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ORIGINAL PAPER

Teachers' Experiences with and Perceptions of Virtual Manipulatives Following the COVID-19 Pandemic

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Abstract

While the use of virtual manipulatives (VM) is rising in classrooms, there is still limited research. regarding teacher experiences with and perceptions of virtual manipulatives. Most of the research regarding teacher perceptions of VM has focused only on short-term uses following professional development sessions and none has highlighted the experiences of teachers using them during emergency remote teaching during COVID-19. The purpose of this study was to explore teacher perceptions and. experiences with virtual manipulatives following emergency remote teaching during COVID-19. To achieve this, the researchers conducted an online survey to gather data on educator's ($n=103$) experiences, perceptions, and usage of virtual manipulatives. The qualitative and quantitative data show that educators feel that VM are a valid and feasible support of mathematics instruction when physical manipulatives are not available. Results regarding usage of virtual manipulatives including frequency of use, standards taught, and types used are presented and discussed.

Keywords Emergency remote teaching · K-12 mathematics education · Technology integration · Virtual manipulatives

Mathematics manipulatives are one of the most common instructional strategies in teaching mathematics (Carbonneau et al., 2018). The use of manipulatives to teach mathematics has been a long-standing best practice (NCTM, 2000). In a math manipulative-based instructional technique students have the opportunity to physically interact with objects to learn targeted information (Carbonneau & Marley, 2012).

While educators have relied on the use of physical mathematics manipulatives for a long time, the use of virtual manipulatives (VM) is rising in classrooms ((Moyer-Packenham & Bolyard, 2016). There is some evidence that teachers who previously did not use VM began using them during emergency remote teaching (ERT) in the COVID-19 pandemic (Schuck & Lambert, 2020). VM have been defined as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer et al., 2002, p. 373). In more recent years this definition has been expanded to include apps for tablets and mobile phones as well. Moyer-Packenham and Bolyard (2016) updated the definition to include VM within apps and they are now defined as "an interactive technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge." (p. 13). This revision includes VM that may "(a) appear in many different technology-enabled environments; (b) be created in any programming language; and (c) be delivered by any technology-enabled device" (Moyer-Packenham & Bolyard, 2016, p.4). Research on the topic of VM has been largely limited to comparing virtual and physical manipulatives (Suh, 2005; Wang & Tseng, 2018). The studies which have focused on teacher perspectives of VM have mostly followed brief professional development sessions or focused on preservice teachers (Akkan & Cakir, 2012; Lin, 2010; Reiten, 2020). The purpose of this study is to gain a deeper understanding of teachers' experiences with and perceptions of VM after being encouraged to use them over a lengthy period of time during emergency remote teaching (ERT).

This study investigated teacher's experiences with and perceptions of VM guided by the Technology Acceptance Model (TAM) (Davis, 1989). The TAM focuses on the

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decision stage of the innovation process and attempts to predict an individual's decision to use technology based on two constructs: perceived usefulness and perceived ease of use (Davis, 1989). This model has been used by other researchers to measure intention to use technology as well as predict actual usage (Waarvik, 2019). The following research questions guided this study:

- RQ1: To what extent do teachers feel the use of virtual manipulatives are a valid and feasible support for mathematics instruction as defined by the Technology Acceptance Model following the COVID-19 pandemic?
  - RQ1.1. What patterns of usage do teachers report regarding virtual manipulatives following ERT?
- RQ2: How do teachers describe their experiences using virtual manipulatives during and after emergency remote teaching (ERT)?

Literature Review

Physical Manipulatives

Leaders in mathematics education have long advocated for the inclusion of multiple means of representation when teaching mathematical concepts. The NCTM (2000) called for students to be active participants in their learning. Many educators have accomplished this by using physical manipulatives. There are a wide variety of physical manipulatives including algeblocks, geometric shapes, base-ten blocks and pattern blocks (Uribe-Flórez & Wilkins, 2017). In a meta-analysis of physical manipulative use in mathematics education, younger children showed an increased benefit from the use of physical manipulatives compared to their older counterparts (Carbonneau et al., 2013). Using physical manipulatives produced a small- to medium sized effect on student learning compared to instruction using only abstract symbols (Carbonneau et al., 2013). Studies since this meta-analysis have confirmed the positive effects of manipulative use on academic achievement. Students who received instruction using physical algebra tiles outperformed those who only used abstract symbols (Larbi & Mavis, 2016). Second-grade students who participated in an intervention using physical manipulatives outperformed those who did not receive the intervention (Liggett, 2017). Finally, there is some evidence that learning preference and mathematics ability may impact the effectiveness of manipulatives (Kablan, 2016).

