

# Earthquake magnitude estimation using the $\tau_c$ and $P_d$ method for earthquake early warning systems

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**Abstract** Earthquake early warning (EEW) systems are one of the most effective ways to reduce earthquake disaster. Earthquake magnitude estimation is one of the most important and also the most difficult parts of the entire EEW system. In this paper, based on 142 earthquake events and 253 seismic records that were recorded by the KiK-net in Japan, and aftershocks of the large Wenchuan earthquake in Sichuan, we obtained earthquake magnitude estimation relationships using the  $\tau_c$  and  $P_d$  methods. The standard variances of magnitude calculation of these two formulas are  $\pm 0.65$  and  $\pm 0.56$ , respectively. The  $P_d$  value can also be used to estimate the peak ground motion of velocity, then warning information can be released to the public rapidly, according to the estimation results. In order to insure the stability and reliability of magnitude estimation results, we propose a compatibility test according to the natures of these two parameters. The reliability of the early warning information is significantly improved though this test.

**Keywords** Earthquake early warning · Earthquake magnitude ·  $\tau_c$  method ·  $P_d$  method

## 1 Introduction

China is one of the most seismically active regions in the world and numerous disastrous earthquakes have occurred

throughout history. In the twentieth century, about one-third of global earthquakes took place in China, resulting in half of the world's death toll (Zhang 2008). Furthermore, more than half of China's land is located in the upper VII degree (Chinese intensity) high intensity area, including 23 capital cities and 2/3 of large cities with populations over a million people. Earthquakes occurring in these areas bring great loss of life and property. In 1976, a  $M_w$  7.5 earthquake struck Tangshan, causing over 240,000 deaths and destruction of the entire city. In May 12, 2008, the great Wenchuan earthquake resulted in 70,000 deaths and 20,000 missing people. Two years later, a  $M_w$  7.1 earthquake occurred in Yushu, Qinghai province, which resulted in nearly 300 deaths. All these painful visions remind us that we should put greater emphasize on earthquake prevention work, and take some effective actions to reduce related losses. Natural disaster itself is inevitable, but if we could take some reasonable prevention measures in advance, losses caused by these disasters will be reduced effectively. Earthquake early warning (EEW) systems are one of the new approaches which can reduce earthquake disaster effectively throughout the world.

By setting real-time transmission seismic monitoring stations near the potential source areas, once an earthquake occurs, we can obtain related information rapidly. Then, earthquake parameters such as epicenter locations and magnitude can be determined in a short time after the event. Based on the basic principle that P-waves travel faster than destructive S-waves and surface waves, and since electromagnetic waves travel faster than seismic waves, EEW systems then can release warning information to nearby cities that may suffer from the earthquakes prior to destructive wave arrival. The “lead-time,” i.e., the time between the warning issue and the arrival of the strongest shaking, may vary from a few seconds to tens of seconds,

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depending on the distance from the epicenter to the target site. People will therefore be able to take some necessary emergency escape measures to protect themselves. For example, they can move out of the most dangerous areas to reduce casualties, slow down or stop the high-speed trains to prevent derailment, turnoff gas to reduce fire, among other precautions. Several countries or regions in the world have set up EEW systems aiming at special establishments, single cities, or even big regions, and many of them have achieved certain actual results (Kamigaichi et al. 2009; Allen et al. 2007; Iglesias et al. 2007; Erdik et al. 2003; Zollo et al. 2009; Hsiao et al. 2009). After the large Wenchuan earthquake, we started to construct a prototype EEW system in Fujian province, southeast coast of China, starting in 2009. The system have been finished and online testing began at the end of 2012. In the Beijing capital region (BCR), a prototype EEW system was constructed as well (Peng et al. 2011). The construction of the EEW system for the BCR was conducted jointly by the Institute of Geophysics at the China Earthquake Administration, and the Department of Geosciences, Taiwan University. The system began to provide an internal prototype of the early warning service in February 2010. In this paper, we focus on earthquake magnitude estimation using the characteristic periods ( $\tau_c$ ) and the peak displacement amplitude ( $P_d$ ) methods in the first 3 s after the arrival of the first P-wave, respectively. Our research results will provide a reference for the whole EEW system construction.

## 2 Method introduction

A complete EEW system should include a real-time earthquake location module, a real-time magnitude estimation module, the ability to estimate seismic intensity in the target areas, and a warning information release module. Among these functional modules, real-time magnitude estimation is the most important and also the most difficult to obtain. It is almost impossible to use conventional methods to calculate earthquake magnitude, as there is always limited information that can be used, due to the high timeliness requirement of EEW system. Therefore, we use special methods to estimate earthquake magnitude rapidly and reliably. Currently, several practical real-time magnitude calculation methods that are widely used can be summarized in three categories: calculations related to predominant periods or frequencies, such as  $\tau_p^{\max}$  and  $\tau_c$ , calculations related to amplitude (i.e.,  $P_d$ ), and calculations related to intensity, such as the  $M_I$  method. In this paper, we mainly introduce two methods that are applied in this paper, namely the  $\tau_c$  and  $P_d$  methods.

