Characterization of Hoek–Brown constant m_i of quasi-isotropic intact rock using rigidity index approach

Seyed Morteza Davarpanah¹ · Mohammad Sharghi² · Balázs Vásárhelyi¹ · Ákos Török¹

Received: 28 September 2020 / Accepted: 3 May 2021 / Published online: 15 June 2021
 © The Author(s) 2021

Abstract

An accurate determination of Hoek–Brown constant m_i is of great significance in the estimation of the failure criteria of brittle rock materials. So far, different approaches such as rigidity index method (R-index), uniaxial compressive strength-based method, and tensile strength-based method, and the combination of these two methods (combination based method) have been proposed to calculate the value of m_i . This paper aims to thoroughly review the previously existing methods to calculate the value of m_i and make comparison between the obtain results to propose the new material constants that provide the best fit with the experimental data. In order to fulfill this goal, a large number of data for different quasi-isotropic intact rock types from the literature were collected and statistically analyzed. Additionally, based on rock types, new material constants are introduced for igneous, sedimentary, and metamorphic rocks. The obtained results proves that for different rock groups (igneous, sedimentary, and metamorphic rocks), R-index method provides the best fit with the experimental data among the others, and it is also independent of rock type. Interestingly enough, there is significant differences in the predicted m_i values using different methods, which is more probably due to the quantity and quality of data used in the statistical analysis.

Keywords Combination method · Hoek–Brown material constant (m_i) · R-index method · Rock types¹ · TS-based method · UCS-based method

Abbreviations

m_i	Hoek–Brown material constant
m_b	Hoek material constant for rock mass
GSI	Geological Strength Index
TS	Tensile strength
UCS	Uniaxial compressive strength
R	Rigidity index
β	Intermediate fracture mechanics parameter
μ	The coefficient of friction for pre-existing sliding crack surfaces
σ_1, σ_3	Major and minor principal stress
σ_c	Uniaxial compressive strength (UCS)
σ_{ci}	Crack initiation stress
σ_t	Tensile strength (TS)

✉ Ákos Török
 torokakos@mail.bme.hu

¹ Department of Engineering Geology and Geotechnics, Budapest University of Technology and Economics, Budapest, Hungary

² Department of Mining Engineering, Sahand University of Technology, Tabriz 0600, Iran

1 Introduction

The Hoek–Brown failure criterion is widely used in rock mechanics and rock engineering practice for determining the strength of brittle intact rock and rock masses. This nonlinear semiempirical failure criterion was introduced by Hoek and Brown [33], and the following form was suggested for intact rock (see also [17]) (Eq. 1):

$$\sigma_1 = \sigma_3 + \sigma_c \left(m_i \frac{\sigma_3}{\sigma_c} + 1 \right)^{0.5} \quad (1)$$

where σ_1 and σ_3 are major and minor principal stress at failure, respectively, m_i is Hoek–Brown material constant, and σ_c is the uniaxial compressive strength of intact rock. According to Eq. (1), two independent parameters are necessary, namely the:

- Uniaxial compressive strength (σ_c)
- Hoek–Brown material constant of the intact rock (m_i)

Although several practical, empirical, and probabilistic approaches have been presented in the literature to address uniaxial compressive strength (UCS) [12, 34, 36, 73, 80], the accurate determination of m_i is still challenging task and is influenced by many factors such as mineral composition, grain size, and cementation of rock. Generally, m_i presents curve-fitting parameter for Hoek–Brown failure envelope [37]. However, researches by Zuo et al. [84] showed that m_i is not a curve-fitting parameter but has physical meanings and can be derived from micro-mechanical principles (Hoek and Martin [38]).

Hoek and Brown [33–35] suggested that these values should be determined by numerous triaxial tests, applying different confining pressures (σ_3) between zero and $0.5 \sigma_c$. These laboratory tests are time-consuming, expensive, and in many cases, there are not enough (or suitable) samples. Singh et al. [70], Peng et al. [55], and Shen and Karakus [65] demonstrated that the reliability of m_i values measured from triaxial test analysis depends on the quality and quantity of test data used in the analysis. They concluded that the range of σ_3 could have a significant influence on the calculation of m_i . It is the reason why several methods were developed for determining the Hoek–Brown constant (m_i) [72].

There are several techniques available to calculate m_i values in the absence of triaxial test. These approaches are entitled, Guidelines [32, 33], R-index [10, 33, 52, 59, 60, 62], UCS-based model [65, 74], tensile strength (TS)-based approach [77], and crack initiation stress-based model [10]. These methods are based on the rock lithological classifications and rock properties that can be easily obtained at an early stage of a project, which can be used in preliminary designs of engineering projects when triaxial test data are not available.

Aladejare and Wang [1] published a paper and introduced a new method for the determination of m_i value. They developed a Bayesian approach for probabilistic characterization of Hoek–Brown constant m_i through Bayesian integration of information from Hoek's guideline chart, regression model, and site-specific UCS data. The UCS data used in this paper were obtained from testing granite samples collected from the Forsmark site, Sweden. In addition, several sets of simulated data are used to explore the evolution of m_i as the number of site-specific data increases.

Wei et al. [78] experimentally investigated the effect of confining pressure σ_3 and critical crack parameter on determination of m_i value. They applied ultrasonic test and load test results of limestone and verified the exponential impact of σ_3 on limestone, while the negative correlation was observed between critical crack parameter and m_i .

Recently, Wen et al. [79] developed an empirical relation for estimation m_i from σ_3 and β , where σ_3 is minor

principal stress and β is an angle between the major principal stress and weak plane based on substantial uniaxial and triaxial compression test data. They considered the effect of anisotropy on the value of Hoek constant (m_i).

In the present study, however, we focused on the quasi-isotropic intact rocks and utilized the existing methods in the literature for predicting m_i values. Furthermore, the detailed comparison is drawn between the obtained values of each approach to calculate which method provides the best estimation for the investigated rocks based on the calculated error function. Accordingly, new material constants are proposed for different igneous, sedimentary, and metamorphic rocks.

2 Determination of Hoek–Brown constant m_i

In the absence of triaxial tests, there are several different methods available to estimate the Hoek–Brown constant m_i , which are referred to as the R-index method [33], UCS-based model [65, 74], TS-based model [77], and combination method [3, 66] below. By using these methods, the H-B constant m_i is obtained for the collected data, the achieved results are compared, and then, the error function was calculated and accordingly considering the lowest error value, the best data fit was presented and finally new material constants introduced.

2.1 Guideline method

Hoek and Brown [33] and Hoek [32] provided values of m_i constants for different types of rocks. The values of m_i are between 7 and 35; however, several factors that influence these values, such as mineral composition, foliation, grain size (texture), and cementation are among others [10].

The updated values of constant m_i for intact rock were collected by [10], using the published values of Hoek [32] (Table 1). Possible data ranges are shown by a variation range value immediately following the suggested m_i value. For example, for sandstones, the m_i values can vary between 13 and 21, and for slates, between 3 and 11.

2.2 Calculation the rigidity of the rock (R-index)

This method was introduced by [33] and also developed by [48, 59–61]. Cai [10] published the paper and showed that Hoek–Brown's strength parameter (m_i) could be determined from the ratio of the uniaxial compressive strength (σ_c) and the tensile strength (σ_t), and the suggested relationship is:

Table 1 The Hoek–Brown constant m_i for intact rocks, by rock group according to the collection of Cai [10]

Rock type	Class	Group	Texture			
			Coarse	Medium	Fine	Very fine
Sedimentary	Clastic		Conglomerate ^a (22 ± 3)	Sandstone (17 ± 4)	Siltstone (7 ± 2)	Claystone (4 ± 2)
			Breccias (19 ± 5)		Greywacke (18 ± 3)	Shales (6 ± 2)
	Non-clastic	Organic	Chalks (7 ± 2)	Sparitic Limestone (10 ± 2)	Micritic Limestone (9 ± 2)	Marls (7 ± 2)
		Carbonates	Crystalline Limestone (12 ± 3)			Dolomites (9 ± 3)
Metamorphic	Non-foliated	Evaporites	Marble (9 ± 3)	Gypsum (8 ± 2)	Anhydrite (12 ± 2)	
				Homfels (19 ± 4)	Quartzites (20 ± 3)	
	Slightly foliated		Migmatite (29 ± 3)	Metasandstone (26 ± 6)		
			Gneiss (28 ± 5)	Amphibolites (26 ± 6)		
Igneous	Foliated ^b	Plutonic	Granite (32 ± 2)	Schists (12 ± 3)	Phyllites (7 ± 3)	Slates (7 ± 4)
			Granodiorite (29 ± 3)	Diorite (25 ± 5)		
		Dark	Gabbro (27 ± 3)	Dolerite (16 ± 5)		
			Norite (20 ± 5)			
	Hypabyssal		Porphyries (20 ± 5)		Diabase (15 ± 5)	Peridotite (25 ± 5)
						Obsidian (19 ± 3)
	Volcanic	Lava		Rhyolite (25 ± 5)	Dacite (25 ± 3)	
				Andesite (25 ± 5)	Basalt (25 ± 5)	
	Pyroclastic		Agglomerate (19 ± 3)	Breccia (19 ± 5)	Tuff (13 ± 5)	

^aConglomerates and breccias may present a wide range of m_i values depending on the nature of the cementing material and the degree of cementation so that they may range from values similar to sandstone to values used for fine-grained sediments

^bThese values are for intact rock specimens tested normal to bedding or foliation. The value of m_i will be significantly different if the failure occurs along a weakness plane

$$m_i \approx R = \frac{\sigma_c}{|\sigma_t|} \quad (2)$$

As can be seen from Fig. 1, when $R > 8$, the error for approximating m_i (Eq. 2) by R (Eq. 4) is less than 1.6%.

It has been observed that when the R is higher than 8, the value of R is nearly equal to Hoek–Brown material constant (m_i). Note, that according to [38] m_i is not a curve fitting parameter, but has a physical meaning and can be derived from micro-mechanics principles. The referred m_i model is express in Eq. (3):

$$m_i = \frac{\mu \sigma_c}{\beta \sigma_t} \quad (3)$$

where σ_c and σ_t are the uniaxial compressive strength (UCS) and tensile strength (TS) of intact rock, respectively. While μ is the coefficient of friction for pre-existing sliding crack surfaces, β is an intermediate fracture mechanics parameter that can be obtained from experimental data. It means when $R > 8$:

$$\frac{\mu}{\beta} \cong 1 \quad (4)$$

In recent years, Hoek and Brown [34] analyzed several published data and proposed the following approximate relationship between the compressive to tensile strength ratio, $\frac{\sigma_c}{|\sigma_t|}$, and the Hoek–Brown parameter m_i (see Fig. 2):

$$\frac{\sigma_c}{|\sigma_t|} = 0.81m_i + 7 \quad (5)$$

$$m_i = 1.235 \frac{\sigma_c}{|\sigma_t|} - 8.642 \quad (6)$$

It means that the Hoek–Brown constant (m_i) can be calculated using the following relationship, using the uniaxial compressive strength (σ_c) and the tensile strength (σ_t) data of the intact rock.

