

Gravitational form factors and nucleon spin structure

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Nucleon scattering by the classical gravitational field is described by the gravitational (energy-momentum tensor) form factors (GFFs), which also control the partition of nucleon spin between the total angular momenta of quarks and gluons. The equivalence principle (EP) for spin dynamics results in the identically zero anomalous gravitomagnetic moment, which is the straightforward analog of its electromagnetic counterpart. The extended EP (ExEP) describes its (approximate) validity separately for quarks and gluons and, in turn, results in equal partition of the momentum and total angular momentum. It is violated in quantum electrodynamics and perturbative quantum chromodynamics (QCD), but may be restored in nonperturbative QCD because of confinement and spontaneous chiral symmetry breaking, which is supported by models and lattice QCD calculations. It may, in principle, be checked by extracting the generalized parton distributions from hard exclusive processes. The EP for spin-1 hadrons is also manifested in inclusive processes (deep inelastic scattering and the Drell–Yan process) in sum rules for tensor structure functions and parton distributions. The ExEP may originate in either gravity-proof confinement or in the closeness of the GFF to its asymptotic values in relation to the mediocrity principle. The GFFs in time-like regions reveal some similarity between inflation and annihilation.

Keywords gravity, form factors, equivalence principle

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1 Introduction

The gravitational form factors (GFFs) are the matrix elements of the energy-momentum tensor (EMT). These objects describing the interaction with fermions probably first appeared in the seminal paper of Kobzarev and

Okun in which the equivalence principle (EP) for spin motion was first identified [1]. Note that as spin is essentially a quantum concept, this paper was probably (one of) the first discussions of the interaction of classical gravity with quantum objects. According to the EP, the anomalous gravitomagnetic moment (AGM), which is the gravitational analog of the anomalous magnetic moment, goes to zero.

This result was soon derived starting from conservation of momentum and angular momentum [2], which manifests the appearance of the EP as a low-energy theorem [3]. Here the axiomatic aspect of the EP is in fact transferred to the postulation that the EMT is a current coupled to gravity.

The particular case of strongly interacting particles was considered by Pagels [4]. Later, when energy-momentum matrix elements were considered as the moments of generalized parton distributions (GPDs), the conservation laws mentioned above led to Ji's sum rules [5, 6], which control the partition of momentum and total angular momentum between quarks and gluons.

The consideration of EMT hadronic matrix elements as couplings to gravity [7, 8] provided another interpre-

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tation of these sum rules and opened the possibility of probing the gravitational couplings of quarks and gluons *separately*. This, with special emphasis on the applications of GFFs to gravitational and cosmological problems, is the main subject of the current paper.

2 GFFs of nucleon and parton angular momenta

Let us define the matrix elements of the EMT as

$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') [A_{q,g}(\Delta^2) \gamma^{(\mu} p^{\nu)} + B_{q,g}(\Delta^2) P^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha / 2M] u(p), \quad (1)$$

where $P^\mu = (p^\mu + (p')^\mu)/2$, $\Delta^\mu = (p')^\mu - p^\mu$, and $u(p)$ is the nucleon spinor. We omit, for the moment, the terms of higher order in Δ , as well as those containing $g^{\mu\nu}$, which will be discussed later. The parton momenta and total angular momenta are just

$$\begin{aligned} P_{q,g} &= A_{q,g}(0), \\ J_{q,g} &= \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)]. \end{aligned} \quad (2)$$

Taking into account conservation of momentum and angular momentum,

$$A_q(0) + A_g(0) = 1, \quad (3)$$

$$A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1, \quad (4)$$

one can see that the difference between partition of the momentum and that of the orbital angular momentum arises entirely from “anomalous” form factors [$B_q(0) = -B_g(0)$].

This property has a seemingly unexpected counterpart in another field of physics [7]. Namely, using the fact that the matrix element (1) describes the interaction of a nucleon with the classical external gravitational field, one arrives at the interpretation of B as an AGM that is the straightforward analog of its electromagnetic (EM) counterpart. The natural extension of the well-known Einstein EP results in zero AGM.

3 Nucleons in an external gravitational field

It is instructive to examine the GFFs (which are straightforward analogs of EM ones) by considering [7] the action of EM and gravity fields uniformly and comparing them. The presentation follows the textbook derivation for the EM case (see [9, 10], section 116).

