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Satellite orbit determination combining C-band ranging and differenced ranges by transfer

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The constraint in the transverse direction of satellite orbit with differenced ranges between master station and slave stations by transfer as an angular observation data is explained in theory. Differenced ranges in combination with C-band ranging by transfer were used in satellite orbit determination. The position error of overlapped orbit differences for combining is less than that for ranging only. The residual of predicted orbit forward 5.5 days for combining is 3.1762 m, while the residual for ranging only with the same duration is 3.5380 m. Both the orbital overlapping and orbit prediction experimentations can testify the constraint in the transverse direction of satellite orbit with differenced ranges, and the results show that the accuracy of orbit determination and orbit prediction is improved by combining differenced ranges and C-band ranging.

differenced ranges by transfer, orbital overlapping, orbit prediction

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The space segment of satellite navigation system of China consists of several geostationary Earth orbit (GEO) satellites. The orbit accuracy of GEO satellites has an influence on positing capability of navigation system directly. Therefore, the research on precise orbit determination of GEO satellites has a great significance.

"Determination of satellite orbit by transfer" was developed by the National Time Service Center, Chinese Academy of Sciences. Owing to the high orbit determination accuracy, the system is suitable for GEO satellite tracking [1]. The accuracy of C-band ranging is better than one centimetre in the direction of line-of-sight. The GEO satellite is placed at an altitude of 36000 km above the equator. The angle of GEO satellite to the ranging stations is only a few degrees. The ranging data is unable to provide constrain in the transverse direction of the satellite orbit owing to the poor observation geometry.

The observation mode of differenced ranges by transfer is the development for C-band ranging by transfer. The principle of the observation mode of differenced ranges is the same as very long baseline interferometry (VLBI). It can provide strongly constrain in the transverse direction of the satellite orbit, perpendicular to the line-of-sight. The improvement of orbit prediction accuracy combining differenced ranges and ranging by transfer has been implemented with short-arc data [2].

In order to explaining the constraint in the transverse direction of satellite orbit by differenced ranges in theory, the relation between the measurement error of angular observation and satellite orbit is analyzed in detail. The experiments of orbital overlapping and orbit prediction are carried out to validate the improvement in accuracy of orbit determination and prediction with differenced ranges.

1 Principle of differenced ranges between master station and slave stations by transfer

Differenced ranges by transfer can be obtained from making

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the differences of the ranges data between the ranges for master station to master station and the ranges for master station to each slave station via satellite transponder. Both of the differenced ranges observations and the VLBI observations are the difference measurement [3]. Therefore differenced ranges as VLBI can provide a constraint in the transverse direction of satellite orbit.

According to the locations of stations, there are 4 baselines between master station and slave stations. Figure 1 shows the stations and the baselines.

The precision of baselines depends on the precision of station coordinates. The precision of the station coordinates is better than 5 mm, while the corresponding precision of baselines is better than 1 cm [2]. The residual of POD with ranging data by transfer with 1 day arc is 6.5 cm, while the residual of POD with differenced ranges by transfer with 1 day arc is 9.4 cm [2]. The observation precision of differenced ranges by transfer is 16.5 mas on the baseline of 1177451.2031 m between Lintong and Shanghai.

Corrected the effects of the instrumentation errors and earth rotation [4–6], the observation equation of differenced ranges can be expressed as

$$R_{i0}(t_0)-R_{00}(t_0)=[R_i^{\mathrm{d}}(t_0)-R_0^{\mathrm{d}}(t_0)]+\Delta T_i. \qquad (1)$$

