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# A review on ductile mode cutting of brittle materials

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**Abstract** Brittle materials have been widely employed for industrial applications due to their excellent mechanical, optical, physical and chemical properties. But obtaining smooth and damage-free surface on brittle materials by traditional machining methods like grinding, lapping and polishing is very costly and extremely time consuming. Ductile mode cutting is a very promising way to achieve high quality and crack-free surfaces of brittle materials. Thus the study of ductile mode cutting of brittle materials has been attracting more and more efforts. This paper provides an overview of ductile mode cutting of brittle materials including ductile nature and plasticity of brittle materials, cutting mechanism, cutting characteristics, molecular dynamic simulation, critical undeformed chip thickness, brittle-ductile transition, subsurface damage, as well as a detailed discussion of ductile mode cutting enhancement. It is believed that ductile mode cutting of brittle materials could be achieved when both crack-free and no subsurface damage are obtained simultaneously.

**Keywords** ductile mode cutting, brittle materials, critical undeformed chip thickness, brittle-ductile transition, subsurface damage, molecular dynamic simulation

## 1 Introduction

Brittle material like glass, silicon, tungsten carbide (WC), germanium and silicon nitride have been widely employed in the industries such as precision engineering, optics, instruments, semi-conductor and micro-electromechanical systems (MEMS) because of its excellent mechanical,

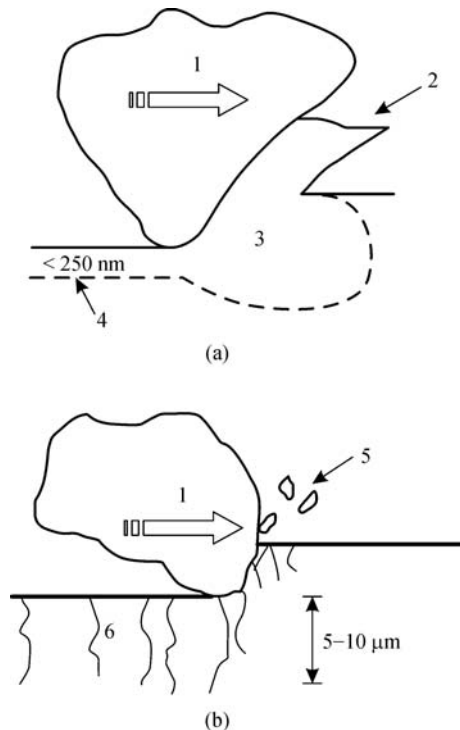
optical, physical and chemical properties. Also, there are rapid growing demands on manufacturing of brittle materials achieving a good quality surface finish, stringent geometry accuracy, and surface integrity with less or free of subsurface damage. Meanwhile, to reduce the manufacturing cost in the production of these components and devices made by brittle materials, effectively machining of these materials is very much demanded. Traditionally, abrasive processes such as grinding, lapping and polishing have been widely used for the final surface finishing of these brittle materials. The demerits associated with these processes include poor grindability, high manufacturing cost, and subsurface damage [1]. Furthermore, the abrasive processes will cause surface flatness deviation due to its uncontrollable material removal resulting in the machined profile inaccuracy [2]. Therefore, after grinding and lapping processes, the chemical-mechanical polishing (CMP) is essential to remove the subsurface damage layer caused by the hard abrasive particles, which makes a very costly production [3]. Also, these abrasive processes especially like CMP are extremely slow, while grinding and lapping processes would impart subsurface damage leading to a degraded surface integrity [4].

In order to improve the surface integrity of these materials, ductile mode cutting (DMC), also called ductile regime cutting or ductile cutting, as a promising technique, has been studied vigorously over the past decades and it is commonly understood that DMC is to remove work materials by plastic flow instead of brittle fracture deriving a damage-free surface. As a result, the subsequent polishing process is no longer necessary or the polishing time can largely reduced because the crack-free surfaces can be directly produced by DMC without subsurface damage or the subsurface damage layer thickness being much smaller, which would significantly reduce the manufacturing time and cost for brittle materials. This advantage cannot be under addressed because in machining even a minor improvement in productivity would lead to a major impact in mass production. A schematic comparison between DMC and BMC (brittle mode cutting) of brittle materials helps to reveal the underlying mechanisms as shown in Fig. 1. The fundamental premise

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1–Abrasive grain; 2–Ductile chip removal; 3–Metallic phase; 4–Machined layer; 5–Brittle chip removal; 6–Microcracks

**Fig. 1** Schematic diagrams of two cutting modes for brittle materials [5]. (a) DMC by removing a ductile metallized layer resulted from the large contact pressure in cutting region; (b) BMC by material fracture leaving subsurface damages, of which the subsurface damage is as deep as  $5\text{--}10\ \mu\text{m}$  due to crack propagations in machining of silicon

of ductile mode cutting states that all brittle material will experience a transition from DMC to BMC when cutting from zero depth of cut (DoC) to a large value regardless its hardness and brittleness. When cutting below the critical undeformed chip thickness (UCT), the energy consumed for crack propagation is larger than that for plastic deformation, DMC will be achieved in brittle materials successfully [6].

The idea of ductile mode machining and its concept appearing in the literature was first reported by King and Tabor [7] in 1954, where the material removal in abrasive wear of rock salt occurred as a result of removing a plastically deformed layer rather than a brittle fracture, although some cracking and fragmentation were still observed. Later in 1976, Huerta and Malkin [8] were the first to show reproducible results of diamond grinding of glass in a ductile mode, which considerably improved the surface quality and machining accuracy. Later, precision grinding of brittle materials in a ductile mode had been extended to others such as silicon and ceramics. Further improvements in ultra-precision machining technology in the 1990s marked the progression for DMC to be applied in more advanced brittle materials such as different types of carbides [9–16]. Ductile mode cutting thus became an

alternative way for finishing of brittle materials as it could produce crack-free mirror surface finish at a much higher efficiency and lower cost than polishing processes owing to its high material removal rate.

This paper will summarize the following key points in DMC of brittle materials: Ductile nature and plasticity of brittle materials, DMC mechanism, and DMC characteristics including critical undeformed chip thickness, chip morphology, surface finish and tool wear, as well as molecular dynamic simulation in cutting of brittle materials. Finally, the last two sessions give an overall discussion and summary of DMC of brittle materials.

## 2 Ductile nature and plasticity of brittle materials

Ductility of a material is defined as the material's ability to undergo permanent deformation through elongation (area reduction in the cross section) or bending without fracturing, while plasticity is defined as the material's deformation, which undergoes non-reversible changes of shape in response to applied forces and/or loading. All materials exhibit the ductile nature no matter how brittle they are, save for the fact that the extent of ductility or plasticity varies for different materials [17]. In evaluating the ductility of a material, an indentation test has been most employed in tandem with other processes such as scratching and grinding.

One of the typical brittle materials, glass, exhibits plasticity or ductile behaviour in the micro-indentation test with concentrated loads at a point, known as microplasticity [18]. Auerbach's law, the linear dependence between cracking load and indenter's diameter, was reported to be a direct consequence of the brittle-ductile transition produced [19]. Indentation on the soda-lime glass at different loads using a Vicker's pyramid indenter indicated that above a certain critical loading cracking was favourable while below the critical loading plastic flow was possible [20]. Indentation method was also used to evaluate the plastic deformation of brittle materials at high hydrostatic pressure [21–24]. A schematic illustration is shown in Fig. 2 for the elastic-plastic behaviour of brittle materials under indentation [21]. Indentation testing at light loadings shows that in the region immediately below the indenter, the material expands and exerts pressure to the surroundings. This creates a uniform hydrostatic pressure around that region and the material flows according to a yielding criterion. An elastic matrix lies beyond this plastically deformed region.

The ductile behaviour of brittle materials below the indenter could be due to phase transformation mechanism, where the characteristic phase of brittle solid transits into a metallic phase under the influence of hydrostatic pressure. This concept was verified by measuring the electrical conductivity of the material near the indenter tip during the

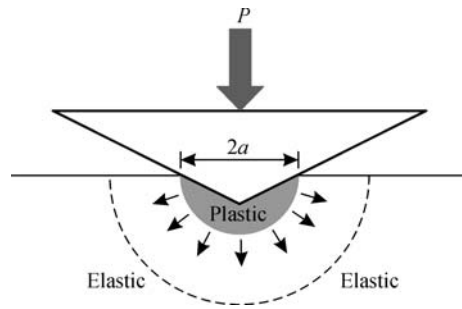


Fig. 2 An indentation model of elastic-plastic behaviour [21]

indentation process of brittle materials. The measurement results revealed a substantial increase in the conductivity of the material below the indenter that can be plastically deformed, which supports the theory of transition to a metallic state [25,26].

