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A modified exponential model for reported death toll during earthquakes*

Xinyan Wu and Jianhua Gu†

Institute of Geophysics, *China Earthquake Administration*, *Beijing* 100081, *China*

Abstract Reliable earthquake death toll estimate can provide valuable references for disaster relief headquarters and civil administration departments to make arrangement and deployment plan during post-earthquake relief work, thus increasing the efficiency of the relief work to a certain extent. In this study, we acquired the death toll data of Wenchuan earthquake, fitted the data using modified exponential curve and compared the result with that of the exponential function. Experimental verification with Chi-Chi earthquake and Kobe earthquake data shows that the fitted result by modified exponential curve is more satisfactory. The final death toll resulting from future destructive earthquakes can be estimated by the acquired fitting function.

Key words: **earthquake; death toll; modified exponential curve**

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1 Introduction

Reducing the casualties is an important goal of the work of earthquake prevention and disaster reduction. Many experts and scholars in China and abroad have carried out explorative research on the law of earthquake death toll. Lomnitz (1970) studied the relationship between occurring time and final death toll of 22 *M*7 to *M*8 earthquakes occurred in Chile from 1570 to 1960. Christokov and Samardjieva (1984) put forward a relationship between death toll and population density and magnitude. Samardjieva and Oike (1992) and Wyss (2005) separately made statistical on data of earthquake death toll for Japan and Himalaya region. Fu et al (1993) had also studied relationship between earthquake death toll and magnitude and occurring time, but he didn’t concern about the time history of the reported death toll from an earthquake.

Since 1995, the time history of the reported death toll from an earthquake has been studied by a number of scholars and gratifying progress has been made. Hong et al (1995) made an analysis and induction about the death toll and building damage from the Hyogo-ken Nanbu, Japan, earthquake; After studying the damage of Kobe earthquake and Lijiang earthquake in Yunnan province, China, Wang and Yang (1997) found that the death tolls from the earthquakes varied with time, and pointed out the significance of time history curve in disaster response and decision making; Gao and Jia (2005) normalized the reported death tolls of dozens of earthquakes, fitted the time-dependent variation in the reported death tolls by quartic polynomial and satisfactorily fitted the relation between earthquake death toll and time after the quake, though the physical significance of the model is uncertain; Liu and Wu (2006) suggested using an exponential model to describe the relation between earthquake death toll and time after the quake.

Based on the death toll data of Wenchuan earthquake collected after the quake, we studied the relation between the number of fatalities (*N*) reported after the event and time (*t*), and obtained a satisfactory result. The method can be used to rapidly estimate the time-related variation in death toll after a strong earthquake, providing references for directing disaster relief work and deploying rescue teams.

2 Data source

The M_S8.0 earthquake, which hit Wenchuan county, Sichuan province at 14:28:04 on May 12, 2008, has caused heavy casualties and property losses to Sichuan and neighboring provinces. Following the quake, we promptly collected the death toll data released by the

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† **Corresponding author. e-mail**: jh.gu@263.net

State Council Information Office and State Council Earthquake Disaster Relief Headquarters (Table 1), and acquired a curve of the number of deaths reported before June 2, 2008, versus time (Figure 1).

Figure 1 shows that N increases with t, reaching 90% of the finally reported figure in two weeks, by June 2, N is very close to the current official figure (excluding those missing) and the growth rate slows down to near zero.

Table 1 Number of reported deaths after Wenchuan earthquake (up to June 2, 2008)

No.	Time	Hours after the shock t/h	Number of deaths N	No.	Time	Hours after the shock t/h	Number of deaths N
1	16:00, May 12	1.5	4	15	12:00, May 21	213.5	41353
2	18:00, May 12	3.5	107	16	10:00, May 22	235.5	51151
3	21:00, May 12	6.5	157	17	19:00, May 22	244.5	55239
4	22:00, May 12	7.5	592	18	12:00, May 23	261.5	55740
5	22:50, May 12	8.3	8533	19	12:00, May 24	285.5	60560
6	17:00, May 13	26.5	11921	20	12:00, May 25	309.5	62664
7	19:00, May 13	28.5	12012	21	12:00, May 26	333.5	65080
8	14:00, May 14	47.5	14866	22	12:00, May 27	357.5	67183
9	16:00, May 15	73.5	19500	23	12:00, May 28	381.5	68109
10	14:00, May 16	95.5	22069	24	12:00, May 29	405.5	68516
11	14:00, May 17	119.5	28881	25	12:00, May 30	429.5	68858
12	14:00, May 18	143.5	32477	26	12:00, May 31	453.5	68977
13	16:00, May 19	169.5	34073	27	12:00, June 1	477.5	69016
14	18:00, May 20	195.5	40075	28	12:00, June 2	501.5	69019