2.3 UCS-based model

Firstly, Shen and Karakus [65] and later Vásárhelyi et al. [74] also emphasized the difficulties in determining the m_i values of rocks. They suggested normalizing the Hoek–

Fig. 1 Relationship between the error in m_i estimate (using Eq. 2) and the strength ratio R [10]

Brown constant ($m_{in} = \frac{m_i}{\sigma_c}$) by using the strength of the rock ($m_{in} = \frac{m_i}{\sigma_c}$), i.e.:

$$m_i = a_c^{b+1} \quad (7)$$

where a and b are constants that depend on rock types.

2.4 TS-based model

The tensile strength-based calculation method was suggested by Wang and Shen [77]. In this method, the m_i value is determined from the tensile strength (σ_t) of the intact rock.

$$m_i = A_t^B \quad (8)$$

Fig. 2 Relationship between $\frac{\sigma_{ci}}{\sigma_t}$ and m_i according to Hoek and Brown [35]

where A and B are material constants that depend on rock types. In general, the constants A and B are curve-fitting parameters.

However, according to the theory of Zuo et al. [84] the m_i value can be estimated from the coefficient of friction for the pre-existing sliding crack surfaces, and fracture mechanics. From this point of view, the constants A and B for the TS-based model seem to have its physical meaning, which have possible relations with the micro-mechanics principles. However, investigating the potential relationships between these constants and other rock properties and explaining the possible physical meanings need to carry out further research to get appropriate amount of reliable data.

2.5 Combination method (UCS and TS)

Analyzing the published data of Sheorey [66], Arshadnejad and Nick [3] suggested a new equation to calculate the Hoek–Brown material constant (m_i) based on uniaxial compressive strength (σ_c) and the tensile strength (σ_t) parameters of the intact rock:

$$m_i = e^{[a(\frac{\sigma_c - b\sigma_t}{\sigma_t})^c]} \quad (9)$$

where a , b , and c are material constants. The suggested constants are summarized in Table 2, according to Arshadnejad and Nick [3].

2.6 Crack initiation stress-based model

According to [10], the Hoek–Brown constant (m_i) depends on the confining pressure. For practical estimate of m_i , it was found that for strong, brittle rocks, applicable to high confining zone the following equation could be used:

$$m_i = 12 \frac{\sigma_c}{\sigma_{ci}} \quad (10)$$

For low confinement stress to tension zone, especially for the tension zone the suggested equation is:

$$m_i = 8 \frac{\sigma_c}{\sigma_{ci}} \quad (11)$$

In these equations, σ_c and σ_{ci} are the uniaxial compressive strength and the crack initiation stress, respectively.

2.7 Bayesian approach

The proposed approach [1] derives the probability density function (PDF) of m_i based on the integration of Hoek's guideline chart, regression model, and site-specific UCS data, under a Bayesian framework. A large number of equivalent samples of m_i are generated from the PDF using Markov Chain Monte Carlo (MCMC) simulation.

Table 2 The suggested constants of different rocks based on Eq. (9) [3]

Rock type	a	B	C
Igneous	1.2	2	0.3
Sedimentary	1.3	2.5	0.26
Metamorphic	1.2	1.5	0.28

3 Experimental data

A wide range of data from literature by [72] were selected and analyzed. The published data of the investigated rocks are illustrated in Appendices 1, 2, and 3 for igneous, sedimentary, and metamorphic rocks, respectively. In these tables, calculated data are also given. Statistical analyses of the measured m_i values for different rock types are summarized in Table 3. In this table, parameters such as number of data sets, maximum (Max.), minimum (min.), average (Ave.), and standard deviation (std.) of measured m_i for three different groups are presented (see the appendix for more details).

4 Analyzing the data

4.1 Comparing the published m_i values with the guideline

Comparing the published values of m_i by [72] with the guidelines [10, 32, 33] given for igneous rocks, the value of m_i for basalt is between 20 and 30, whereas the minimum published value of m_i for Basalt is 4.31 (see Appendix 1) which does not fall within the expected range. The minimum published value of m_i for basalt is even less than m_i value for volcanic tuffs. The latter one is between 8 and 13. The published values of m_i for granite are between 11.68 and 34.03, but the expected range based on the guidelines is between 29 and 35. The same discrepancy was observed for gabbro. The expected range based on the guideline values is between 24 and 30, whereas the published results are in between 15.31 and 20.74.

For diorite, the value of m_i based on the guidelines is between 20 and 30; however, according to published data, the value of m_i is between 6.1 and 11. For granodiorite, based on the guideline, the value of m_i changes between 26 and 32, while based on published results, it is between 11 and 18.15. For diabase, according to the guidelines, the value of m_i is between 10 and 20; nevertheless, based on published data it is between 15.3 and 20.3. For andesite and rhyolite, the value of m_i based on the guidelines is between 20 and 30; however, the published values are 6.22 and 5.43

for andesite and rhyolite, respectively. For agglomerate, the value of m_i is between 16 and 22, but the published value is 7.92, which does not fit the guideline range.

For sedimentary rocks, the minimum value of m_i for shale is 3.76 which is close to the estimated range of m_i which is between 4 and 8 (see Appendix 2). Also, the maximum published value of m_i for sandstone is 35.11 which is much higher than the estimated range of m_i value and based on guideline which varies between 13 and 21. Moreover, for dolomite, the value of m_i based on guidelines is between 6 and 12, whereas the value of m_i based on published data is between 7.8 and 17.5. For limestone, the value of m_i based on guideline is between 7 and 15; however, based on published data, the value of m_i is between 5.3 and 14.6.

For metamorphic rocks, it is interesting to know that both maximum and minimum published values are for slate with the amounts of 1.42 and 29.54, respectively (see Appendix 3). However, based on guideline the values of m_i for slate changes between 3 and 11. It means that the range of published value for slate does not fit the estimated range given by the guidelines. For gneiss, the value of m_i based on guidelines is between 23 and 33; however, based on published data, the value ranges between 5.3 and 27. For schist, the published value of m_i is 20.42, but the values based on guideline vary between 9 and 15. For quartzite, the published value of m_i is between 7.36 and 22.7; however, based on guideline it is between 17 and 23. Also, the values of m_i were calculated according to [10].

The differences between the published values of m_i and the guideline values are displayed in Table 4 and Fig. 3. As shown, the published values of m_i do not fit well with the guideline. As it is clear from the graph that there is a good consistency between published experimental data for m_i and R-index method, whereas other methods have significant errors in estimation of m_i for all studied rock samples. The obtained results based on different proposed calculation method are presented in Appendices 4, 5, and 6 for igneous, sedimentary, and metamorphic rocks, respectively. It should be noted that the constant parameters were derived from the existing Eqs. 2, 6, 7, 8, and 9, respectively.

4.2 Evaluation of R-index model

According to our analyses for determination of m_i in respect to Eq. 2 (R-index method) and Eq. 5, it was observed that m_i value is independent of rock type. The correlations between m_i and the ratio between uniaxial compressive strength (UCS) and tensile strength (TS), which is called R-index method, for different rocks (sandstone, shale, slate, and gneiss) are presented in Fig. 4.

Table 3 Summary of the measured m_i for different group of rocks

Rock type	Number of data sets	Measured m_i				Source
		min	Max	Ave	std	
Igneous	32	4.31	34.03	15.42	7.13	[6, 8, 9, 13, 20, 28, 43, 50, 51, 63, 76] (see data in Appendix 1)
Sedimentary	107	3.76	35.11	12	6.32	[2, 6–8, 14, 16, 18, 20–26, 29, 30, 39–41, 44–46, 47, 49, 53, 56–58, 60, 64, 67–69, 71, 75, 83] (see data in Appendix 2)
Metamorphic	46	1.43	32.33	13.36	10.07	[4, 5, 6, 8, 9, 11, 15, 19, 20, 22, 31, 39, 44, 47, 54, 63, 69] (see data in Appendix 3)

Table 4 Maximum and minimum values of m_i based on different methods for different rocks

Rock type	Rock name	Guideline		R-index		UCS based		TS based		Combination method		Published data		Source (for published data)
		min	Max	min	max	min	max	min	max	min	max	min	max	
Igneous	Diabase	10	20	15.31	20.32	6.17	18.99	10.06	12.72	13.61	18.84	15.31	20.2	[6, 8]
	Diorite	20	30	6.11	11	9.55	22.5	7.21	14.26	5.53	9.62	6.1	11	[6, 50]
	Gabbro	24	30	15.04	20.73	14.73	15.4	11.08	14.22	13.34	19.29	15.03	20.74	[9]
	Granite	29	35	11.69	34.08	8.92	22.8	7.42	15.42	10.22	36.95	11.68	34.03	[6, 8, 20, 28, 43, 50, 51, 63]
	Granodiorite	26	32	11.01	18.15	9.1	12.2	13.14	13.2	9.65	16.49	11.04	18.15	[6, 13]
Sedimentary	Quartzdiorite	–	–	11.9	20.96	8.08	31.69	7.38	16.02	10.41	19.56	11.9	20.97	[6, 9, 20]
	Dolomite	6	12	7.86	17.5	21.49	23.51	5.23	9.18	6.95	15.81	7.86	17.49	[8, 60]
	Limestone	7	15	5.32	14.67	19.8	22.11	6.18	13.05	4.9	13	5.32	14.66	[6, 20, 22, 39, 71]
	Sandstone	13	21	3.97	35.13	17.82	22.23	5.48	29.87	3.83	38.53	3.98	35.11	[2, 6, 7, 14, 16, 18, 20, 21, 23–26, 40–42, 44–46, 49, 53, 56–58, 63, 67, 69, 75, 83]
	Shale	4	8	3.76	24.96	17.82	22.13	6.52	29.87	3.65	24.3	3.76	25.31	[47, 67, 68]
Metamorphic	Coal	–	–	4.21	27.86	17.89	20.17	8.99	30.4	4.02	28.03	4.21	28.02	[29, 64]
	Slate	3	11	1.43	31.04	7.2	22.27	3.83	48.42	1.98	32.44	1.43	30.97	[4, 15, 19, 47]
	Gneiss	23	33	5.34	32.31	8.07	22.46	4.68	11.44	4.92	34.28	5.34	32.33	[9, 39]
	Quartzite	17	23	7.36	30.1	8.45	21.1	5.66	10.43	6.55	31.1	7.36	30.1	[6, 8, 11, 69]
	Marble	6	12	6.07	8.52	19.94	21.35	7.66	11.24	5.5	7.5	6.08	8.53	[20, 22, 31, 44, 54, 63]

As shown for different rock types, correlation value $R^2 = 1$ and it is not influenced by rock type.

4.3 Evaluation of UCS-based model

The relationship between m_i and uniaxial compressive strength (UCS) for the investigated rock samples (UCS-based method) shows an opposite result, compared to R-index method. It is evident that based on this approach, no correlation was found for the studied rock samples (Fig. 5).

4.4 Evaluation of TS-based model

Based on our analyses for determination of m_i value by applying TS-based method, the established correlations for the investigated rock types were weak except for slate where the value of $R^2 = 0.66$ (Fig. 6).

In order to examine the relationship between m_i and rigidity of rock (R-index method) for different rock groups, analyses were also carried out based on three different rock types (igneous, sedimentary, and metamorphic) and the results are illustrated in Fig. 7. Additionally, the error function was calculated for each rock type. The calculated

Fig. 3 Comparison of published values of m_i [66] with the guidelines [4, 8, 9], R-index, UCS-based, TS-based, and combination methods: **a** igneous rocks; **b** sedimentary rocks; **c** metamorphic rocks

Fig. 4 Correlation between $\frac{\sigma_c}{\sigma_t}$ and m_i for different rock types: **a** sandstone; **b** shale; **c** slate; **d** gneiss

amount of error function represents the value of $\left(\frac{\mu}{\beta}\right)$ as mentioned earlier in Eq. (3).

