

# Recent progress in theoretical studies of nuclear magnetic moments

ZHAO EnGuang<sup>1,2</sup>

<sup>1</sup>*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China;*

<sup>2</sup>*Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China*

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Nuclear magnetic moment is highly sensitive to the underlying structure of atomic nuclei and therefore serves as a stringent test of nuclear models. The advanced nuclear structure models have been successful in analyzing many nuclear structure properties, but they still cannot provide a satisfactory description of nuclear magnetic moments. Recently attempts to summarize the present understanding on nuclear magnetic moments in both relativistic and non-relativistic theoretical models have been made. The detailed contents are covered in the issue entitled “Nuclear magnetic moments and related topics” (in *Sci China Phys Mech Astron*, Vol. 54, No. 2, 2011). In this paper some of the related achievements will be highlighted.

**nuclear magnetic moments, status and progress, relativistic and non-relativistic many-body models**

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Nuclear magnetic moment is an important physical observable that reflects the interplay between collective and single-particle degrees of freedom in atomic nuclei. It therefore provides a stringent test of various nuclear structure models. A concise but interesting history and present understanding of nuclear magnetic moments have been provided in [1].

Since the successes of the nuclear shell model established in 1949 by Mayer and Jensen for the explanation of the magic numbers ( $Z$  or  $N = 2, 8, 20, 28, 50, 82, \dots$ ), the understanding of the magnetic moment of an odd- $A$  nucleus has been done in the extreme single-particle picture which leads to the well known Schmidt values [2]. It was observed in the early 1950s [3], however, that almost all nuclear magnetic moments are sandwiched between the two Schmidt lines.

The pion, predicted by Yukawa in 1935, and discovered experimentally by Powell in 1947, was pointed out to be very important for understanding nuclear magnetic moments by Miyazawa in 1951 [4] and by Villars in 1952 [5] via the one-pion exchange currents, which can be understood as a medium correction in comparison with the free nuclear magnetic moments. Besides the pion effect, the first-order configuration mixing was pointed out to be also important in the

odd- $A$  nuclei with a  $jj$ -closed core by Arima and Horie in 1954 [6,7]. This effect is also called the first-order core polarization or Arima-Horie effect. However, for the nuclei with a  $LS$ -closed core  $\pm 1$  nucleon, the first-order configuration mixing does not contribute to nuclear magnetic moments. In order to understand the difference between Schmidt values and experiment data in this type of nuclei, it was realized that one has to take into account the second-order configuration mixing, which is also called the tensor correlation. The isoscalar magnetic moments provide us the best evidence of the tensor correlations. There were also lots of discussion on whether the  $\Delta$ -hole mixing can explain the magnetic moments [8–10].

In the past decades, covariant density functional theory (CDFT) has been successfully applied to describe the nuclear structure over the whole periodic table [11–15]. However, the relativistic description of the magnetic moment is still unsatisfactory. By taking into account the renormalized currents by the random phase approximation (RPA) or applying the self-consistent deformed CDFT with the time-odd fields, the isoscalar magnetic moments in the nuclei with a  $LS$ -closed core  $\pm 1$  nucleon could be reproduced quite well. Unfortunately, these effects cannot remove the discrepancy existing in the isovector magnetic moments [16–21]. To eliminate

email: egzhao@mail.itp.ac.cn

this discrepancy, one-pion exchange current corrections have been included in the relativistic model, which were found to be significant. However they lead to a larger disagreement with data. Recently, the second-order configuration mixing has been considered in the fully self-consistent relativistic theory and it turned out to be important for improving the description of the isovector magnetic moments [22].

In addition, many models have been further extended to describe magnetic moment of nuclear ground-state or gyromagnetic ratio ( $g$ -factor defined as a ratio of magnetic moment to the angular momentum) of nuclear excited states [23–36]. In most of these models, however, nuclear magnetic moments were calculated by adjusting model parameters to reproduce the experimental data or by adopting a model-space-dependent effective orbital/spin  $g$ -factor. Although good agreement with the experimental data could be achieved in this way, a *quantitative* and *universal* description of nuclear magnetic moments would definitely requires further theoretical investigations.