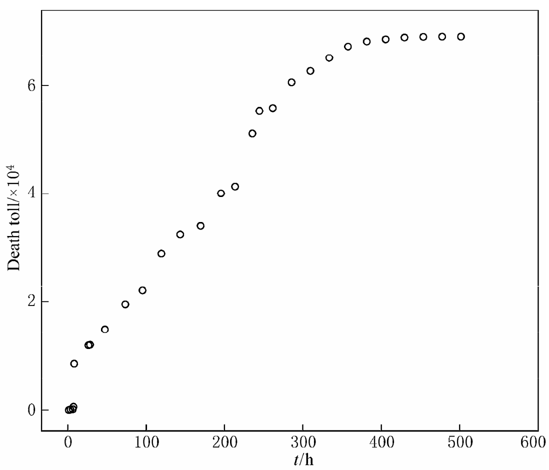

Figure 1 Time history of the reported deaths during Wenchuan earthquake.

3 Method

Liu and Wu (2006) proposed using an exponential model to describe the relation between the number of deaths from an earthquake and time after the quake, and believed that the larger the number of unfound deaths is, the easier it will be to find more deaths. Let N_0 be the final number of deaths, then $N_0–N$ is the number of unfound deaths. If the disaster relief efficiency is a constant, α, then $\alpha(N_0–N)$ is the number of deaths found in one unit of time, i.e.

$$\frac{\Delta N}{\Delta t}=\alpha(N_0-N)\,. \tag{1}$$

By finding the primitive function of the above equation, we get the relation between N and t

$$N=N_0(1-\mathrm{e}^{-at})\,. \tag{2}$$

The resultant time history curve of the reported deaths from 21 earthquakes in China and abroad (Gao and Jia, 2005) shows that the death toll rises rapidly within the first few days after the quake. With the passing of time and deepening of relief work, the possibility of finding survivors phases down and the growth rate of death toll slows down to saturation value. This pattern is reflected by the variation curve of death tolls at different times after the Wenchuan earthquake shown in Figure 1, though the maximum growth rate appears at 235.5 h after the quake instead of at the first few time points. In view of such an extremely huge earthquake, the above phenomenon is justifiable. As traffic and communications were cut off after the earthquake, it was impossible to report all the casualties to higher authorities within a short period of time, and the official figures released in the first few days could only involve damage and losses of some of the disaster-hit areas. Only after all-around rescue operations were carried out by large numbers of relief teams in the hard-hit areas, could the casualty figures be completely presented.

The above rule is completely in conformity with the phenomenon described by modified exponential curve in the long-term time series prediction model, therefore, we can fit the relation between N and t using this curve.

$$N = N_0 + a \cdot b^t, \tag{3}$$

where N_0, a and b are unknown constants, $N_0>0$, $a\neq 0$, $0<b\neq 1$. N_0 is the final number of possible deaths, and b is the average growth rate.

In fact, equation (2) is a specific form of equation (3) and equation (3) is the generalization of equation (2). Equation (3) takes b while equation (2) takes $e^{-\alpha}$ as the base. In equation (3), all the three coefficients are to be determined by the sequence of number itself, and they obviously approximate the data more closely than those in equation (2).