### 3 Ductile mode cutting mechanism

The concept of DMC material removal is based on the hypothesis that all brittle material will experience a brittle-ductile mode transition in cutting with an UCT below the critical value. Some mechanisms in DMC are described below in details.

One understanding of material removal mechanism can be illustrated by indentation-sliding analysis [20,27]. The material removal happens in four stages. (a) Material under indenter started to subject an elastic deformation. This creates a small elastic deformation zone due to high hydrostatic pressure below the indenter. (b) Radial/median cracks formed on a plane at the elastic-plastic boundary when further increasing the loading. (c) Lateral cracks formed in an addition to radial/median cracks, which spread outward from the deformation zone, beneath the indentation surface, and may interact with the radial system. (d) When severely loaded those cracks turn upward to intersect the free surface, thereby causing severe disruption of the pattern by chipping. Residual stresses are the main cause of lateral cracking and eventually resulted in the material removal by fracturing. In the nanometer scale cutting of silicon using a diamond tool, this mode of material removal must be avoided as much as possible to eliminate brittle fracture and consequent micro-crack formation on or near the machined surface.

More work on DMC of brittle materials has been reported, but the nature of brittle-ductile transition is not very clear. A systematic study of the machining mechanism is of theoretical significance and practical value-added. Many studies have been developing into understanding the brittle-ductile transition phenomenon and revealing their mechanism. One view of brittle-ductile transition is based on cleavage fracture due to pre-existing flaws [28]. Also, a larger DoC would definitely result in a

larger UCT, which may cause the material removal in the brittle-ductile transition manner. Nakasuji et al. [28] and Shimada et al. [22] proposed a possible material removal mechanism, which can be classified into two modes in cutting of brittle materials. One is due to plastic deformation in the characteristic slip direction and another is due to brittle fracture on the characteristic cleavage plane. When DoC scale becomes smaller, such as in the sub-micrometer or nanometer range, both stresses  $\sigma_c$  and  $\tau_c$  increase to the same order as a perfect material's intrinsic strength. Thus, plastic deformation takes place before cleaving.

During the process of indentation, pyramidal indenter is categorized as sharp-type indenter and spherical indenter as blunt-type. If indentation-sliding is applied to simulate ultra-precision cutting, grinding, or polishing, all these indenters do fall into the category of sharp indenters as its edge radius or grit size is extremely small to be ignored. Shaw [29] proposed a material removal mechanism that heavy extrusion happened ahead of a large edge radius tool and Komanduri [30] proposed a mechanism like in grinding to cut using tools with a large negative rake angle. The cutting modes were studied through grooving tests on an inclined plane using a solid cutter [31]. As shown in Fig. 3 where DBT is the abbreviation for "ductile-to-brittle transition", the critical DoC in grooving of tungsten carbide was obtained in average of  $4.76 \mu\text{m}$ . Puttick et al. [32] proposed similar models but including the case of nanometric cutting of a nominally brittle material like silicon. Acoustic emission (AE) signals were monitoring in micro grinding of glass and germanium [33] because of brittle materials having distinct AE energy when machining in different modes. This bond-breaking energy in DMC is greater than that in BMC; hence, the AE energy could be utilized to characterize the brittle-ductile transition.

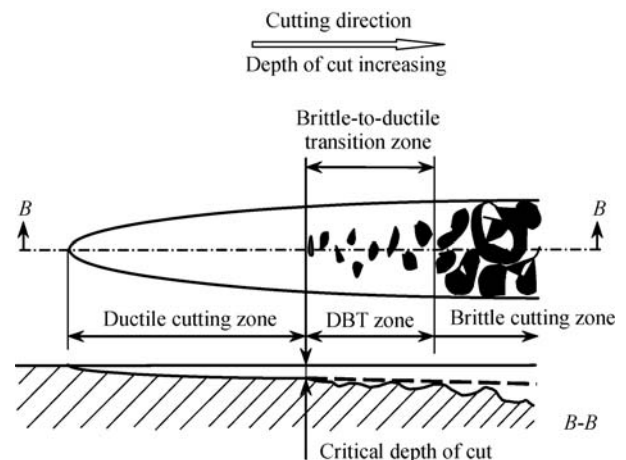


Fig. 3 Schematic diagram illustrated brittle-ductile transition in grooving [31]

Nanoscratch test was conducted on the (111) plane of single crystal GGG ( $\text{Gd}_3\text{Ga}_5\text{O}_{12}$ ) along  $\langle 110 \rangle$  direction with different indentation depth [34]. Its material removal mechanism can be divided into four stages: A plastic flow zone formed through the combination of “polycrystallization of nanocrystalline” and “amorphous transformation” when the normal scratch force is very small; micro crack zone formed showing bifurcation and deflection in propagation when increasing the normal force; median crack generated by the severe slip of crystal planes further increasing normal force; and transverse cracks generated when unloading. The scratch normal force and velocity would affect the residual depth of the scratch, deformation of micro-fracture and burrs in the scratch of glass BK7 as well [35]. In partial ductile mode of silicon, the self-healing of microcracks, microfractures and small spillings could take place by filling the defect cavities with the ductile metallic silicon phase in reality [36]. The material removal mechanism in diamond turning of reaction-bonded silicon carbide involves ductile cutting, cleavage cracking and SiC grain dislodgement [37]. The material removal manner depends on the SiC grain size and depth and the grain boundaries bonding strength. A slip model based on the slip orientation factor was proposed to qualitatively describe the ductile mode turning mechanism of single crystal silicon [38]. The crystallographic orientation also changed with DoC due to the difference in material removal mechanism between plastic deformation and brittle fracture.

When cutting of brittle materials at a DoC being sufficiently small, its tool cutting edge radius  $R$  normally in micron scale, will be at the same order with the used DoC  $a_o$ . Thus, the actual cutting edge will be the arc edge, and the straight cutting edge will not be involved in cutting regardless its nominal rake angle  $\gamma$  being positive or negative. In fact, its actual working rake angle  $\gamma_{ne}$  is always large negative, which is resulting in a large compressive stress in the cutting region. In this scenario, work material fracturing due to pre-existing defects will be suppressed by the large cutting compressive stress undertook in the cutting region, meanwhile plastic deformation will dominate the chip formation [13]. DMC is like the plastic extrusion taken place ahead of the tool. As a result, DMC will be achieved successfully in cutting of brittle materials accordingly.

## 4 Ductile mode cutting characteristics

### 4.1 Critical undeformed chip thickness

The critical UCT in DMC is the chip thickness in which permanent material removal takes place without fracturing or cracking in cutting of brittle materials. Equations have also been reported to determine this critical UCT. Given by Biffano et al. [39], a model was derived based on Griffith

fracture criterion. The critical UCT  $d_c$  is expressed by

$$d_c = \psi \frac{E}{H} \left( \frac{K_c}{H} \right)^2, \quad (1)$$

where  $E$  is the material's Young's modulus,  $H$  is hardness,  $K_c$  is fracture toughness, and  $\psi$  is material's brittle-ductile transition factor, which is varied for different brittle materials [40].

Similarly, Blake and Scattergood [41] and Blackley and Scattergood [42] also developed a turning model considering the damaged depth, as shown in Fig. 4. Ultra-precision cutting of glass was conducted using a diamond tool with nose radius of 0.8 mm to evaluate its cutting performance [43]. Figure 5 showed scanning electron microscope (SEM) photograph of the groove surface achieved in turning of soda-lime glass, and further confirmed the above turning model, where the smooth center area was achieved in DMC and two fractured side areas were obtained in BMC. With this model, Eq. (2) was derived to determine the critical UCT  $d_c$  and damage depth  $y_c$  with the given tool radius  $R$ , tool feed  $f$  and the ductile-brittle transition location  $Z_e$ .

$$\frac{Z_e^2 - f^2}{R^2} = \frac{d_c^2}{f^2} - 2 \frac{d_c + y_c}{R}. \quad (2)$$

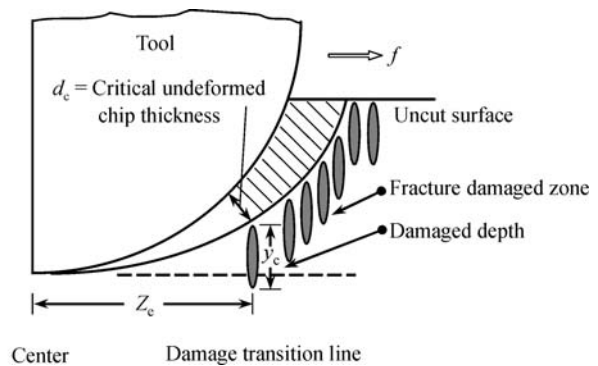


Fig. 4 A turning model considering the damaged depth [43]