Let us start with the more common case of the interaction with an electromagnetic field, which is described

by the matrix element of the EM current,

$$M = -\langle P' | J_q^\mu | P \rangle A_\mu(\Delta). \quad (5)$$

This matrix element at zero momentum transfer is fixed by the fact that the interaction is due to the *local* U(1) symmetry, whose *global* counterpart produces the conserved charge [and of course depends on the normalization of eigenvectors $\langle P | P' \rangle = (2\pi)^3 2E \delta(\mathbf{P} - \mathbf{P}')$].

$$\langle P | J_q^\mu | P \rangle = 2e_q P^\mu. \quad (6)$$

Thus, in the rest frame, the interaction is defined completely by the scalar potential:

$$M_0 = -\langle P | J_q^\mu | P \rangle A_\mu = -2e_q M \phi(\Delta). \quad (7)$$

At the same time, the interaction with the weak classical gravitational field is

$$M = -\frac{1}{2} \sum_{q,G} \langle p' | T_{q,G}^\mu | p \rangle h_{\mu\nu}(\Delta), \quad (8)$$

where h is the deviation of the metric tensor from its Minkowski value. The relative factor 1/2, which will play a crucial role, arises from the fact that the variation of the action with respect to the metric produces an EMT with the coefficient 1/2, whereas the variation with respect to a classical source A^μ produces a current without this coefficient. It is this coefficient that guarantees the correct value in the Newtonian limit, which is fixed by the *global* translational invariance

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2P^\mu P^\nu, \quad (9)$$

which, together with the approximation for h (where the factor of 2 has a geometrical origin) [11, 12],

$$h_{00}(x) = 2\phi(x), \quad (10)$$

results in the rest frame expression:

$$M_0 = -\frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu}(\Delta) = -2M \cdot M \phi(\Delta), \quad (11)$$

where we used the same notation for the gravitational and scalar EM potentials and identified the normalization factor as $2M$ in order to make the similarity between (7) and (11) obvious. One can see that the interaction with a gravitational field is described by the charge, which is equal to the particle mass; this is just the EP. It appears here as a low-energy theorem rather than a postulate. The similarity with the EM case allows one to clarify the origin of this theorem, suggesting that the interaction with gravity is due to the *local* counterpart of *global* symmetry, although it may be proved starting just from the Lorentz invariance of the soft graviton ap-

proximation [3]. Here the axiomatic aspect of the EP is in fact transferred to the postulation of the EMT as a current coupled to gravity.

The situation for the terms linear in Δ is different for electromagnetism and gravity. Whereas these terms are defined by the specific dynamics in the EM case, producing the anomalous magnetic moment, the similar contribution in the gravitational case is entirely fixed by angular momentum conservation (4).¹ This means that, in terms of the gravitational interaction, the AGM of any particle is identically equal to zero.

One can generalize this pioneering result of Kobzarev and Okun and show [7] that it is not restricted to the nucleon or spin-1/2 Dirac particle. The presence of Dirac spinors in the parametrization (1) is actually not crucial. To show this, it is convenient to use the equation of motion in order to attribute all the Δ dependence to the *anomalous* form factor $P^\mu \bar{u} \sigma^{\nu\alpha} u \Delta_\alpha$. After the linear Δ dependence is extracted, the spinors can be taken at the same momentum, which is a convenient choice for the average one, P , and calculation of the matrix element is reduced to the trace of the density matrix:

$$\begin{aligned} \bar{u}(P) \sigma^{\nu\alpha} \Delta_\alpha u(p) &= \text{Tr} \rho(P) i \sigma^{\nu\alpha} \Delta_\alpha \\ &= \text{Tr} \frac{1}{2} (\hat{P} + M) (1 + \hat{S} \gamma_5) i \sigma^{\nu\alpha} \Delta_\alpha \\ &= 2i \epsilon^{\rho\sigma\nu\alpha} P^\rho S^\sigma \Delta^\alpha. \end{aligned} \quad (12)$$

By considering the matrix element of the projection of the Pauli–Lubanski operator, the constraint (4) may now be easily generalized to a particle of *any* spin; thus, the *total* conserved EMT of all the constituents is

$$\langle p' | \sum T^{\mu\nu} | p \rangle = 2P^\mu P^\nu + i P^{(\mu} \epsilon^{\nu)\sigma\rho\alpha} P^\rho S^\sigma \Delta^\alpha / M. \quad (13)$$

As in the spin-1/2 case, S is the average spin of the states $|P\rangle, |P'\rangle$ (they should enter in the symmetric way because of the positive charge parity of the EMT).