Virtual Manipulatives

While physical manipulatives have been embraced by educators of mathematics, VM are gaining in popularity (Moyer-Packenham & Bolyard, 2016). There is limited evidence that educators were motivated to try VM during the COVID-19 pandemic. (Schuck & Lambert, 2020). VM have been defined as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer et al., 2002, p. 373). VM were initially developed as Flash or Java programs, but now are predominantly HTML5 or app based (Shin et al., 2021).

There is much evidence that the use of VM leads to improved student outcomes (Bouck et al., 2017; Hensberry et al., 2015; Lin, 2010; Moyer-Packenham et al., 2012; Park, 2019; Satsangi et al., 2018; Wang & Tseng, 2018). Using a repeated measures design, Hensberry et al., (2015) found that students who used VM to learn fractions had a large growth in learning gains. Using a virtual manipulative with ten frames led to improved scores on a place value assessment in first grade students, especially for students with weaker counting skills (Flevares et al., 2022).

VM have also been used to improve learning outcomes for students with disabilities. Visual models have been recommended by researchers to support students with disabilities in learning concepts and skills that are necessary to solve abstract and symbolic mathematical problems (Shin et al., 2017; Bouck et al., 2017) found that students with disabilities experienced an increase in accuracy in solving equivalent fraction problems following an intervention using explicit instruction with VM. In a study by Jimenez and Besaw (2020), students with autism and moderate intellectual disability participated in story-based virtual manipulative lessons and gained early numeracy skills.

Most of the research on VM focuses on their use in a few main content areas. Many studies on VM focus on their use in teaching content related to fractions (Bouck et al., 2017; Hansen et al., 2016; Hensberry et al., 2015; Shin et al., 2017). Other research has focused on the use of VM in teaching place value using virtual base-10 blocks (Flevares et al., 2022; Moyer-Packenham & Bolyard, 2016).

As referenced in the updated definition, VM can be either web-based or app-based. As the previous definition limited VM to being only web-based, most of the literature on VM focuses only on web-based manipulatives. Some of the web-based manipulatives that have been studied include PhET Simulations, Conceptua Math, Illuminations, National Library of VM, Fun Fractions, ABCya, Brainingcamp, and iTalk2Learn (Hansen et al., 2016; Hensberry et al., 2015; Shin et al., 2017). However, there are many studies on using apps for educational purposes, as well as a growing body of literature regarding app-based VM. Some of the app-based manipulatives that have been studied include Fraction Tiles App by Brainingcamp, GoWorksheetMaker, Montessori Numbers, Pink Tower, Intro to Math, Friends of Ten, Hungry Guppy, Fingu, 100s Board, Math Motion Zoom, Number

Lines, Place Value Cards, Skip Counting Beads, and Spin for Beans 50 which was adapted from a curricular game (Bouck et al., 2017; Flevares et al., 2022; Jimenez & Besaw, 2020; Moyer-Packenham & Bolyard, 2016).

In many studies, researchers have noted they believe the reason VM lead to improved student achievement is that VM often provide feedback to the student (Bouck & Sprick, 2018; Hensberry et al., 2015; Reiten, 2020; Lin, 2010; Satsangi et al., 2018; Wang & Tseng, 2018).

Student Perceptions of Virtual Manipulatives

There is limited research on student perceptions of VM. In a survey conducted on fourth grade students following several days using the PhET Simulation Fractions Intro and Fraction Lab, the majority of students liked using the computer to work on fractions, felt that the manipulatives improved their understanding and were easy to use, and would like to use manipulatives to learn other mathematical concepts (Hensberry et al., 2015). Students have also reported that math is more fun and experience higher engagement when using VM (Hensberry et al., 2015; Jimenez & Besaw, 2020). There is some evidence that VM are experienced differently by students based on their valuation of mathematics and their level of mathematics anxiety. Adolescents that believe mathematics is important were more likely to believe that VM are beneficial for problem solving processes (Lee & Chen, 2010). Adolescents that had less math anxiety were more likely to believe that VM were easier to use (Lee & Chen, 2010).