The obtained results of regression analyses based on proposed approaches for determination of m_i are summarized in Table 5 (according to rock group) and Table 6

(according to different investigated rock samples), and the statistical results are illustrated in Table 7.

In the proposed method by Arshadnejad and Nick [3], (combination method), (i), (j), and (k) are presented as material constants. Specifically explaining, based on their analyses for different rock groups (igneous, sedimentary,

Fig. 5 Correlation between UCS and m_i for different rock types: **a** sandstone; **b** shale; **c** slate; **d** gneiss

Fig. 6 Correlation between tensile strength (TS) and m_i for different rock types: **a** sandstone; **b** shale; **c** slate; **d** gneiss

and metamorphic), there are different material constants. See Table 2 for their obtained results.

4.5 Evaluation of combination method (UCS and TS methods)

We analyzed the data set based on Eq. 9 as proposed by [3]. The results are depicted in Fig. 8. The graphs clearly illustrate that the Hoek constant (m_i) is independent of rock type. Accordingly, it is worth to mention the point that the intact rocks are known to contain a wide-ranging uncertainty owing to inherent heterogeneities. For rock mass classification in respect to uncertainties, *GSI* (Geological Strength Index) provides good classification of rock mass category as it relies on structure as well as surface conditions of existing discontinuities which are highly associated with high uncertainties in rock mass. Ván and Vásárhelyi [72] investigated the sensitivity of Hoek material constant for rock mass (m_b) in relation to *GSI*. Based on their measurement, they calculated that 10% deviation in the *GSI* value; the relative sensitivity of m_b is at least double the uncertainties of the *GSI* values and may be 7 times higher in case of large disturbance parameters and low and high *GSI* values.

5 Discussion

When triaxial test data are not available, m_i values can be estimated by six existing methods entitled, guidelines [32, 33], R-index [10, 33, 52, 59, 60, 62], UCS-based model [65, 74], TS-based model [77], crack initiation stress-based model [10], combination based model [3, 66], and Bayesian approach [1] to evaluate the prediction performance of existing methods, large database of triaxial tests for eight common rock types (slate, shale, limestone, quartzdiorite, sandstone, quartzite, gneiss, and granite) from literature was selected and analyzed as indicated in Appendices 1, 2, and 3.

In the present research, to ensure the quantity and quality of data, the statistical analysis with the help of SPSS software was performed on quasi-isotropic rocks. The results are presented in Fig. 9. As shown, normal distribution function and standard deviation were calculated for each rock group. According to Fig. 9, for igneous rock, the calculated standard deviation is 7.21 with the mean value of 15.36, for sedimentary rock is 6.32 with the mean value of 12, and for metamorphic rocks is 10.07 with the mean value of 13.37. Interestingly enough, metamorphic rocks exhibit higher standard deviation, which can be associated with the foliated texture of the investigated metamorphic rocks.

Similarly, Aladejare and Wang [1] investigated the values of m_i for 30,000 equivalent granite samples based on Bayesian approach and realized that the histogram peaks at

a m_i value of around 30, and roughly about 27,479 out of the total 30,000 samples representing about 91.5% are within m_i range of 18 and 48. Therefore, the 90% inter-perceptual range of m_i is roughly between m_i values of 18 and 48 (Fig. 9d). In comparison with our results, however, Wen et al. [79] published an interesting paper about anisotropic rocks, which has drawn our attention to their new empirical relation. The large number of the data set (738) of different rock types was analyzed. They observed that the more anisotropic the rock is, the more significant decrease in m_i value is evident. Using this relation, the H–B strength criterion is modified, and its performance is evaluated in two examples. The results illustrate the improved ability of the modified criterion to determine the strength of metamorphic and sedimentary rocks.

Based on the empirical model by Wen et al. [79], most of the estimated values of m_i are within the upper/lower limit of 90%, which accounts for 96.6% of all the data, and that all the estimated values are within the upper/lower limit of 80%. This result indicates that the developed relation estimates of m_i agree well with the experimental observations, demonstrating that the developed relation has an excellent ability to estimate m_i accurately. The comparison graph between our results and recently published paper by Wen et al. [79] is presented in Fig. 10.

Moreover, Wen et al. [80–82] thoroughly investigated the impact of minor principal stress (σ_3) on rock ductile–brittle behavior. Based on their measurement, under a low minor principal stress, rocks experience dilatancy and brittleness that causes the exciting microcrack inside the rock to coalesce, which result in an increase in volume; In

Fig. 7 Correlation between the ratio of uniaxial compressive strength (UCS) and tensile strength (TS) and m_i for: **a** igneous; **c** sedimentary; **e** metamorphic rocks and calculated error function respected (**b**, **d**, **f**)

Table 5 Calculated values of m_i by different methods (Eqs. 2, 5, 7, 8, 9, 10)

Rock type	R-index $m_i = a\left(\frac{\sigma_c}{\sigma_t}\right) + b$			Hoek and Brown $m_i = c\left[1.23\left(\frac{\sigma_c}{\sigma_t}\right) - 8.64\right] + d$			UCS-based method $m_i = e\sigma_c^f$			TS-based method $m_i = g_t^h$			Combination method $m_i = e\left[i\left(\frac{a-d}{c}\right)^k\right]$			
	a	b	R^2	c	d	R^2	e	f	R^2	g	h	R^2	i	j	k	R^2
Igneous	1	− 0.17	1	0.81	6.88	1	0.29	0.73	0.19	29.14	− 0.3	0.04	1.08	2	0.34	0.99
Sedimentary	1	− 0.17	1	0.81	6.88	1	15.16	− 0.07	0.7	21.81	− 0.34	0.38	1.08	2	0.34	0.99
Metamorphic	1	− 0.37	1	0.82	6.76	1	5.96	0.07	0.35	36.91	− 0.6	0.52	0.95	2	0.4	0.99
All types	1	− 0.21	1	0.81	6.86	1	10.98	0.004	0.64	22.8	− 0.34	0.32	1.07	2	0.37	0.99

Table 6 Determination of m_i based on existing methods

Rock type	Origin	Name	R-index method $m_i = a\left(\frac{\sigma_c}{\sigma_t}\right) - b$			UCS-based method $m_i = c\sigma_c^d$			TS-based method $m_i = e_t^f$		
			a	b	R^2	c	d	R^2	e	f	R^2
Igneous	Plutonic	Granite	1	0.08	1	0.0005	1.87	0.46	436.97	− 1.17	0.89
		Quartzdiorite	1	0.12	1	2.72	0.24	0.3	−	−	−
Sedimentary	Clastic	Sandstone	1	0.15	1	−	−	−	20.19	− 0.25	0.23
		Shale	1	0.28	1	−	−	−	17.78	− 0.37	0.47
Metamorphic	Biochemical	Limestone	1	0	1	2.18	0.33	0.1	29.45	− 0.52	0.35
		Gneiss	1	0.21	1	0.29	0.68	0.54	−	−	−
		Quartzites	1	0.14	1	0.12	0.86	0.91	−	−	−
		Slate	1	0.42	1	−	−	−	33.31	− 0.65	0.67

Table 7 Statistical results of predicted m_i values using different approaches

Rock type	Estimation method	Predicted m_i			
		Min	Max	Ave	SD
Igneous	R-index	4.31	34.08	15.36	7.21
	UCS method	6.17	31.69	16.36	6.15
	TS method	7.21	16.02	11.98	2.41
	Combination method	4.1	36.95	14.3	7.65
Sedimentary	R-index	3.76	35.11	12	6.32
	UCS method	17.82	23.51	20.3	1.12
	TS method	5.23	30.4	12.58	5.15
	Combination method	3.65	38.53	10.95	6.47
Metamorphic	R-index	1.43	32.33	13.37	10.07
	UCS method	7.2	22.46	14.99	6.74
	TS method	3.83	48.42	9.4	6.92
	Combination method	1.98	34.29	13.18	10.44

contrast, applying the high minor principal stress, the rock undergoes ductility which leads to microcrack opening.

In the recently published paper by He et al. [27], they suggested that the constant m_i of the H–B criterion can be continuously estimated during drilling. Therefore, a

method to estimate the constant m_i from drilling data was proposed. Based on the proposed method for m_i determination, for the granite, slate, the obtained values of m_i in their work were lower than the suggested values for granite

Fig. 8 Calculated values of m_i based on combination method for: **a** igneous rocks; **b** sedimentary rocks; **c** metamorphic rocks; **d** all rocks

and sandstone from [33], and they were higher than the suggested value for slate and limestone.

To establish the correlations for specific rock types, Shen and Karakus [65], Wang and Shen [77] considered five of the most common rock types (coal, granite, limestone, marble, and sandstone) from the database in the Rocscience [61], in which there are at least 12 groups of data with 115 triaxial tests available. Results illustrate that four rock types have trends of decreasing m_i with increasing σ_{ci} ; however, such a correlation for limestone was not observed. It is worth noting that this finding is also in agreement with our obtained results for limestone. This may be due to the fact that limestone has a wide array of test data with different compositions and cementations; for example, there are different guidelines-based m_i values for three types of limestone, crystalline ($m_i = 12 \pm 3$), sparitic ($m_i = 10 \pm 2$), and micritic ($m_i = 9 \pm 2$), in the Hoek guidelines [32]. Therefore, the data sets for limestone are quite widely scattered compared with those for the other four rock types.

The observed value of m_i for limestone is between 5.3 and 14.6; In contrast, according to guideline, the m_i value of limestone is between 7 and 15, according to R-index method m_i is between 5.32 and 14.67, according to UCS-based method m_i is between 19.8 and 22.1, based on TS-based method m_i is between 6.18 and 13.05, and based on combination method m_i is between 4.9 and 13. Thus,

R-index method gives the best estimate of m_i value among the others. However, based on [65], UCS-based method provides a better prediction of m_i value than the guidelines-based and R-index methods and according to [77], TS-based model exhibits the best performance for granites, limestone, and marble.

Additionally, the observed value of m_i for slate is between 1.42 and 30.97; however, based on guideline, the m_i value of slate is between 3 and 11, based on R-index method m_i is between 1.59 and 31.23, based on UCS-based method m_i is between 7.2 and 22.3, based on TS-based method m_i is between 3.82 and 48.42, and based on combination method m_i is between 1.98 and 32.44. So, R-index method gives the best estimate of m_i value among the others. The achieved value of m_i is inconsistent with the obtained value of m_i by [16], which they used the internal friction angel method for estimating the value of m_i for dolomite, slate, dike, and granodiorite. The obtained values of m_i were 6.28, 5.1, 12.8, and 13.7, respectively.

Moreover, the observed value of m_i for shale is between 3.76 and 25.31, but, according to guideline, the m_i value of shale is between 4 and 8, according to R-index method m_i is between 3.7 and 24.9, according to UCS-based method m_i is between 17.82 and 22.1, based on TS-based method m_i is between 6.52 and 29.87, and based on combination method m_i is between 3.65 and 24.3. Therefore, R-index

Fig. 9 Histograms of the predicted m_i values for: **a** igneous; **b** sedimentary; **c** metamorphic rocks considering the new material constants; **d** equivalent samples for Hoek–Brown constant m_i from Bayesian approach

method gives the best estimate of m_i value among the others.