As the form factors in the spin-1/2 case differ from those for the matrix element of the vector current J^μ by the common factor P^ν , one may define the *gyrogravitomagnetic ratio* in the same way as the common gyromagnetic ratio, and it should have the Dirac value $g = 2$ for a particle of any spin J :

$$\mu_G = J, \quad (14)$$

which coincides with the standard Dirac magnetic moment up to the interchange $e \leftrightarrow M$, making the Bohr

magneton equal to 1/2.

However, the situation changes if one defines the gyrogravitomagnetic moment as a response to an external gravitomagnetic field. The ϵ tensor in coordinate space produces the curl, and the gravitomagnetic field acting on the particle spin is equal to

$$\mathbf{H}_J = \frac{1}{2} \text{rot} \mathbf{g}; \quad \mathbf{g}_i \equiv g_{0i}, \quad (15)$$

where the factor 1/2 is just the normalization factor in Eq. (10) mentioned above. The relevant off-diagonal components of the metric tensor may be generated by rotation of a massive gravity source [11, 12].

This field also induces another effect: the straightforward analog of the Lorentz force [11, 12] produced by the first (spin-independent) term in (14). In that case, the gravitomagnetic field for low-velocity particles (this restriction is actually inessential, as we can always perform a Lorentz boost, reducing the particle velocity sufficiently) is

$$\mathbf{H}_L = \text{rot} \mathbf{g} = 2\mathbf{H}_G. \quad (16)$$

Consider now the motion of the particle in the gravitomagnetic field. The effect of the Lorentz force is reduced according to the Larmor theorem (which is also valid for low velocities) to rotation with the Larmor frequency

$$\omega_L = \frac{H_L}{2}. \quad (17)$$

This is also the frequency of the *macroscopic* gyroscope dragging. At the same time, the *microscopic* particle dragging frequency is

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L. \quad (18)$$

The common frequency for microscopic and macroscopic gyroscopes is just the Larmor frequency; thus, the effect of the gravitomagnetic field is equivalent to frame rotation. This should be considered as a (post-Newtonian) manifestation of the EP.

Our approach clarifies the origin of this equality as a cancellation of the geometrical factor 1/2 in Eq. (12) and the “quantum” value 2 of the gyrogravitomagnetic ratio. Note that for a free particle, the latter coincides with the usual gyromagnetic ratio, and such a cancellation provides an interesting connection between geometry, the EP, and the special renormalization properties (cancellation of the strongest divergencies) for particles with $g = 2$.² Another interesting connection is provided by the fact that it is just the deviation from $g = 2$ that

¹) The reason is that the structure of the Poincaré group is more extensive than that of the $U(1)$ group.

²) The gravitational interaction of particles with $g = 2$ was discussed in Ref. [13] as providing the most elegant form for the interaction.

determines the Gerasimov–Drell–Hearn sum rule for a particle with arbitrary spin [14].

The equality of precession of classical and quantum rotators was confirmed by analysis of the Dirac equations in external gravitational and inertial fields starting from the rotating frame [15] and in weak gravity [16] and later by generalization to the case of rotating bodies in strong and even arbitrary fields [17–19]. During this analysis, it was observed [16] that the definition of the EP as equal motion of a test body in a gravity field and the non-inertial frame, which is frequently given in textbooks, is actually valid only for nonrelativistic motion. This can easily be shown using the GFF method.

The particle momentum evolution is fully determined by the forward matrix element fixed by momentum conservation (9), and the matrix element (11) for a particle at rest takes the form

$$\mathcal{M} = M^2 h_{00}(q). \quad (19)$$

The coincidence of the (00) components of the metric in the gravitational field and accelerated frame proves the EP.