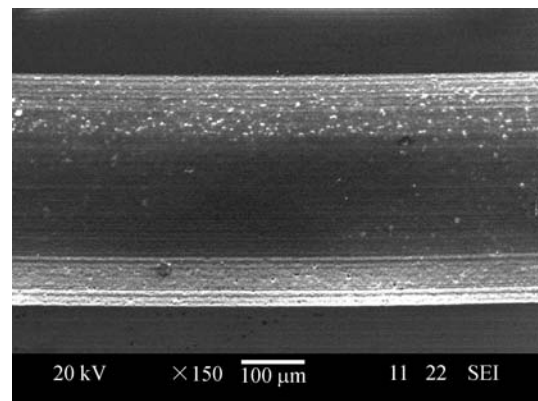
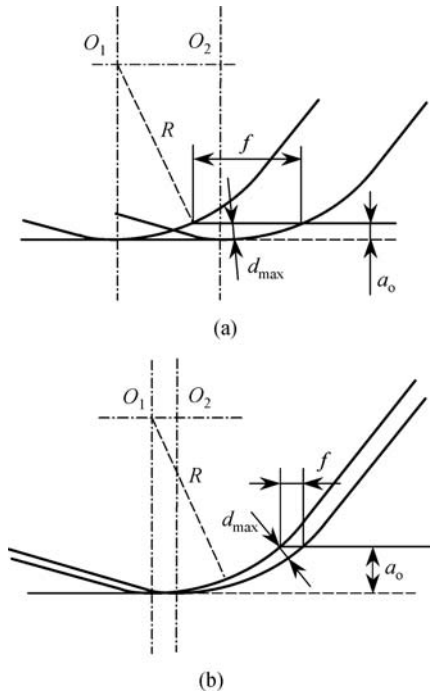


Fig. 5 SEM photograph of the groove surface achieved in turning of soda-lime glass [43]

Machining experiments were performed to develop a model calculating the chip thickness [14,33,44]. As shown in Fig. 6, the maximum UCT  $d_{\max}$  is given by Ref. [44].

$$d_{\max} = R - \sqrt{R^2 + f^2 - 2f\sqrt{2Ra_0 - a_0^2}}, \quad (3)$$

where  $a_0$  is the depth of cut.



**Fig. 6** Schematic illustration of DMC chip formation [43].

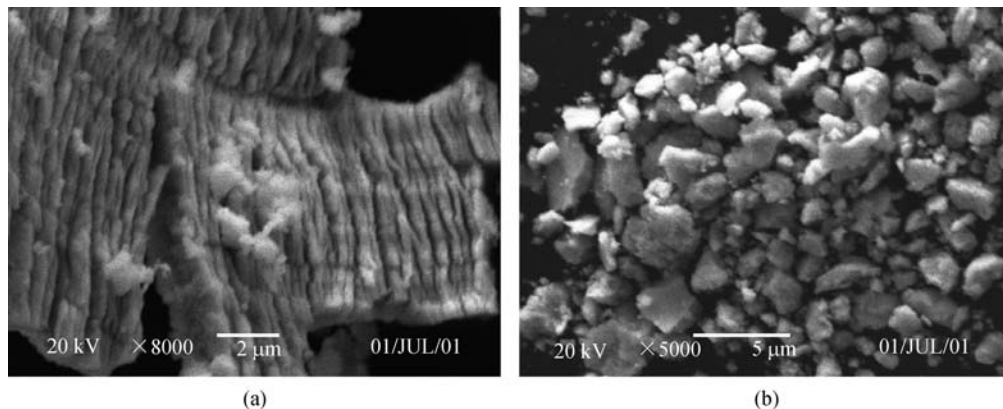
(a)  $\sqrt{2Ra_0 - a_0^2} \leq f$ ; (b)  $\sqrt{2Ra_0 - a_0^2} > f$

Although some works have been done to determine the critical UCT in DMC, so as to avoid experimental trial and error, more work needs to be done in order to devise a more accurate computation to describe of this critical cutting parameter.

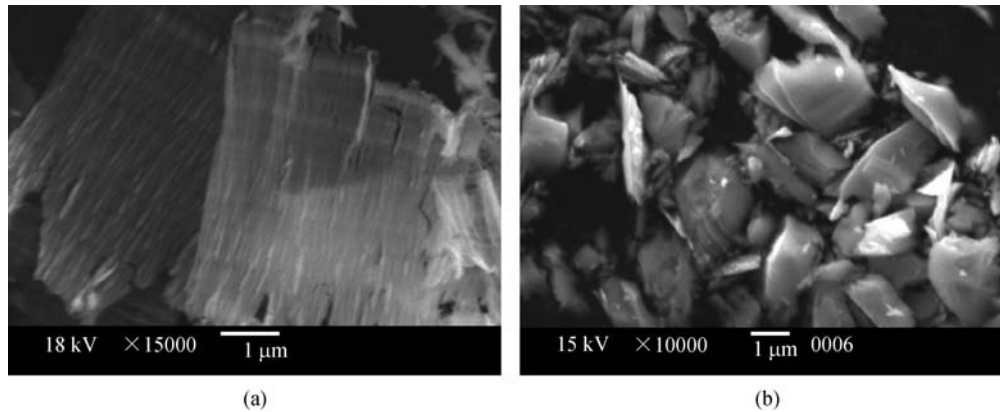
## 4.2 Chip morphology

In DMC, one key characteristic feature is the nature of chip formation. Actually, the apparent departure of DMC from brittle mode machining is its characteristics of chip morphology. In this connection, a considerable number of publications dealt with the studies of chip formation mechanism and chip structure in the course of DMC [14,45]. Figure 7 [14] showed the SEM photographs of chip formed in machining of tungsten carbide, where layer-type chips was obtained when machining with a maximum UCT  $d_{\max}$  of 920 nm (Fig. 7(a)), and particle-type chips obtained when machining under a maximum UCT  $d_{\max}$  of 1164 nm (Fig. 7(b)). Figure 8 [45] showed the SEM photographs of chip formed in machining of single crystal silicon, where layer-type chips was obtained when machining with a maximum UCT of 20 nm (Fig. 8(a)), and particle-type chips obtained when machining under a maximum UCT of 690 nm (Fig. 8(b)). The experimental results on cutting of silicon and tungsten carbide apparently confirmed that DMC can be achieved.

A comprehensive investigation was done to study the chip formation and chip structure in turning of silicon. The studies were conducted using SEM, TEM (transmission electron microscope) and Raman micro-spectroscopy. Three different cutting chip structures, fully lamellar amorphous structure, amorphous structure with traces of crystals, and partially amorphous with residues of crystalline material, were analyzed using electron diffraction [46]. Moreover, it was indicated that up to five structural phases of silicon may be revealed in one chip particle. Based on this analysis, an assumption was formulated that the mechanism of the material removal may vary from shear to extrusion depending on the position of the contacting silicon along the tool cutting edge. This, in turn, was caused by the different combination of shear and compressive stresses in the contact zone. It was also detected that chips removed during cutting of single crystalline silicon consisted of nano-needles, nano-ribbons, and nano-fibers [47]. The shape and size of these



**Fig. 7** SEM photos of chips formation in cutting of WC [14]. (a)  $d_{\max} = 920$  nm; (b)  $d_{\max} = 1164$  nm



**Fig. 8** SEM photos of chip formation in turning of silicon [45]. (a)  $d_{\max} = 20$  nm; (b)  $d_{\max} = 690$  nm

three types of chips depended on the cutting depth and geometric parameters of the cutting edge. Electron diffraction analysis showed that needle like chips had a somewhat amorphized crystal structure, while nano-ribbon and nano-fiber type chips were almost fully transformed into an amorphous phase. This study showed the possibility of the effective and inexpensive use of ductile mode cutting method to produce mechanically flexible nano-ribbons and fibers for nano- and micro-mechanical and electronic devices. It should be noted that in studying of the chip phase composition as well as the machined surface of brittle materials, the Raman spectroscopy is widely used these days [5,48,49] instead of the TEM which was widely used in earlier studies [50,51] on DMC of semiconductor materials. This is due to the fact that Raman spectroscopy makes it possible to diagnose virtually all existing phases of brittle materials, a labor intensive procedure for the sample preparation is not required and the investigations can be conducted at ordinary conditions without the need of vacuum chambers.