Here $R_{i0}(t_0)-R_{00}(t_0)$ is the observed value of the differenced ranges, namely the difference of time delays on the path between the time delay of the signals from master station to master station via satellite and that of the signals from master station to slave station via satellite. $R_i^{\mathrm{d}}(t_0)-R_0^{\mathrm{d}}(t_0)$ is the calculated value, namely the difference of time delays between the time delay of the signals from satellite to master station and that of the signals from satellite to slave station at the moment t_0 when the signals from master station are received by the satellite. The subscript "0" indicates master station at Lintong, and "*i*" is slave station in order as Shanghai, Changchun, Kunming, and Urumqi. The superscript "d" is for the downlink. ΔT_i is the clock offset at slave station *i* with respect to the master station, and it can be obtained from TWSTFT [7]:

$$R_i^d(t_0)-R_0^d(t_0)=$$
$$\sqrt{[x_i-X(t_0)]^2+[y_i-Y(t_0)]^2+[z_i-Z(t_0)]^2}$$
$$-\sqrt{[x_0-X(t_0)]^2+[y_0-Y(t_0)]^2+[z_0-Z(t_0)]^2}, \qquad (2)$$

where (x_i, y_i, z_i) is the coordinates of slave station, and (x_0, y_0, z_0) is the coordinates of master station. $[X(t_0), Y(t_0), Z(t_0)]$ is the position of satellite at the moment t_0.

Ionosphere correction adopts the prediction model which proposed by Li et al. [8] and troposphere correction adopts Marini-Murray model. The detailed principle of differenced ranges between master station and slave stations by transfer can be referred to [2].

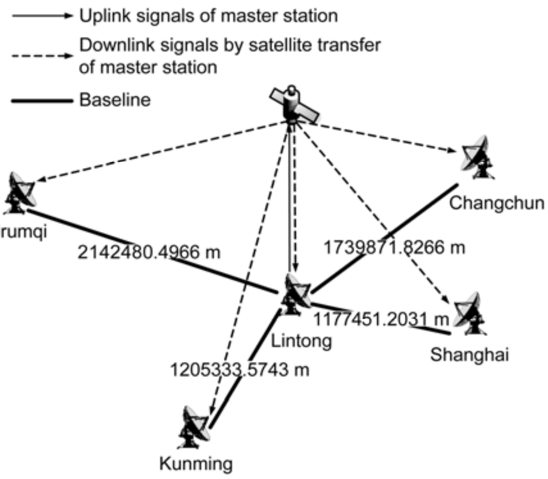

Figure 1 Principle of differenced ranges between master station and slave stations by transfer.

2 The relation between the measurement error of angular observation and satellite orbit

The ranging mainly provides a strong constraint in the radial direction, while the differenced ranges are able to strongly constrain the orbit in the transverse direction. The accuracy of satellite orbit can be improved by differenced ranges data [9].

The report given by Petrachenko [10] mentioned the relation between the measurement error of angular observation and satellite orbit. Figure 2 shows the geometry vector relation between observation direction, baseline and time delay. According to Figure 2, we get

$$\begin{cases} \vec{R}_1=\vec{x}+\vec{R}_2, \\ \tau=\dfrac{1}{c}\sqrt{\vec{R}_1\cdot\vec{R}_1}-\dfrac{1}{c}\sqrt{\vec{R}_2\cdot\vec{R}_2}. \end{cases} \qquad (3)$$

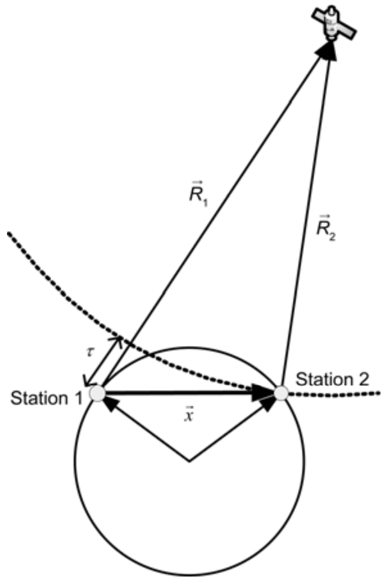

Figure 2 Geometry principle of angular observation.

Here, $\vec{R}_1$ is the observation vector of station 1; $\vec{R}_2$ is the observation vector of station 2; $\vec{x}$ is the vector of baseline from station 1 to station 2; τ is the difference time delay of observation between station 1 and station 2; c is the speed of light.