### 2.1 $\tau_p^{\max}$ and $\tau_c$ method

In 2003, Allen and Kanamori first introduced a dominant period character calculation method, referred to as the  $\tau_p^{\max}$  method. The  $\tau_p^{\max}$  value can be extracted from real-time velocity records. Allen and Kanamori (2003), Kanamori (2005), Olson and Allen (2005), Wu and Kanamori (2005, 2008), and Wu and Li (2006) conducted a series of studies on this method. The dominant period character,  $\tau_p^{\max}$ , calculation equation is as follow:

$$\tau_i^p = 2\pi\sqrt{\frac{X_i}{D_i}}, \quad (1)$$

where  $X_i = \alpha X_{i-1} + x_i^2$ ,  $D_i = \alpha D_{i-1} + (dx/dt)_i^2$ ,  $\tau_i^p$  is the predominant period measured at  $i$  s,  $x_i$  is the ground motion velocity,  $X_i$  is ground motion velocity derivative square value after smoothing,  $\alpha$  is a constant smoothing factor which equals 0.999. Wolfe's (2006) research indicates that the  $\tau_p^{\max}$  parameter is a nonlinear function of spectral amplitude and period that gives greater weight to higher amplitudes and higher frequencies in the spectrum. As a result of unilateral weighting, the stability of this method will be highly affected. According to Wu's research, the accuracy and reliability of this method is not only affected by sampling rate, but also by pretreatment processes. When different low-pass filters and time window length are chosen, the predominant period calculation result will change as a result.

$\tau_c$  is the average period of ground motion within some specified time window. It was first introduced by Kanamori (2005) and is a modified version of the method originally developed by Nakamura (1988). The  $\tau_c$  parameter is calculated from the first several seconds of P-wave data as follows:

$$\tau_c = \frac{2\pi}{\sqrt{r}} \quad r = \frac{\int_0^{\tau_0} \dot{u}^2(t) dt}{\int_0^{\tau_0} u^2(t) dt}, \quad (2)$$

where  $u$  is the high-pass filtered displacement of the vertical component ground motion and  $\dot{u}$  is the velocity differentiated from  $u$ . The integration is taken over the time interval  $[0, \tau_0]$  after the station is triggered. If  $\tau_0$  is too long, the response time that can be used after received the warning information will be shortened. However, if  $\tau_0$  is too short, earthquake magnitude cannot be estimated accurately. Kanamori et al. (1997) considered all these factors and suggested setting  $\tau_0$  to 3 s. He also believes that we can estimate earthquake magnitude accurately according to the  $\tau_c$  value, as fault rupture has been basically completed for earthquakes below  $M_w$  6.5 within this time. For greater earthquakes up to  $M_w$  7.0–7.5, the  $\tau_c$  value can also be used to make a quick judgment of whether the

earthquake is destructive or not. The  $\tau_c$  method is one of the main adopted methods used in this paper.

### 2.2 $P_d$ method

As is well known, the amplitude of ground motion will gradually attenuate with the increase of epicenter distance when traversing the earth’s crust due to factors such as geometric diffusion, reflection, and refraction. Analogously, if the amplitude of the P-wave (or few seconds after the P-wave) is known, earthquake magnitude can be calculated according to this attenuation relation. But as we have mentioned, there is a significant difference between the warning magnitude calculated from the initial amplitude and traditional earthquake magnitude calculations due to the limited available information at this time. Related research results (Ma 2008) indicate that peak displacement shows a more robust correlation with earthquake magnitude than initial velocity or initial acceleration amplitude, and the displacement parameter is frequently adopted to calculate the magnitudes of earthquakes monitored by global early warning systems. Normally, the displacement amplitude is extracted from the vertical component in order to minimize the effect of the S-wave arrival.

$P_d$  is the maximum displacement amplitude during the initial 3 s of the P-wave arrival, when filtered by a one-way high-pass Butterworth filter with a cutoff frequency of 0.075 Hz (Wu et al. 2007). Using a large number of records from southern California, Japan and Taiwan, Wu et al. (2007) obtained a relationship between  $P_d$  and  $M_w$ ; their research also indicate that there is a good correlation between  $P_d$ , peak ground motion of velocity (PGV), and PGA, and therefore,  $P_d$  can also be a useful criterion for seismic intensity estimation of warning target-area. Lancieri and Zollo (2008) carried out further investigation of this method. They suggested that earthquake magnitude can also be calculated using the peak amplitude of the P-wave initial displacement,  $P_d$ , in 2–4 s time window, or S-wave initial displacement,  $S_d$ , in 1–2 s time window. Other scholars (Ma 2008) also studied this method, and indicated that  $P_d$  is a robust parameter of earthquake magnitude. Therefore, in this paper, we also adopt it as one of the main methods to determine earthquake magnitude.