At the same time, for a moving particle, the space components of the metric $h_{zz} = h_{xx} = h_{yy} = h_{00}$ (see, e.g., [11, 12])³⁾ also contribute. As a result, the matrix elements in the gravitational field (M_g) and in the accelerated frame (M_a) differ by the obvious kinematic factor:

$$M_g = (\epsilon^2 + (\mathbf{p})^2) h_{00}(q), \quad M_a = \epsilon^2 h_{00}(q). \quad (20)$$

The ratio of these matrix elements is

$$R = \frac{\epsilon^2 + (\mathbf{p})^2}{\epsilon^2}. \quad (21)$$

This expression for ultrarelativistic particles provides just the well-known factor of 2 that is the difference between the Einsteinian and Newtonian expressions for light deflection. This means that the problem being discussed, despite some misleading statements in the literature, is familiar to experts. This also demonstrates that the coupling to the EMT is the real content of the EP.

4 Helicity in a gravitational field and possible cosmological implications

The crucial factor of 1/2 makes the evolution of the particle helicity rather different in magnetic and gravitomagnetic fields. The spin of a (Dirac) particle in a magnetic field is dragged with the cyclotron frequency, which is twice the Larmor frequency. It coincides with the frequency of the velocity precession; thus, helicity is

conserved. At the same time, the gravitomagnetic field causes the velocity to be dragged twice as fast as the spin, changing the helicity. This factor of 2, however, is precisely that required to realize the possibility of reducing the entire effect of the gravitomagnetic field to frame rotation. Whereas the spin vector is the same in the rotating frame and is dragged only by rotation of the coordinate axis, the velocity vector is transformed and makes an additional contribution, providing the factor of 2 to Coriolis acceleration. The geometrical factor of 1/2 connects these phenomena with the EP.

Note that all of these considerations are essentially based on the smallness of the particle velocity achieved by the Lorentz boost as mentioned above and therefore do not lead to loss of generality. Let us consider a massive particle scattered by a rotating astrophysical object. The effect of the gravitomagnetic field is reduced to rotation of the local comoving frame, which becomes inertial at large distances before and after scattering. Consequently, the helicity is not changed by the gravitomagnetic field, which is confirmed by explicit calculation of the Born helicity-flip matrix element in the case of a massive neutrino [7, 17].

It may seem that the EP should entirely exclude the possibility of the helicity flip in scattering by a gravity source. This is, however, not the case if the usual Newtonian-type “gravitoelectric” force is considered [7]. Its action is also reduced to local acceleration of the comoving frame, in which the helicity of the particle is not altered. However, the comoving frame after scattering differs from the initial one by the corresponding velocity $\delta\mathbf{v} = \int \mathbf{a} dt$. The corresponding boost to the original frame generally changes the helicity of the massive particle (the similar effect for the gravitomagnetic field is just rotation by the angle $\delta\Omega = \int \boldsymbol{\omega} dt$ and does not affect the helicity). The same boost may be considered as the source of the well-known deflection of particle momentum $\delta\phi \approx |\delta\mathbf{v}|/|\mathbf{v}|$ leading to the Newtonian expression, as this consideration applies to a nonrelativistic particle. The average helicity of the completely polarized beam after such scattering may be estimated in the semiclassical approximation as $\langle P \rangle \approx \cos\phi \approx 1 - \phi^2/2$. According to the correspondence principle, this quantity may be expressed as

$$\langle P \rangle = \frac{d\sigma_{++} - d\sigma_{+-}}{d\sigma_{++} + d\sigma_{+-}} \approx 1 - 2 \frac{d\sigma_{+-}}{d\sigma_{++}}, \quad (22)$$

where $d\sigma_{+-} \ll d\sigma_{++}$ are the helicity-flip and non-flip cross sections, respectively. Comparing the classical and quantum expressions for $\langle P \rangle$, one obtains

³⁾ Note that this expression can also be obtained using the GFFs just by contracting (9) with the graviton propagator.

$$\frac{d\sigma_{+-}}{d\sigma_{++}} \approx \frac{\phi^2}{4}. \quad (23)$$

To check this simple approach, one may calculate this ratio for a Dirac particle scattered by a gravitational source. In the Born approximation, the result is easy to find:

$$\frac{d\sigma_{+-}}{d\sigma_{++}} = \frac{\tan^2 \frac{\phi}{2}}{(2\gamma - \gamma^{-1})^2}. \quad (24)$$

This expression coincides with the estimate (23) when the deflection angle is small and the particle is slow ($\gamma = E/m \rightarrow 1$), whereas for fast particles,

$$\frac{d\sigma_{+-}}{d\sigma_{++}} \approx \frac{\phi^2}{16\gamma^2}. \quad (25)$$