#### 4.3 Subsurface damage

Subsurface integrity is another key characteristic feature dominating the DMC of brittle materials. In diamond cutting of single crystalline silicon, there is a machining-induced subsurface damage exhibiting four features of amorphization, poly-crystallization, dislocation, and internal microcracking [52,53]. Near crack free-surface is transformed into an amorphous phase above a dislocation layer, which is mainly caused by the high compressive stress in the cutting zone [52]. Internal microcracks form earlier than surface microcracks. The amorphous layer thickness and dislocation density are very much depended on DoC and tool rake angle because they attribute the high compressive stress [53]. A subsurface damage analysis method was developed to determine the subsurface damage layer thickness by analyzing the surface damaged region with sinusoidal wave along radial direction [54].

Typically, the depth of subsurface damage on a silicon wafer induced by an ultra-precision grinding is around 1–3  $\mu\text{m}$  [55]. The average subsurface damage depth imparted by a fine grinding process of silicon wafers is up to 6  $\mu\text{m}$  [56]. Those damaged layers have to be removed subsequently by employing heavy CMP, which makes the production extremely slow and very costly.

#### 4.4 Surface finish

Surface finish is the nature of a surface, which is another key characteristic feature in DMC. Mirror finish is desired for improved functioning of the optical components made by brittle materials.

Surface characteristics produced by DMC of brittle materials were experimentally studied extensively. It was found surface roughness obtained in DMC of silicon was much better than that generated from the grinding [55]. Very smooth surfaces and continuous chips can be achieved in DMC of silicon using an external high hydrostatic pressure of 400 MPa with a diamond tool having an edge radius at nanometer scale [21]. A surface roughness value of below 10 nm was obtained in DMC of silicon wafers as shown in Fig. 9 [57]. Nakasuji et al. [28] achieved a surface roughness value of less than 20 nm on optical materials by diamond turning. The surface roughness  $R_{\max}$  of 20 nm was achieved at a DoC of 100 nm in cutting of silicon using a large negative rake angle [58]. A surface roughness of  $R_a = 14.5$  nm in ultra-precision turning of glass ZKN7 was obtained using a tool with nanometric edge radius [6]. Liu et al. [43] achieved  $R_a$  of 20.3 nm in groove machining of soda-lime glass using a computerized numerical control turning machine by a diamond tool with a comparatively larger cutting edge involvement. Schinker [59] machined optical glass indicated that the quality of surface finish was decided by several factors such as microshear pattern, subsurface residual stress, microripple pattern, thermal induced deviation in physical properties of glass and different

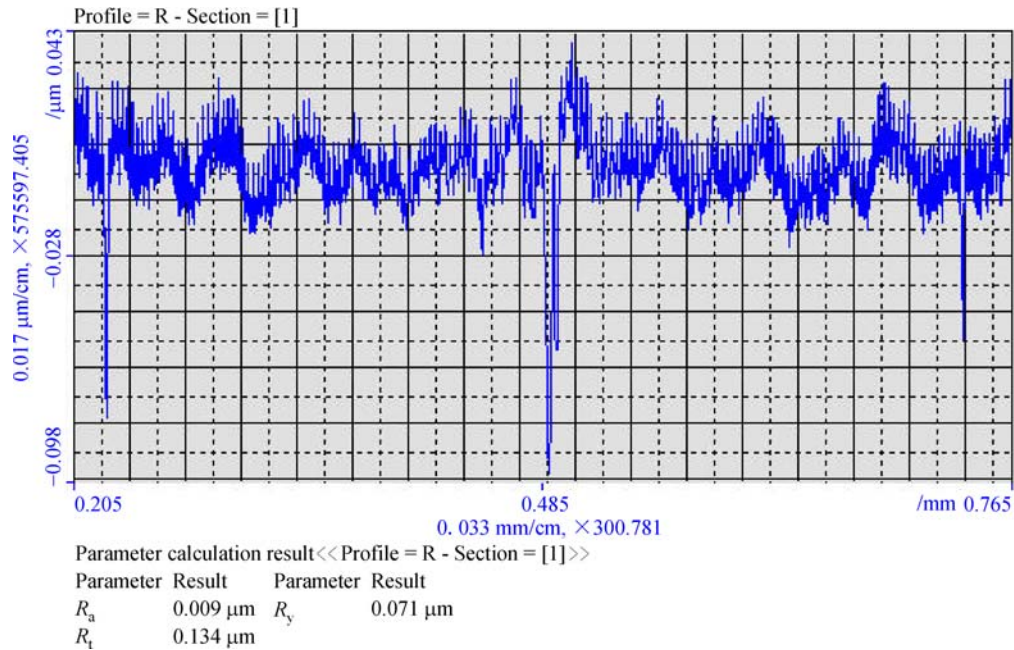


Fig. 9 Measured surface roughness in cutting of single crystal silicon wafer [57]

microcrack system. To achieve an optimum level of surface quality for a given cutting speed in diamond turning of glass, DoC must be sufficiently low.

SEM examination on machined surfaces obtained in high speed micro milling of WC showed in Fig. 10(a), proved that DMC can be achieved with a small UCT [60]. Cutting with a large UCT would lead to a fractured surface as shown in Fig. 10(b). SEM and AFM (atomic force microscope) examinations on machined surfaces obtained in cutting of silicon showed in Figs. 11(a) and 11(c), respectively [4], also proved that DMC can be achieved with a small UTC. Cutting with a large UTC would lead to a fractured surface as shown in Figs. 11(b) and 11(d). TEM examination of the nano-machined silicon showed that even the surface damaged in DMC was more homogeneous than that in grinding [33].

Further studies indicated that the tool sharpness, i.e., tool cutting edge radius, was a major factor affecting the quality of surface finish obtained in DMC [4]. Due to the similarity in material removal mechanism, similar factors are assumed to have a controlling influence on the achievable surface quality in DMC of silicon, glass, tungsten carbide and other brittle materials. It was well established that the machined surface roughness was largely influenced by UCT which is controlled by feed rate and DoC. Therefore, by controlling machining parameters and selected proper cutting tool geometries accordingly, DMC can be successfully achieved with damage-free surfaces in cutting of brittle materials. This will largely reduce the overall processing time and eliminate the subsequent abrasive-based surface finishing processes such as CMP.

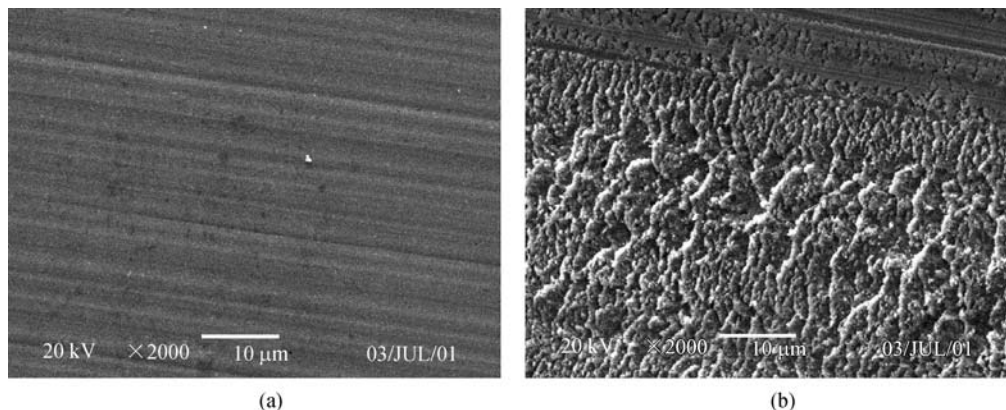
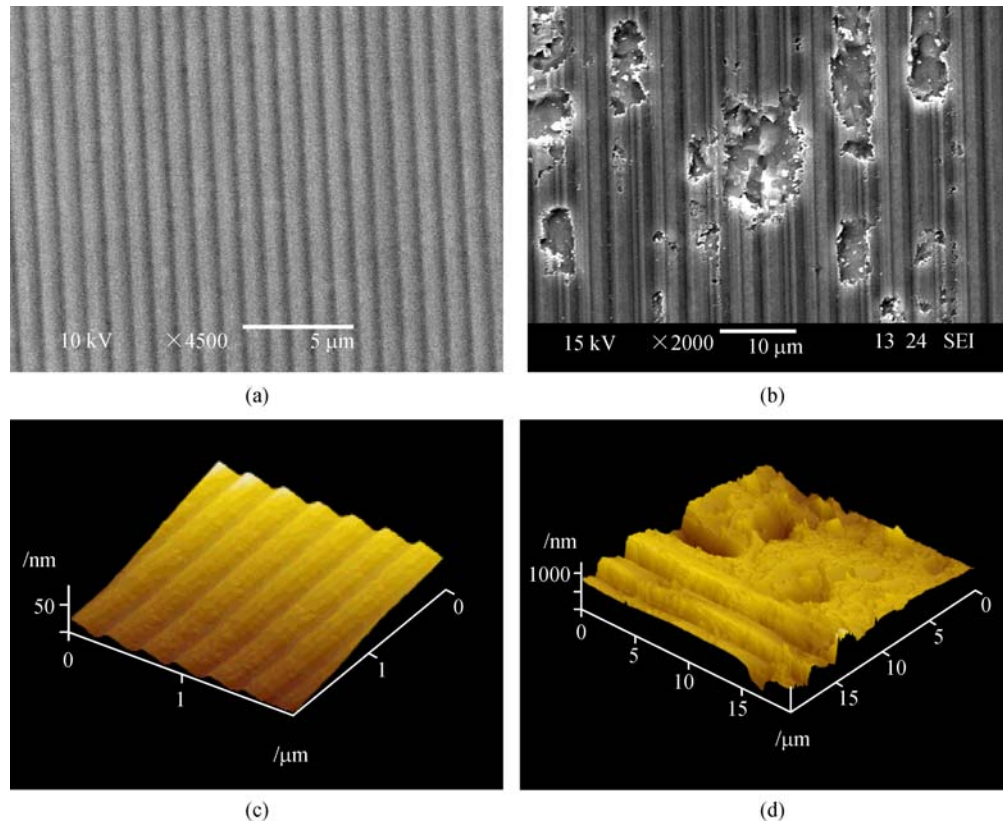