### 3 Data

In order to estimate earthquake magnitude accurately using the initial P-wave information, a certain quantity of seismic records is needed. Magnitude intervals of statistical samples, and event numbers in each magnitude class should be as complete as possible. In this paper, we collected 55 events recorded by the KiK-net network in Japan ( $M_{jma}$

4.0–7.3) and 87 Wenchuan aftershock events ( $M_L$  3.5– $M_s$  6.3), the distinctions between different magnitude units are ignored, and all values are referred to in general, as  $M$ . Records of the Chi–Chi mainshock and the Wenchuan mainshock are also used in order to test the reliability of relationships obtained in this study, but they were not used in the calculation. Details of these events’ distribution, magnitude, and location are shown in Figs. 1 and 2. After some background data processing, including a baseline correction, we adopted the real-time simulation algorithm proposed by Jin et al. (2003) to transform acceleration

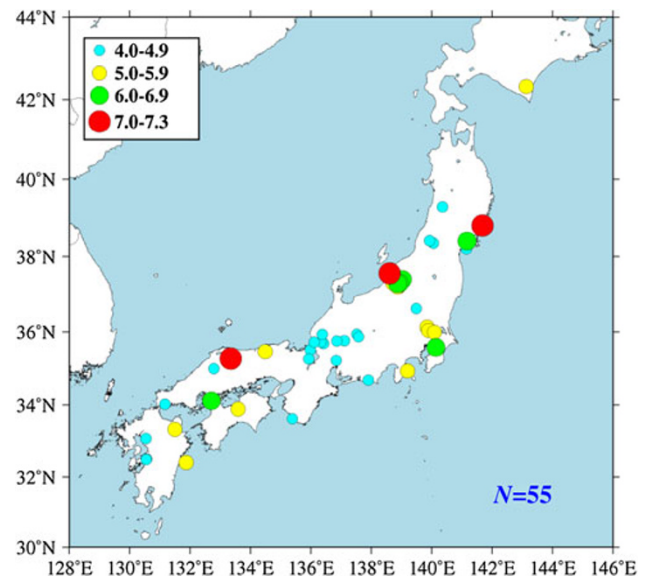


Fig. 1 Plot of the KiK-net earthquake events used in this study

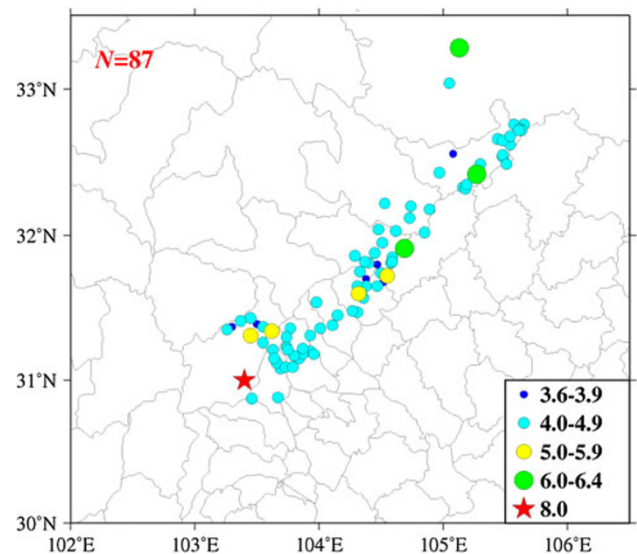


Fig. 2 Wenchuan aftershock events used in this study

records into velocity and displacement so as to satisfy the requirements of the algorithm. Next, we manually picked the P-wave arrival time on the vertical component of acceleration records in each station.

P-waves are the most developed, and easily identified, on the vertical component. Therefore, we usually extract related information from the vertical component such as P-wave arrival time, azimuth angle, and first motion direction. Similarly, we can extract some characteristic parameters that reflect magnitude from the vertical component, e.g., the initial peak displacement and the predominant period. In this paper, we mainly utilize vertical component records for our investigation. According to the earthquake travel-time equation, seismic phases will become extremely complicated due to phenomena such as reflection, and refraction when the distance to the epicenter exceeds 100 km and the  $P_g$  phase is not the first arrival phase. These factors yield results in which the relevant feature parameter cannot be picked up reliably in the estimation of earthquake magnitude, thereby leading to erroneous results. Therefore, in order to get a reliable magnitude estimation result, and also fulfill the rapid time requirements of the EEW system, only those records within 30 km of the epicenter are selected. Within this range, P-waves are relatively simple and easy to recognize. These results are obtained promptly, thereby satisfying the need for rapid results when using the EEW system. In other words, the features extracted from these stations can insure the reliability and accuracy of earthquake magnitude estimation results. Due to the complexity of the seismic source, even for the same event, characteristic parameters extracted from different stations will be different, resulting in large scatter among measurements. However, the use of the average value for the same event will improve the discrepancy of estimation results, and this can also effectively reduce the uncertainty of seismic magnitude estimation results caused by the natural variability of early warning parameters at different stations.