Teacher Perspectives on Virtual Manipulatives

Much of the research on VM is focused on comparing them to physical manipulatives (Wang & Tseng, 2018). The studies which have focused on teacher perspectives have mostly followed brief professional development sessions or focused on preservice teachers (Akkan & Cakir, 2012; Lin, 2010; Reiten, 2020). When asked about the reasons they choose to include VM in their instruction, many educators stated that the immediate feedback these manipulatives could provide was a key reason they include them in their instruction (Reiten, 2020; Satsangi et al., 2018). Additional reasons for using VM included providing students with opportunities to connect multiple representations, providing students with differentiated instruction, increasing engagement and exploration, providing opportunities for students to use and develop mathematical models, reduce cognitive load, providing additional practice opportunities, and aiding students in visualizing mathematical concepts (Reiten, 2020). In a study where teachers co-created a virtual manipulative with researchers to meet the needs of their students all teachers expressed interest in the app they created and would recommend it to colleagues (Hansen et al., 2016).

Emergency Remote Teaching in the COVID-19 Pandemic

While there is a body of literature on distance learning and online education, there is still much to be learned about ERT and its lasting effects on education (Hodges et al., 2020; Ferri et al., 2020). It is important to note that distance learning and online education are distinct from ERT in that ERT is not standard procedure and is in response to a crisis (Hodges et al., 2020). Studying ERT and its effects on education can help educators to better understand for possible future instances of ERT and meet the needs of students who have been impacted by ERT (Ferri et al., 2020). Studies on mathematics education during ERT have not researched the use of VM, instead focusing on the use of screencasting and static notes (Ardiç, 2021; Lishchynska & Palmer, 2021). There is some evidence that ERT increased and diversified instructional technology usage in mathematics teachers (Ardiç, 2021). There are a few papers that reference the usage of VM during COVID-19, but none evaluate the effectiveness of these tools or their acceptance by teachers or students (Borba, 2021; Livy et al., 2022). This study is concerned with teachers' experiences with perceptions of VM after being encouraged to use them over a lengthy period of time during emergency remote teaching.

Technology Acceptance Model

The Technology Acceptance Model (TAM) was selected to guide the current study. TAM was developed by Davis (1989) to demonstrate the factors that help explain whether potential users of technology will use the technology. The model is based on research that suggests that the more complex a technology is, the more likely it is to be overlooked in favor of technology that is perceived as easier to use (Davis, 1989). The model purports that there are two main variables that lead to a technology being accepted by a user, perceived usefulness and perceived ease of use (Davis, 1989). Perceived usefulness is defined as "the degree to which a person believes that using a particular system would enhance his or her job performance" (Davis, 1989, p. 320). In education, a teacher would perceive a technology to be useful if it would enhance student learning outcomes. Perceived ease of use is defined as "the degree to which a person believes that using a particular system would be free of effort" (Davis, 1989, p. 320).

TAM and Technology Integration in Education

Although the TAM was originally used to predict behavior of end users in business settings there are now several studies that have demonstrated its application in predicting the behavior of teachers regarding usage of educational technology (Scherer et al., 2019; Teo et al., 2008; Waarvik, 2019). Many studies that use TAM as a theoretical framework focus on the student as the end user rather than the teacher (Waarvik, 2019). However, the TAM is a good fit for examining the intentions of educators rather than students because students do not usually get to select the types of educational technology used in classrooms. This study seeks to understand teachers' experiences with and perceptions of VM using the TAM.

Methodology

This exploratory study utilized a convergent, parallel, mixed-methods design to examine K-12 teacher's experiences with and perceptions of VM following ERT during the COVID-19 pandemic. A cross-sectional survey instrument collected complementary quantitative and qualitative data to develop a more complete understanding of the phenomenon (Creswell & Plano Clark, 2011). A survey was used for data collection because of the ease and affordability of surveys, and quick turnaround on data collection.

Participants

This study used convenience sampling and recruited 104 teachers who teach math in K-12 settings through two venues. Initial participants were recruited through professional learning networks (i.e., Facebook, Twitter, Reddit) with the goal of recruiting 100 participants, justified by Waarvik (2019). The challenge of recruiting sufficient participants led to the expansion of the recruiting venue and to include teachers who did not use VM to gain an understanding of their perceptions and experiences during ERT. The researchers expanded recruitment to directly contact teachers from local school districts using their district email addresses. The response rate for teachers who were emailed directly was 2.1%. The majority of participants were recruited from a variety of school districts in the Pacific Northwest. An effort was made to include both urban and rural districts.