Fig. 10 SEM photographs obtained surfaces in cutting of WC [60]. (a)  $d_{\text{max}}$  of 644 nm; (b)  $d_{\text{max}}$  of 1164 nm



**Fig. 11** SEM and AFM photos obtained surfaces in turning of silicon [4,57]. (a)  $d_{\max}$  of 65 nm; (b)  $d_{\max}$  of 348 nm; (c)  $d_{\max}$  of 65 nm; (d)  $d_{\max}$  of 348 nm

#### 4.5 Tool wear

One challenge in DMC of brittle materials is the tool wear which not only increases the manufacturing cost but also effects chip formation and machined surface integrity [61]. The tool wear becomes compounded when cutting with large tool nose radii [44,62], thus reducing the tool wear is really significant, so as to achieve the desired surface roughness and dimensional tolerance and/or accuracy [63]. Yan et al. [64] explained the diamond cutting behaviour of silicon at a DoC less than 1  $\mu\text{m}$ . Tool wear mainly happened on tool flank face having the common wear patterns [65–68]. Similarly, the tool cutting edges underwent two processes in nanoscale cutting of silicon wafers: Wear on main cutting edge increasing its sharpness but not changing its shape so as to enhance chip formation in DMC, which can be attributed to the increment of compressive stress in the cutting zone; and micro/nanogrooves generated at flank face forming some micro-cutting edges [61]. The effect of diamond crystallographic orientation on DMC of silicon clearly indicated that wear resistance and tool life were greater for the tool rake face being on crystallographic orientation  $\{110\}$  than that on crystallographic orientation  $\{100\}$  and  $\{111\}$  [57]. Using molecular dynamics (MD) simulation studying the tool wear mechanism demonstrated the presence of grooves on

flank face in DMC of silicon [69,70]. The outcome indicated that a temperature increase in the cutting zone might soften the diamond along the tool flank face. A large hydrostatic pressure in cutting of silicon also leads to a phase transformation from the single crystalline to the amorphous phase, in which interatomic bond lengths may not be the same. In some groups of atoms in the amorphous phase, interatomic bond lengths may be even smaller than in the crystalline silicon. These discrete groups of atoms may be much harder than the initial crystalline silicon and may act as hard abrasive particles in the amorphous phase.

Furthermore, cutting distance, chip size, rake angle and cutting speed had an adverse effect on tool wear compared with coolant and side rake angle [62]. It was further explained that length of cut was the sole parameter that had a very significant effect on tool wear.

## 5 Molecular dynamic simulation

MD simulation has played a significant role in helping to solve a myriad of machining challenges associated with brittle materials at the atomic- or nano-scale level, thereby providing more deep understanding into the various machining process that cannot be readily attained through either theoretical or experimental studies [71]. Pioneering



work in MD simulation was started in the late 1950's [72,73]. MD simulation was reported in the late 1980's to model nano-scale cutting in ultra-precision machining [74–77]. It is also worth noting that choosing of an appropriate and accurate potential energy function determines the simulation quality, as well as affecting its computing time. The validity of the potential function should be checked for properties like lattice constant, cohesive energy and elastic constants before they can be considered as reasonably valid.

Figure 12(a) shows a schematic representative model used in MD nano-metric cutting simulation which is deemed appropriate for such work, and Fig. 12(b) shows a MD simulation outcome [40,78]. Here, work material is modelled as deformable and cutting tool is modelled as either infinitely hard [78,79] or considered deformable bodies [80]. It is ideal to model both the work material and cutting tool as deformable bodies to facilitate their tribological interaction. The model uses negative tool rake angle [30,81] and also should have a finite tool edge radius [42,78,82–84] as it is considered suitable for cutting of brittle materials. MD simulation is becoming a widely used studying method to understand the material deformation mechanisms rapidly. Special phenomena such as chip formation, work material deformation and phase transformation during nanometer scale cutting can be observed at the nanometer level [85–88].

Simulation based on renormalized molecular dynamics indicated that DMC of single crystalline silicon with defect-free can always be achieved in an absolute vacuum [89]. MD simulation demonstrated that the amorphous phase transformation in cutting of silicon is a key mechanism for inelastic deformation, and crack propagation can be avoided by stable shearing under a compressive stress [90]. MD simulations show that the high-pressure phase transformation (HPPT) of silicon results in a metallization to form a metastable phase, which persists only when the cutting tool is able to retain sufficient stress [91]. However, currently MD simulations suffer from the spurious effects of high cutting speeds and the accuracy of the simulation results has yet to be fully explored [91].

## 6 Discussion

All brittle materials would experience a brittle-ductile mode transition when cutting with an increasing DoC from zero. According to Eq. (1) based on Griffith fracture propagation criterion, the critical undeformed chip thickness  $d_c$  can be predicted by the material Young's modulus, hardness and fracture toughness, which would be a certain value for a given material. The theoretical  $d_c$  value for DMC of tungsten carbide was calculated to be 2.114  $\mu\text{m}$  [92]. Grooving and machining tests were conducted to identify the experimental  $d_c$  value for DMC of tungsten carbide, soda-lime glass and single crystal silicon wafer. It was 2.485  $\mu\text{m}$  for tungsten carbide obtained by grooving using a cubic boron nitride tool with the cutting edge radius of 5.8  $\mu\text{m}$  and speed of 144 m/min [92], 560 nm for soda-lime glass obtained by grooving using a single crystalline diamond tool at cutting diameter of 38 mm with the speed of 1000 r/min [43], and 40 nm for single crystal silicon wafer obtained by turning using a single crystalline diamond tool with the cutting speed of 1000 r/min and feed rate of 5  $\mu\text{m}/\text{revolution}$  [45], respectively. However, the critical UCT  $d_c$  value obtained from both theoretical prediction and experimental results for DMC of brittle materials is very small, at micron, sub-micron or even nanometer level, which largely constrains their actual industry application.

Naturally, questions on how to overcome this problem are surfacing up. Noted that the key parameter for DMC is the critical UCT  $d_c$ , as DMC can be achieved on brittle materials when cutting below the critical value. In practical application aspect, the  $d_c$  value is expected to be as large as possible. Thus, are we able to increase the  $d_c$  value for a given material? And to what extent?