As N_0, a and b are unknown, the 3-sum algorithm in statistics is suitable for equation (3) (Jia, 2006). By the so-called 3-sum algorithm, the observed data are divided equally into three sections (each containing n data), summation is performed for each section and then the three equations, each of which contains three unknown parameters, are solved. The expressions for the parameters are as follows:

$$\begin{cases} b = \left(\dfrac{S_3 - S_2}{S_2 - S_1}\right)^{\frac{1}{n}} \\ a = (S_2 - S_1)\dfrac{b-1}{b(b^n - 1)^2} \\ N_0 = \dfrac{1}{n}\left(S_1 - 1\dfrac{ab(b^n - 1)}{b-1}\right) \end{cases}, \tag{4}$$

where S_1, S_2 and S_3 are the total sums of the observed values, respectively.

Following is the fitting and comparison of the death toll from Wenchuan earthquake using the two methods respectively.

4 Results

4.1 Fitted result by exponential function

Based on Wenchuan earthquake death toll data as of June 2, we carried out computation with SPSS15.0 statistical software and got the following exponential model:

$$N_{\text{Wenchuan}} = N_0(1 - e^{-at}), \tag{5}$$

where N_0 and a have their errors of 88877±11557 and (0.004±0.0 005) h, respectively, at 95% confidence level.

Substituting $t = 717.5$ into the model, we get the predicted death toll figure of 81 663 at 12:00 on June 11, as compared with the actual figure of 69 146, the relative error being 18%.

Using the death toll data as of May 21, we would get the errors 51993±1 7447 and 0.007±0.004/h for N_0 and a, respectively, at 95% confidence level.

Substituting $t = 717.5$ into the model, we get the predicted death toll figure of 51 651 at 12:00 June 11, nearly 20 000 fewer than the actual fatalities, the relative error being −25%.

4.2 Fitted result by modi fied exponential curve

Data fitting for Wenchuan earthquake using modified exponential curve requires equal interval between time points of the death toll figure. Figure 1 shows the time interval of most data from two days after the quake is around 24 hours. In order to maintain consistency of the cut-off time, we conducted data processing by linear interpolation, and $\Delta t = 24$. The 21 data are divided equally into 3 sections, $n = 7$. Table 2 displays the interpolated death toll from Wenchuan earthquake and first-order difference link relative.

Table 2 Number of deaths from Wenchuan earthquake after interpolation and analytical data

No.	Hours after the shock t/h	Number of deaths after interpolation N	First-order difference link relative	No.	Hours after the shock t/h	Number of deaths after interpolation N	First-order difference link relative
1	21.5	11149		12	285.5	60560	1.309
2	45.5	14565		13	309.5	62664	0.437
3	69.5	18638	1.192	14	333.5	65080	1.148
4	93.5	21856	0.790	S_2		376331	
5	117.5	28313	2.007	15	357.5	67183	0.870
6	141.5	32176	0.598	16	381.5	68109	0.440
7	165.5	34073	0.491	17	405.5	68516	0.440
S_1		160770		18	429.5	68858	0.840
8	189.5	38875	2.531	19	453.5	68977	0.348
9	213.5	41353	0.516	20	477.5	69016	0.328
10	237.5	52059	4.320	21	501.5	69019	0.077
11	261.5	55740	0.344	S_3		479678	

By calculating first-order difference link relative of the death toll at each time point, we found that the number fluctuates invariably around a constant, suggesting rationality of the modified exponential curve model.

Substituting relevant data into the above equation yields:

$$\begin{cases} b = 0.900 \\ a = -88085 \\ N_0 = 82123 \end{cases}, \quad (6)$$

Then we have the modified exponential model of the death toll from Wenchuan earthquake

$$N = 82123 - 88085 \times 0.900^t . \quad (7)$$

By calculating the predicted value and relative error ε_t of each time point using the above model, we get the mean absolute percentage error $\bar{\varepsilon}$ for examining predictive validity of the model:

$$\bar{\varepsilon} = \frac{1}{21}\sum_{t=1}^{21}\frac{|\varepsilon_t|}{N_t} \times 100\% = 10\%.$$

The result defines $\bar{\varepsilon}$ as 10%, indicating that precision of the model prediction has reached certain extent and the model can be used for trend projection.

Substituting $t = 30$ into the model, we get the predicted death toll figure of 78 389 at 12:00 on June 11, as compared with the actual figure of 69 146, the relative error being 13%.