Furthermore, the observed value of m_i for sandstone is between 3.97 and 35.1; nevertheless, according to guideline, the m_i value of sandstone is between 13 and 21, according to R-index method m_i is between 3.9 and 35, according to UCS-based method m_i is between 17.82 and 22.23, based on TS-based method m_i is between 5.48 and 29.87, and based on combination method m_i is between 3.83 and 38.5. As a result, R-index method gives the best estimate of m_i value among the others. Nevertheless, based on [77], UCS-based model gives the best prediction for sandstone.

As well as that, the observed value of m_i for quartzite is between 7.36 and 30.1, while, according to guideline, the m_i value of quartzite is between 17 and 23, according to

R-index method m_i is between 7.36 and 30.14, according to UCS-based method m_i is between 8.45 and 21.1, based on TS-based method m_i is between 5.66 and 10.43, and based on combination method m_i is between 6.55 and 31.10. Hence, R-index method gives the best estimate of m_i value among the others.

In addition, the observed value of m_i for gneiss is between 5.34 and 32.33, whereas, according to guideline, the m_i value of gneiss is between 23 and 33, according to R-index method m_i is between 5.3 and 32.3, according to UCS-based method m_i is between 8.07 and 22.46, based on TS-based method m_i is between 4.68 and 11.44, and based on combination method m_i is between 4.92 and 34.28. Therefore, R-index method gives the best estimate of m_i value among the others.

Fig. 10 Comparison between m_i values estimated via the proposed relation and measured values for different rock types (red rhombus symbol is related to quasi-isotropic intact igneous rocks; green triangle symbol is related to quasi-isotropic intact sedimentary rocks; blue cross symbol is related to quasi-isotropic intact metamorphic rocks; and small orange square symbol is related to anisotropic intact rocks (Wen et al. [79])) (color figure online)

Finally, the observed value of m_i for granite is between 11.68 and 34.03; on the other hand, according to guideline, the m_i value of granite is between 29 and 35, according to R-index method m_i is between 11.7 and 34.1, according to UCS-based method m_i is between 8.92 and 22.8, based on TS-based method m_i is between 7.42 and 15.42, and based on combination method m_i is between 10.22 and 36.95. Thus, R-index method gives the best estimate of m_i value among the others.

6 Conclusion

This paper comprehensively reviewed the proposed methods for determination of m_i value using the triaxial data set published by [66]. New linear and nonlinear correlations were found. New material constants were suggested for igneous, sedimentary, and metamorphic rocks such as granite, quartzdiorite, sandstone, shale, limestone, gneiss, quartzite, and slate. Furthermore, the error function was calculated for each rock type. According to our analyses, R-index approach with new material constants provides the best fit for all the studied lithotypes including igneous, sedimentary, and metamorphic rocks. Moreover, it is worth

mentioning that comparison graph between out achieved results for quasi-isotropic rocks and anisotropic rocks was developed. The data scattering of quasi-isotropic rocks differs from the anisotropic rocks in that quasi-isotropic rocks exhibit normal distribution in narrower range between 1.43 and 35.11, whereas anisotropic rocks display wider range of data distribution between 1 and 80.

The combination methods (UCS) and (TS) provide the best fit for investigated rock data, when the determination coefficient and error function are considered. For various rock types such as igneous, sedimentary, and metamorphic rocks, the different approaches resulted in non-uniform correlations. For instance, TS-based approach works well for the granite and gives estimation fit with $R^2 = 0.89$. For quartzites, UCS-based approach displays the best approximation with $R^2 = 0.91$, and for slate, TS-based model exhibits the best correlation with $R^2 = 0.67$.

R-index method, the value of m_i is not influenced by rock type. In other words, this method gives the best estimation for all the investigated rock types and is independent of rock type. However, in all the other approaches, the rock type plays a significant role in correlation values and results.

Appendix 1: Published values of triaxial parameters for Hoek–Brown criterion using data set for igneous rocks [66]

No	Rock name	σ_c (MPa)	σ_t (MPa)	m_i	$\frac{\sigma_c}{\sigma_t}$	$\frac{m_i}{\sigma_c}$	Source
1	Agglomerate tuff	92	11.43	7.926	8.05	0.09	Betourney et al. [6]
2	Tuff	125.2	5.44	22.963	23.01	0.18	Wang and Kemeny [76]
3	Andesite	201.9	31.64	6.225	6.38	0.03	Betourney et al. [6]
4	Basalt	79.1	17.45	4.313	4.53	0.05	Betourney et al. [6]
5	Diabase	322.9	15.85	20.324	20.37	0.06	Betourney et al. [6]
6	Diabase	532	34.6	15.310	15.38	0.03	Brace [8]
7	Diorite	67.8	10.82	6.103	6.27	0.09	Betourney et al. [6]
8	Diorite	124.3	18.57	6.548	6.69	0.05	
9	Diorite	279.7	25.21	11.004	11.09	0.04	Mogi [50]
10	Gabbro	379.1	25.1	15.033	15.10	0.04	Broch [9]
11	Gabbro	226.9	10.92	20.738	20.78	0.09	
12	Granite	241.3	11.34	21.227	21.28	0.09	Betourney et al. [6]
13	Granite	318.2	10.31	30.816	30.86	0.10	Brace [8]
14	Granite	260	18.56	13.936	14.01	0.05	Franklin and Hoek [20]
15	Granite	220.6	8.28	26.589	26.64	0.12	Heard et al. [28]
16	Granite	193.1	13.83	13.884	13.96	0.07	Johnson et al. [43]
17	Granite	115.7	9.83	11.681	11.77	0.10	
18	Granite	163.5	10.92	14.900	14.97	0.09	
19	Granite	306.5	23.24	13.115	13.19	0.04	Mogi [50]
20	Granite	337.7	15.45	21.812	21.86	0.06	Mogi [51]
21	Granite	93.8	2.75	34.034	34.11	0.36	Schwartz [63]
22	Granite breccia	334.9	21.06	15.837	15.90	0.05	Betourney et al. [6]
23	Granodiorite	113.1	10.16	11.043	11.13	0.10	Betourney et al. [6]
24	Granodiorite	259.1	14.23	18.150	18.21	0.07	Dayre and Giraud [13]
25	Lamprophyre	116.3	14.02	8.174	8.30	0.07	Betourney et al. [6]
26	Quartzdiorite	174.7	9.53	18.274	18.33	0.10	Betourney et al. [6]
27	Quartzdiorite	173.4	14.47	11.903	11.98	0.07	
28	Quartzdiorite	98.6	7.35	13.343	13.41	0.14	
29	Quartzdiorite	273.8	15.71	17.371	17.43	0.06	Broch [9]
30	Quartzdiorite	209.7	9.98	20.965	21.01	0.10	
31	Quartzdiorite	300.2	23.57	12.658	12.74	0.04	Franklin and Hoek [20]
32	Rhyolite	106.4	18.96	5.430	5.61	0.05	Betourney et al. [6]

Appendix 2: Published values of triaxial parameters for Hoek–Brown criterion using data set for sedimentary rocks [66]

No	Rock name	σ_c (MPa)	σ_t (MPa)	m_i	$\frac{\sigma_c}{\sigma_t}$	$\frac{m_i}{\sigma_c}$	Source
1	Dolomite	145.3	18.2	7.859	7.98	0.05	Brace [8]
2	Dolomite	524.5	64.22	8.044	8.17	0.02	

No	Rock name	σ_c (MPa)	σ_t (MPa)	m_i	$\frac{\sigma_c}{\sigma_t}$	$\frac{m_i}{\sigma_c}$	Source
3	Dolomite	218.7	12.46	17.493	17.55	0.08	Mogi [60]
4	Limestone	65.9	4.47	14.663	14.74	0.22	Betourney et al. [6]
5	Limestone	128.8	9.85	12.992	13.08	0.10	
6	Limestone	94.9	13.15	7.076	7.22	0.07	Franklin and Hoek [20]
7	Limestone	53.6	7.84	6.686	6.84	0.12	
8	Limestone	217.9	39.62	5.319	5.50	0.02	Gnirk and Cheathan [22]
9	Limestone	45	6.26	7.038	7.19	0.16	Horino and Ellickson [39]
10	Limestone	54.7	9.3	5.714	5.88	0.10	
11	Limestone	58.7	10.63	5.345	5.52	0.09	
12	Limestone	58	9.56	5.905	6.07	0.10	
13	Limestone	63.4	8.65	7.197	7.33	0.11	
14	Limestone	49.2	6.09	7.957	8.08	0.16	
15	Limestone	100.3	9.45	10.517	10.61	0.10	Stowe [71]
16	Sandstone	85.2	9.87	8.520	8.63	0.10	Aldritch [2]
17	Sandstone	75.5	8.72	8.543	8.66	0.11	
18	Sandstone	149.9	6.51	22.996	23.03	0.15	Betourney et al. [6]
19	Sandstone	129.9	18.15	7.017	7.16	0.05	Borecki et al. [7]
20	Sandstone	112.9	14.68	7.561	7.69	0.07	
21	Sandstone	109	8.0	13.367	13.44	0.12	Dlugosz et al. [14]
22	Sandstone	21.7	0.88	24.537	24.66	1.13	Dunikowski [16]
23	Sandstone	152.4	16.54	9.110	9.21	0.06	Everling [18]
24	Sandstone	74.2	5.98	12.33	12.41	0.17	Franklin and Hoek [20]
25	Sandstone	74.6	4.28	17.378	17.43	0.23	
26	Sandstone	211.7	18.09	11.618	11.70	0.05	
27	Sandstone	41.5	2.92	14.123	14.21	0.34	Glushko and Kirmichanskiy [21]
28	Sandstone	65.4	5.79	11.206	11.30	0.17	Gowd and Rummel [23]
29	Sandstone	93.9	3.78	24.761	24.84	0.26	Gustkewicz [24]
30	Sandstone	42.6	1.22	35.014	34.92	0.82	
31	Sandstone	150.6	14.8	10.079	10.18	0.07	Hareland et al. [25]
32	Sandstone	75.4	5.25	14.288	14.36	0.19	
33	Sandstone	93.3	9.74	9.474	9.58	0.10	
34	Sandstone	10	0.4	25.314	25.00	2.53	Harza Engg Co. [26]
35	Sandstone	163	10.41	15.602	15.66	0.10	Hoshino et al. [40]
36	Sandstone	173.7	14.9	11.570	11.66	0.07	Hossaini and Vutukuri [41]
37	Sandstone	236.1	56.05	3.975	4.21	0.02	Ilnitskaya et al. [42]
38	Sandstone	76.9	6.26	12.191	12.28	0.16	Kovári and Tisa [44]
39	Sandstone	72.1	5.48	13.082	13.16	0.18	
40	Sandstone	110.7	7.26	15.174	15.25	0.14	Kuntysh [45]
41	Sandstone	98.8	8.09	12.125	12.21	0.12	Kwasniewski [46]
42	Sandstone	103.6	10.63	9.643	9.75	0.09	
43	Sandstone	104.2	13.39	7.652	7.78	0.07	
44	Sandstone	44.2	3.39	12.972	13.04	0.29	Misra [49]
45	Sandstone	61	5.75	10.510	10.61	0.17	
46	Sandstone	48.2	2.87	16.759	16.79	0.35	
47	Sandstone	99.5	7.39	13.379	13.46	0.13	
48	Sandstone	162.1	16.47	9.741	9.84	0.06	
49	Sandstone	102.1	5.82	17.498	17.54	0.17	
50	Sandstone	110.3	6.33	17.382	17.42	0.16	
51	Sandstone	86.7	4.38	19.734	19.79	0.23	