Differentiating eq. (3) we get

$$\begin{cases} \mathrm{d}\vec{R}_1 = \mathrm{d}\vec{R}_2 = \mathrm{d}\vec{R}, \\ \mathrm{d}\tau = \dfrac{\mathrm{d}\vec{R}}{c} \cdot \left(\dfrac{\vec{R}_1}{\sqrt{\vec{R}_1 \cdot \vec{R}_1}} - \dfrac{\vec{R}_2}{\sqrt{\vec{R}_2 \cdot \vec{R}_2}} \right). \end{cases} \tag{4}$$

The formula in the parentheses of the second equation in eq. (4) is the difference of the two unit vectors in the direction of line-of-sight of the two stations. Angular observation can provide a constraint in the plane that constituted by both stations and satellite. Orbit determination can be carried out with differenced ranges in two intersected baselines; therefore, eq. (4) is reduced to a problem in plane structured by satellite and the two stations.

If we define that

$$\begin{cases} \mathrm{d}\vec{R} = \Delta R \cdot \vec{I}_R + \Delta\varepsilon \cdot L \cdot \vec{I}_b, \\ \dfrac{\vec{R}_1}{\sqrt{\vec{R}_1 \cdot \vec{R}_1}} = \cos A \cdot \vec{I}_b + \sin A \cdot \vec{I}_R, \\ \dfrac{\vec{R}_2}{\sqrt{\vec{R}_2 \cdot \vec{R}_2}} = -\cos B \cdot \vec{I}_b + \sin B \cdot \vec{I}_R. \end{cases} \tag{5}$$

Here $\vec{I}_b$ is the unit vector of the baseline. $\vec{I}_R$ is the unit vector that is perpendicular to the baseline. ΔR is the change of satellite position that is perpendicular to the baseline. $\Delta\varepsilon$ is the angle change of satellite position that parallel to the baseline. X is the length of the baseline. L is the distance from the satellite to the baseline.

From Figure 3, $A+B+\theta=180°$. Then eq. (4) becomes

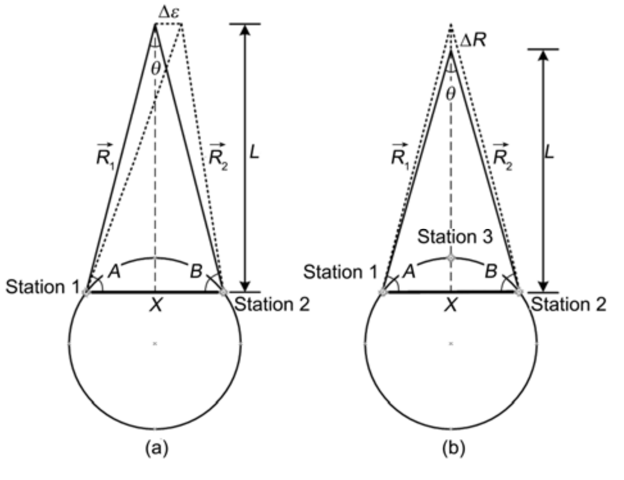

Figure 3 Principle of time delay error relates to satellite position errors in transverse (a) and radial (b) directions.

$$\Delta\tau = \frac{2 \cdot \Delta\varepsilon \cdot L}{c} \cdot \sin\frac{\theta}{2} \cdot \sin\left(A + \frac{\theta}{2}\right) - \frac{2 \cdot \Delta R}{c} \cdot \cos\left(A + \frac{\theta}{2}\right) \cdot \sin\frac{\theta}{2}, \tag{6}$$

where $\Delta\tau$ is the concrete express of $\mathrm{d}\tau$. If $A+\theta/2=\pi/2$, the satellite is right over the two observation stations. The angle of the satellite to the area is only a few degrees for the observation stations are placed in the area of China, and consequently $\sin(\theta/2)$ and $\tan(\theta/2)$ are almost equal. The change in time delay ($\Delta\tau_1$) due to transverse direction error in position ($\Delta\varepsilon$) can be written approximately as

$$\Delta\tau_1 = \frac{\Delta\varepsilon \cdot X}{c}. \tag{7}$$

In case of GEO satellites, the distance from the earth center to the satellite is 4.2×10^7 m, and the error in position of satellite (Δ) is

$$\Delta \approx \Delta\varepsilon \cdot L = \frac{\Delta\tau_1 \cdot c \cdot 4.2\times10^7}{X}.$$