In practical use of the method, we could put fewer data in the model, so as to predict the possible death toll in three weeks or even in one month. Let's take the data before May 21 ($n = 3$) as an example. After calculation, we obtain the following modified exponential model of death toll:

$$N = 94486 - 89348 \times 0.944^t , \quad (8)$$

where $\bar{\varepsilon}$ is 8%, which is also within the permissible range.

Substituting $t = 30$ into the model, we get the predicted death toll figure of 78 628 at 12:00 on June 11, the relative error being 13.7%.

Figures 2 and 3 show the fitted versus actual numbers of deaths from Wenchuan earthquake using exponential function and modified exponential curve respectively. The fitted results and relative errors by the two methods show the fewer data used in fitting, the larger the relative error of the estimated death toll; the more data used (i.e. the later the cut-off time), the closer the estimated number to the actual figure. This is consistent with the actual situation.

5 Comparative analysis of Chi-Chi and Kobe earthquakes

In order to illustrate the feasibility of the fitting methods of modified exponential curve and exponential function, we made a comparison by adding the data of Chi-Chi (Shi, 2006) and Kobe (Hong, 1995) earthquakes. Table 3 lists the fitted numbers of deaths from all three

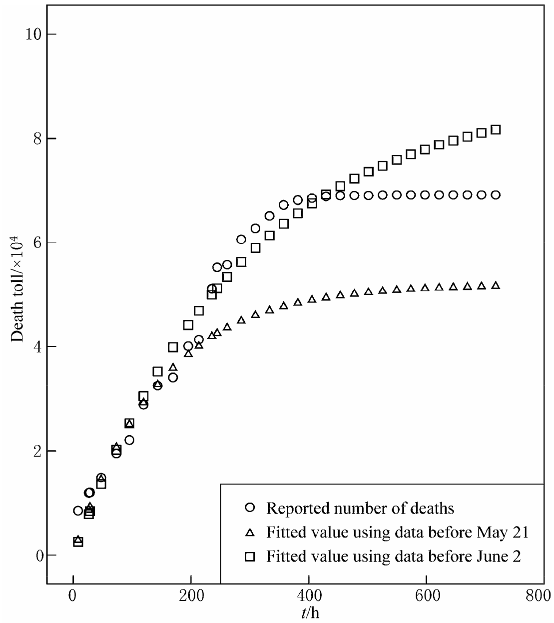

Figure 2 The fitted versus actual numbers of deaths from Wenchuan earthquake using exponential function.

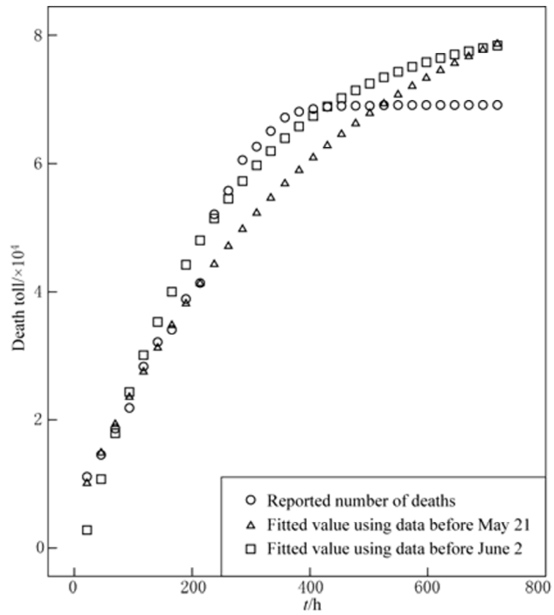

Figure 3 The fitted versus actual numbers of deaths from Wenchuan earthquake using modified exponential curve.

earthquakes respectively using the two functions and the inspection results. Figures 4 and 5 display the fitted versus actual numbers of deaths from Chi-Chi and Kobe earthquakes respectively, using modified exponential curve.

Table 3 Fitted numbers of deaths for all three earthquakes using the two functions and the inspection results.