The effect of tool sharpness (or called cutting edge radius) on ductile mode cutting has been studied in some extent [40,45,57]. The undeformed chip thickness for DMC of silicon has not to be larger than its cutting tool edge radius [57]. The critical UCT for DMC of silicon was also investigated using different diamond tool edge radii [40,45]. It was found increasing cutting tool edge radius

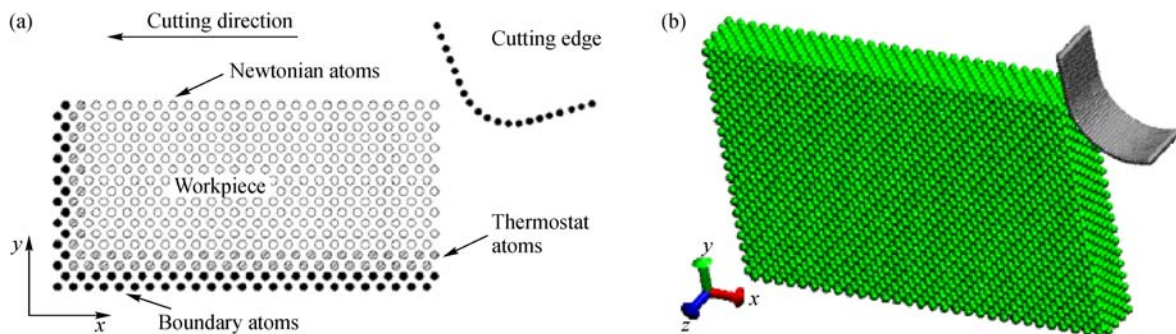


Fig. 12 Molecular dynamic simulation model of nanoscale DMC [40,78]

could derive a larger critical undeformed chip thickness [40]. But there is an upper bound of cutting edge radius for DMC of silicon wafers [45]. When cutting of silicon using a tool edge radius beyond the limit, DMC would not be achievable [45]. Likely, the compressive stress in cutting region decreases with increasing of cutting edge radius. As such, any way to increase compressive stress in the cutting region?

A customized stage having an external hydrostatic pressure attachment was used to cut silicon at the nanometric scale with diamond tools [21]. Very smooth surfaces and continuous chips, i.e., DMC, were achieved with an UCT of 50 nm, which is larger than the critical UCT of 40 nm [45] (both studies of Refs. [21,45] using the off-the-shelf single crystal diamond tools supposed having the same range cutting edge radius, other diamond tools with larger tool edge radii used in Ref. [45] were unusual and particularly polished in the lab, and their edge radii were measured using an indentation method [93]). As a large hydrostatic pressure leads to a transformation in chip formation zone from the single crystalline phase to the amorphous phase [69,70]. And the high hydrostatic pressure results in a plastic deformation of brittle materials [22–24]. This also proved that applying an external high hydrostatic pressure would increase the  $d_c$  value so as to improve DMC performance.

Some attempts have also been conducted on hybrid machining to help achieve ductile mode cutting [94–102]. The critical DoC for ductile-brittle transition in machining of WC and glass using the ultrasonic vibration tool was found being much larger than that with the conventional stationary tool, which was increased to about 3.5 and 7 times, respectively [94,95]. A model was developed for brittle materials to predict the  $d_c$  value for ultrasonic vibration cutting and experimental results well verified the predicted  $d_c$  value, of which the  $d_c$  value for DMC of silicon wafers was increased to around 2 times of that obtained without ultrasonic vibration assistance [97]. Laser assisted machining was studied on single crystal silicon to explore the machining mechanism with different conditions, resulting in a greater DoC for DMC of silicon than that without laser heating [98]. A larger critical DoC was also achieved in DMC of nanocrystalline hydroxyapatite with thermal assisted compared with that obtained without thermal assistance [99]. Thermal softening of brittle materials would result in a larger DoC, as HPPT implies that all materials are removed in the ductile mode [98]. Surface modification by hydrogen ion implantation can be employed for enhancing for DMC of single crystal silicon [101,102].

Extensive experimental studies have proved that in DMC of brittle materials, there are three key apparent features: Continuous chips formed, smooth and crack-free surface, and no subsurface damage obtained. There are also two unapparent features derived from ductile mode cutting: Residual stress and subsurface microstructure,

which most likely would change the materials' mechanical, optical, physical and chemical properties. Thereafter, it would largely limit their promising industrial applications. But the research on residual stress obtained in DMC of brittle materials is not sufficient and need to be further exploration.

Although some models and cutting mechanisms have been developed to illustrate the brittle-ductile transition in ductile mode cutting, it is still yet not clearly understood. The ductile mode cutting mechanism needs to be broadly investigated through extensive theoretical, experimental and simulation studies to have a comprehensive understanding the chip formation and damage-free surface generation. Hybrid manufacturing and/or machining processes have evidenced the somehow improvement in DMC of brittle materials. Novel and breakthrough technologies on hybrid manufacturing/machining processes need to be innovated and developed to largely improve DMC performance and brittle materials' machinability. Also, the research on DMC should be extended to more advanced and new emerged brittle materials. It will help to eliminate manufacturing barriers effectively and bloom the industrial demands significantly.

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## 7 Summary

This paper has attempted to provide a generic overview of DMC of brittle materials which attracted more and more studies from academics in the past decades. In fact, substantial studies have been conducted and demonstrated that DMC can be successfully accomplished under certain cutting conditions achieving smooth and damage-free surface on different brittle materials including silicon, soda-lime and BK7 glass, tungsten carbide, etc. The technical bottleneck for industrial applications of brittle materials is their extremely poor machinability, particularly for DMC. The ultimate goal of DMC is to improve brittle materials' machinability as much as possible, such that more and more brittle materials could be widely utilized in the engineering and industries, which would certainly benefit from their excellent mechanical, optical, physical and chemical properties. Therefore, more studies in the future need to be conducted on DMC of brittle materials focusing on machinability improvement, of which hybrid manufacturing/machining processes should be paid more attention.

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## References

1. Venkatesh V C, Inasaki I, Toenshof H K, et al. Observations on polishing and ultraprecision machining of semiconductor substrate

