

Numerical simulation of Urumqi Glacier No. 1 in the eastern Tianshan, central Asia from 2005 to 2070

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Due to climate changes, most of the alpine glaciers have retreated dramatically during the past decades. Thus it is significant to predict the alpine glacier variability in the future for a better understanding of the impact of climate changes on water resource. In this paper, we perform the numerical simulation on Urumqi Glacier No.1 in the eastern Tianshan, central Asia (hereafter Glacier No.1 for short) by considering both the mass balance and ice flow. Given the shape of the Glacier No.1, the velocity of the glacier is obtained by solving a two-dimensional nonlinear Stokes equation and simulated result is in agreement with the observation. In order to predict the variability of Glacier No.1 in the next decades, a climatic scenario is constructed with a temperature rise rate as $0.17^{\circ}\text{C}/10\text{ a}$ and precipitation as constant during the period of 2005–2070. The simulation shows that, the glacier terminus will retreat slowly and the glacier will thin dramatically before 2040, while after year 2040, the glacier terminus retreat will accelerate. This study confirms the increasing retreat rate of alpine glaciers under global warming.

alpine glacier, numerical simulation, prediction

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As the solid water reservoir, the alpine glaciers are very sensitive to climatic fluctuations. In the past decades, most of the alpine glaciers around the world have suffered from degradation, which can be considered as a direct evidence of global warming. The glacial meltwater play an important role in hydrological cycle and sea level variation [1–5]. In the arid and semi-arid regions like the northwestern China, the glacial meltwater is also an important water resource. It is estimated that glacial meltwater accounts for 22% of river runoff in the northwestern China. Some branches of the Tarim River even receive more than 40% of water from glacier runoff [6]. Thus, the prediction of the alpine glacier variability is not only of significant scientific importance, but also valuable for evaluating the water resource sustainability, especially in arid regions.

The prediction of glacier variability depends on accurate understanding of physical procedures related to glacier var-

iation before the model establishment. The glacier variability include two parts, one is the snow or ice accumulation and ablation on the glacier surface, and the other is the ice redistribution due to the ice flow which is the key point to the glacier change simulation. When studying ice flow with years, ice can be considered as an incompressible non-Newtonian fluid that is governed by the mass and momentum conservation in the limit of a stationary nonlinear Stokes flow, in which the shear strain rate and shear stress can be described by the Green's law [7]. It was very difficult to simulate the glacier variability by solving the full Stokes equation due to insufficient understanding of glacier changes and limited calculation capability in the early years. Therefore, some studies simulated ice flow by solving simplified Stokes equation using finite difference method [8–11]. The improvement of numerical calculation capability has led to great progress in simulating ice flow, as the full Stokes equation can be solved using more advanced finite element method that enables the prediction of alpine

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glacier change [12,13].

The western China holds the largest number of alpine glaciers outside the Polar Regions. Recently, continued shrinkage of these glaciers has been widely attributed to atmospheric warming according to observations on the glacier terminus and mass balance. A few studies estimated the variability of glaciers in China in the 21st century using the statistical analysis [14,15]. Other studies discussed the response of alpine glaciers in China to climate change using the simplified flow models [16–19]. However, little is done in describing the numerical simulation of glacier variability based on its physical processes, thus preventing full understanding of the alpine glacier changes in China.

This paper aims to simulate the ice flow of Glacier No.1 by solving a two-dimensional Stokes equation. The coupled ice flow model with the mass balance model is then used to predict the variation of Glacier No.1 in a climate scenario for the next decades.

1 Method and data

Due to gravity, glacier flows downslope, with its velocity mainly controlled by the shape of the glacier. Snowfall exceeds snow melt (accumulation) in the upper reach of the glacier, while ice melt dominates in the lower reach of the glacier (ablation). This results in changes of the glacier shape, which, in return, leads to variation of ice velocity. As shown in Figure 1, glacier in the accumulation zone flows down to compensate the melting ice in the ablation zone. When the loss of ice in the ablation zone surpasses its gain from the accumulation zone, the glacier will retreat and vice versa. Thus, to acquire a holistic picture of the variability of a glacier, both the mass balance and ice flow velocity should be considered.

From the mass conservation perspective, the vertically integrated continuity equation, describing how the change in ice thickness H is related to flux divergence and specific mass balance b , can be written as

$$\frac{\partial H}{\partial t} = b - \nabla \cdot (\vec{V}H). \quad (1)$$

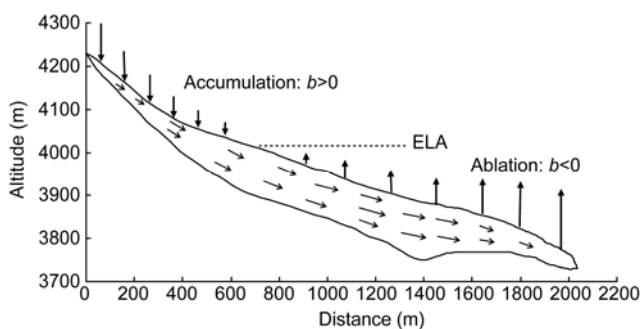


Figure 1 Sketch map of accumulation, ablation and ice flow in Glacier No.1. ELA means equilibrium line altitude.