A time delay error of 1 ns corresponds to an orbit error of about 6 m for the GEO satellite with a baseline of 2000 km. Correspondingly, a time delay error of 1 ns corresponds to an orbit error of about 4 m for the GEO satellite with a baseline of 3000 km. Therefore, with the lengthening baseline under the same time delay error, the smaller orbit error can be obtained.

According to eq. (6), the change in time delay ($\Delta\tau_2$) due to radial direction error in position (ΔR) can be written as

$$\Delta\tau_2 = -\frac{2 \cdot \Delta R}{c} \cdot \cos\left(A + \frac{\theta}{2}\right) \cdot \sin\frac{\theta}{2}. \tag{8}$$

If A is close to 90° and the baseline is not too long, the relation between the change in time delay ($\Delta\tau_2$) and the radial direction error in position (ΔR) is

$$\Delta\tau_2 = \frac{\Delta R}{c} \cdot \frac{X^2}{2L^2}. \tag{9}$$

If A is 90°, $\Delta\tau_2$ will be zero. $\Delta\tau_2$ is anti-symmetric and the anti-symmetric center is 0.

From eqs. (7) and (9), we can come to a conclusion that differenced ranges as angular observation can provide strong constraint in transverse direction than that in radial direction. In statistical orbit determination, even differenced ranges by transfer measurements for one baseline can be used [11]. There are 4 baselines between master station and slave stations. Therefore the baselines can constrain different directions of the satellite's motion and improve the accuracy of orbit determination.

3 Experiments and data analysis

To confirm the improvement of satellite orbit using differenced ranges combined with C-band ranging data, the orbital overlapping and orbit prediction experimentations are carried out. The experimentations for Sino-1 satellite are implemented during the observation period of June 7–10, 2005. The instrumentations time delay is determined in the duration of first 10 minutes each hour.

3.1 Accuracy analysis of orbital overlapping

Orbital overlapping is an effective way to evaluate the orbit accuracy [12]. Orbital overlapping experimentations were carried out to confirm the improvement of orbit determination precision combining differenced ranges and C-band ranging.

Two types of observation data, differenced ranges combined with ranging data of June 7–10, 2005 were employed to determine orbit with a 1.5 days arc. Table 1 shows the residuals (RMS) of POD with a 1.5 days arc observation using ranging data only and jointly with differenced ranges, respectively.

If the orbit determination strategy is reasonable, the position error of the overlapping arc will be small [13,14]. The analysis is made on position errors of overlapped orbit difference with 0.5 day arc. Figure 4 shows the detailed overlapped orbit difference. The position accuracy of the two overlapping with the combination of ranging data and differenced ranges is higher than that with only ranging data. Table 2 statistics the orbital overlapping accuracy using only ranging data and the combination of ranging data and

Table 1 Residuals of 1.5 days arc observation (unit: m)

Date	Ranging data only		Ranging data combined with differenced ranges				
			ranging data		differenced ranges		combined
	ranging data number	RMS	ranging data number	RMS	differenced ranges number	RMS	RMS
Period of June 7 at 0:00 to June 8 at 12:00	539452	0.082	539433	0.092	428376	0.161	0.127
Period of June 8 at 0:00 to June 9 at 12:00	536097	0.090	536078	0.099	425143	0.144	0.121
Period of June 9 at 0:00 to June 10 at 12:00	528848	0.090	528848	0.098	424267	0.164	0.132

Table 2 Statistics of orbital overlapping accuracy using only ranging data and the combination of the two types of data (unit: m)

