisation directions when the GPR array and the conventional common-offset GPR are E_z linearly polarised transmission (vertically polarised transmission wave). It can be seen from Fig. 18 that in the GPR array observation mode based on the plane beam signal, the wave front of each component of the electric field presents the shape of a plane wave envelope in space, and the reflected wave after the E_x orthogonal polarisation component (Fig. 18b) is scattered by the metal ball energy is evenly distributed in all directions in space. The response of the metal sphere to the plane wave is to form a spherical wave front. The E_y orthogonal polarisation component and the E_z co-polarisation component have waveform distortions due to frequency dispersion effects induced by the metal sphere.

Fig. 17 Metal pipeline model E_z component scattering field

However, they still exhibit strong scattering energy in space, and prominent abnormal features occur at the target location. In the conventional common-offset radar observation mode, the E_y orthogonal polarisation component has no obvious echo signal. The echo energy of the E_z co-polarisation component is weak. Although the E_x orthogonal polarisation component produces a spherical reflection signal, the response to the characteristics of the metal sphere is relatively fuzzy, and the abnormal background features cannot be highlighted.

Electromagnetic waves consist of electric and magnetic fields that mutually perpendicular in space. Figure 19 depicts snapshot of the vertical wave field along each polarisation component of the conventional common-offset GPR and GPR array magnetic fields at 45 ns. Similarly, under the GPR array observation method, the wave front of the orthogonal polarisation component of H_y (Fig. 19d) propagates downwards in the form of a plane beam. It has the same wave field characteristics as the E_x polarisation component of the electric field (Fig. 18b) on the horizontal plane. The H_z co-directional polarisation component (Fig. 19f) due to the vertical co-directional polarisation at the position of the metal ball and left-handed and right-handed components form a symmetrically distributed polarisation field, and the wave field is clear and stable. To a certain extent reflects the contour of the target body along the vertical direction. The polarised components of the conventional common-offset GPR magnetic field are weak in terms of energy intensity. The wave field characteristics of the H_x orthogonal polarisation component (Fig. 19a) and the E_y orthogonal polarisation component (Fig. 18c) are similar. There is no obvious echo signal. The wave field characteristics of the orthogonal polarisation component of H_y (Fig. 19c) and the co-polarisation component of E_x (Fig. 18a) are similar, and a spherical echo signal is generated. The H_z co-polarisation component (Fig. 19e) also produces a symmetrical polarised wave field at the position of the metal ball. However, the wave field energy is relatively weak, and the target body contour is blurred.

In the GPR array observation mode based on a plane beam signal, the wave envelopes of the co-polarised components and cross-polarised components emitted in the vertical direction propagate in space as plane beams, and the wave field amplitude each polarisation component is relatively strong. The method can highlight the abnormal characteristics of the target body relative to the background medium and highlight the effective part of the signal. In the conventional common-offset GPR observation mode, only cross-polarised components (E_x , H_y) emitted in the vertical direction form reflected echoes at the target. Its signal energy is weak, and it is affected by various noise signals, increasing the difficulty of data interpretation.

Fig. 18 Recording diagram of each component of TE wave of common-offset GPR and GPR array at 45 ns

Fig. 19 Recording diagram of each component of TM wave of common-offset GPR and Array GPR at 45 ns

Conclusion

This article addresses a signal integration technology using multiple transmitters and multiple receiver antenna arrays based on the MIMO radar signal diversity technology and applies it to GPR. The integrated signal presents a plane wave envelope state in space. Furthermore, by constructing different numerical models to analyse the wave field characteristics of the GPR array based on the plane beam signal and study the response law of the target, the following conclusions are obtained.