No	Rock name	σ_c (MPa)	σ_t (MPa)	m_i	$\frac{\sigma_c}{\sigma_t}$	$\frac{m_i}{\sigma_c}$	Source
52	Sandstone	72.7	3.8	19.065	19.13	0.26	Murrel [53]
53	Sandstone	28.9	3.93	7.216	7.35	0.25	Ramamurthy [56]
54	Sandstone	111.9	14.21	7.748	7.87	0.07	Ramez [57]
55	Sandstone	116.4	5.86	19.813	19.86	0.17	Rao et al. [58]
56	Sandstone	104.9	5.88	17.793	17.84	0.17	
57	Sandstone	119.2	7.18	16.551	16.60	0.14	
58	Sandstone	54.8	1.56	35.107	35.13	0.64	Schwartz [63]
59	Sandstone	70.8	7.68	9.115	9.22	0.13	Sheorey et al. [67]
60	Sandstone	64.8	5.02	12.824	12.91	0.20	Singh et al. [69]
61	Sandstone	62.9	9.64	6.370	6.52	0.10	Vutukuri and Farough Hossani [75]
62	Sandstone	45	3.04	14.754	14.80	0.33	Wilhelmi and Somerton [83]
63	Siltstone	65.2	6.73	9.582	9.69	0.15	Hobbs [30]
64	Shale	220.6	33.85	6.364	6.52	0.03	McLamore and Gray [47]
65	Shale	184.7	25.29	7.164	7.30	0.04	
66	Shale	154	17.56	8.655	8.77	0.06	
67	Shale	84.5	5.35	15.737	15.79	0.19	
68	Shale	185	26.47	6.848	6.99	0.04	
69	Shale	190.2	27.28	6.830	6.97	0.04	
70	Shale	175	21.34	8.078	8.20	0.05	
71	Shale	193.9	26.86	7.082	7.22	0.04	
72	Shale	112	23.52	4.555	4.76	0.04	
73	Shale	107.9	26.9	3.762	4.01	0.03	
74	Shale	78.8	15.73	4.808	5.01	0.06	
75	Shale	57.4	11.15	4.959	5.15	0.09	
76	Shale	66.8	13.15	4.886	5.08	0.07	
77	Shale	93.2	22.07	3.986	4.22	0.04	
78	Shale	99.3	21.88	4.318	4.54	0.04	
79	Shale	58	8.08	7.043	7.18	0.12	
80	Shale	80.4	13.76	5.672	5.84	0.07	Sheorey et al. [67]
81	Shale	66.9	4.21	15.826	15.89	0.24	Singh [68]
82	Shale	25.9	2.23	11.509	11.61	0.44	
83	Shale	28.7	3.72	7.568	7.72	0.26	
84	Shale	10	0.4	25.314	25.00	2.53	
85	Coal	14.1	0.93	15.1232	15.16	1.07	Hobbs [29]
86	Coal	23.6	2.26	10.334	10.44	0.44	
87	Coal	58.9	13.27	4.213	4.44	0.07	
88	Coal	36.5	4.13	8.728	8.84	0.24	
89	Coal	30.3	3.45	8.673	8.78	0.29	
90	Coal	40.1	3.96	10.034	10.13	0.25	
91	Coal	28.2	1.63	17.300	17.30	0.61	
92	Coal	26.2	2.1	12.401	12.48	0.47	
93	Coal	10.8	0.59	18.403	18.31	1.70	
94	Coal	10.6	0.38	28.021	27.89	2.64	
95	Coal	32.3	3.17	10.101	10.19	0.31	
96	Coal	31.3	3.47	8.915	9.02	0.28	
97	Coal	18.7	1.46	12.763	12.81	0.68	
98	Coal	15.6	0.61	25.643	25.57	1.64	
99	Coal	35.6	3.97	8.871	8.97	0.25	
100	Coal	33	3.13	10.420	10.54	0.32	

No	Rock name	σ_c (MPa)	σ_t (MPa)	m_i	$\frac{\sigma_c}{\sigma_t}$	$\frac{m_i}{\sigma_c}$	Source
101	Coal	38	4.13	9.090	9.20	0.24	Shea-Albin et al. [64]
102	Coal	17.2	1.33	12.940	12.93	0.75	
103	Coal	19.6	1.35	14.483	14.52	0.74	
104	Coal	32.5	3.29	9.784	9.88	0.30	
105	Coal	25.1	2.12	11.792	11.84	0.47	
106	Coal	28.9	1.87	15.382	15.45	0.53	
107	Coal	17.1	0.97	17.579	17.63	1.03	

Appendix 3: Published values of triaxial parameters for Hoek–Brown criterion using data set for metamorphic rocks [66]

No	Rock name	σ_c (MPa)	σ_t (MPa)	m_i	$\frac{\sigma_c}{\sigma_t}$	$\frac{m_i}{\sigma_c}$	Source
1	Schist	133.6	6.59	20.246	20.27	0.15	Barat [5]
2	Slate	148.6	18.64	7.844	7.97	0.05	Attewell and Sandford [4]
3	Slate	108.7	28.66	3.528	3.79	0.03	
4	Slate	53.4	27.5	1.428	1.94	0.03	
5	Slate	62.3	21.7	1.980	2.87	0.03	Donath [15]
6	Slate	98	41.56	1.933	2.36	0.02	
7	Slate	129.4	39.96	2.930	3.24	0.02	
8	Slate	178.3	19.97	8.819	8.93	0.05	
9	Slate	57.8	1.86	30.965	31.08	0.54	
10	Slate	14.5	0.64	22.700	22.66	1.57	
11	Slate	44.2	4.6	9.504	9.61	0.22	
12	Slate	68.1	4.03	16.853	16.90	0.25	
13	Slate	155.1	10.85	14.217	14.29	0.09	
14	Slate	167.6	12.22	13.644	13.72	0.08	Fayed [19]
15	Slate	242	39.35	5.988	6.15	0.02	McLamore and Gray [47]
16	Slate	181.9	40.14	4.311	4.53	0.02	
17	Slate	99.2	30.5	2.945	3.25	0.03	
18	Slate	106.3	31.44	3.085	3.38	0.03	
19	Slate	124	27.16	4.345	4.57	0.04	Broch [9]
20	Slate	162.6	34.66	4.479	4.69	0.03	
21	Slate	220.3	44.09	4.797	5.00	0.02	
22	Gneiss	315.1	17.58	17.865	17.92	0.06	
23	Gneiss	75.3	13.63	5.343	5.52	0.07	
24	Gneiss	221.7	13.61	16.233	16.29	0.07	
25	Gneiss	195.4	29.86	6.389	6.54	0.03	
26	Gneiss	197.7	22.29	8.759	8.87	0.04	
27	Gneiss	106.4	11.17	9.423	9.53	0.09	
28	Gneiss	272.8	11.32	24.061	24.10	0.09	
29	Gneiss	234.8	7.8	30.063	30.10	0.13	
30	Gneiss	222.2	7.54	29.438	29.47	0.13	Horino and Ellickson [39]
31	Gneiss	224.8	7.66	29.301	29.35	0.13	
32	Gneiss	212.5	6.57	32.329	32.34	0.15	

No	Rock name	σ_c (MPa)	σ_t (MPa)	m_i	$\frac{\sigma_c}{\sigma_t}$	$\frac{m_i}{\sigma_c}$	Source
33	Gniess	252.8	9.7	26.014	26.06	0.10	
34	Gniess	254.2	10.93	23.229	23.26	0.09	
35	Gniess	226.4	7.21	31.387	31.40	0.14	
36	Gniess	267	8.98	29.700	29.73	0.11	
37	Quartzite	144.5	19.28	7.363	7.49	0.05	Betourney et al. [6]
38	Quartzite	657.6	21.82	30.102	30.14	0.05	Brace [8]
39	Quartzite	219	13.88	15.710	15.78	0.07	Chan et al. [11]
40	Quartzite	111.8	8.6	12.925	13.00	0.12	Singh et al. [69]
41	Marble	94.3	12.13	7.652	7.77	0.08	Franklin and Hoek [20]
42	Marble	91.2	10.56	8.525	8.64	0.09	Gnirk and Cheatham [22]
43	Marble	132	21.18	6.075	6.23	0.05	Hoek [31]
44	Marble	104.6	16.33	6.251	6.41	0.06	Kovári and Tisa [44]
45	Marble	109.2	15	7.145	7.28	0.07	Ouyang and Elsworth [54]
46	Marble	49.9	6.91	7.087	7.22	0.14	Schwartz [63]

Appendix 4: Measured and calculated values of m_i based on different methods for igneous rocks

No	Rock name	m_i (measured)	$\frac{\sigma_c}{\sigma_t}$	m_i (R-index (Cai))	m_i (UCS based)	m_i (TS based)	m_i (Hoek–Brown (2019))	m_i (combination method)
1	Agglomerate tuff	7.93	8.05	7.92	7.69	14.03	1.26	7.00
2	Tuff	22.96	23.01	22.97	21.27	12.20	19.67	21.88
3	Andesite	6.23	6.38	6.22	13.54	10.34	-0.79	5.63
4	Basalt	4.31	4.53	4.31	6.90	12.36	-3.06	4.10
5	Diabase	20.32	20.37	20.32	18.99	12.72	16.42	18.84
6	Diabase	15.31	15.38	15.31	6.17	10.06	10.27	13.61
7	Diorite	6.10	6.27	6.11	9.55	14.26	-0.93	5.53
8	Diorite	6.55	6.69	6.54	21.32	12.13	-0.41	5.88
9	Diorite	11.00	11.09	11.00	22.50	7.21	5.01	9.62
10	Gabbro	15.03	15.10	15.04	14.73	11.08	9.94	13.34
11	Gabbro	20.74	20.78	20.73	15.40	14.22	16.92	19.29
12	Granite	21.23	21.28	21.23	16.25	14.06	17.53	19.86
13	Granite	30.82	30.86	30.83	19.50	12.13	29.32	32.14
14	Granite	13.94	14.01	13.94	8.92	11.68	8.59	12.29
15	Granite	26.59	26.64	26.60	22.13	10.57	24.13	26.39
16	Granite	13.88	13.96	13.89	21.92	8.86	8.53	12.24
17	Granite	11.68	11.77	11.69	21.15	9.96	5.84	10.22
18	Granite	14.90	14.97	14.91	21.67	9.61	9.78	13.21
19	Granite	13.12	13.19	13.11	22.64	7.42	7.58	11.52
20	Granite	21.81	21.86	21.81	22.80	8.53	18.24	20.52
21	Granite	34.03	34.11	34.08	20.84	15.42	33.31	36.95
22	Granite breccia	15.84	15.90	15.84	16.21	14.54	10.92	14.13
23	Granodiorite	11.04	11.13	11.04	9.10	13.14	5.05	9.65
24	Granodiorite	18.15	18.21	18.15	12.20	13.20	13.76	16.49