The variable \vec{V} in eq. (1) is the velocity at the time t . This equation means the variation of the thickness of a glacier depends on both the mass balance and ice flow flux divergence.

1.1 Ice flow equation

Glacier ice is generally treated as an incompressible, heat-conducting non-Newtonian fluid. Then glacier flow can be described by the Stokes equation:

$$\rho \frac{d\vec{V}}{dt} = \nabla \cdot (2\eta\vec{D} - p\vec{I}) - \rho\vec{g}, \quad (2)$$

$$\nabla \cdot \vec{V} = 0, \quad (3)$$

where p is the pressure. \vec{I} is the identity stress tensor.

$\vec{D} = \frac{1}{2}(\nabla\vec{V} + \nabla\vec{V}^T)$ is the strain rate tensor. $\eta = \frac{1}{2}A^{-\frac{1}{n}}$

$(\sqrt{0.5 \times (\vec{D} \cdot \vec{D})})^{\frac{1-n}{n}}$ is the viscosity coefficient, whereby n is

the flow index and can be taken as 3, and A is the flow parameter related to glacial temperature. In this paper, A equals to $2.22 \times 10^{-24} \text{ Pa}^{-n} \text{ s}^{-1}$.

1.2 Mass balance equation

Mass balance is the algebraic sum of accumulation and ablation on the glacial surface. As shown in Figure 1, when accumulation is larger than ablation, mass balance is positive and *vice versa*. There are two types of mass balance models, one is the statistical degree-day model and the other is the energy-mass balance model which describes the energy balance process at the glacial surface. However, the parameters of energy-mass balance model are difficult to obtain on the alpine glaciers, thus confining the application of the model to most glaciers on high altitudes. In contrast, the parameters of the degree-day mass balance model are easy to get and bear definite physical meanings. That may be why the degree-day mass balance model has been so widely applied to the alpine glaciers in the mass balance simulation [20,21]. The general degree-day mass balance model is shown as:

$$b = \int_t [(1-f) \cdot m + P_s] dt, \quad (4)$$

$$m = \text{DDF} \times \text{PDD} = \text{DDF} \times \sum_{i=1}^n H_i \cdot T_i, \quad (5)$$

where f is the refrozen ratio taken to be 0.1. m is the water equivalent of melting ice, which is the product of degree-day factor (DDF) and positive accumulated temperature (PDD) during a certain period. T_i is the daily mean temperature. H_i is the logical variable. When $T_i \geq 0$, $H_i=1$

and when $T_1 < 0$, $H_i = 0$. P_s is the solid precipitation at a certain altitude of a glacier, and can be computed by [22]:

$$P_s = \begin{cases} P & T < T_s, \\ \frac{T_1 - T}{T_1 - T_s} P & T_s \leq T \leq T_1, \\ 0 & T > T_1, \end{cases} \quad (6)$$

where P is the precipitation total at a certain altitude, T_s and T_1 are the critical temperature of solid and liquid precipitation, and equal to 0°C and 2°C , respectively.

1.3 Data

When it comes to numerical modeling of a glacier, parameters such as degree-day factor, precipitation, temperature and glacial shape are essential for eqs. (1)–(6). However, most observations on glacier variability in the western China are focused on mass balance and the terminus, and are temporally inconsistent. Fortunately, there are few glaciers such as Glacier No.1 that have been systematically monitored since the 1960s. The observation results [18,19,23–27] can meet the demand for the numerical simulation. For example, the variation of degree-day factor with altitudes has been studied by Cui et al. [24] and the shape of the glacier has been surveyed by Li et al. [19].

Air temperature and precipitation amount along the slope of Glacier No.1 is derived from the data at Daxigou meteorological station, taken the temperature and precipitation gradient with altitudes as $-0.6^\circ\text{C}/100\text{ m}$ and $22\text{ mm}/100\text{ m}$ [28], respectively. The station (3539 m a.s.l.) is 2.5 km away from the glacier.

To predict the change of Glacier No.1, a climate scenario is assumed in reference to the warming rate during 1959 to 2004 recorded at Daxigou meteorological station, i.e., temperature increases at a linear trend of $0.17^\circ\text{C}/10\text{ a}$ during 2005–2070. The assumed temperature linear trend is consistent with the B1 scenario provided by IPCC [3]. There is no significant variation trend of precipitation amount at Daxigou from 1959 to 2004, the precipitation in the Glacier No.1 area is therefore assumed to be constant in the next decades.