Date	Mode	Delta *R*	Delta *T*	Delta *N*	Delta POS
Period of June 8 at 0:00 to June 8 at 12:00	ranging data only	0.125	0.535	1.384	1.489
	combined POD	0.131	0.241	0.729	0.779
Period of June 9 at 0:00 to June 9 at 12:00	ranging data only	0.608	0.517	0.807	1.135
	combined POD	0.631	0.427	0.640	0.995

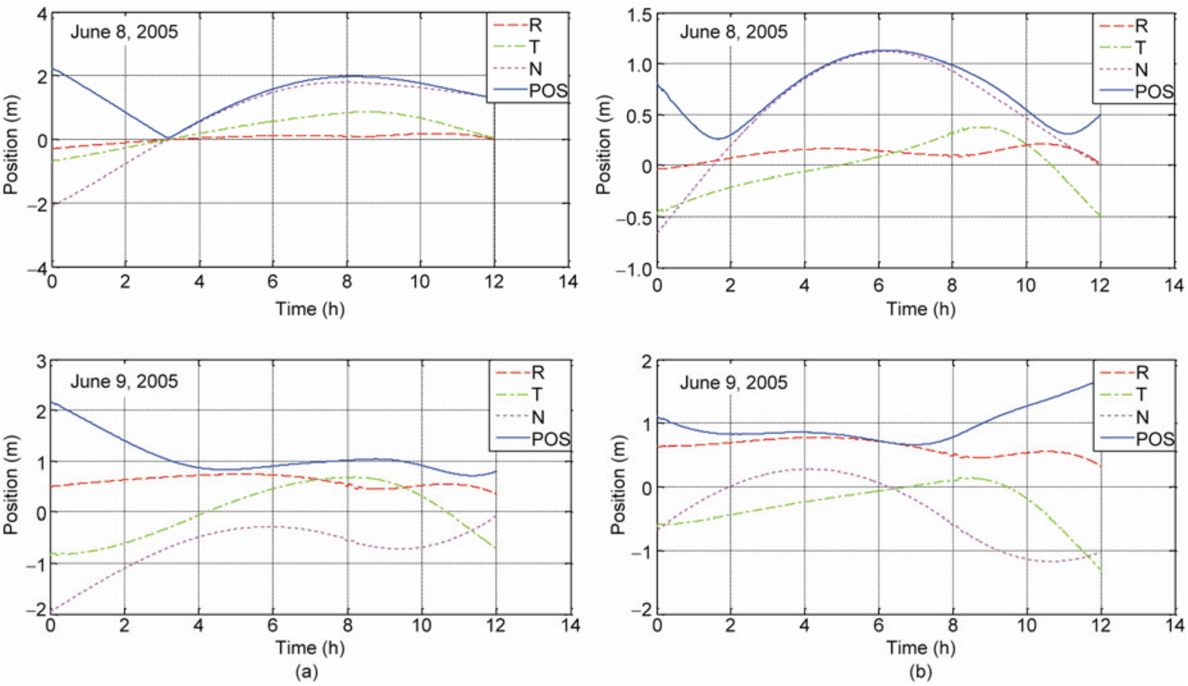

Figure 4 Orbital differences between POD using only ranging data and combined POD in the radial, transverse and normal directions. (a) POD using only ranging data; (b) combined POD.

differenced ranges. The orbital overlapping accuracy in the transverse direction and the normal direction are also improved with the constraint of differenced ranges in the transverse direction and the normal direction.

3.2 Long arc orbit prediction experimentations

C-band ranging data and differenced ranges by transfer are employed in long arc orbit prediction experimentations. The two types of observation data of June 7–8, 2005 were employed to determine orbit with 1.5 days arc and predict the orbit with 5.5 days arc.

The orbit determination with 1.5 days arc using only ranging data is implemented, and the estimated parameters are 6 orbital elements, 1 transponder delay parameter, 1 solar radiation pressure parameter. The orbit determination with 1.5 days arc observation data using the combination of ranging data and differenced ranges is also implemented, and the estimated parameters are 6 orbital elements, 1 transponder delay parameter, 1 solar radiation pressure parameter, 4 baseline system error parameters.