Demographics

103 responses were kept for the data analysis. One response was eliminated because they indicated they did not teach mathematics during 2020. One participant declined to answer demographic questions but answered all other questions so their response was kept. Of the kept responses, 80.6% were female, 14.6% were male, 2.9% declined to answer, and 1% identified as non-binary. A plurality of participants were in the 40–49 age range (31.1%), followed by the 50–59 age range (26.2%), the 30–39 age range (24.3%), the 21–29 age range (8.7%), and 60 + age range (8.7%). As far as teaching experience, 8.7% of teachers had zero to three years of experience, 11.7% of teachers had four to six years of experience, 11.7% had seven to nine years of experience, and 67% had 10 + years of experience. With respect to grade level taught, 8.7% were kindergarten teachers, 49.5% were elementary teachers, 14.6% were middle school teachers, and 26.2% were high school teachers.

Instrumentation and Data Collection

The online survey created in Google Forms began with a brief description of VM including the definition and visual examples. The survey contained 19 items that included questions about participant demographics and their teaching environment (5 questions), perceptions of VM based on the TAM (6 questions), and questions related to their usage of VM (4 questions). The survey used a variety of item types, including five-point Likert-scale items, closed-ended prompts, and open-ended prompts. Six items of the survey instrument measured TAM modeled after an instrument validated by Waarvik (2019). Of these items, two were intended to measure perceived ease of use (*I am knowledgeable about managing virtual manipulatives in my classroom, I understand how to implement virtual manipulatives in my classroom*) and two were designed to measure perceived usefulness (*The use of virtual manipulatives in the classroom would increase my effectiveness as a teacher, The use of virtual manipulatives in the classroom would increase my students' learning*), and two were intended to measure preference for virtual manipulatives (*I like to use virtual manipulatives in my classroom, If I could, I would use more virtual manipulatives in my classroom*). One open-ended question asked teachers to describe their experience using VM during ERT if they were used, or about the reasons they chose not to use them during ERT. An early draft of the survey was sent to four members of the potential audience for feedback and the survey was revised for clarity accordingly. The tests of internal reliability for items measuring TAM constructs reveal that Cronbach's alpha coefficients are at 0.90, 0.81, and 0.91 for *perceived ease of use*, *preference for virtual manipulatives*, and *perceived usefulness* respectively. Given that there were only two items for each construct, the internal consistency of the instrument is considered to be adequate (Taber, 2018).

Each participant accessed the survey through a link embedded in the recruitment email or the social media post and results are anonymous. Data collection continued until the number of respondents exceeded 100, at which point data analysis began.

Data Analysis

Data analysis began with descriptive statistics for frequency to describe the participants. Next descriptive statistics (frequency and mean) were calculated for Likert-scale questions. Descriptive statistics (frequency) were also used for the questions regarding usage.

This study utilized latent content analysis for the qualitative data on teacher experiences with and perceptions of VM following the COVID-19 pandemic ERT (Downe-Wamboldt, 1992). Latent content analysis began with two initial readings of the entire data set. The first author used memoing to record notes that were identified during the initial reading. The first author created a codebook consisting of themes and descriptions after initial review of the data and based on the theoretical framework that guides this research. 30% of the open-ended responses were randomly selected and coded by both authors individually based on the codebook. The interrater reliability (Kappa = 0.64) between two coders indicates a substantial agreement (McHugh, 2012). The differences in coding were resolved through discussion and the codebook was refined to reflect the evolved understanding. The first author coded the rest of the data individually based on the refined codebook.

Results

RQ1: To What Extent Do Teachers Feel the Use of Virtual Manipulatives are a Valid and Feasible Support for Mathematics Instruction as Defined by the Technology Acceptance Model Following the COVID-19 Pandemic?

Table 1 below provides the descriptive statistics of participants' responses on six questions corresponding to the three constructs of TAM (i.e., Perceived Ease of Use, Perceived Usefulness, and Preference for Virtual Manipulatives). The means of the six questions range from 3.2 to 3.8. For each construct of TAM, the participant's Likert scores were averaged to give a participant a score for the construct. The average Perceived Ease of Use (PEOU) score was 3.7 (SD = 0.97), Perceived Usefulness (PU) score was 3.3 (SD = 0.96), and the Preference for Virtual Manipulatives (PFVM) score was 3.3 (SD = 0.89).

RQ1.1. What Virtual Manipulatives do Teachers Report Using Most Frequently?

Out of 102 teachers, 55 reported using VM at least once per week, while 47 reported using VM less than once per week. Of the 55 teachers who used VM at least once per week, most reported using VM three times a week. The following table shows the frequency of usage. The following table provides greater detail regarding usage (Table 2).