- materials. *CIRP Annals-Manufacturing Technology*, 1995, 44(2): 611–618
2. Tönshoff H K, Schmieden W V, Inasaki I, et al. Abrasive machining of silicon. *CIRP Annals-Manufacturing Technology*, 1990, 39(2): 621–635
  3. Pei Z J, Fisher G R, Liu J. Grinding of silicon wafers: A review from historical perspectives. *International Journal of Machine Tools and Manufacture*, 2008, 48(12–13): 1297–1307
  4. Liu K, Zuo D W, Li X P, et al. Nanometric ductile cutting characteristics of silicon wafer using single crystal diamond tools. *Journal of Vacuum Science & Technology B, Nanotechnology and Microelectronics: Materials, Processing, Measurement, and Phenomena*, 2009, 27(3): 1361–1366
  5. Domnich V, Gogotsi Y. Phase transformations in silicon under contact loading. *Reviews on Advanced Materials Science*, 2002, 3: 1–36
  6. Fang F Z, Chen L J. Ultra-precision cutting for ZKN7 glass. *CIRP Annals-Manufacturing Technology*, 2000, 49(1): 17–20
  7. King R F, Tabor D. The strength properties and frictional behaviour of brittle solids. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 1954, 223 (1153): 225–238
  8. Huerta M, Malkin S. Grinding of glass: The mechanics of the process. *Journal of Engineering for Industry*, 1976, 98(2): 459–467
  9. Foy K, Wei Z, Matsumura T, et al. Effect of tilt angle on cutting regime transition in glass micromilling. *International Journal of Machine Tools and Manufacture*, 2009, 49(3–4): 315–324
  10. Ono T, Matsumura T. Influence of tool inclination on brittle fracture in glass cutting with ball end mills. *Journal of Materials Processing Technology*, 2008, 202(1–3): 61–69
  11. Matsumura T, Ono T. Cutting process of glass with inclined ball end mill. *Journal of Materials Processing Technology*, 2008, 200 (1–3): 356–363
  12. Takeuchi Y, Sawada K, Sata T. Ultraprecision 3D micromachining of glass. *CIRP Annals-Manufacturing Technology*, 1996, 45(1): 401–404
  13. Liu K, Li X P, Liang S Y. The mechanism of ductile chip formation in cutting of brittle materials. *International Journal of Advanced Manufacturing Technology*, 2007, 33(9–10): 875–884
  14. Liu K, Li X P, Liang Y S. Nanometer-scale ductile cutting of tungsten carbide. *Journal of Manufacturing Processes*, 2004, 6(2): 187–195
  15. Arif M, Rahman M, Wong Y S. Analytical model to determine the critical feed per edge for ductile-brittle transition in milling process of brittle materials. *International Journal of Machine Tools and Manufacture*, 2011, 51(3): 170–181
  16. Arif M, Rahman M, Wong Y S. Ultraprecision ductile mode machining of glass by micromilling process. *Journal of Manufacturing Processes*, 2011, 13(1): 50–59
  17. Swain M V. Microfracture about scratches in brittle solids. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 1979, 366(1727): 575–597
  18. Dolev D. A note on plasticity of glass. *Journal of Materials Science Letters*, 1983, 2(11): 703–704
  19. Finnie I, Dolev D, Khatibloo M. On the physical basis of Auerbach's law. *Journal of Engineering Materials and Technology*, 1981, 103(2): 183–184
  20. Lawn B R, Evans A G. A model for crack initiation in elastic/plastic indentation fields. *Journal of Materials Science*, 1977, 12 (11): 2195–2199
  21. Yan J, Yoshino M, Kuriyagawa T, et al. On the ductile machining of silicon for micro electro-mechanical systems (MEMS), optoelectronic and optical applications. *Materials Science and Engineering: A*, 2001, 297(1–2): 230–234
  22. Shimada S, Ikawa N, Inamura T, et al. Brittle-ductile transition phenomena in microindentation and micromachining. *CIRP Annals-Manufacturing Technology*, 1995, 44(1): 523–526
  23. Bridgman P, Šimon I. Effects of very high pressures on glass. *Journal of Applied Physics*, 1953, 24(4): 405–413
  24. Sun Y L, Zuo D W, Wang H Y, et al. Mechanism of brittle-ductile transition of a glass-ceramic rigid substrate. *International Journal of Minerals Metallurgy and Materials*, 2011, 18(2): 229–233
  25. Clarke D R, Kroll M C, Kirchner P D, et al. Amorphization and conductivity of silicon and germanium induced by indentation. *Physical Review Letters*, 1988, 60(21): 2156–2159
  26. Gridneva I V, Milman Y V, Trefilov V I. Phase transition in diamond-structure crystals during hardness measurements. *Physica Status Solidi (a)*, 1972, 14(1): 177–182
  27. Lawn B R, Wilshaw R. Indentation fracture: Principles and applications. *Journal of Materials Science*, 1975, 10(6): 1049–1081
  28. Nakasuji T, Kodera S, Hara S, et al. Diamond turning of brittle materials for optical components. *CIRP Annals-Manufacturing Technology*, 1990, 39(1): 89–92
  29. Shaw M C. *New theory of grinding*. Institution of Engineers, Australia: Mechanical & Chemical Engineering Transactions, 1972, 73–78
  30. Komanduri R. Some aspects of machining with negative rake tools simulating grinding. *International Journal of Machine Tool Design and Research*, 1971, 11(3): 223–233
  31. Liu K, Li X P, Rahman M, et al. A study of the cutting modes in grooving of tungsten carbide. *International Journal of Advanced Manufacturing Technology*, 2004, 24(5–6): 321–326
  32. Puttick K E, Whitmore L C, Chao C L, et al. Transmission electron microscopy of nanomachined silicon crystals. *Philosophical Magazine A*, 1994, 69(1): 91–103
  33. Bifano T G, Yi Y. Acoustic emission as an indicator of material-removal regime in glass micro-machining. *Precision Engineering*, 1992, 14(4): 219–228
  34. Li C, Zhang F H, Meng B B, et al. Research of material removal and deformation mechanism for single crystal GGG ( $Gd_3Ga_5O_{12}$ ) based on varied-depth nanoscratch testing. *Materials & Design*, 2017, 125: 180–188
  35. Li C, Zhang F H, Ding Y, et al. Surface deformation and friction characteristic of nano scratch at ductile-removal regime for optical glass BK7. *Applied Optics*, 2016, 55(24): 6547–6553
  36. Kovalchenko A M, Milman Y V. On the cracks self-healing mechanism at ductile mode cutting of silicon. *Tribology International*, 2014, 80: 166–171
  37. Yan J, Zhang Z, Kuriyagawa T. Mechanism for material removal in diamond turning of reaction-bonded silicon carbide. *International Journal of Machine Tools and Manufacture*, 2009, 49(5): 366–374

38. Shibata T, Fujii S, Makino E, et al. Ductile-regime turning mechanism of single-crystal silicon. *Precision Engineering*, 1996, 18(2–3): 129–137
39. Bifano T G, Dow T A, Scattergood R O. Ductile-mode grinding: A new technology for machining brittle materials. *Journal of Engineering for Industry*, 1991, 113(2): 184–189
40. Arefin S, Li X P, Cai M B, et al. The effect of the cutting edge radius on a machined surface in the nanoscale ductile mode cutting of silicon wafer. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, 2007, 221(2): 213–220
41. Blake P N, Scattergood R O. Ductile-mode machining of germanium and silicon. *Journal of the American Ceramic Society*, 1990, 73(4): 949–957
42. Blackley W, Scattergood R O. Ductile-regime machining model for diamond turning of brittle materials. *Precision Engineering*, 1991, 13(2): 95–103
43. Liu K, Li X P, Liang S Y, et al. Nanometer-scale, ductile-mode cutting of soda-lime glass. *Journal of Manufacturing Processes*, 2005, 7(2): 95–101
44. Liu K, Li X P, Rahman M, et al. CBN tool wear in ductile cutting of tungsten carbide. *Wear*, 2003, 255(7–12): 1344–1351
45. Liu K, Li X P, Rahman M, et al. A study of the effect of tool cutting edge radius on ductile cutting of silicon wafers. *International Journal of Advanced Manufacturing Technology*, 2007, 32(7–8): 631–637
46. Jasinevicius R G, Duduch J G, Pizani P S. Structure evaluation of submicrometre silicon chips removed by diamond turning. *Semiconductor Science and Technology*, 2007, 22(5): 561–573
47. Yan J W, Gai X H, Kuriyagawa T. Fabricating nano ribbons and nano fibers of semiconductor materials by diamond turning. *Journal of Nanoscience and Nanotechnology*, 2009, 9(2): 1423–1427
48. Jasinevicius R G, Porto A J V, Duduch J G, et al. Multiple phase silicon in submicrometer chips removed by diamond turning. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 2005, 27(4): 440–448
49. Tanikella B V, Somasekhar A H, Sowers A T, et al. Phase transformations during microcutting tests on silicon. *Applied Physics Letters*, 1996, 69(19): 2870–2872
50. Morris J C, Callahan D L, Kulik J, et al. Origins of the ductile mode in single-point diamond turning of semiconductors. *Journal of the American Ceramic Society*, 1995, 78(8): 2015–2020
51. Puttick K E, Whitmore L C, Zhdan P, et al. Energy scaling transitions in machining of silicon by diamond. *Tribology International*, 1995, 28(6): 349–355
52. Yan J, Asami T, Harada H, et al. Fundamental investigation of subsurface damage in single crystalline silicon caused by diamond machining. *Precision Engineering*, 2009, 33(4): 378–386
53. Yan J, Asami T, Harada H, et al. Crystallographic effect on subsurface damage formation in silicon microcutting. *CIRP Annals*, 2012, 61(1): 131–134
54. Yu D P, Wong Y S, Hong G S. A novel method for determination of the subsurface damage depth in diamond turning of brittle materials. *International Journal of Machine Tools and Manufacture*, 2011, 51(12): 918–927
55. Liu K, Li X P, Rahman M, et al. Study of surface topography in nanometric ductile cutting of silicon wafers. In: *Proceedings of Electronics Packaging Technology Conference*. Singapore: IEEE, 2002, 200–205
56. Pei Z J, Billingsley S R, Miura S. Grinding induced subsurface cracks in silicon wafers. *International Journal of Machine Tools and Manufacture*, 1999, 39(7): 1103–1116
57. Arefin S, Li X P, Rahman M, et al. The upper bound of tool edge radius for nanoscale ductile cutting of silicon wafer. *International Journal of Advanced Manufacturing Technology*, 2007, 31(7–8): 655–662
58. Shibata T, Ono A, Kurihara K, et al. Cross-section transmission electron microscope observations of diamond-turned single-crystal Si surfaces. *Applied Physics Letters*, 1994, 65(20): 2553–2555
59. Schinker M G. Subsurface damage mechanisms at high-speed ductile machining of optical glasses. *Precision Engineering*, 1991, 13(3): 208–218
60. Liu K, Li X P, Rahman M. Characteristics of high speed micro cutting of tungsten carbide. *Journal of Materials Processing Technology*, 2003, 140(1–3): 352–357
61. Li X P, He T, Rahman M. Tool wear characteristics and their effects on nanoscale ductile mode cutting of silicon wafer. *Wear*, 2005, 259(7–12): 1207–1214
62. Born D K, Goodman W A. An empirical survey on the influence of machining parameters on tool wear in diamond turning of large single-crystal silicon optics. *Precision Engineering*, 2001, 25(4): 247–257
63. Zong W J, Sun T, Li D, et al. XPS analysis of the groove wearing marks on flank face of diamond tool in nanometric cutting of silicon wafer. *International Journal of Machine Tools and Manufacture*, 2008, 48(15): 1678–1687
64. Yan J W, Syoji K, Tamaki J. Some observations on the wear of diamond tools in ultra-precision cutting of single-crystal silicon. *Wear*, 2003, 255(7–12): 1380–1387
65. Sharif Uddin M, Seah K H W, Li X P, et al. Effect of crystallographic orientation on wear of diamond tools for nanoscale ductile cutting of silicon. *Wear*, 2004, 257(7–8): 751–759
66. Uddin M S, Seah K H W, Rahman M, et al. Performance of single crystal diamond tools in ductile mode cutting of silicon. *Journal of Materials Processing Technology*, 2007, 185(1–3): 24–30
67. Wilks J. Performance of diamonds as cutting tools for precision machining. *Precision Engineering*, 1980, 2(2): 57–72
68. Paul E, Evans C J, Mangamelli A, et al. Chemical aspects of tool wear in single point diamond turning. *Precision Engineering*, 1996, 18(1): 4–19
69. Cai M B, Li X P, Rahman M. Characteristics of “dynamic hard particles” in nanoscale ductile mode cutting of monocrystalline silicon with diamond tools in relation to tool groove wear. *Wear*, 2007, 263(7–12): 1459–1466
70. Cai M B, Li X P, Rahman M. Study of the mechanism of groove wear of the diamond tool in nanoscale ductile mode cutting of monocrystalline silicon. *Journal of Manufacturing Science and Engineering*, 2007, 129(2): 281–286
71. Komanduri R, Raff L M. A review on the molecular dynamics simulation of machining at the atomic scale. *Simulation, Proceedings of the Institution of Mechanical Engineers Part B: Journal of*