We also acknowledge that records should have a certain signal-to-noise ratio (SNR) in order to obtain reliable results, that is to say all selected records should be sufficiently accurate. For large earthquakes, near-field stations can easily meet the SNR requirements; using the SNR to filter data is mainly to insure the reliability of magnitude estimation for moderate and small earthquakes. Therefore, following the approach of Wu and Li (2006), we selected records of large earthquakes of which amplitude of 3 s acceleration records after P-wave trigger exceeds 5 gal and the ratio of the average amplitude within this window time length to the average before triggering exceeds 10. According to this selection principle, 253 vertical seismic records were selected (Fig. 3).

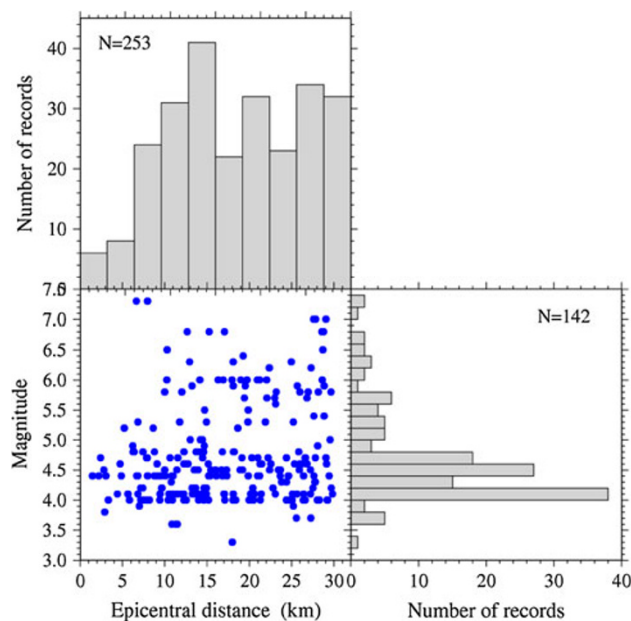


Fig. 3 Events and records used in this study

## 4 Statistical results

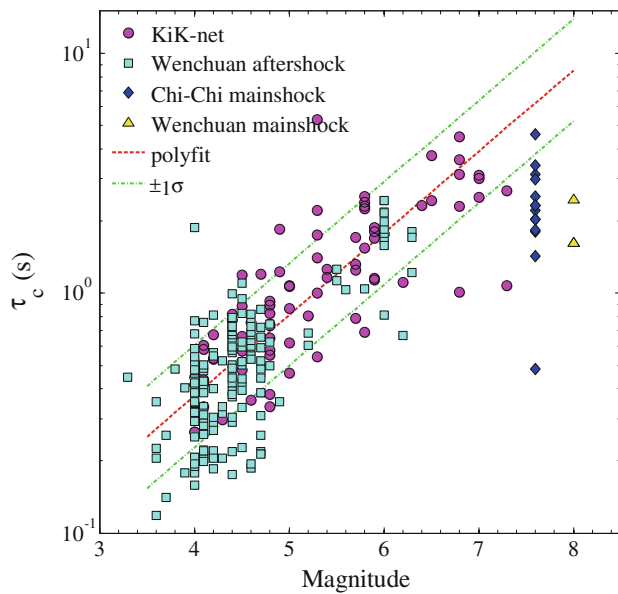
Based on the above data, we extracted the predominant period,  $\tau_c$ , and the initial peak displacement amplitude,  $P_d$ , from each record. We then derived two relationships between these two characteristic parameters, and determined the earthquake magnitude by statistical regression. The regression formulae are  $M = a + b \log_{10}(\tau_c)$  and  $M = a + b \log_{10}(D) + c \log_{10}(P_d)$ , where,  $M$  is earthquake magnitude,  $D$  is the distance from the epicenter to the recording station, and  $a$ ,  $b$ ,  $c$  are coefficients. Results are given in the following paragraphs.