The array antenna observation model can weaken the influence of the directional instability of the antenna itself on the profile observation. Whether it is a simple ideal electric dipole point source or an antenna entity with a specific structure, the antenna receiving and sending signals are affected by the underground background medium parameters, antenna position, far and near fields, polarisation direction, and structure. The pattern of a single antenna unit has a high and low gain in a specific direction. Therefore, when the antenna is used to measure different positions along the survey line, the target's incident signal strength and the receiving antenna can receive the target signal strength due to the different angles of the target to the antenna. It is also different so that the observation profile contains the information of the characteristics of the transmit and receive antenna patterns so that the profile data are discontinuous in the horizontal direction. However, in the near field, the GPR array transmitting wave field based on the plane beam signal is a plane waveform. If a uniform plane wave illuminates the target, the incident wave field is independent of the radiation direction of a single antenna unit. Therefore, the horizontal data profile is stable and clear, and the data profile can highlight the abnormal background features.

When the GPR array detects the target body, the range of one scan is extensive, which can significantly improve the detection efficiency. In the observation method that multiple transmitting antennas simultaneously excite pulse signals of the same frequency, the energy of the echo signal increases significantly due to the superposition of energy. Compared with the single-transmit and single-receive GPR detection method, the GPR array observation method based on plane beam signals is used. Since the transmitted signal is completely coherent, the target response signal also has great coherence, so the signal-to-noise ratio of the received signal and the dynamic range have been greatly improved.

The GPR array observation method based on plane beam signals can combine transmitting antennas and receiving antennas arbitrarily. Each receiving antenna receives

superimposed signals from different transmitting antennas. In the plane beam-based GPR array observation mode, electromagnetic waves can be better polarised. It can describe the target object's spatial position and morphological characteristics, contains richer target body information, and provides more ideal detection data. The combination of transmitting antenna and receiving antenna can be diversified. Therefore, it is expected to achieve a full polarisation effect.

In general, the GPR array observation method based on plane beam signals solves the problems of insufficient data quality and low detection efficiency caused by the lack of system detection in the traditional single-transmit and single-receive GPR. It not only improves the detection efficiency and signal-to-noise ratio, and realises multiple coverages of underground space data, but also displays the visible cross-sectional position of the underground target body, which helps to accurately distinguish the geological structure and accurately locate the target body, ensuring the target response signal the authenticity, stability, and accuracy of target parameters.

Finally, the GPR array is still in the research stage. In order to obtain a better data collection effect and obtain more physical parameters related to the medium, the author will continue to deepen the research on the array GPR.

Appendix

Three-dimensional FDTD principle

Divide the solved model area into countless Yee units as shown in Fig. 1. When the FDTD method is used to simulate the propagation of electromagnetic waves in three-dimensional space, Faraday's law of electromagnetic induction and Ampere's loop law can be evolved to:

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu} \nabla \times \mathbf{E} - \frac{\rho}{\mu} \mathbf{H} \quad (19)$$

$$\frac{\partial \mathbf{E}}{\partial t} = -\frac{1}{\epsilon} \nabla \times \mathbf{H} - \frac{\sigma}{\epsilon} \mathbf{E}. \quad (20)$$

Coupled partial differential equations can be obtained in three-dimensional space:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu} \left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \rho H_x \right) \quad (21)$$

$$\frac{\partial Hy}{\partial t} = \frac{1}{\mu} \left(\frac{\partial Ez}{\partial x} - \frac{\partial Ex}{\partial z} - \rho Hy \right) \quad (22)$$

$$\frac{\partial Hz}{\partial t} = \frac{1}{\mu} \left(\frac{\partial Ex}{\partial y} - \frac{\partial Ey}{\partial x} - \rho Hz \right) \quad (23)$$

$$\frac{\partial Ey}{\partial t} = \frac{1}{\varepsilon} \left(\frac{\partial Hx}{\partial z} - \frac{\partial Hz}{\partial x} - \sigma Ey \right) \quad (24)$$

$$\frac{\partial Ez}{\partial t} = \frac{1}{\varepsilon} \left(\frac{\partial Hy}{\partial x} - \frac{\partial Hx}{\partial y} - \sigma Ez \right) \quad (25)$$

$$\frac{\partial Ex}{\partial t} = \frac{1}{\varepsilon} \left(\frac{\partial Hz}{\partial y} - \frac{\partial Hy}{\partial z} - \sigma Ex \right), \quad (26)$$