Experimentally, advances in modern experimental techniques and sensitive detectors have made it even possible to measure, with a reasonable accuracy, magnetic moments of short-lived nuclear states [37–40].

In order to draw more attention to the status of nuclear magnetic moment studies and also to introduce the major achievements on the related subjects, the editorial board of *Science China Physics Mechanic and Astronomy* has invited a number of major theoretical nuclear physicists in this field to contribute to a special issue entitled “Nuclear magnetic moments and related topics” (in *Sci China Phys Mech Astron*, Vol. 54, No. 2, 2011). This paper attempts to summarize the progress on theoretical studies of nuclear magnetic moment and the related topics.

## 1 Remarks and discussion

### 1.1 Arima-Horie effect on nuclear magnetic moments

In the extreme single-particle shell model, magnetic moment of an odd- $A$  nucleus is carried only by one valence nucleon, which leads to the well known Schmidt values. It was observed in the early 1950s [3], however, that almost all nuclear magnetic moments are sandwiched between the two Schmidt lines, and that some of them, like  $^{17}\text{F}$  or  $^{15}\text{N}$ , show only small deviations from the Schmidt values, while others, like  $^{209}\text{Bi}$  or  $^{207}\text{Tl}$ , show very large deviations. In this extreme single-particle picture, one expects that the valence proton particle (or proton hole) in the latter nuclei moving independently around the core of  $^{208}\text{Pb}$  should be similar to that in the former nuclei moving around  $^{16}\text{O}$ . Therefore, it is impossible to interpret such differences between the two groups of nuclei within this model.

In 1954 Arima and Horie pointed out a very distinct difference between these two groups of nuclei [6]. Nuclei in the former group are  $LS$ -closed, i.e. the spin-orbit partners

$j = \ell \pm 1/2$  of the core are completely occupied. Therefore they are not expected to be excited strongly by a M1 external field. As for the latter group (like  $^{208}\text{Pb}$ ), their cores are  $jj$ -closed, i.e. one of the spin-orbit partners is open, and therefore nucleons in the core can be strongly excited to the empty spin-orbit partner by the M1 external field. This M1 giant resonance state of the core can be excited by the interaction with the valence nucleon [41]. This is the idea of Arima-Horie effect on nuclear magnetic moments. Besides, the second-order core polarization and the meson exchange current (MEC) were found to be also very important in explanation of the discrepancy between the Schmidt values and the experimental data [42–44]. It has been shown that the total effects of second-order core polarization and MEC give corrections, which improve the description of isovector magnetic moments by the Schmidt values [45, 46]. A recent Green’s function Monte Carlo calculation of magnetic moments and M1 transitions for  $A \leq 7$  nuclei demonstrated again the importance of the MEC contributions to nuclear isovector magnetic moments [32].

In [1], Arima presented a brief review of this history as well as the present understandings of nuclear magnetic moments and Gamow-Teller transitions. The roles of configuration mixing, MEC and relativistic effects have been addressed. The quenching of isoscalar spin matrix elements and the recent measurement of the Gamow-Teller strength in (p,n) and (n,p) reactions on  $^{90}\text{Zr}$  pointed out the importance of the tensor correlations.

### 1.2 Nuclear magnetic moments from covariant density functional theories

In the past decades, the covariant density functional theory or relativistic mean-field (RMF) approach incorporating important relativistic effects has been used extensively in the analysis of structure properties. With a few universal parameters, it has already achieved great successes not only in describing many nuclear phenomena for both stable and exotic nuclei [11–15], but also in reproducing the elemental abundance distributions in both solar system and ultra-metal-poor stars [47–53]. However, a straightforward application of the single-particle RMF model with only time-even fields cannot reproduce the magnetic moment of nuclear ground state, even for near  $LS$  double-closed shell nuclei [16–21]. The underlying reason is due to the small Dirac effective mass ( $M^* \sim 0.6M$ ) in the RMF approach which results in the enhancement of the Dirac current. The solution of this problem lies in treating the response of the nuclear core to the unpaired valence nucleon properly. One way is to treat the polarization effect of the unpaired nucleon on the core by allowing excitations from the core and thus creating particle-hole vibration. The coupling of a single-particle state in a nuclear medium to such a vibration state by meson exchanges in the framework of relativistic RPA could restore the single-particle electromagnetic current to its free-nucleon value [20, 54, 55]. A more