2 Numerical simulation of ice flow

In order to simulate the ice flow of Glacier No.1, we need to solve eqs. (2) and (3). Observation shows that the annual mean ice flow velocity of Glacier No.1 is less than 8 m/a,

which means $\rho \frac{d\bar{V}}{dt}$ in eq. (2) can be ignored compared to

the other two items. Then, eq. (2) is simplified into stress equilibrium equation. The pressure at the glacial surface equals to that in the atmosphere. The bottom of the glacier

could be frozen, thus the bed rock can be taken as a fixed boundary, which means the velocity at the bottom of the glacier (V_b) is 0. On the other hand, the bottom layer of the glacier may slide along the bed rock in the function of $v(x,t)$. Therefore, the boundary conditions of eqs. (2) and (3) are expressed as

$$\text{At the surface: } (-p\bar{I} + 2\eta\bar{D}) \cdot \bar{n} = p_{\text{air}} \cdot \bar{n}, \quad (7)$$

$$\text{At the bottom: } V_b = v(x,t), \quad (8)$$

where p_{air} is the air pressure, \bar{n} is the outward unit vector.

It is very difficult and currently unable to solve the non-linear equations (2) and (3) in three dimensions due to the complication of numerical simulation and lack of the initial and boundary conditions. Therefore, in this paper we only try to solve eqs. (2) and (3) in two dimensions at the vertical profile along the main streamline of the glacier in Figure 2.

Figure 2(a) is the triangular mesh deformation of the vertical profile with refinement meshes in the lower reach of the glacier considering its complicated variation. There are 4919 meshes totally. Our study shows that in the lower reach of the glacier, movement at the glacier bottom, with the average velocity as around 2.5 m/a, accounts for about 50% of the entire glacier flow [18]. Glacier flow at the bottom is therefore essential in the consideration. Figure 2(b) shows the velocity field by solving eqs. (2) and (3) and (7) and (8). Figure 3 compares the simulated surface flow velocity in Figure 2(b) with the observation, demonstrating a good agreement between the two. We therefore conclude that it is feasible to simulate the glacier flow based on the Stokes equation.

3 Numerical simulation of future variation of Glacier No.1

To numerically simulate future variation of Glacier No.1, we integrate eqs. (1)–(8), and practice the following technological process in the simulation (Figure 4).

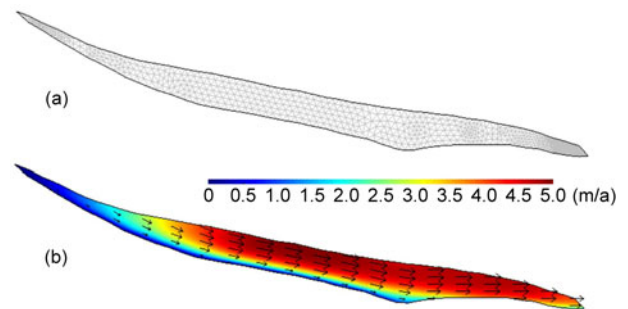


Figure 2 The triangular mesh deformation in the vertical profile of Glacier No.1 (a) and the simulated velocity field (b).

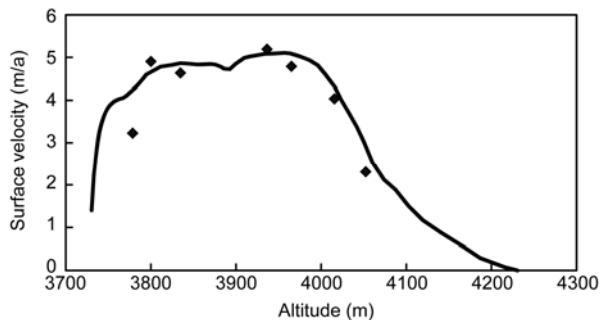


Figure 3 Comparison of the simulated (line) and observed (dot) surface velocity.

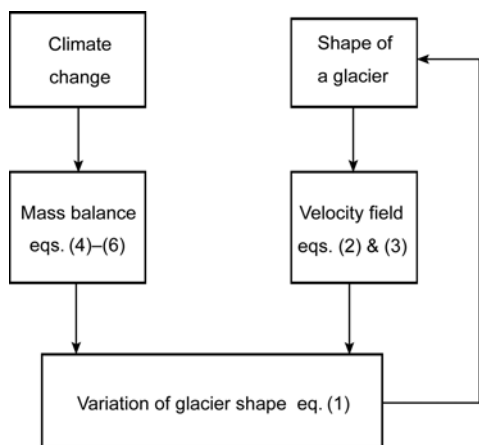


Figure 4 This simulation process to integrated eqs. (1)–(8) for the Glacier variability.