No	Rock name	m_i (measured)	$\frac{\sigma_c}{\sigma_t}$	m_i (R-index (Cai))	m_i (UCS based)	m_i (TS based)	m_i (Hoek–Brown (2019))	m_i (combination method)
25	Lamprophyre	8.17	8.30	8.17	12.14	14.82	1.56	7.21
26	Quartzdiorite	18.27	18.33	18.28	8.08	13.07	13.91	16.62
27	Quartzdiorite	11.90	11.98	11.90	16.86	16.02	6.10	10.41
28	Quartzdiorite	13.34	13.41	13.34	13.92	12.75	7.86	11.73
29	Quartzdiorite	17.37	17.43	17.37	10.64	14.61	12.80	15.68
30	Quartzdiorite	20.97	21.01	20.96	31.69	11.99	17.20	19.56
31	Quartzdiorite	12.66	12.74	12.66	22.61	7.38	7.03	11.10
32	Rhyolite	7.36	7.49	5.43	14.36	13.24	0.58	4.99

Appendix 5: Measured and calculated values of m_i based on different methods for sedimentary rocks

No	Rock name	m_i (measured)	$\frac{\sigma_c}{\sigma_t}$	m_i (R-index (Cai))	m_i (UCS based)	m_i (TS based)	m_i (Hoek–Brown (2019))	m_i (combination method)
1	Dolomite	7.86	7.98	7.86	21.49	8.06	1.18	6.95
2	Dolomite	8.04	8.17	8.04	23.51	5.23	1.41	7.10
3	Dolomite	17.49	17.55	17.50	22.12	9.18	12.95	15.81
4	Limestone	14.66	14.74	14.67	20.33	13.05	9.49	12.99
5	Limestone	12.99	13.08	13.00	21.31	9.95	7.44	11.41
6	Limestone	7.08	7.22	7.08	20.86	9.02	0.24	6.31
7	Limestone	6.69	6.84	6.69	20.04	10.77	-0.23	6.00
8	Limestone	5.32	5.50	5.32	22.11	6.18	-1.88	4.90
9	Limestone	7.04	7.19	7.05	19.80	11.63	0.20	6.29
10	Limestone	5.71	5.88	5.71	20.07	10.15	-1.41	5.22
11	Limestone	5.35	5.52	5.34	20.17	9.70	-1.85	4.92
12	Limestone	5.91	6.07	5.90	20.15	10.06	-1.18	5.37
13	Limestone	7.20	7.33	7.19	20.28	10.41	0.38	6.41
14	Limestone	7.96	8.08	7.96	19.92	11.74	1.30	7.03
15	Limestone	10.52	10.61	10.52	20.94	10.10	4.41	9.19
16	Sandstone	8.52	8.63	8.52	20.70	9.95	1.98	7.49
17	Sandstone	8.54	8.66	8.54	20.53	10.38	2.01	7.51
18	Sandstone	23.00	23.03	22.98	21.54	11.47	19.68	21.89
19	Sandstone	7.02	7.16	7.02	21.32	8.07	0.16	6.26
20	Sandstone	7.56	7.69	7.56	21.11	8.68	0.82	6.71
21	Sandstone	13.37	13.44	13.37	21.06	10.64	7.89	11.75
22	Sandstone	24.54	24.66	24.62	18.81	22.79	21.69	23.87
23	Sandstone	9.11	9.21	9.11	21.56	8.33	2.69	7.98
24	Sandstone	12.33	12.41	12.33	20.50	11.81	6.62	10.80
25	Sandstone	17.38	17.43	17.37	20.51	13.25	12.80	15.68
26	Sandstone	11.62	11.70	11.62	22.06	8.08	5.75	10.16
27	Sandstone	14.12	14.21	14.14	19.69	15.11	8.84	12.48
28	Sandstone	11.21	11.30	11.21	20.32	11.94	5.25	9.79
29	Sandstone	24.76	24.84	24.80	20.84	13.83	21.91	24.10

No	Rock name	m_i (measured)	$\frac{\sigma_c}{\sigma_t}$	m_i (R-index (Cai))	m_i (UCS based)	m_i (TS based)	m_i (Hoek–Brown (2019))	m_i (combination method)
30	Sandstone	35.01	34.92	34.89	19.72	20.38	34.31	38.20
31	Sandstone	10.08	10.18	10.08	21.55	8.66	3.88	8.81
32	Sandstone	14.29	14.36	14.29	20.53	12.35	9.03	12.62
33	Sandstone	9.47	9.58	9.47	20.83	9.99	3.14	8.30
34	Sandstone	25.31	25.00	24.96	17.82	29.87	22.11	24.30
35	Sandstone	15.60	15.66	15.59	21.66	9.77	10.62	13.89
36	Sandstone	11.57	11.66	11.57	21.76	8.64	5.70	10.12
37	Sandstone	3.98	4.21	3.97	22.23	5.48	-3.46	3.82
38	Sandstone	12.19	12.28	12.20	20.55	11.63	6.47	10.68
39	Sandstone	13.08	13.16	13.08	20.46	12.17	7.54	11.49
40	Sandstone	15.17	15.25	15.18	21.09	11.05	10.11	13.48
41	Sandstone	12.13	12.21	12.13	20.92	10.65	6.38	10.62
42	Sandstone	9.64	9.75	9.64	20.99	9.70	3.35	8.44
43	Sandstone	7.65	7.78	7.65	21.00	8.96	0.93	6.78
44	Sandstone	12.97	13.04	12.96	19.77	14.35	7.40	11.38
45	Sandstone	10.51	10.61	10.51	20.22	11.97	4.41	9.19
46	Sandstone	16.76	16.79	16.73	19.89	15.20	12.02	15.03
47	Sandstone	13.38	13.46	13.39	20.93	10.99	7.92	11.77
48	Sandstone	9.74	9.84	9.74	21.66	8.35	3.47	8.52
49	Sandstone	17.50	17.54	17.49	20.97	11.92	12.94	15.80
50	Sandstone	17.38	17.42	17.37	21.08	11.58	12.79	15.67
51	Sandstone	19.73	19.79	19.74	20.73	13.14	15.71	18.20
52	Sandstone	19.07	19.13	19.08	20.47	13.80	14.89	17.48
53	Sandstone	7.22	7.35	7.22	19.19	13.64	0.41	6.43
54	Sandstone	7.75	7.87	7.75	21.10	8.78	1.05	6.86
55	Sandstone	19.81	19.86	19.81	21.16	11.90	15.79	18.28
56	Sandstone	17.79	17.84	17.78	21.01	11.88	13.30	16.11
57	Sandstone	16.55	16.60	16.54	21.20	11.09	11.78	14.83
58	Sandstone	35.11	35.13	35.10	20.07	18.73	34.57	38.53
59	Sandstone	9.12	9.22	9.11	20.44	10.84	2.70	7.99
60	Sandstone	12.82	12.91	12.83	20.31	12.54	7.24	11.26
61	Sandstone	6.37	6.52	6.37	20.27	10.03	-0.61	5.74
62	Sandstone	14.75	14.80	14.74	19.80	14.90	9.57	13.05
63	Siltstone	9.58	9.69	9.58	20.32	11.34	3.28	8.39
64	Shale	6.35	6.52	6.36	22.13	6.52	-0.62	5.74
65	Shale	7.16	7.30	7.17	21.86	7.20	0.34	6.38
66	Shale	8.66	8.77	8.66	21.58	8.16	2.15	7.61
67	Shale	15.74	15.79	15.73	20.69	12.27	10.79	14.02
68	Shale	6.85	6.99	6.85	21.86	7.09	-0.04	6.13
69	Shale	6.83	6.97	6.83	21.90	7.02	-0.06	6.11
70	Shale	8.08	8.20	8.08	21.77	7.64	1.45	7.13
71	Shale	7.08	7.22	7.08	21.93	7.06	0.24	6.31
72	Shale	4.56	4.76	4.55	21.10	7.39	-2.78	4.29
73	Shale	3.76	4.01	3.76	21.05	7.05	-3.71	3.65
74	Shale	4.81	5.01	4.81	20.59	8.48	-2.48	4.50
75	Shale	4.96	5.15	4.95	20.14	9.54	-2.31	4.61
76	Shale	4.89	5.08	4.88	20.35	9.02	-2.39	4.55
77	Shale	3.99	4.22	3.99	20.83	7.55	-3.45	3.83
78	Shale	4.32	4.54	4.32	20.93	7.57	-3.06	4.10

No	Rock name	m_i (measured)	$\frac{\sigma_c}{\sigma_t}$	m_i (R-index (Cai))	m_i (UCS based)	m_i (TS based)	m_i (Hoek–Brown (2019))	m_i (combination method)
79	Shale	7.04	7.18	7.04	20.15	10.65	0.19	6.28
80	Shale	5.67	5.84	5.67	20.62	8.88	-1.45	5.18
81	Shale	15.83	15.89	15.83	20.36	13.32	10.91	14.12
82	Shale	11.51	11.61	11.53	19.05	16.57	5.65	10.08
83	Shale	7.57	7.72	7.59	19.18	13.90	0.85	6.73
84	Shale	25.31	25.00	24.96	17.82	29.87	22.11	24.30
85	Coal	15.12	15.16	15.10	18.25	22.37	10.01	13.40
86	Coal	10.33	10.44	10.35	18.92	16.49	4.20	9.04
87	Coal	4.21	4.44	4.21	20.17	8.99	-3.18	4.02
88	Coal	8.73	8.84	8.72	19.51	13.41	2.23	7.66
89	Coal	8.67	8.78	8.67	19.26	14.27	2.16	7.62
90	Coal	10.03	10.13	10.03	19.64	13.61	3.82	8.77
91	Coal	17.30	17.30	17.24	19.16	18.45	12.64	15.54
92	Coal	12.40	12.48	12.40	19.06	16.91	6.71	10.86
93	Coal	18.40	18.31	18.25	17.92	26.14	13.88	16.59
94	Coal	28.02	27.89	27.86	17.89	30.40	25.67	28.03
95	Coal	10.10	10.19	10.09	19.34	14.69	3.89	8.82
96	Coal	8.92	9.02	8.91	19.30	14.24	2.45	7.82
97	Coal	12.76	12.81	12.73	18.62	19.16	7.11	11.16
98	Coal	25.64	25.57	25.53	18.38	25.85	22.82	25.02
99	Coal	8.87	8.97	8.86	19.48	13.60	2.39	7.77
100	Coal	10.42	10.54	10.45	19.37	14.75	4.33	9.13
101	Coal	9.09	9.20	9.09	19.57	13.41	2.68	7.97
102	Coal	12.94	12.93	12.86	18.51	19.78	7.27	11.28
103	Coal	14.48	14.52	14.45	18.68	19.68	9.22	12.77
104	Coal	9.78	9.88	9.78	19.35	14.50	3.51	8.55
105	Coal	11.79	11.84	11.76	19.01	16.86	5.92	10.28
106	Coal	15.38	15.45	15.39	19.19	17.60	10.37	13.68
107	Coal	17.58	17.63	17.57	18.50	22.05	13.04	15.88

Appendix 6: Measured and calculated values of m_i based on different methods for metamorphic rocks