- Engineering Manufacture, 2001, 215(12): 1639–1672
72. Alder B J, Wainwright T E. Studies in molecular dynamics I: General method. *Journal of Chemical Physics*, 1959, 31(2): 459–466
  73. Alder B J, Wainwright T E. Studies in molecular dynamics II: Behavior of a small number of elastic spheres. *Journal of Chemical Physics*, 1960, 33(5): 1439–1451
  74. Belak J F, Boercker D B, Stowers I F. Simulation of nanometer-scale deformation of metallic and ceramic surfaces. *MRS Bulletin*, 1993, 18(5): 55–60
  75. Belak J F, Stowers I F. A molecular dynamics model of the orthogonal cutting process. In: *Proceedings of American Society of Photoptical Engineers Annual Conference*. Rochester: Lawrence Livermore National Lab, 1990, 76–80
  76. Stowers I F, Belak J F, Lucca D A, et al. Molecular dynamics simulation of the chip forming process in single crystal copper and comparison with experimental data. In: *Proceedings of American Society of Photoptical Engineers Annual Conference*. Stanford, 1991, 100–104
  77. Goel S, Luo X, Reuben R L. Wear mechanism of diamond tools against single crystal silicon in single point diamond turning process. *Tribology International*, 2013, 57: 272–281
  78. Cai M B, Li X P, Rahman M. Molecular dynamics modelling and simulation of nanoscale ductile cutting of silicon. *International Journal of Computer Applications in Technology*, 2007, 28(1): 2–8
  79. Cai M B, Li X P, Rahman M. Study of the temperature and stress in nanoscale ductile mode cutting of silicon using molecular dynamics simulation. *Journal of Materials Processing Technology*, 2007, 192–193: 607–612
  80. Zhang Z G, Fang F Z, Hu X T, et al. Molecular dynamics study on various nanometric cutting boundary conditions. *Journal of Vacuum Science & Technology B, Nanotechnology and Microelectronics: Materials, Processing, Measurement, and Phenomena*, 2009, 27(3): 1355–1360
  81. Komanduri R, Chandrasekaran N, Raff L M. Effect of tool geometry in nanometric cutting: A molecular dynamics simulation approach. *Wear*, 1998, 219(1): 84–97
  82. Cai M B, Li X P, Rahman M. Study of the mechanism of nanoscale ductile mode cutting of silicon using molecular dynamics simulation. *International Journal of Machine Tools and Manufacture*, 2007, 47(1): 75–80
  83. Cai M B, Li X P, Rahman M, et al. Crack initiation in relation to the tool edge radius and cutting conditions in nanoscale cutting of silicon. *International Journal of Machine Tools and Manufacture*, 2007, 47(3–4): 562–569
  84. Cai M B, Li X P, Rahman M. Molecular dynamics simulation of the effect of tool edge radius on cutting forces and cutting region in nanoscale ductile cutting of silicon. *International Journal of Manufacturing Technology and Management*, 2005, 7: 455–466
  85. Zhang L C, Tanaka H. Atomic scale deformation in silicon monocrystals induced by two-body and three-body contact sliding. *Tribology International*, 1998, 31(8): 425–433
  86. Zhang L C, Tanaka H. On the mechanics and physics in the nano-indentation of silicon monocrystals. *JSME International Journal. Series A, Solid Mechanics and Material Engineering*, 1999, 42(4): 546–559
  87. Cheong W C D, Zhang L C. Molecular dynamics simulation of phase transformations in silicon monocrystals due to nano-indentation. *Nanotech*, 2000, 11(3): 173–180
  88. Cai M B, Li X P, Rahman M. High-pressure phase transformation as the mechanism of ductile chip formation in nanoscale cutting of silicon wafer. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, 2007, 221(10): 1511–1519
  89. Inamura T, Shimada S, Takezawa N, et al. Brittle/ductile transition phenomena observed in computer simulations of machining defect-free monocrystalline silicon. *CIRP Annals*, 1997, 46(1): 31–34
  90. Tanaka H, Shimada S, Anthony L. Requirements for ductile-mode machining based on deformation analysis of mono-crystalline silicon by molecular dynamics simulation. *CIRP Annals-Manufacturing Technology*, 2007, 56(1): 53–56
  91. Goel S, Luo X, Agrawal A, et al. Diamond machining of silicon: A review of advances in molecular dynamics simulation. *International Journal of Machine Tools and Manufacture*, 2015, 88: 131–164
  92. Liu K, Li X P. Ductile cutting of tungsten carbide. *Journal of Materials Processing Technology*, 2001, 113(1–3): 348–354
  93. Li X P, Rahman M, Liu K, et al. Nanoprecision measurement of diamond tool edge radius for wafer fabrication. *Journal of Materials Processing Technology*, 2003, 140(1–3): 358–362
  94. Liu K, Li X P, Rahman M, et al. Study of ductile mode cutting in grooving of tungsten carbide with and without ultrasonic vibration. *International Journal of Advanced Manufacturing Technology*, 2004, 24(5–6): 389–394
  95. Moriwaki T, Shamoto E, Inoue K. Ultraprecision ductile cutting of glass by applying ultrasonic vibration. *CIRP Annals-Manufacturing Technology*, 1992, 41(1): 141–144
  96. Liu K, Li X P, Rahman M. Characteristics of ultrasonic vibration assisted ductile cutting of tungsten carbide. *International Journal of Advanced Manufacturing Technology*, 2008, 35(7–8): 833–841
  97. Zhang X Q, Arif M, Liu K, et al. A model to predict the critical undeformed chip thickness in vibration-assisted machining of brittle materials. *International Journal of Machine Tools and Manufacture*, 2013, 69: 57–66
  98. Ravindra D, Ghantasala M K, Patten J. Ductile mode material removal and high-pressure phase transformation in silicon during micro-laser assisted machining. *Precision Engineering*, 2012, 36(2): 364–367
  99. Ma J F, Pelate N, Lei S T. Thermally assisted high efficiency ductile machining of nanocrystalline hydroxyapatite: A numerical study. *Ceramics International*, 2013, 39(8): 9377–9384
  100. Zheng H Y, Liu K. *Handbook of Manufacturing Engineering and Technology: Machinability of Engineering Materials*. London: Springer, 2014, 2: 899–939
  101. Fang F Z, Chen Y H, Zhang X D, et al. Nanometric cutting of single crystal silicon surfaces modified by ion implantation. *CIRP Annals-Manufacturing Technology*, 2011, 60(1): 527–530
  102. To S, Wang H, Jelenković E V. Enhancement of the machinability of silicon by hydrogen ion implantation for ultra-precision micro-cutting *International Journal of Machine Tools and Manufacture*, 2013, 74: 50–55