where $\mathbf{E}$ is the electric field strength, $\mathbf{H}$ denotes the magnetic field strength, ε is the dielectric constant, σ is the conductivity, μ is the magnetic permeability, and ρ is the reluctivity for calculating the magnetic loss. The Yee difference solution to six coupled partial differential equations is produced, and a difference grid is established in the problem domain. The grid nodes correspond to a set of corresponding integer labels on a one-to-one basis, as shown in Fig. 1:

$$(i, j, k) = (i\Delta x, j\Delta y, k\Delta z). \quad (27)$$

The value of any function $F^n(x, y, z)$ at this point at time $n\Delta t$ can be expressed as:

$$F^n(i, j, k) = F(i\Delta x, j\Delta y, k\Delta z), \quad (28)$$

where Δx , Δy , Δz are the spatial steps of the rectangular grid along the x -, y -, and z -directions, respectively, and Δt is the time step. Yee uses the central difference to replace the differential coordinates in time and space, resulting in second-order accuracy:

$$\frac{\partial F(i, j, k)}{\partial x} = \frac{F^n\left(i + \frac{1}{2}, j, k\right) + F^n\left(i - \frac{1}{2}, j, k\right)}{\Delta x} + O(\Delta x^2) \quad (29)$$

$$\frac{\partial F(i, j, k)}{\partial t} = \frac{F^{n+\frac{1}{2}}(i, j, k) + F^{n-\frac{1}{2}}(i, j, k)}{\Delta t} + O(\Delta t^2). \quad (30)$$

In the time domain, the electric field and magnetic field are calculated alternately and iteratively with a difference of half a step. The six coupled partial differential equations are turned into difference equations, such that at time step n :

$$\begin{aligned} \frac{\partial H_x^n\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right)}{\partial t} &= \frac{H^{n+\frac{1}{2}}\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right) - H^{n-\frac{1}{2}}\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right)}{\Delta t} \\ H_x &= \frac{H^{n+\frac{1}{2}}\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right) + H^{n-\frac{1}{2}}\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right)}{2} \\ \frac{\partial E_y}{\partial z} &= \frac{E_y^n\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right) - E_y^n\left(i, j + \frac{1}{2}, k\right)}{\Delta z} \\ \frac{\partial E_z}{\partial y} &= \frac{E_z^n\left(i, j, k + \frac{1}{2}\right) - E_z^n\left(i, j + 1, k + \frac{1}{2}\right)}{\Delta y}. \end{aligned} \quad (31)$$

By substituting Eq. (31) into the partial differential Eq. (21) and simplifying to obtain the recurrence equation of the magnetic field in its x -component, we obtain:

$$\begin{aligned} H_x^{n+\frac{1}{2}}\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right) &= \frac{1 - \frac{\rho\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right)\Delta t}{2\mu\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right)}}{1 + \frac{\rho\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right)\Delta t}{2\mu\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right)}} H_x^{n-\frac{1}{2}}\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right) \\ &\quad + \frac{\Delta t}{\mu\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right)} \times \frac{1}{1 + \left\{ \rho\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right)\Delta t / \left[2\mu\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right) \right] \right\}} \\ &\quad \times \left\{ \frac{E_y^n\left(i, j + \frac{1}{2}, k + \frac{1}{2}\right) - E_y^n\left(i, j + \frac{1}{2}, k\right)}{\Delta z} \right. \\ &\quad \left. + \frac{E_z^n\left(i, j, k + \frac{1}{2}\right) - E_z^n\left(i, j + 1, k + \frac{1}{2}\right)}{\Delta y} \right\}. \end{aligned} \quad (32)$$

In the same way, the z -component and y -component of the magnetic field are obtained.