### 4.1 $\tau_c$ method

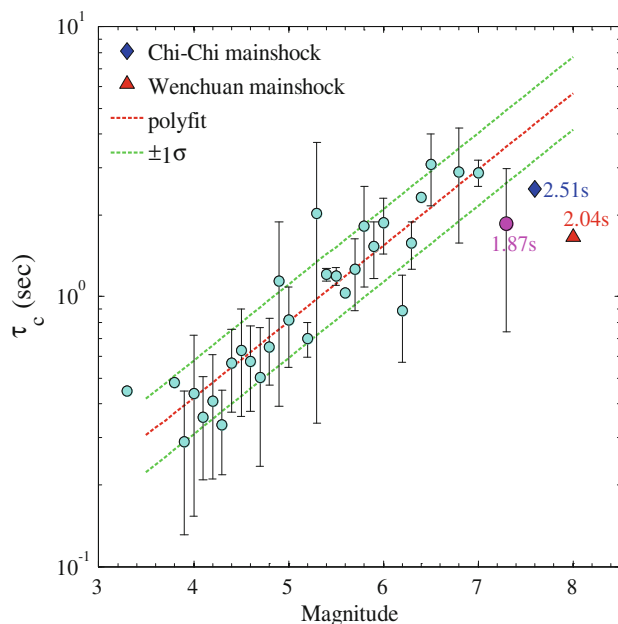
$$\log_{10}(\tau_c) = 0.34 \times M - 1.79 \pm 0.21, \quad (3)$$

$$M = 2.16 \times \log_{10}(\tau_c) + 5.22 \pm 0.65. \quad (4)$$

As shown in Fig. 4 and Eqs. (3) and (4), the period parameter  $\tau_c$  is closely related to the earthquake magnitude, and the standard deviation of magnitude estimation using this method is  $\pm 0.65$  units for all events. Despite some specific values that deviate from the trend, most features extracted from KiK-net records and Wenchuan aftershock records are all have obvious correlation with earthquake magnitude. Just as mentioned above,  $\tau_c$  values extracted from different stations vary significantly, even in the same event. Therefore, in this paper, we take the average of  $\tau_c$  results for each magnitude bin and obtain the relationship between the average  $\tau_c$  value and earthquake magnitude. Results are shown in Fig. 5 and Eqs. (5) and (6).



**Fig. 4** Statistical relationships between  $\tau_c$  and earthquake magnitude



**Fig. 5** Statistical relations between average value of  $\tau_c$  and magnitude

$$\log_{10}(\tau_c) = 0.28 \times M - 1.48 \pm 0.14, \tag{5}$$

$$M = 2.94 \times \log_{10}(\tau_c) + 5.30 \pm 0.46. \tag{6}$$

It can be seen from Fig. 5 that when the average value of  $\tau_c$  was used to calculate the earthquake magnitude, the discreteness of the statistical relationship was improved, and that the standard deviation of the magnitude calculation reduces to  $\pm 0.46$  units. As there are only two station records that can be used for  $M_{jma}$  6.2, the average value of  $\tau_c$  for this

magnitude deviates substantially. In Figs. 4 and 5, we also added the first two stations’ calculation results from the Wenchuan main shock (PXZ and DXY stations), and 13 stations’ calculation results from the Chi–Chi main shock (within 30 km of the epicenter). We took the average value of each event separately, and they do not participate in the statistical results of the relevant equations. From the above analysis, we find that when the  $M$  exceeds 7.0 (including an event with  $M_{jma}$  7.3 recorded by KiK-net,  $\tau_c$  average value is 1.87 s), an obvious magnitude “saturation” phenomenon will exist. If we still use the period parameter  $\tau_c$  in the calculation, then the earthquake magnitude will be underestimated significantly. Kanamori (2005) did a numerical simulation to study the  $\tau_c$  method using the Sato rupture model, and his result shows that the  $\tau_c$  method is only effective for earthquakes under  $M_w$  6.5. As there is only 3 s information available, it’s impossible to make an exact estimation of earthquake magnitude for larger earthquakes. In other words, “saturation” phenomena will definitely occur.

We can also conclude that differences in the  $\tau_c$  value extracted from different regions are quite small. Wu and Li (2006) and Wu and Kanamori (2008) obtained a relationship between the average value of  $\tau_c$ , and  $M_w$ , of the first six stations that were initially triggered in every event using 54 seismic events ( $M_w$  4.1–8.3) that occurred in Taiwan, southern California, and Japan. The relationship is:

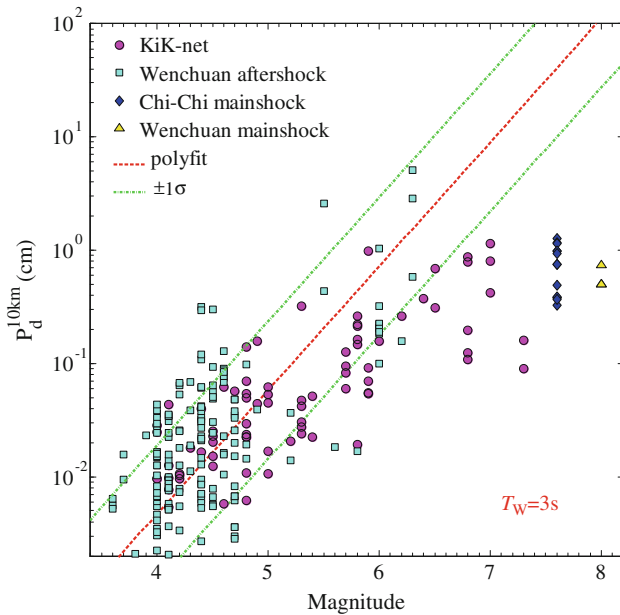
$$M_w = 3.373 \times \log_{10}(\tau_c) + 5.787 \pm 0.412. \tag{7}$$