This effect should, in particular, lead to the helicity flip of any massive neutrino. It is very small when scattering by a single object is considered but may be enhanced when a neutrino propagates in the Universe. This effect can be especially strong for anisotropic Bianchi universes [21]. The anisotropy of the Universe leads to precession of spin (and, according to the EP, as discussed above, to the helicity flip) with a characteristic timescale on the order of a very early Universe age. This should turn Majorana neutrinos into antineutrinos and Dirac ones into their sterile counterparts. The resulting fraction of sterile Dirac neutrinos should be, generally speaking, of order 1. At the same time, modest fine-tuning on the order of a percent may lead to an excess of sterile neutrinos over active ones by two orders of magnitude, which might be sufficient to explain the deficit of neutrinos that prevents them from constituting dark matter.

Manifestation of the post-Newtonian EP is especially interesting when the gravitoelectric component is absent. In contrast to the EM case, one cannot realize this situation by canceling the contributions of positive and negative charges. At the same time, one may consider instead the interior of a rotating shell (the Lense–Thirring effect). Especially interesting is the case of the shell constituting a model of the Universe (see Ref. [22], Section 21.12; Ref. [23], Section 9.7), the mass and radius of which are of the same order (which probably should categorize such models as open universes), when the dragging frequency may be equal to the shell rotation frequency, which is just Mach’s principle. Note that the low-energy theorem and emerging EP, which guarantee a unique precession frequency for all quantum and classical rotators, is a necessary counterpart of Mach’s principle. Otherwise, rotation of the Universe as a whole may be detected from the disagreement between classical and quantum rotators.

5 Extension of EP

The relation of GFFs to the moments of GPDs opens, in principle, the possibility of extracting the separate couplings of quarks and gluons to gravity using experimental data and nonperturbative quantum chromodynamics (NPQCD) calculations [7]. One may pass then to the extended EP (ExEP) [24, 25], which states that the individual contributions of quarks and gluons to the AGM of the nucleon are zero in NPQCD.

This property is violated in perturbation theory (PT) [26–29]. This is not surprising qualitatively, as quarks and gluons correspond to the matrix elements of non-conserved operators that cannot be separated owing to operator mixing, whereas beyond the PT, one may consider the dominant nonperturbative (up to PT corrections) separation.

The reason for PT violation of the ExEP value was attributed [26, 27] to the use of the unsubtracted dispersion relations for the relevant form factors. In the case of performing the subtraction, it results in zero AGM but leads to the absence of a smooth transition of the electron mass $m_e \rightarrow 0$. At the same time, such a limit is unphysical in the case of (almost) massless quarks because of spontaneous chiral symmetry breaking, and the ExEP may be restored [24, 25].

This picture is already supported by the well-known fact of approximate cancellation of the proton and neutron anomalous magnetic moments. Their singlet combination differs from the AGM because they correspond to the first and second moments, respectively, of the GPDs E . This picture is also supported by the lattice data, including the most recent data [30]. However, interesting support is coming from the consideration of spin-1 hadrons.

5.1 EP for deuterons and vector mesons

As we discussed above, the concepts of the AGM and (Ex)EP may be easily generalized [7] to particles of any spin. As in the case of spin-1/2 particles, the smallness of the AGM should correspond to the smallness of the isoscalar magnetic moment; for ρ mesons, it should lead [31] to $g \approx 2$. Recently, the gyromagnetic ratio of charged ρ mesons was calculated in lattice QCD [32], and a value fairly close to the ExEP-supported factor of 2 was obtained.

There are more spin degrees of freedom for vector mesons (and, say, deuterons), which leads to additional GFFs [33]. The tensor polarization is P -even, unlike the vector ones. It does not require the Levi-Civita tensor;

thus, EP is also demonstrated [34] for forward matrix elements.

These considerations lead to the zero sum rule for the sum of the second moments of the quark and gluon tensor spin structure functions derived many years ago [35, 36], and the suggested demonstration [34] of the EP is related to them as that of Kobzarev and Okun is related to Ji's sum rules:

$$\begin{aligned} \sum_q \langle P, S | T_i^{\mu\nu} | P, S \rangle_{\mu^2} &= 2P^\mu P^\nu (1 - \delta(\mu^2)) \\ &+ 2M^2 S^{\mu\nu} \delta_1(\mu^2), \\ \langle P, S | T_g^{\mu\nu} | P, S \rangle_{\mu^2} &= 2P^\mu P^\nu \delta(\mu^2) - 2M^2 S^{\mu\nu} \delta_1(\mu^2). \end{aligned} \quad (26)$$

$$(27)$$

The ExEP, in turn, indicates that it is valid separately for quarks and for gluons; thus, $\delta(\mu^2) \approx 0$. The available data on the deuteron tensor spin structure function [37] provide just the required singlet combination for quarks and indicate that the second moment is more compatible with zero than the first one. This means that collective gluons in the deuteron are suppressed in comparison to the collective sea, and the ExEP explains this fact.