When solving, the electric field leads the magnetic field by half a time step.

$$\begin{aligned} \frac{\partial E_x}{\partial t} &= \frac{E_x^{n+1}\left(i + \frac{1}{2}, j, k\right) - E_x^n\left(i + \frac{1}{2}, j, k\right)}{\Delta t/2} \\ E_x &= \frac{E_x^{n+1}\left(i + \frac{1}{2}, j, k\right) + E_x^n\left(i + \frac{1}{2}, j, k\right)}{2} \\ \frac{\partial H_z}{\partial y} &= \frac{H_z^{n+1/2}\left(i + \frac{1}{2}, j + \frac{1}{2}, k\right) - H_z^{n+1/2}\left(i + \frac{1}{2}, j - \frac{1}{2}, k\right)}{\Delta y} \\ \frac{\partial H_y}{\partial z} &= \frac{H_z^{n+1/2}\left(i + \frac{1}{2}, j, k + \frac{1}{2}\right) - H_z^{n+1/2}\left(i + \frac{1}{2}, j, k - \frac{1}{2}\right)}{\Delta y} \end{aligned} \quad (33)$$

By substituting Eq. (33), we obtain the electric field recurrence equation in the x -direction:

$$\begin{aligned}
 E_x^{n+1}\left(i + \frac{1}{2}, j, k\right) &= \frac{1 - \frac{\sigma\left(i + \frac{1}{2}, j, k\right)\Delta t}{2\varepsilon\left(i + \frac{1}{2}, j, k\right)}}{1 + \frac{\sigma\left(i + \frac{1}{2}, j, k\right)\Delta t}{2\varepsilon\left(i + \frac{1}{2}, j, k\right)}} \cdot E_x^n\left(i + \frac{1}{2}, j, k\right) + \frac{\Delta t}{\varepsilon\left(i + \frac{1}{2}, j, k\right)} \\
 &\times \frac{1}{1 + \left\{ \sigma\left(i + \frac{1}{2}, j, k\right)\Delta t / \left[2\varepsilon\left(i + \frac{1}{2}, j, k\right) \right] \right\}} \\
 &\times \left\{ \frac{H_z^{n+1/2}\left(i + \frac{1}{2}, j + \frac{1}{2}, k\right) - H_z^{n+1/2}\left(i + \frac{1}{2}, j - \frac{1}{2}, k\right)}{\Delta y} \right. \\
 &\left. + \frac{H_y^{n+1/2}\left(i + \frac{1}{2}, j, k - \frac{1}{2}\right) - H_y^{n+1/2}\left(i + \frac{1}{2}, j, k + \frac{1}{2}\right)}{\Delta z} \right\}. \quad (34)
 \end{aligned}$$

According to Eq. (34), the electric field strength is related to the conductivity and permittivity of the medium. The electric field strength at the next moment depends on that at the previous moment and the magnetic field strength that is adjacent in space and differs in time by half a time step. We then calculate the three-component recurrence equations of the electric and magnetic fields along x -, y -, and z -directions, respectively, and multiply the electric field strength of the corresponding component by the harmonic function to obtain the transient expression of the polarisation field on this component.

Principle of perfect matching layer with complex frequency shift based on recursive convolution

The wave equation changes with complex coordinates in the PML layer. Taking the x -direction as an example, the transformation relationship of the frequency domain coordinates in PML is derived as follows:

$$\tilde{x}(x) = x - \frac{i}{\omega} \int_0^x dx(s)ds \quad (35)$$

The partial derivative of x is

$$\partial \tilde{x} = \frac{i\omega}{i\omega + dx} \partial x = \frac{1}{sx} \partial x, \quad (36)$$

where sx is the complex stretch function; x is the original coordinate; $\tilde{x}$ denotes the coordinate after transformation; ω

is the circular frequency; and dx represents the attenuation coefficient.