Due to different seismic records used in research, there are some differences between these three relations. Under the same circumstances, earthquake magnitudes calculated from the equation in this paper will be a bit smaller than the equations recommended by Wu. Meanwhile, it can be observed that slopes of these three lines are basically consistent, which proves once again that the  $\tau_c$  parameter does not show a strong correlation with the specific considered area.

#### 4.2 $P_d$ method

In this paper, we refer the method of Wu and Li (2006) and use the same definition to calculate the  $P_d$  parameter.  $P_d$  is the maximum displacement amplitude within 3 s after initial P-wave arrival. Earthquake location results are also needed when using this method, which is different from the  $\tau_c$  method, so in order to insure the reliability of the statistical relationship, we make a careful check of each event, and make sure that they all have accurate qualities. Then, we obtained a relationship between  $P_d$  and earthquake magnitude, as shown in Fig. 6.

$$M = 0.91 \times \log_{10}(P_d) + 0.48 \times \log_{10}(D) + 5.65 \pm 0.56. \tag{8}$$



**Fig. 6** Statistical relations between  $P_d$  and magnitude

In Fig. 6,  $P_d^{10km}$  is the  $P_d$  value normalized at a reference distance of 10 km. Magnitude estimation variance of using this method is  $\pm 0.56$  units—it's close to the  $\tau_c$  method. Also as shown in Fig. 6, most  $P_d$  values are within the scope of  $\pm 1\sigma$  for earthquakes lower than  $M$  6.5. However, when  $M$  exceeds 6.5, an obvious “saturation” phenomenon occurs, and earthquake magnitude will be underestimated at this time. Lancieri and Zollo (2008) pointed out that not only does the  $P_d$  parameter have this problem, but when the  $S_d$  parameter is used, the “saturation” phenomenon also exists, but it occurs at larger magnitudes. Different “saturation” phenomena occur when calculating magnitude by displacement amplitude of P- and S-waves, which can be explained by isochrones theory. Generally speaking, the isochrones area should be more than or equal to the ruptured fault area in the same period of time (this depends on the relative position between the seismograph and fault). Within the short time after P- or S-wave generation, isochrones cover the fault area that radiated high-frequency ground motion. But as S-wave propagates more slowly than P-wave, the fault area surrounded by S-wave isochrones is definitely larger than the fault area surrounded by P-wave isochrones at the same time, and therefore the saturated magnitude of  $S_d$  will be larger than  $P_d$ .

The study of Murphy and Nielsen (2009) indicates that just 3 s after a station is triggered, the isochrones cover 80 % of the whole rupture zone, and this can also explain the statistical relationship between  $P_d$  and earthquake magnitude. On the other hand, this also proves that under-sampling of the fault is the main cause of the magnitude “saturation” in the estimation process. When

the time window length expands to 4 s, or when we use the  $S_d$  parameter, we can sample larger faults, then the saturation magnitude can expand to 7.5. But for even bigger earthquakes, we cannot estimate the magnitude accurately by the initial P–S-wave displacement amplitude.

As we have previously mentioned, according to Wu's research, there is a good correlation between  $P_d$ , PGV, and PGA, but particularly for PGV. This relationship can be used to make a quick judgment of whether or not the earthquake is expected to be destructive. Through the relationship between  $P_d$  and PGV, we can estimate the seismic intensity in each station rapidly and develop a relevant algorithm including an “onsite warning” model (Kanamori 2005; Zollo et al. 2010). Here, we take the similar approach and obtain a statistical relationship between displacement amplitude  $P_d$  and PGV. Results are shown in Fig. 6 and Eq. (9).

$$\log_{10}(\text{PGV}) = 0.65 \times \log_{10}(P_d) + 0.79 \pm 0.40. \quad (9)$$

It can be seen from the above results that there is a good correlation between  $P_d$  and PGV, and that the relation has no regional bias. Even if the time-window length is only 3 s, PGV can be accurately estimated by the  $P_d$  value (variance of the statistical equation is  $\pm 0.40$ ). But from the above figure, we also observed that for large earthquakes (such as Chi–Chi and Wenchuan earthquakes), the relations between  $P_d$  and PGV may be very different for moderate and small events, which may be due to the saturation effect on the  $P_d$  value. We need more large events to verify our conjecture, in order to obtain a new relationship that is appropriate for large earthquakes. In brief,  $P_d$  is not only limited to earthquake magnitude estimation, but can also be used in continuous prediction of ground motion intensity (Fig. 7).