Table 1 Descriptive statistics of the technology acceptance model related questions

Construct	Question	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree	M	SD
Perceived Ease of Use	I am knowledgeable about managing virtual manipulatives in my classroom.	4 (3.9%)	14 (13.6%)	14 (13.6%)	55 (53.4%)	16 (15.5%)	3.6	1.03
	I understand how to implement virtual manipulatives in my classroom.	4 (3.9%)	10 (9.7%)	11 (10.7%)	59 (57.3%)	19 (18.4%)	3.8	0.99
Preference for Virtual Manipulatives	I like to use virtual manipulatives in my classroom.	3 (2.9%)	16 (15.5%)	28 (27.2%)	45 (43.7%)	11 (10.7%)	3.4	0.98
	If I could, I would use more virtual manipulatives in my classroom.	8 (7.8%)	18 (17.5%)	37 (35.9%)	26 (25.2%)	14 (13.6%)	3.2	1.12
Perceived Usefulness	The use of virtual manipulatives in the classroom would increase my effectiveness as a teacher.	5 (4.9%)	17 (16.5%)	41 (39.8%)	34 (33.0%)	6 (5.8%)	3.2	0.95
	The use of virtual manipulatives in the classroom would increase my students' learning.	3 (2.9%)	11 (10.7%)	37 (35.9%)	42 (40.8%)	10 (9.7%)	3.4	0.91

Table 2 Days per week using VM

Days per week using virtual manipulatives	Frequency (Percentage)
Less than once per week	47 (45.6%)
1	15 (14.6%)
2	13 (12.6%)
3	20 (19.4%)
4	2 (2%)
5	4 (3.9%)
5+	1 (1%)
Did not respond	1 (1%)

For this question, participants were given a checklist of VM with common manipulatives as well as an option to add their own, and an option for none. Regarding the manipulative type used, applets within textbooks or curriculum websites were the most popular with 29 participants using these types of manipulatives. Table 3 below presents the top 10 most frequently reported VM. The suggested age range for the VM is also provided. Also of note was that several participants reported creating their own manipulatives (7). Platforms used for creating VM included Google Slides, Jamboard, Seesaw, and Desmos.

Teachers were also surveyed regarding the Common Core mathematics content and practice standards they taught using VM. Regarding mathematics content standards, teachers reported using VM to teach Geometry (57.3%), Numbers and Operations (52.4%), Measurements & Data (45.6%), and Operations and Algebraic Thinking (44.7%) most frequently. Table 4 presents the frequency and percentage of the response on the common core math content standards teachers teach with VM.

Table 3 Top 10 VM reported in quantitative data

Manipulative Type	Recommended Age Range	Frequency
Applets within textbook/curriculum	K-12	29
ABCya	PreK-6	24
Math playground	K-6	23
Desmos	6–8	23
Geogebra	6–12	19
Math learning center	K-5	19
NLVM	K-12	16
Didax	K-12	14
Toy theater	K-3	14
None	N/A	13

Table 4 Common core standards taught using VM

Common core mathematics content standards	Frequency (%)
Counting & Cardinality	29 (28.2%)
Operations & Algebraic thinking	46 (44.7%)
Number & Operations	54 (52.4%)
Measurements & Data	47 (45.6%)
Geometry	59 (57.3%)
Ratios & Proportional relationships	16 (15.5%)
The Number System	20 (19.4%)
Expressions & Equations	23 (22.3%)
Statistics & Probability	15 (14.6%)
Number and Quantity (High School)	8 (7.8%)
Algebra (High School)	22 (21.4%)
Functions (High School)	19 (18.4%)
Modeling (High School)	12 (11.7%)
Statistics & Probability (High School)	12 (11.7%)
My school does not follow common core standards.	5 (4.9%)
I have not used virtual manipulatives.	19 (18.4%)

Regarding mathematics practice standards, teachers reported using VM to teach *model with mathematics* (68%), *make sense of problems and persevere in solving them* (51.5%), *use appropriate tools strategically* (51.5%), and *reason abstractly and quantitatively* (36.9%) most frequently. See Table 5 for greater detail on the findings regarding mathematics practices standards most frequently taught through the use of VM.

Teachers were also asked about their intentions for future use. The final Likert-Scale item read: *After the pandemic I am more likely to use virtual manipulatives.* About half of teachers felt that they would be more likely to use virtual manipulatives in the future, selecting either agree or strongly agree (48.6%), followed by those who neither agreed nor disagreed (30.1%), and then by those who disagreed (21.4%) (Table 6).

RQ2 (Qualitative): How Do Teachers Describe Their Experiences Using Virtual Manipulatives During and After Emergency Remote Teaching (ERT)?