The general form of the complex frequency shift stretching function obtained from Eq. (36) is:

$$sx = Kx + \frac{dx}{\alpha x + i\omega}. \quad (37)$$

We use $\bar{s}x(t)$ to represent the inverse Fourier transform of $\frac{1}{sx}$ to convert frequency domain coordinates into time-domain coordinates:

$$\partial \tilde{x} = \bar{s}x(t) * \partial x. \quad (38)$$

We apply an inverse Fourier transform to Eq. (41) and get:

$$\bar{s}x(t) = \frac{\delta(t)}{Kx} - \frac{dx}{K_x^2} H(t) e^{-(dx/Kx + \alpha x)t}, \quad (39)$$

where $\delta(t)$ is the impulse function; $H(t)$ is the unit step function.

Let

$$\zeta \tilde{x}(t) = -\frac{dx}{K_x^2} H(t) e^{-(dx/Kx + \alpha x)t}. \quad (40)$$

Then:

$$\partial \tilde{x} = \frac{1}{Kx} \partial x + \zeta x(x) * \partial x. \quad (41)$$

The convolution at discrete time $n\Delta t$ is:

$$\begin{aligned}
 \psi_x^{n\Delta t} &= (\zeta x * \partial x)^{n\Delta t} = \int_0^{n\Delta t} (\partial x)^{n\Delta t - \tau} \zeta x(\tau) d\tau \\
 &= \sum_{m=0}^{n-1} \int_{m\Delta t}^{(m+1)\Delta t} (\partial x)^{(n\Delta t - \tau)} \zeta x(\tau) d\tau. \quad (42)
 \end{aligned}$$

When the electric and magnetic fields of electromagnetic waves are calculated using alternating grids:

$$\begin{aligned}
 \psi_x^{n\Delta t} &= \sum_{m=0}^{n-1} (\partial x)^{((n-(m+1/2))\Delta t)} \int_{m\Delta t}^{(m+1)\Delta t} \zeta x(\tau) d\tau \\
 &= \sum_{m=0}^{n-1} Zx(m) (\partial x)^{((n-(m+1/2))\Delta t)}, \quad (43)
 \end{aligned}$$

where

$$\begin{aligned}
Zx(m) &= \int_{m\Delta t}^{(m+1)\Delta t} \zeta x(\tau) d\tau \\
&= -\frac{dx}{Kx} \int_{m\Delta t}^{(m+1)\Delta t} e^{-(dx/Kx+\alpha x)\tau} d\tau \\
&= \alpha x e^{-(dx/Kx+\alpha x)m\Delta t}
\end{aligned} \quad (44)$$

and

$$\alpha x = \frac{dx}{Kx(dx + Kx\alpha x)}(bx - 1); bx = e^{-(dx/Kx+\alpha x)\Delta t}. \quad (45)$$

Equation (42) can be simplified to:

$$\psi_x^{n\Delta t} = bx\psi_x^{(n-1)\Delta t} + \alpha x(\partial x)^{(n-1/2)\Delta t}. \quad (46)$$

We perform normal iterative calculations in the model area and use Eq. (41) to perform coordinate transformation calculations in PML:

$$\partial \bar{x} = \frac{1}{Kx} \partial x + \psi x. \quad (47)$$

Among them, ψx can be obtained by use of the recursive formula in (46). We conduct coordinate transformation on Eqs. (50) and (51) so that the recurrence formula can be calculated alternately in time:

$$\partial_x^{n-1/2} = \frac{1}{Kx} \partial_x^{n-1/2} + \psi_x^{n-1/2} = \frac{1}{Kx} \partial_x^{n-1/2} + \frac{\psi_x^n + \psi_x^{n-1}}{2}. \quad (48)$$

Substituting Eq. (46) into Eq. (48) gives:

$$\partial_x^{n-1/2} = \frac{1}{Kx} \partial_x^{n-1/2} + \frac{(bx + 1)\psi_x^{n-1} + \alpha x(\partial x)^{n-1/2}}{2}. \quad (49)$$