No	Rock name	m_i (measured)	$\frac{\sigma_c}{\sigma_t}$	m_i (R-index (Cai))	m_i (UCS based)	m_i (TS based)	m_i (Hoek–Brown (2019))	m_i (combination method)
1	Schist	20.25	20.27	20.22	8.41	11.73	16.30	18.73
2	Slate	7.84	7.97	7.85	8.47	6.23	1.17	6.94
3	Slate	3.53	3.79	3.53	8.29	4.80	-3.97	3.46
4	Slate	1.43	1.94	1.43	7.88	4.92	-6.25	-
5	Slate	1.98	2.87	2.52	7.97	5.68	-5.11	2.59
6	Slate	1.93	2.36	1.93	8.23	3.83	-5.74	1.98
7	Slate	2.93	3.24	2.93	8.39	3.92	-4.66	2.95
8	Slate	8.82	8.93	8.82	8.58	5.98	2.34	7.74

No	Rock name	m_i (measured)	$\frac{\sigma_c}{\sigma_t}$	m_i (R-index (Cai))	m_i (UCS based)	m_i (TS based)	m_i (Hoek–Brown (2019))	m_i (combination method)
9	Slate	30.97	31.08	31.04	7.93	25.31	29.58	32.44
10	Slate	22.70	22.66	22.61	7.20	48.42	19.23	21.46
11	Slate	9.50	9.61	9.50	7.78	14.60	3.18	8.32
12	Slate	16.85	16.90	16.84	8.02	15.82	12.14	15.13
13	Slate	14.22	14.29	14.22	8.49	8.66	8.94	12.56
14	Slate	13.64	13.72	13.64	8.54	8.06	8.23	12.01
15	Slate	5.99	6.15	5.99	22.27	6.19	-1.08	5.44
16	Slate	4.31	4.53	4.31	21.83	6.15	-3.07	4.10
17	Slate	2.95	3.25	2.94	20.92	6.76	-4.64	2.96
18	Slate	3.09	3.38	3.09	21.03	6.69	-4.48	3.09
19	Slate	4.35	4.57	4.35	21.25	7.03	-3.02	4.13
20	Slate	4.48	4.69	4.48	21.66	6.47	-2.87	4.23
21	Slate	4.80	5.00	4.80	22.13	5.95	-2.49	4.49
22	Gneiss	17.87	17.92	17.87	8.93	6.46	13.41	16.19
23	Gneiss	5.34	5.52	5.34	8.07	7.54	-1.84	4.92
24	Gneiss	16.23	16.29	16.23	8.71	7.55	11.40	14.52
25	Gneiss	6.39	6.54	6.39	8.63	4.68	-0.59	5.76
26	Gneiss	8.76	8.87	8.76	8.64	5.59	2.27	7.69
27	Gneiss	9.42	9.53	9.42	8.27	8.51	3.08	8.25
28	Gneiss	24.06	24.10	24.06	22.46	9.49	21.00	23.19
29	Gneiss	30.06	30.10	30.07	22.23	10.78	28.39	31.06
30	Gneiss	29.44	29.47	29.44	22.14	10.91	27.61	30.18
31	Gneiss	29.30	29.35	29.31	22.16	10.85	27.46	30.01
32	Gneiss	32.33	32.34	32.31	22.07	11.44	31.14	34.29
33	Gneiss	26.01	26.06	26.02	22.34	10.01	23.42	25.64
34	Gneiss	23.23	23.26	23.21	22.35	9.61	19.97	22.17
35	Gneiss	31.39	31.40	31.37	22.17	11.08	29.98	32.91
36	Gneiss	29.70	29.73	29.70	22.43	10.28	27.93	30.54
37	Quartzite	7.36	7.49	7.36	8.45	6.11	5.8	6.54
38	Quartzite	30.10	30.14	30.10	9.40	5.66	28.43	31.11
39	Quartzite	15.72	15.78	15.71	8.70	7.46	10.77	14.00
40	Quartzite	12.93	13.00	12.92	21.10	10.43	7.35	11.34
41	Marble	7.65	7.77	7.65	20.85	9.27	0.92	6.77
42	Marble	8.53	8.64	8.52	20.80	9.72	1.98	7.50
43	Marble	6.08	6.23	6.07	21.35	7.66	-0.97	5.50
44	Marble	6.25	6.41	6.25	21.00	8.37	-0.76	5.65
45	Marble	7.15	7.28	7.14	21.07	8.62	0.31	6.37
46	Marble	7.09	7.22	7.08	19.94	11.24	0.24	6.32

Acknowledgements The project presented in this article is supported by National Research, Development, Innovation Office (NKFIH 124508). The financial support of TKP2020 Institution Excellence Subprogram, Grant No. TKP2020 BME-IKA-VÍZ of the National Research Development and Innovation Office of Hungary is acknowledged.

Funding Open access funding provided by Budapest University of Technology and Economics.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

- Aladejare A, Wang Y (2019) Probabilistic characterization of Hoek–Brown constant m_i of rock using Hoek's guideline chart, regression model and uniaxial compression test. *Geotech Geol Eng* 37:5045–5060. <https://doi.org/10.1007/s10706-019-00961-7>
- Aldrich MJ (1969) Pore pressure effects on Berea sandstone subjected to experimental deformation. *Geol Soc Am Bull* 80:1577–1586. [https://doi.org/10.1130/0016-7606\(1969\)80](https://doi.org/10.1130/0016-7606(1969)80)
- Arshadnejad S, Nick N (2016) Empirical models to evaluate of “ m_i ” as an intact rock constant in the Hoek–Brown rock failure criterion. 19th Southeast Asian Geotechnical Conference & 2nd AGSSEA Conference (19SEAGC & 2AGSSEA) Kuala Lumpur, June 2016.
- Attewell PB, Sandford MR (1974) Intrinsic shear strength of a brittle anisotropic rock—I: experimental and mechanical interpretation. *Int J Rock Mech Min Sci Geomech Abstr* 11:423–430. [https://doi.org/10.1016/0148-9062\(74\)90453-7](https://doi.org/10.1016/0148-9062(74)90453-7)
- Barat D (1995) Personal communication from C.M.R.I., Dhanbad (from Sheorey, 1997)
- Betournay MC, Gorski B, Labrie D, Jackson R, Gyenge M (1991) New considerations in the determination of Hoek and Brown material constants. In: Wittke W (ed) 7th Int. Cong. Rock Mech. (ISRM), Aachen, vol 1, pp 195–200
- Borecki M, Kwasniewski M, Pacha J, Oleksy S, Berszakiewicz Z, Guzik (1982) Triaxial compressive strength of two mineralogic/diagenetic varieties of coal. measure, fine-medium grained Pniówek and Anna sandstones tested under confining pressure up to 60 MPa. *Prace Instytutu PBKIO Politechniki Śląskiej*, 119/2, Gliwice
- Brace WF (1964) Brittle fracture of rocks. In: Judd WR (ed) *State of stress in the Earth's Crust*. Elsevier, pp 111–174
- Broch E (1974) The influence of water on some rock properties. In: *Advances in rock mechanics*. 3rd Int. Cong. Rock Mech., Denver, vol 2, pp 33–38
- Cai M (2010) Practical estimates of tensile strength and Hoek–Brown strength parameter m_i of brittle rocks. *Rock Mech Rock Eng* 43:167–184. <https://doi.org/10.1007/s00603-009-0053-1>
- Chan SSM, Crocker TJ, Wardell GG (1972) Engineering properties of rocks and rock masses in the deep mines of the Coeur d'Alene Mining District, Idaho. *Trans Soc Min Eng AIME* 252:353–361
- Davarpanah M, Somodi G, Kovács L, Vásárhelyi B (2020) Experimental determination of the mechanical properties and deformation constants of Mórággy Granitic Rock Formation (Hungary). *Geotech Geol Eng* 38:3215–3229. <https://doi.org/10.1007/s10706-020-01218-4>
- Dayre M, Giraud A (1986) Mechanical properties of granodiorite from laboratory tests. *Eng Geol* 23:109–124. [https://doi.org/10.1016/0013-7952\(86\)90033-5](https://doi.org/10.1016/0013-7952(86)90033-5)
- Długosz M, Gustkiewicz J, Wysocki A (1981) Apparatus for investigation of rocks in a triaxial state of stress. Part II. Some investigation results concerning certain rocks. *Archiwum Gornictwa* 26:29–41
- Donath FA (1964) Strength variations and deformational behaviour in anisotropic rock. In: Judd WR (ed) *State of stress in the Earth's Crust*. Elsevier, New York, pp 281–297
- Dunikowski A, Korman S, Kohsling J (1969) Laboratory test indices of physico-mechanical properties of rocks in three-axial state of stress. *Przegląd Gorniczy* 25:523–528
- Eberhardt E (2012) The Hoek–Brown failure criterion. *Rock Mech Rock Eng* 45:981–988. <https://doi.org/10.1007/s00603-012-0276-4>
- Everling G (1960) Rock mechanical investigations and basis for determination of rock pressure according to deformation of drill holes. *Gluckauf* 96:390–409
- Fayed LA (1968) Shear strength of some argillaceous rocks. *Int J Rock Mech Min Sci* 5(1):79–85. [https://doi.org/10.1016/0148-9062\(68\)90024-7](https://doi.org/10.1016/0148-9062(68)90024-7)
- Franklin JA, Hoek E (1970) Developments in triaxial testing technique. *Rock Mech* 2:223–228. <https://doi.org/10.1007/BF01245576>
- Glushko VT, Kirnichanskiy GT (1974) Engineering—geological prognosticating of stability of the openings in deep coal mines. Nedra, Moscow
- Gnirk PF, Cheatham JB (1965) An experimental study of single bit tooth penetration into dry rock at confining pressures of 0–5000 psi. *J Soc Pet Eng* 5:117–130
- Gowd TN, Rummel F (1980) Effect of confining pressure on the fracture behavior of a porous rock. *Int J Rock Mech Min Sci Geomech Abstr* 17:225–229. [https://doi.org/10.1016/0148-9062\(80\)91089-X](https://doi.org/10.1016/0148-9062(80)91089-X)
- Gustkiewicz J (1984) Personal communication from M.A. Kwasniewski, Gliwice, 1995 (from Sheorey, 1997)
- Hareland G, Polston CE, White WE (1993) Normalised rock failure envelope as a function of grain size. *Int J Rock Mech Min Sci Geomech Abstr* 30:715–717. [https://doi.org/10.1016/0148-9062\(93\)90012-3](https://doi.org/10.1016/0148-9062(93)90012-3)
- Harza Engineering Co. (1976) Comprehensive ground control study of a mechanized longwall operation, final report, 2. USBM OFR 5(2)-775(2)-77 (published as NTIS report)
- He M, Zhang Z, Zheng J. et al. (2020) A new perspective on the constant m_i of the Hoek–Brown failure criterion and a new model for determining the residual strength of rock. *Rock Mech Rock Eng*. 53:3953–3967. <https://doi.org/10.1007/s00603-020-02164-6>
- Heard HC, Abey AE, Bonner BP, Schock RN (1974) Mechanical behaviour of dry Westerly granite at high confining pressure. Lawrence Livermore Laboratory Rept. No. UCRL 51642, 14 p
- Hobbs DW (1964) The strength and the stress-strain characteristics of coal in triaxial compression. *J Geol* 72:214–231. <https://doi.org/10.1086/626977>
- Hobbs DW (1970) Behaviour of broken rocks under triaxial compression. *Int J Rock Mech Min Sci* 7:125–148. [https://doi.org/10.1016/0148-9062\(70\)90008-2](https://doi.org/10.1016/0148-9062(70)90008-2)
- Hoek E (1983) Strength of jointed rock masses. *Geotechnique* 33:87–223. <https://doi.org/10.1680/geot.1983.33.3.187>