Teachers were asked one open-ended question which aimed at answering the research question: *How do teachers describe their experiences using virtual manipulatives during and after emergency remote teaching (ERT)?* The prompt was: *In a few sentences, please describe your experience using virtual manipulatives during and after emergency remote teaching (ERT). If you decided not to use virtual manipulatives during this time, please describe what impacted that decision.* The data analysis resulted in seven themes, including *teacher usage, content taught,*

Table 5 Mathematics practice standards taught using VM

Common core mathematical practice standards	Frequency (%)
Make sense of problems and persevere in solving them.	53 (51.5%)
Reason abstractly and quantitatively.	38 (36.9%)
Construct viable arguments and critique the reasoning of others.	20 (19.4%)
Model with mathematics.	70 (68%)
Use appropriate tools strategically.	53 (51.5%)
Attend to precision.	28 (27.2%)
Look for and make use of structure.	33 (32%)
Look for and express regularity in repeated reasoning.	26 (25.2%)
My school does not follow common core standards.	5 (4.9%)
I have not used virtual manipulatives.	20 (20%)
Did not respond	3(2.9%)

Table 6 Intention to use VM following COVID-19

	Strongly Disagree	Disagree	Neither agree nor disagree	Agree	Strongly Agree	M	SD
After the pandemic I am more likely to use virtual manipulatives.	7 (6.8%)	15 (14.6%)	31 (30.1%)	39 (37.9%)	11 (10.7%)	3.3	0.96

manipulative type used, perceived ease of use, barriers/cons of using, and *student experience.*

Teacher Usage

The *teacher usage* theme described usage of VM before, during, and after ERT. The theme was also applied to conditions or preferences of using VM. Some educators stated that they began using VM due to the COVID-19 pandemic. Others were experienced users prior to ERT. Some educators indicated that they had continued to use VM after the return to face-to-face instruction, while others indicated that they abandoned VM when they returned to school. Some educators indicated that they had a preference for using VM in face-to-face instruction rather than trying to teach students to use these tools remotely. Other educators felt that VM were a good substitute for physical manipulatives that were unavailable, such as in ERT, but when they could they would prefer to use physical manipulatives. Teachers also described their usage of VM including using them for modeling for students, or linking them to a class website for students to access independently. Sample responses that reflected this category included:

- "Virtual manipulatives became an essential part of my teaching practice after transitioning to online learning (ERT)."
- "I used virtual manipulatives stuff during Zoom instruction with success."
- "I have used virtual manipulative *[sic]* for at least the past 10 years for demonstration in my classes, so it was natural to keep using them during our emergency remote teaching as well as the times were in a hybrid model with half of the students in class and half at home."
- "I have used virtual manipulatives to demonstrate concepts and have shared access to the links and apps to provide students independent access to use as needed."

Content Taught

Educators identified several different content areas of mathematics that they taught through using VM. The most frequently mentioned content was algebra and equations ($n = 7$), followed by fractions ($n = 4$), and place value/base-10 ($n = 3$). Some responses that reflected this category included:

- "I used virtual manipulatives to model fraction and equation concepts over Zoom."
- "I used the geoboard for Pythagorean Theorem and algebra tiles for solving equations and distributive property."
- "Since going back to in-person learning (after ERT), I still use them, especially to show quick examples to the students (e.g., for our unit adding & subtracting fractions) and to support student understanding but am using less now that we are back in person."
- "There are some things I really prefer using physical manipulatives for, but I do like virtual fraction bars and base ten blocks better than physical ones."

Manipulative Type Used

The theme *manipulative type used* refers to any specific VM mentioned by teachers in their response. The most popular type of VM referenced by participants was Desmos ($n = 16$), followed by Geogebra ($n = 5$), algebra tiles ($n = 5$), applets within textbook or curriculum ($n = 4$), and self-created (3). Some responses that reflected this category included:

- "I used Desmos frequently and found it a positive experience for both myself and my students."
- "I have used Desmos, Geogebra, and virtual fraction representation regularly this year to engage students in the mathematics of our course and to address gaps in student learning due to COVID-19."
- "I used many provided through our school's curriculum. I also used my own by manipulating different figures on Google Jamboard."