## 5 Compatibility test of two parameters

Once the EEW information is released to the public, it will certainly have a significant influence on social order, people's daily lives, and normal production. Therefore, the reliability of warning information is very important. By investing in new technology, we can improve the accuracy of early warning information. Wu and Kanamori (2008) believe that both the  $\tau_c$  and  $P_d$  parameters can be rough criteria used to release preliminary warning alerts. They found that if  $\tau_c > 1$  s, and  $P_d > 0.5$  cm at the same site, then the earthquake is likely to be a destructive one, and warning information should be released quickly. A similar approach has been used in a recent paper by Zollo et al. (2010), who developed a “threshold based” methodology to EEW. Their method is based on the combined use of  $P_d$  and  $\tau_c$  for the definition of local alert levels.

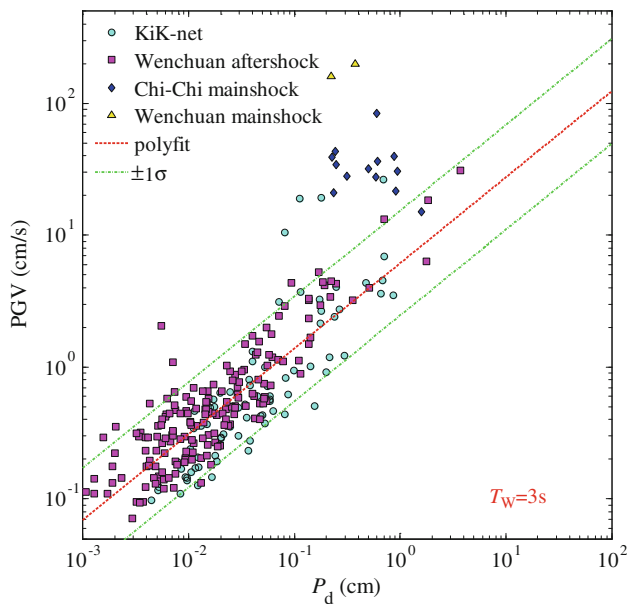


Fig. 7 Statistical result between  $P_d$  and PGV

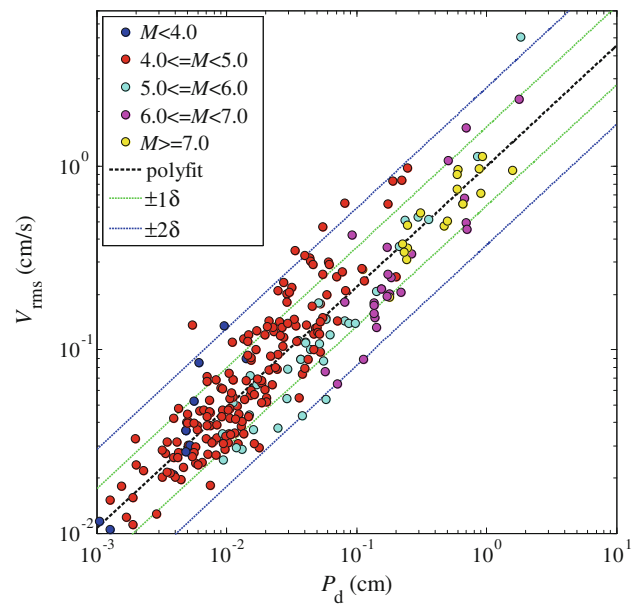


Fig. 9 Statistical correlation among of  $P_d$  and  $V_{rms}$

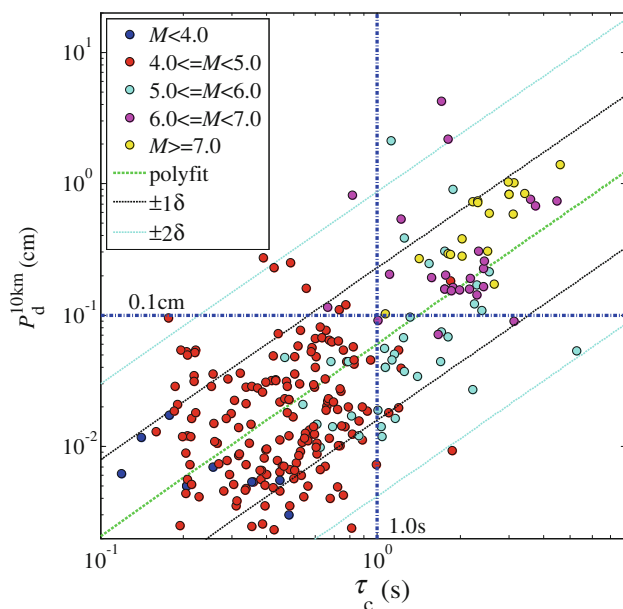


Fig. 8 Statistical relations between  $P_d$  and  $\tau_c$

From relevant analysis in this paper, we can conclude that,  $\tau_c$  is a good estimator of the period and frequency characteristics of ground motion, while  $P_d$  provides a good estimate of the amplitude and intensity characteristics of ground motion. For a large enough earthquake, these two types of ground motion characteristic parameters must be fully compatible. That is, the frequency spectrum of the earthquake should be wide enough, and the intensity of the earthquake should be large enough as well. If the  $P_d$ - $\tau_c$  compatibility test is not satisfied, warning information will