5.2 Cosmological implications of ExEP

One may of course think that the smallness of the individual quark and gluon AGMs for nucleons and the closeness of the quark gravigyromagnetic ratio to 2 for vector mesons is occasional. It is possible, nevertheless, to speculate [39] about their possible relevance to very general problems of gravity and cosmology.

Consider the very strong gravity field near a black hole. Assume also the semiclassical picture in which the average angular momenta of quarks and gluons precess independently. If their angular velocities differ, the field may deconfine hadrons. If, however, they precess in accordance with the ExEP, the confinement may be gravity-proof, which is attractive because of the well-known black hole complementarity [39].

Another possibility may be the closeness of the ExEP values to the asymptotic ones. Assuming that QCD evolution is growing backward in the expanding Universe, one should recall that this process is “antikinetic” [40]. As a result, too large deviations from the asymptotic values at large scales should result in violation of positivity at lower scales. Assuming the existence of a multiverse, one may expect that the probability of obtaining a particular deviation from the asymptotic values should be its decreasing function. As a result, Vilenkin's mediocrity principle (see, e.g., Ref. [41]) should favor smaller deviations from the asymptotic values.

6 Quadrupole and time-like GFFs

Let us now discuss the quadrupole GFF, which is related in terms of the GPDs to the elegant Polyakov–Weiss D-term [42] appearing in analyticity-based analysis as a subtraction constant [43, 44]:

$$\langle P + q/2 | T^{\mu\nu} | P - q/2 \rangle = C(q^2)(g^{\mu\nu} q^2 - q^\mu q^\nu) + \dots, \quad (28)$$

where the gravitoelectric and gravitomagnetic form factors (1) are now dropped. C has definite (positive) sign in all known cases, including hadrons (see [42], which also discusses general stability arguments), photons [45], and Q-balls [46, 47].

It is interesting to compare [48] this decomposition to the vacuum matrix element when one has the well-known cosmological constant:

$$\langle 0 | T^{\mu\nu} | 0 \rangle = A g^{\mu\nu}. \quad (29)$$

One may relate the matrix element (29) to the vacuum one in two-dimensional transverse space orthogonal to P and q to obtain the effective two-dimensional “cosmological constant”:

$$A = C(q^2)q^2. \quad (30)$$

The positive C leads to a negative cosmological constant in the scattering process (when $q^2 < 0$) and to a positive one in the annihilation process described by time-like GFFs when $q^2 > 0$. There seems to be some relation between annihilation and inflation!

It is of course very interesting whether the real cosmological constant in our Universe may be understood as emerging from annihilation at extra dimensions. This is qualitatively similar to brane cosmology, and one should stress that in this scenario, the Big Bang is due to one-graviton annihilation. The specification of extra-dimensional states that provide the cosmological constant in our space, which has the mass dimension 4, remains to be investigated.

7 Discussion and conclusions

The GFFs provide a complementary interpretation, stemming from the EP, of some basic elements of the nucleon's spin structure, such as Ji's sum rules.

Scattering amplitude analysis involving these form factors constitutes a complementary approach to gravitational phenomena involving elementary particles. In particular, the helicity dynamics of massive Dirac neutrinos in an anisotropic Universe may lead to formation of dark

matter from their sterile components.

From the other side, the GFFs provide a unique option for extracting information about the separate couplings of quarks and gluons to gravity from experiments and (mostly) from NPQCD. This knowledge is not incompatible with the approximate validity of the ExEP separately for quarks and gluons. It may be valid only in NPQCD and may possibly lead to gravity-proof confinement.

It is quite interesting to consider GFFs in time-like regions, where they indicate some similarity between inflation and annihilation.

Further tests of the ExEP may include lattice calculations and experiments, in particular more accurate measurement of the deuteron tensor spin structure function.

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