Perceived Ease of Use

The *perceived ease of use* theme was based on the TAM and was used for participant responses indicating that they felt VM were easy to use for both themselves and their students. The majority of participants that referenced ease of use in their response indicated that VM were difficult to use for both the teacher and the student ($n = 9$). Teachers stated that the large range of VM required a lot of time for teachers to determine which VM were of high quality and worth taking the time to train students to use. Teachers also mentioned that it can be difficult to introduce VM to students who are unfamiliar with them in a remote learning environment like ERT. Some responses specifically mentioned that younger students struggled to use VM. However, some teachers did feel that VM were easy to use ($n = 3$). Some responses that represent this category include:

- "During ERT they were difficult because the students were unfamiliar with how to use the *[sic]*, It has been better when we can discover them together in the class room."
- "I used very little virtual manipulatives during ERT for 1st grade. They are difficult for younger students to use without first teaching them."
- "At the beginning it was horrible. Steep learning curve. It took many hours of preparation."
- "Using them was simple and students caught on well."

Perceived Usefulness

The *perceived usefulness* theme was based on the TAM and was used for participant responses indicating that they felt VM were useful or explained how they experienced the usefulness of VM. Some participants stated that VM were helpful and useful. Participants stated that VM were useful as a replacement for unavailable or impractical physical manipulatives. Many participants shared that VM were useful for providing models or visual aides to students, particularly for special education students or English language learners. Other participants appreciated that VM allowed them to provide as close to an in-person experience as possible, increase student engagement, collect data for assessments, see student work, and give feedback. Some responses that reflected this category included:

- "My students do not generally have physical manipulatives at home so this is a good option for more hands on learning."
- "It was a great visual aid. I had a lot of ELs that needed to see what I was saying."
- "I was very thankful that I had these tools available to me. When I couldn't use physical tools or students were at home, they still had access to learning with these tools."

Barriers/Cons of Using

The theme *barriers to use* was used to describe reasons teachers gave for not utilizing VM. One example of a barrier was the amount of time required to sort through the various VM, set them up, or create them from scratch. Teachers also referenced a lack of access, technology support, and a lack of training on proper usage of VM. Another barrier referenced by teachers was student's inability to access remote learning. Additional barriers referenced by teachers included their belief that student screen time should be decreased and that VM lack the kinesthetic aspect of VM. Some teachers shared that they held pedagogical beliefs that young students could not use VM, or contrastingly that older students did not benefit from the use of manipulatives. Cons of using referred to negative effects teachers perceived as occurring due to the use of VM. Some teachers shared that their students have a hard time staying on task or making connections to abstract mathematical concepts when using VM compared to physical manipulatives. Some statements that reflect this category include:

- "Because of this, and the lack of physical connection, my students struggled more than usual connecting manipulative work to iconic and symbolic work."
- "I don't use them much now that we are in person as we are trying to limit screen use."
- "I kept my teaching very basic. I had seven of my 25 students that weren't able to access remote learning. I did send review packets to those students."

- "I wanted to us NLVM, but couldn't get it to work on our laptops. I recently tried again, but we need something installed. I asked our tech, but have not heard back."

Teacher Perceptions of Student Experience

The theme *teacher perceptions of student experience* included responses where teachers described how their students felt about or responded to VM. Teachers describe their students as more successful after gaining experience with or having explicit instruction in using VM. Many responses indicated that VMs work better for some students than for others, based on mathematics ability level, learning needs, or student behavior. Some teachers reported that VM helped build conceptual understanding in students and that they enjoyed using them. Some teachers also stated that many students did not participate in remote learning or use VM when provided with access. Some statements that reflect this theme included:

- "Virtual manipulatives were good for students who have a solid concrete understanding of manipulatives and values. However, if students did not have that concrete understanding, they struggled to use them. When we returned face to face, I intentionally taught students to use the virtual manipulatives and they did better when we had to pivot to emergency virtual weeks in 2020–2021."
- "I mainly used Zearn. I found that it helped many students move from direct modeling to iconic math."
- "Students seem to enjoy using virtual manipulatives just as much as physical manipulatives."

Discussion

The purpose of this exploratory mixed-methods study was to expand our understanding of the experiences with and perceptions of K-12 teachers regarding VM following ERT during the COVID-19 pandemic. Survey responses of 103 participants were analyzed and reported.

The quantitative data indicates that teachers have demonstrated a slight acceptance of virtual manipulatives. Teachers indicated that they perceived VMs as being easy to use and felt confident using them in their instruction (M = 3.7. SD = 0.97). Teachers also indicated a slight preference for using VMs in their instruction (M = 3.3, SD = 0.96). They also indicated that they believed that VMs were useful for instruction (M = 3.4, SD = 0.89). Regarding usage, most teachers (54.4%) used VM at least once per week. The most frequently used VM were applets within the textbook or curriculum, ABCya, Math Playground and Desmos. VM were most frequently used to teach geometry, numbers and operations, measurements and data, and operations and algebraic thinking. About half of teachers indicated that they would be more likely to use VMs in the future (48.6%).