not be released to the public. This test has the effect of eliminating explosions and other false triggers. The compatible relationship between  $\tau_c$  and  $P_d$  is shown in Fig. 8 and formula (10).

$$\log_{10}(P_d^{10km}) = 1.44 \times \log_{10}(\tau_c) - 1.03 \pm 0.58. \quad (10)$$

Figure 8 shows the relationship between  $\tau_c$  and  $P_d$ , and most of the data are within the range of two times the variance. Based on variance analysis theory, we can make the following judgments: data within  $\pm 1\sigma$  can be judged as “deterministic” events, those data within  $\pm 1\sigma$  to  $\pm 2\sigma$  can be judged as “possible” events, and data beyond  $\pm 2\sigma$  are considered “unlikely” events. Most interference or abnormal signals are eliminated using this test. During research, we also noticed that for earthquakes greater than  $M$  6.0, the  $\tau_c$  values are all exceed 1.0 s, while all  $P_d$  values exceed 0.1 cm, which are slightly different values than Wu’s results. One reason for this may be that we selected smaller magnitude events than Wu, and we will look into this matter in further research.

By the definition of  $P_d$ , it just reflects the peak intensity of the initial 3 s but not the whole average intensity. In order to insure the reliability of early warning information, we propose the following average intensity parameter of ground motion: the root mean square of velocity,  $V_{rms}$ , which is also computed in a 3 s time window:

$$V_{rms} = \frac{1}{n} \sqrt{\sum_{i=1}^n V_{i,ew}^2 + \sum_{i=1}^n V_{i,ns}^2 + \sum_{i=1}^n V_{i,ud}^2}. \quad (11)$$

The variables  $V_{ew}$ ,  $V_{ns}$ , and  $V_{ud}$  are the velocity records in the EW, NS, and UD components, respectively; the

subscript  $i$  in each expression is the mean data point of the record. Therefore, we use  $V_{\text{rms}}$  to reflect the average intensity of ground motion after triggering. After that, we obtain the relationship between  $P_d$  and  $V_{\text{rms}}$ , as shown in Fig. 9 and Eq. (12).

$$\log_{10}(V_{\text{rms}}) = 0.64 \times \log_{10}(P_d) - 0.03 \pm 0.20. \quad (12)$$

Because  $P_d$  and  $V_{\text{rms}}$  are amplitude-related parameters, correlation between these two parameters is obviously better than the correlation between  $\tau_c$  and  $P_d$ . Similarly, data within  $\pm 1\sigma$  are termed “deterministic” events, those data within  $\pm 1\sigma$  to  $\pm 2\sigma$  are “possible” events, and those data beyond  $\pm 2\sigma$  are “unlikely” events. Through a compatibility test, we can effectively eliminate the abnormal interferences and make sure that warning information have a certain degree of reliability and accuracy, and therefore, the efficiency of the EEW system can be improved.

## 6 Conclusions

As a new and effective way to mitigate earthquake disasters, EEW systems have received more and more attention over the past two decades. Several countries or regions have set-up their own EEW systems, or are still in the testing stages. In this paper, we focus on the earthquake magnitude estimation method used in EEW systems. Based on 142 earthquake events and 253 records, we extracted two characteristic parameters,  $\tau_c$  and  $P_d$ , and obtained relationships between these two parameters and earthquake magnitude. In order to insure the stability and reliability of magnitude estimation results, we proposed the compatibility test method according to the natures of these parameters, such as  $\tau_c$  is a period parameter and  $P_d$  is a peak intensity parameter, and  $V_{\text{rms}}$  is an average intensity parameter. From the analysis results presented in this paper, we can conclude that earthquake magnitudes can be estimated using these two above parameters.

What needs to be stressed is that, factors such as source processes, spreading medium, site condition, and instrument response, are all involved in earthquake magnitude determination. Information that can be used in early warning is rather limited, so the combined use of amplitude and period parameters is expected to provide more stable and reliable magnitude estimations. In addition, process of earthquake is very complicated, and single period parameters or amplitude parameters can reflect earthquake scale to some extent. But earthquake magnitude estimation results using these two parameters may be very different, therefore, how to comprehensively apply the two parameters more reasonably or develop new feature parameter is a problem that deserves further study.

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