general discussion starting from a Ward identity, in which the coupling to a vibration state represents a vertex correction, arrived at the same conclusion [56]. In Landau-Migdal quasi-particle approach or in the language of quantum liquids, the effective single-particle currents in nuclei or the “back-flow” effect were also introduced to resolve this problem [57].

Thanks to the development of numerical computation, the fully self-consistent RMF calculations of  $A \pm 1$  nucleons become possible. Then the time-odd fields generated by the unpaired valence nucleon could be treated properly. To include the time-odd components self-consistently in the RMF approach, the spherical symmetry must be broken at the mean-field level. After taking into account the time-odd nuclear magnetic potential in axially [58–60] or triaxially [61] deformed RMF models, the isoscalar magnetic moments of  $LS$  double-closed shell  $\pm 1$  nucleon systems can be reproduced well. However, there are still several problems to be solved in this framework. One of them is the restoration of rotational symmetry broken by the time-odd fields at mean-field level. A significant progress has been made in the implementation of angular momentum projection based on the RMF approaches [62–66] in the past decade. Due to the numerical complexity, these implementations are currently restricted to even-even nuclei. The extension of such kind of calculations for odd- $A$  nuclei requires further efforts.

Another problem is to remove the discrepancy existing in the isovector magnetic moments as there is no vertex corrections for the isovector part of the currents. To eliminate the remaining discrepancy, similar as the previous non-relativistic studies, the MEC correction was performed in the relativistic models [67, 68]. Unfortunately, although the MEC correction was found to be significant, the agreement with the data became worse.

In [22], using the single-particle wave function of Dirac spinor and the two-body residual interaction derived from a covariant energy density functional, a step further was made to incorporate the second-order core polarization correction to nuclear magnetic moments of nuclei with a  $LS$ -closed core  $\pm 1$  nucleon and with  $A = 15, 17, 39$  and  $41$ . The second-order core polarization was found to contribute significantly to nuclear magnetic moments. It is the cancelation between the second-order core polarization and the one-pion exchange current corrections that improves the relativistic description of isovector magnetic moments.

### 1.3 $g$ -Factor of nuclear low-lying excited states

The renormalization of the orbital  $g$ -factor  $g_\ell$  in nuclei is a fascinating subject in nuclear physics. It has an impact not only on nuclear magnetic moments, but also on electric and magnetic sum rules for nuclear collective excitations. The relation between  $g_\ell$  and the E1 sum rule in the region of the giant dipole resonance (GDR) has been investigated [69]. This relation, which is much more general than the original derivation in the Fermi gas model, is consistent with experimental

data. The relation between  $g_\ell$  and the recently determined M1 sum rule for the scissors mode in deformed nuclei, however, remains a puzzle which has to be examined in future works.

Nuclear shell model provides a firm framework for studying low-lying states in nuclei. However, the configuration space of shell models is too huge to be handled for medium-mass and heavy nuclei. In order to study the properties of low-lying states, one usually has to truncate the shell model space. Pair approximation is one of the ideas along this line. The nucleon pair approximation of the nuclear shell model, including its history and physical foundation as well as its validity and applications to the energy spectra were discussed in [70]. The electromagnetic moments of a few nuclei with mass number around  $A \sim 210$  region were calculated by implementing the recently developed technique of diagonalizing the shell model Hamiltonian in the nucleon pair basis.

