Correction to: No-arbitrage commodity option pricing with market manipulation

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The paper [1] contains two mistakes. We are grateful to Man Kit Tsui for his questions and remarks, leading us to identify them.

The first mistake concerns the well-posedness of the Riccati equation (2.9) for the coefficient D . In the paper [1] we claim that there exists a solution to that equation over the whole interval $[0, T]$. Now, while this is true when $\lambda \geq 0$, this is not necessarily so when $\lambda < 0$. Indeed, in the latter situation in order to avoid any explosion before the final time T , the following extra condition has to be imposed

$$a - (2/\kappa)\lambda^2 a^4 \geq 0, \quad (0.1)$$

which corresponds to the sufficient condition $\Re(C) \geq 0$ in [2, Lemma 10.12(b)], where $\Re$ denotes the real part and $C = a - (2/\kappa)\lambda^2 a^4$ in our case. Clearly, the same observation applies to the Riccati equations (3.8) and (4.7). Consequently, when $\lambda < 0$ the extra assumption (0.1) has to be added to the statements of Theorems 2.1, 3.1 and 4.1. With such an additional assumption, the proofs will stay the same. Notice that Section 5 (Numerical illustration) is not affected by this change, as when $\lambda < 0$ the numerical values that were chosen satisfy the new condition (0.1).

The second mistake regards the notion of solution in the three models. Let us focus on the first model for the sake of illustration. A careful inspection of the proof of Theorem 2.1,

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in particular the use of the martingale optimality principle, reveals that what we have found is not the solution to the optimization problem (2.3) in [1], but to a different problem, which we describe here. First, let us give the correct definition of admissible controls, which should replace [1, Definition 2.1]:

Definition 0.1 We say that any quadruplet (u, z, h, Δ) is *admissible* if the following properties are satisfied:

- (i) $(u_t, z_t)_{t \in [0, T]}$ are progressively measurable processes with values in $\mathbb{R} \times (-1, \infty)$ such that

$$\mathbb{E} \left[\int_0^T (u_t^2 + z_t^2) dt \right] < \infty, \quad \mathbb{E} \left[\int_0^T q_t^2 (1 + z_t) dt \right] < \infty;$$

- (ii) there exists a unique equivalent martingale measure $\mathbb{Q}^{u, z}$ for the price process $\tilde{S}$, equivalently for $(q_t)_{t \in [0, T]}$, with $h_T \in L^1(\mathbb{P}) \cap L^1(\mathbb{Q}^{u, z})$;
 (iii) $(\Delta_t)_{t \in [0, T]}$ is a real-valued progressively measurable process satisfying

$$\mathbb{E} \left[\int_0^T (|\Delta_t u_t| + |\Delta_t|^2 (1 + z_t)) dt \right] < \infty,$$

and such that the following holds $\mathbb{Q}^{u, z}$ -a.s.

$$h_t = \mathbb{E}^{\mathbb{Q}^{u, z}} [h_T | \mathcal{F}_t] = \mathbb{E}^{\mathbb{Q}^{u, z}} [h_T] + \int_0^t \Delta_s dq_s,$$

for all $t \in [0, T]$.

The set of all admissible quadruplets will be denoted by $\mathcal{A}$.

The solution we have obtained in Theorem 2.1 is a quadruplet $(\hat{u}, \hat{z}, \hat{h}, \hat{\Delta})$ such that:

1. $(\hat{u}, \hat{z}, \hat{h}, \hat{\Delta})$ belongs to $\mathcal{A}$, where the set $\mathcal{A}$ has been defined above;
2. given $(\hat{h}, \hat{\Delta})$, the pair $(\hat{u}, \hat{z})$ solves the optimization problem

$$\sup_{(u, z, \hat{h}, \hat{\Delta}) \in \mathcal{A}} \mathbb{E} \left[\int_0^T \left(P_t - \frac{\kappa}{2} u_t^2 - \frac{g}{2} z_t^2 - \lambda \hat{\Delta}_t u_t \right) dt \right].$$

Observe that this notion of solution can be interpreted as an equilibrium between the production and the investment divisions, hence it is consistent with the discussion before Definition 2.1 in [1]. The same changes have to be made in the other two models. While for the second model, the modifications are straightforward, for the third one it is worth giving some detail. We use the following notation for the producer and the trader payoffs, respectively:

$$J_{\text{pr}}(u, z, h, \Delta) = \mathbb{E} \left[\int_0^T \left(q_t (S_t - a q_t) - \frac{\kappa}{2} u_t^2 - \lambda (\mu - a u_t) \Delta_t \right) dt \right],$$

$$J_{\text{tr}}(u, z, h, \Delta) = \mathbb{E} \left[\int_0^T \left(-\frac{g}{2} z_t^2 + \lambda (\mu - a u_t) \Delta_t \right) dt \right].$$

First, the notion of admissibility in Definition 4.1 has to be changed in a similar way as above. Second and more importantly, Definition 4.2 has to be replaced by the following one:

Definition 0.2 We say that the quadruplet $(\hat{u}, \hat{z}, \hat{h}, \hat{\Delta}) \in \mathcal{A}$ is a Nash equilibrium if

$$\tilde{J}_{\text{pr}}(\hat{u}, \hat{z}, \hat{h}, \hat{\Delta}) \geq \tilde{J}_{\text{pr}}(u, \hat{z}, \hat{h}, \hat{\Delta}), \quad \tilde{J}_{\text{tr}}(\hat{u}, \hat{z}, \hat{h}, \hat{\Delta}) \geq \tilde{J}_{\text{tr}}(\hat{u}, z, \hat{h}, \hat{\Delta}), \quad (0.2)$$

for all deviations u, z such that both $(u, \hat{z}, \hat{h}, \hat{\Delta})$ and $(\hat{u}, z, \hat{h}, \hat{\Delta})$ belong to $\mathcal{A}$.

We stress once more that all the results in the numerical illustration section as well as all the comments and interpretations therein are not affected by these changes.

References

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