32. Hoek E (2007) Practical rock engineering—chapter 11: rock mass properties. <https://www.rocscience.com/learning/hoek-corner>
33. Hoek E, Brown ET (1980) Underground excavation in rock. Institution of Mining and Metallurgy, London
34. Hoek E, Brown ET (1997) Practical estimates of rock mass strength. *Int J Rock Mech Min Sci* 34(8):1165–1186. [https://doi.org/10.1016/S1365-1609\(97\)80069-X](https://doi.org/10.1016/S1365-1609(97)80069-X)
35. Hoek E, Brown ET (2019) The Hoek–Brown Failure criterion and GSI—2018 edition. *J Rock Mech Geotech Eng* 11:445–463. <https://doi.org/10.1016/j.jrmge.2018.08.001>
36. Hoek E, Carter TG, Diederichs MS (2013) Quantification of the geological strength index chart. In: 47th US rock mechanics/geomechanics symposium, San Francisco, June 2013
37. Hoek E, Marinos P (2000) Predicting tunnel squeezing problems in weak heterogeneous rock masses. *Tunnels Tunn Int* 32(11):45–51
38. Hoek E, Martin CD (2014) Fracture initiation and propagation in intact rock—a review. *J Rock Mech Geotech Eng* 6(4):287–300. <https://doi.org/10.1016/j.jrmge.2014.06.001>
39. Horino FG, Ellickson ML (1970) A method for estimating strength of rock containing planes of weakness. *USBM Rep Inv* 7449, 26 p
40. Hoshino K, Koide H, Inami K, Iwamura S, Mitsui S (1972) Mechanical properties of Japanese tertiary sedimentary rocks under high confining pressures. *Rept Geol Survey* 244
41. Hossaini SME, Vutukuri VS (1993) On the accuracy of multi-failure triaxial test for the determination of peak and residual strength of rocks. In: Szwedzicki (ed) *Aust. Conf. Geotech. Instrumentation and Monitoring in Open Pit and underground Mining*, Kalgoorlie, pp 223–228
42. Illnitskaya EI, Teder RI, Vatolin ES, Kuntyshev MF (1969) Properties of rock and methods of their determination. Nedra, Moscow
43. Johnson B, Friedman M, Hopkins TN (1987) Strength and microfracturing of Westerly granite extended wet and dry at temperatures to 800 °C and pressures to 200 MPa. In: Farmer IW, Daeman JJK, Desai CS, Glass CE, Newman SP (eds) *28th US Symp. Rock Mech.*, Tucson, pp 399–412
44. Kovári K, Tisa A (1975) Multiple failure state and strain controlled triaxial tests. *Rock Mech* 7:17–33. <https://doi.org/10.1007/BF01239232>
45. Kuntyshev MF (1964) Investigation of methods of determining the basic physico-mechanical characteristics of rocks, used while solving the problem of rock pressure. *Cand. Techn. Sci. Thesis*, Moscow
46. Kwasniewski MA (1983) Deformational and strength properties of the three structural varieties of carboniferous sandstones. In: *5th Int. Cong. Rock Mech. (ISRM)*, 1, Balkema, Rotterdam, pp A 105–A 115.
47. McLamore R, Gray KE (1967) The mechanical behaviour of anisotropic sedimentary rocks. *Trans Am Soc Mech Eng Ser B* 89:62–76
48. Mehrishal A, Sharifzadeh M, Shahryar K (2015) Estimating of Hoek–Brown m_i using internal friction angle. In: *Proceedings of the 24th International Mining Congress of Turkey, IMCET 2015*, pp 488–493
49. Misra B (1972) Correlation of rock properties with machine performance. Ph.D. thesis, Leeds University
50. Mogi K (1965) Deformation and fracture of rocks under confining pressure, (2): Elasticity and plasticity of some rocks. *Bull Earthquake Res Inst (Tokyo Univ)* 42:349–379
51. Mogi K (1966) Some precise measurements of fracture strength of rocks under uniform compressive stress. *Rock Mech Eng Geol* IV:41–55
52. Mostyn G, Douglas K (2000) Strength of intact rock and rock masses. In: *Proceedings of international conference on geotechnical and geological engineering*, vol 1. Technomic Publishing, Lancaster, pp 1389–1421
53. Murrel SAF (1965) The effect of triaxial stress systems on the strength of rock at atmospheric temperature. *Geophys J* 10:231–281. <https://doi.org/10.1111/j.1365-246X.1965.tb03155.x>
54. Ouyang Z, Elsworth D (1991) A phenomenological failure criterion for brittle rock. *Rock Mech Rock Eng* 24:133–153. <https://doi.org/10.1007/BF01042858>
55. Peng J, Rong G, Cai M, Wang X, Zhou C (2013) An empirical failure criterion for intact rocks. *Rock Mech Rock Eng* 47(2):347–356. <https://doi.org/10.1007/s00603-013-0514-4>
56. Ramamurthy T (1989) Personal communication from I. I. T., Delhi (from Sheorey, 1997)
57. Ramez MRH (1967) Fractures and strength of a sandstone under triaxial compression. *Int J Rock Mech Min Sci* 4:257–268. [https://doi.org/10.1016/0148-9062\(67\)90010-1](https://doi.org/10.1016/0148-9062(67)90010-1)
58. Rao KS, Rao GV, Ramamurthy T (1983) Strength anisotropy of a Vindhyan sandstone. In: *Indian Geotech Conf, Madras*, vol 1, pp 141–148
59. Read S, Richards L (2014) Correlation of direct and indirect tensile tests for use in the Hoek–Brown constant m_i . In: *Rock engineering and rock mechanics: structures in and on rock masses*. Taylor and Francis, London
60. Richards L, Read S (2011) A comparative study of m_i , the Hoek–Brown constant for intact rock material. In: *Proceedings 45th US rock mechanics/geomechanics symposium*. American Rock Mechanics Association, San Francisco
61. Rocscience (2012) RocData. <http://www.rocscience.com/products/4/RocDataæ> (March 7, 2014)
62. Sari M (2010) A simple approximation to estimate the Hoek–Brown parameter m_i for intact rocks. In: *Rock mechanics in civil and environmental engineering*. Taylor and Francis, London
63. Schwartz AE (1964) Failure of rock in the triaxial shear test. In: *Proceedings of the 6th rock mechanics symposium*. University of Missouri, USA
64. Shea-Albin VR, Hansen DR, Gerlick RE (1991) Elastic wave velocity and attenuation as used to define phases of loading and failure in coal. *USBM Rept Inv* 9355, 43 p
65. Shen J, Karakus M (2014) Simplified method for estimating the Hoek–Brown constant for intact rocks. *J Geotech Geoenviron Eng* 140(1):0401–04025. [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0001116](https://doi.org/10.1061/(ASCE)GT.1943-5606.0001116)
66. Sheorey PR (1997) Empirical rock failure criteria. Central Mining Research Institute, India
67. Sheorey PR, Biswas AK, Choubey VD (1989) An empirical failure criterion for rocks and jointed rock masses. *Engg Geol* 26:141–159. [https://doi.org/10.1016/0013-7952\(89\)90003-3](https://doi.org/10.1016/0013-7952(89)90003-3)
68. Singh SK (1995) Personal communication from C.M.R.I., Dhanbad (from Sheorey, 1997)
69. Singh M, Sabu AK, Srivastava RK, Tiwari RP (1992) Evaluation and applicability of strength for rocks: sandstones and quartzites of Mirzapur region. In: *Asian Regional Symp. Rock Slopes*, Oxford and IBH, New Delhi, pp 117–124
70. Singh M, Raj A, Singh B (2011) Modified Mohr–Coulomb criterion for non-linear triaxial and polyaxial strength of intact rocks. *Int J Rock Mech Min Sci* 48(4):546–555. <https://doi.org/10.1016/j.jrmms.2011.02.004>
71. Stowe RL (1969) Strength and deformation properties of granite, basalt, limestone and tuff at various loading rates. *U.S Army Corps Eng., Waterways Exp. Stn., Vicksburg Miss., Misc. Paper C-69-1*
72. Ván P, Vászárhelyi B (2014) Sensitivity analysis of GSI based mechanical parameters of the rock mass. *Period Polytech Civil Eng* 58(4):379–386. <https://doi.org/10.3311/PPci.7507>

73. Vásárhelyi B, Kovács D (2016) Empirical methods of calculating the mechanical parameters of the rock mass. *Period Polytech Civ Eng* 61(1):39–50. <https://doi.org/10.3311/PPci.10095>
74. Vásárhelyi B, Kovács L, Török A (2016) Analysing the modified Hoek–Brown failure criteria using Hungarian granitic rocks. *Geomech Geophys Geo-energ Geo-resour* 2:131–136. <https://doi.org/10.1007/s40948-016-0021-7>
75. Vutukuri VS, Farough Hossani SM (1993) Correlation between the effect of confining pressure on compressive strength in triaxial test and the effect of dia/height ratio on compressive strength in unconfined compression test. In: Peng SS (ed) 12th Conf. Ground Control in Mining, Morgantown, pp 316–321
76. Wang R, Kemeny JM (1995) A new empirical failure criterion for rock under polyaxial compressive stresses. In: Daemen JJK, Schultz RA (eds) 35th US Symp. Rock Mech., Balkema, Rotterdam, pp 453–458
77. Wang W, Shen J (2017) Comparison of existing methods and a new tensile strength based model in estimating the Hoek–Brown constant m_i for intact rocks. *Eng Geol* 224:87–96. <https://doi.org/10.1016/j.enggeo.2017.05.008>
78. Wei X, Zuo J, Shi Y, Liu H, Jiang Y, Liu C (2020) Experimental verification of parameter m in Hoek–Brown failure criterion considering the effects of natural fractures. *J Rock Mech Geotech Eng* 12(5):1036–1045. <https://doi.org/10.1016/j.jrmge.2020.01.006>
79. Wen T, Tang H, Huang L, Hamza A, Wang Y (2020) An empirical relation for parameter m_i in the Hoek–Brown criterion of anisotropic intact rocks with consideration of the minor principal stress and stress-to-weak-plane angle. *Acta Geotech*. <https://doi.org/10.1007/s11440-020-01039-y>
80. Wen T, Tang HM, Wang YK, Ma JW (2020) Evaluation of methods for determining rock brittleness under compression. *J Nat Gas Sci Eng*. <https://doi.org/10.1016/j.jngse.2020.103321>
81. Wen T, Tang H, Huang L, Wang YK, Ma JW (2020) Energy evolution: a new perspective on the failure mechanism of purple-red mudstones from the Three Gorges Reservoir area, China. *Eng Geol*. <https://doi.org/10.1016/j.enggeo.2019.105350>
82. Wen T, Tang HM, Wang YK (2020) Brittleness evaluation based on the energy evolution throughout the failure process of rocks. *J Pet Sci Eng*. <https://doi.org/10.1016/j.petrol.2020.107361>
83. Wilhelmi B, Somerton WH (1967) Simultaneous measurement of pore and elastic properties of rocks under triaxial stress conditions. *J Soc Pet Engrs* 7:283–294
84. Zuo J, Liu H, Li H (2015) A theoretical derivation of the Hoek–Brown failure criterion for rock materials. *J Rock Mech Geotech Eng* 7(4):361–366. [https://doi.org/10.1016/S1365-1609\(00\)00049-6](https://doi.org/10.1016/S1365-1609(00)00049-6)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.