The orbit prediction experimentations with the two orbit determination strategies are carried out. Figure 5 shows the comparison of residuals (O-C) with orbit determination of 1.5 days arc and orbit prediction of 5.5 days arc using only ranging data and the combination of ranging data and differenced ranges.

The residuals of orbit prediction using only ranging data and the combination of ranging data and differenced ranges is 3.5380 and 3.1762 m, respectively. The comparison of residuals between the two orbit prediction strategies indicates that the orbit prediction accuracy can be improved using the combination of ranging data and differenced ranges.

4 Conclusions

The improvement of orbit determination accuracy and orbital prediction accuracy are discussed in the paper. The orbit determination using only differenced ranges between master station and slave stations by transfer is under the effect of the systematic errors of differenced ranges. The orbit determination using the combination of C-band ranging data and differenced ranges by transfer is proposed and implemented.

The relation between the measurement error of angular observation and satellite orbit position indicates that differenced ranges as angular observation data can provide a constraint in the transverse direction of satellite orbit. The accuracy of orbit determination and orbit precision is limited by the length of baselines between master station and slave stations. In order to improve the orbit accuracy, the baselines of differenced ranges should be lengthened.

The orbital overlapping experimentations shows that the orbit determination using ranging data combined with differenced ranges can provide high POD position accuracy, and the errors in the transverse direction also can be constrained. The orbit prediction experimentations indicates that residuals of long-arc orbit prediction using combined POD is better than that only using ranging data.

The differenced ranges observation mode between master station and slave stations by transfer is a new development based on the C-band ranging tracking system. The accuracy of orbit determination and orbit prediction can be improved with the combination of ranging data and differenced ranges. The combined orbit determination approach is of great value for application in the field of deep space tracking of craft and orbit determination and orbit prediction of navigation system.

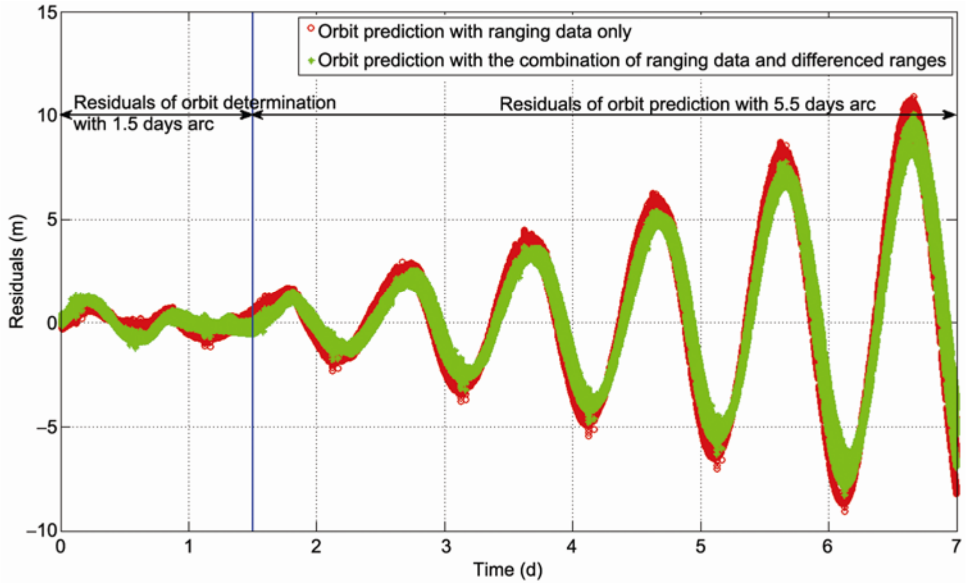

Figure 5 Comparison of residuals (O-C) with observation arc of 1.5 days and orbit prediction arc of 5.5 days.

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