Events	N	Fitted parameters by exponential function and inspection results				Fitted parameters by modified exponential curve and inspection results				
		N_0	a/h	$\bar{\varepsilon}$	ε_t	N_0	a	B	$\bar{\varepsilon}$	ε_t
Wenchuan	69146	88877±11557	0.004±0.0005	12%	18%	82123	−88085	0.900	10%	13%
Chi-Chi	2079	2040±74	0.059±0.009	3.9%	−2.16%	2055	−1353	0.728	4.8%	−2.07%
Kobe	5102	5209±137	0.019±0.001	10.5%	2%	5097	−7078	0.708	5%	−0.078%

Note: Data up to June 2; relative error ε_t refers to error between the estimated number and actual figure at 12:00, June 2.

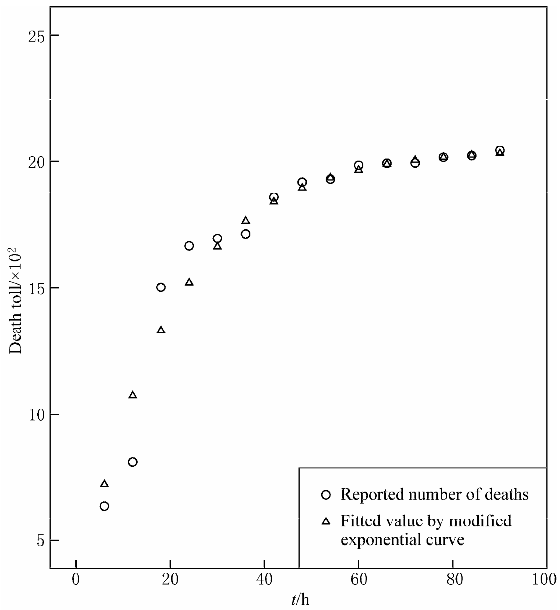

Figure 4 The fitted versus actual numbers of deaths from Chi-Chi earthquake using modified exponential curve.

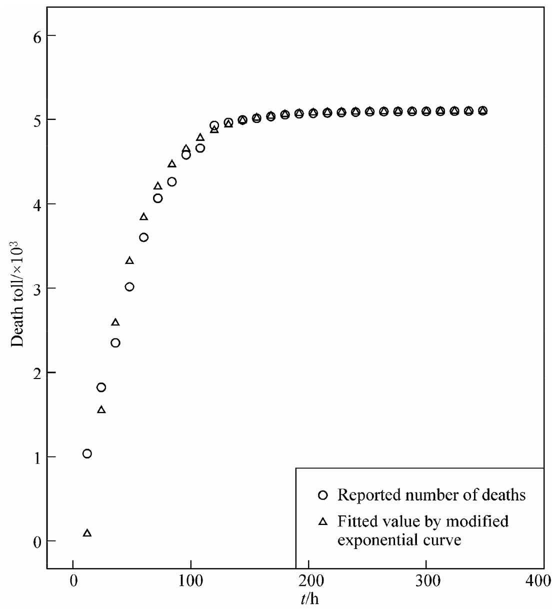

Figure 5 The fitted versus actual numbers of deaths from Kobe earthquake using modified exponential curve.

As regards relative errors between estimated and actual numbers of deaths for all three earthquakes, the relative error between the fitted result and actual figure given by modified exponential curve is much smaller than that given by exponential function, and the mean absolute percentage error $\bar{\varepsilon}$ of two of the earthquakes (without Chi-Chi) obtained by exponential function is remarkably higher than that obtained by modified exponential curve. Both indicate that modified exponential curve features higher precision than exponential function.

Table 3, Figures 4 and 5 comprehensively show that modified exponential curve is more applicable to earthquake death toll estimation than exponential function.

6 Discussion and conclusions

Modified exponential curve fitting method is applicable to death toll estimation at least three days after an earthquake when relatively more data are available, and the more data involved in fitting, the closer the estimated number to the actual figure. In terms of relative errors, the fitted result by modified exponential curve is more satisfactory than the result from exponential function. If death toll information with shorter time interval (e.g. 12 hours or even 6 hours) is available, we can make the estimation within a shorter period of time.

Death toll data fit for Chi-Chi, Kobe and Wenchuan earthquakes shows that modified exponential curve can satisfactorily describe the time history of death toll from an earthquake, which thereby provides references for disaster relief headquarters in decision making.

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