According to formula (49), $\partial_x^{n-1/2}$ can be solved by recursion. The electric field recursion formula under spatial discretisation, according to Yee's grid, the discretisation of formula (49) in time and space can be obtained:

$$\begin{aligned}
&\epsilon r \epsilon 0 \frac{E_z^{n+1}(i, j, k) - E_z^n(i, j, k)}{\Delta t} + \sigma \frac{E_z^{n+1}(i, j, k) - E_z^n(i, j, k)}{2} \\
&= \frac{H_y^{n+1/2}(i, j, k) - H_y^{n+1/2}(i-1, j, k)}{Kx\Delta x} \\
&\quad - \frac{H_x^{n+1/2}(i, j, k) - H_x^{n+1/2}(i-1, j, k)}{Ky\Delta y} \\
&\quad + \sum_{m=0}^{N-1} Z0x(m) \frac{H_y^{n-m+1/2}(i, j, k) - H_y^{n-m+1/2}(i-1, j, k)}{\Delta x} \\
&\quad + \sum_{m=0}^{N-1} Z0y(m) \frac{H_x^{n-m+1/2}(i, j, k) - H_x^{n-m+1/2}(i-1, j, k)}{\Delta y}.
\end{aligned} \quad (50)$$

In the above formula, the $Z0i(k)$ term contains the calculation of convolution, and the discrete convolution calculation is very complicated, but at the same time the $Z0i(k)$ term is in simple exponential form, so their sum can be obtained by recursive convolution, introducing a new set of auxiliary expressions ϕi :

$$\begin{aligned}
\phi_{ezi}^{n+1/2}(i, j, k) &= b i \phi_{ezi}^{n-1/2}(i, j, k) + a i [H_i^{n+1/2}(i, j, k) \\
&\quad - H_i^{n+1/2}(i-1, j, k)] / \Delta i \quad (i = x, y).
\end{aligned} \quad (51)$$

Substitute ϕi into the recursive expression to the electric field Ez component after finishing Eq. (51):

$$\begin{aligned}
E_z^{n+1}(i, j, k) &= \frac{1 - \frac{\sigma(m)\Delta t}{2\epsilon(m)}}{1 + \frac{\sigma(m)\Delta t}{2\epsilon(m)}} E_z^n(i, j, k) \\
&\quad + \frac{\frac{\Delta t}{\epsilon(m)}}{1 + \frac{\sigma(m)\Delta t}{2\epsilon(m)}} [\phi_{ezx}^{n+1/2}(i, j, k) - \phi_{ezy}^{n+1/2}(i, j, k)] \\
&\quad + \frac{\frac{\Delta t}{\epsilon(m)}}{1 + \frac{\sigma(m)\Delta t}{2\epsilon(m)}} \left[\frac{H_y^{n+1/2}(i, j, k) - H_y^{n+1/2}(i-1, j, k)}{Kx\Delta x} \right. \\
&\quad \left. - \frac{H_x^{n+1/2}(i, j, k) - H_x^{n+1/2}(i-1, j, k)}{Ky\Delta y} \right].
\end{aligned} \quad (52)$$

Equation (52) is the conductivity of σ and ϵ is the permittivity of the medium, $m = (i, j, k)$. In the simulation, the K value of the simulation area is set to 1, and the K value is gradually changed in the PML layer. And the layer is a finite thickness unit, σK and α change monotonously in the PML layer:

$$\sigma z(z) = \frac{\sigma \max |z - z0|^m}{d^m} \quad (53)$$

$$Kz(z) = 1 + (K \max - 1) \frac{|z - z0|^m}{d^m} \quad (54)$$

$$\alpha z(z) = \frac{\alpha \max |z - z0|}{d}, \quad (55)$$

where d is the thickness of the PML layer, $z0$ is the interface position of the PML layer close to the FDTD area, and $m=4$ is used in the simulation. The best value of $\sigma \max$ can be taken as $\sigma \max = \frac{m+1}{\sqrt{\epsilon} 150 \pi \delta}$, where δ is the cell size, $K \max = 5 \sim 11$, $\alpha \max = 0 \sim 0.05$. According to the frequency band of the ground penetrating radar, $K \max$ takes about 5, and $\alpha \max$ takes 0.008 as an appropriate value.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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