Extension of mean-field approaches to describe the  $g$ -factor of nuclear excited states requires the restoration of rotational symmetry breaking in the mean-field approximation. Recently, the self-consistent beyond mean-field study of  $g$ -factor for nuclear low-lying excited states was carried out in  $^{24}\text{Mg}$  [71]. The nuclear wave functions were constructed by configuration mixing of relativistic mean-field states projected on good angular momentum. In this approach, there is no need to introduce effective charge or effective orbital and spin  $g$ -factor for neutron and proton since the full configuration was used. The available experimental  $g$  factor and spectroscopic quadrupole moment have been reproduced quite well. Furthermore, the calculated  $g$  factors have been found to be almost the same for the low-lying excited states with different angular momenta and close to the empirical value  $g_R = Z/A$  of rigid rotor. It indicates that the dominant configurations are quite similar for these low-spin yrast states in  $^{24}\text{Mg}$ .

### 1.4 Some related topics

In the special issue, some related topics, e.g. the phase transition of nuclear shape, masses of nuclei and the application of the nucleon pair approximation are discussed. Here some comments are given and the emphasis is on the results mainly from Chinese research groups.

Atomic nuclei display a variety of different equilibrium shapes – spherical, axially deformed, or soft with respect to triaxial deformations. The transitions in nuclear shapes, also referred as quantum phase transition (QPT) reveal the changes of dominant configurations or distinctly different shapes in nuclear states. In the last decade, QPTs in nuclei have attracted a lot of attention both in theory and experiment [72–76]. Meanwhile, several Chinese groups have published their results on this subject too [77–87].

Masses of atomic nuclei are of primary importance as they not only allow the determination of the existence limits of nuclei and provide the essential information about neutron-proton (np) pairing, but also serve as an important key to re-

veal the origin of proton-rich nuclei.

Although around ten global models have been developed to reproduce measured masses and to predict unknown masses far from the valley of stability, they are presently limited to an accuracy at the level of 400–600 keV [88]. Recently, the semi-empirical macroscopic-microscopic mass formula is further improved by considering some residual corrections and the rms deviation from 2149 known nuclear masses is significantly reduced to 336 keV [89, 90]. On the other hand, various local mass relations or formulae have been often demonstrated to have a better accuracy. For instance, in [91] the Coulomb displacement energy (CDE) was computed in the RMF model and the rms deviation with respect to all the available CDEs with  $Z \geq 8$  was improved by more than a factor of 5 in comparison with the corresponding rms value for absolute masses. Another highlight of local mass relations that has been extensively investigated in the last few years is the residual proton-neutron interactions [92–94]. With the help of local mass relations, the accuracy and predictive power of some global mass models can be significantly improved [95].

The increasing interest in nuclei far from the stability line demands special attention to the pairing correlations. The comparison between the calculated results with both microscopic and phenomenological nuclear pairing interactions was made in [96]. The parameters in the isospin- and density-dependent zero-range pairing interaction [97] were readjusted by fitting neutron gaps from a microscopic calculations [98]. For the pairing in nuclei, Chen et al. proposed the nucleon pair approximation model which is well applicable to even-even nuclei [99]. This model was refined and generalized to a unified approach which can be used to both even and odd nuclei [100]. In recent years, these models are extensively used in many respects of nuclear processes and considerable progresses were obtained [101–108].

The description of deformed dripline nuclei requires dedicated efforts in treating both deformation and continuum effects properly. Attempts along this line have been made in the past decades [109–115].

For the above topics, some review articles and comments can also be found in [116–118].

## 2 Summary and perspectives

In summary, the progress of theoretical studies on nuclear magnetic moments and the recent developments on the related subjects have been reviewed. The emphasis has been put on those topics covered in the issue entitled “Nuclear magnetic moments and related topics” (in *Sci China Phys Mech Astron*, Vol. 54, No. 2, 2011).

Theoretical description of nuclear magnetic moments is one of the long-standing subjects. The magnetic dipole moments of most atomic nuclei throughout the periodic table still remain unexplained and the underlying physics mecha-

nism is not fully understood. We are looking forward to more research contributions to this important subject in the future.

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