The qualitative data indicated that teachers perceived VM as being useful, particularly in conditions when physical manipulatives were impractical or unavailable. Teachers also indicated additional reasons that VM being useful and these reasons are mostly aligned with those found in Reiten (2020). The majority of teachers whose responses referenced ease of use shared that VM were difficult for teachers and students to use during ERT. Teachers also shared barriers of using virtual manipulatives which included both first and second order barriers. First-order barriers are external and refer to issues such as lack of access, time, training, and support (Ertmer, 1999). Second-order barriers are internal and refer to issues such as teacher's pedagogical beliefs, beliefs regarding technology, and ability to adapt to change (Ertmer, 1999). The majority of barriers teachers reported were first order barriers. Teachers shared that student experience with VM varied based on their mathematics ability, behavior, and learning needs. This is consistent with the literature that student attitudes towards learning mathematics impact their perceptions of the efficacy of VM (Lee & Chen, 2010).

The qualitative data included slight differences from the quantitative data regarding reported usage. In the open-ended questions teachers most frequently reported using Desmos, Geogebra, Algebra Tiles, applets within textbook or curriculum and self-created manipulatives. The content they reported using VM most frequently to teach as algebra and equations, fractions, and place value/base-10. The use of the checkboxes for the quantitative usage data may have helped participants recall their usage. Much of the current literature regarding VM currently focuses on their use in teaching fractions but this data may encourage researchers to investigate the usage of virtual manipulatives in teaching algebra and geometry to better align with current practice. After a review of the literature (e.g., Reiten, 2020), the researchers expected for the majority of teachers to share that the feedback provided by VM was a key reason for implementing them in instruction. However, only a few responses mentioned this. It may be that the use of feedback is an important feature for teachers using VM in face-to-face instruction prior to COVID-19, but that in remote or hybrid environments it is less of a consideration.

Additionally, the qualitative data seemed to reveal that teachers found VM more difficult to use than the quantitative data indicated. The majority of responses in the *ease of use* theme indicated that VM were difficult to use for both teacher and student. This may differ by the mathematics ability of the student, the school setting, the teacher or student's previous experience with technology or another uncovered variable. It may also be that teachers who had difficulty in using VM were more likely to report it in the open-ended response. The open-ended responses also provide some context for the lower PEOU scores given by some participants. This slight discrepancy indicates that further research is needed in this area.

Limitations

This study recruited participants from both large urban districts and small rural districts. There was not a survey question clarifying if teachers taught in urban/rural settings and this may have had an impact on their ability to use VM as a few teachers referenced. Future studies may look at this as a factor that may affect teacher's decisions to use VM.

Another limitation is that some educators are unfamiliar with VM. This may have been a factor in this study's low response rate. Additionally, when looking at participant responses it appears that four participants may have confused VM with virtual learning, or ERT. The researchers attempted to mitigate this as best as possible by providing a definition and a few visual examples of VM at the beginning of the survey.

As with any self-reported survey data, the data relies on participants being truthful regarding their answers. An attempt was made to obtain only responses from practicing K-12 teachers by recruiting from private groups for these educators as well as sending the survey directly to district email addresses. Furthermore, educators who were interested in taking a survey on VM may have had strong feelings about VM in one way or another and more neutral educators may have been less inclined to participate. Additionally, many teachers have reported feeling overworked following the COVID-19 pandemic and as a result may have been less inclined to participate in the survey which could account for the lower response rate (Pressley, 2021). While it is important to acknowledge a response bias, it is nearly unavoidable in survey-based research. Finally, future researchers may seek to validate the instrument.

Future Directions

Most of the current literature focuses on studying the use of VMs to teach fractions (Bouck et al., 2017; Hansen et al., 2016; Hensberry et al., 2015; Shin et al., 2017), but teachers in this study more frequently use VMs to teach geometry during and after ERT so perhaps more research is needed in this area. Future research may also investigate differences in experiences between teachers who use VM to model mathematics concepts to students rather than students using VM to actively participate in lessons. Additionally, this study did not elicit student perceptions of VMs but tried to increase our understanding of student's experiences from their teachers' perspectives. Future studies should be conducted to better understand students' perceptions of VM during and after ERT. Finally, this study only sought to describe teacher perceptions and experiences about VM using the TAM as a framework, but future research could consider using the TAM to predict intention to use VM.

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s11528-022-00796-9.

Declarations

Conflict of Interest The authors declare that they have no conflict of interest.

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