The variations of Glacier No.1 during 2005–2070 are simulated under the climate scenario constructed in section 1.3. The glacier variation with time is shown in Figure 5. Before 2040, the glacier would retreat rather gently but with a rapid thinning rate in the lower reach of the glacier, while after 2040 glacier retreat would accelerate with continuous thinning. The simulation indicates the glacier retreat as ~290 m during 2005–2040, with the rate increasing to ~950 m during 2040–2070. By the Year 2070, ice will have melted away below 4000 m a.s.l., leaving the remaining part

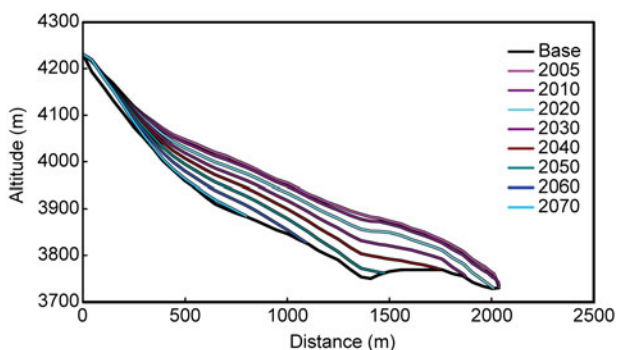


Figure 5 Variation of the Glacier No.1 with time under a reconstructed climatic scenario.

of the glacier above 4000 m a.s.l as a cirque glacier.

4 Discussion and conclusions

The variations of a glacier include two parts according to eq. (1), one is the mass balance on the glacial surface, and the other is the ice convergence and divergence due to ice flow. The mass balance can be measured directly by the stakes on the glacier surface. However, the convergence and divergence occurring in the glacier interior can only be obtained through calculation. Let $\Delta h = -\nabla \cdot (\vec{V}H)$ in eq. (1), then Δh is the elevation variation of the glacier surface with the convergence and divergence. When Δh is positive, it means that the elevation of the glacier surface increases due to the convergence and vice versa. Figure 6 shows the comparison between the calculated Δh and the measured mass balance b with altitudes in 2005, demonstrating identical values of Δh with b . This means that the loss of ice in the ablation zone could be compensated through ice flow by the mass gain in the accumulation zone. Specifically, Δh is negative and b is positive above 4000 m a.s.l., indicating positive mass balance increases the glacial surface elevation while mass divergence due to ice flow decreases the elevation. Below 4000 m a.s.l., on the other hand, Δh is positive and b is negative, indicating negative mass balance decreases the glacial surface elevation while mass convergence due to ice flow increases the elevation. The shape of a glacier is therefore constantly changing.

The flow velocity of Glacier No.1 decreased during 1981–2007 [26]. According to the numerical simulation, with the thinning and retreat of Glacier No.1, the flow velocity would have decreased during 2005–2070, resulting in less ice transportation from the accumulation zone to the ablation zone. As shown in Figure 5, due to the thick ice layer and fast flow in 2005, the strong downward transportation of ice compensate for the loss of ice in the ablation zone, leading to slow glacier retreat before Year 2040. Although the recession of terminus is slow, the supplement from upper reach of the glacier is smaller than the mass

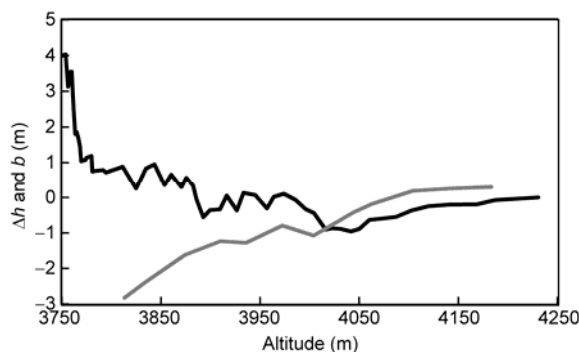


Figure 6 Comparison of the mass balance (gray line) and Δh (black line) with altitudes in 2005.

balance, resulting in rapid thinning of the glacier. After Year 2040, ice flow velocity decreases with thinning glacier and decreasing accumulation coverage, leading to further thinning of the glacier in the ablation zone and accelerated glacier terminus recession, as the mass gain in the accumulation zone fails to supplement timely the loss in the ablation zone. Thus if the global warming continues, the alpine glaciers' retreat will become a more and more serious issue.

In summary, to comprehensively study and predict the alpine glacier variability and its response to climate change, both mass balance and ice flow should be considered. The development of numerical computation skills and software enables the solution of the Stokes equation. Although viscosity coefficient is a function of temperature, it is set to be constant regardless of the variation of glacier temperature in this simulation. Glacier is a three-dimensional fluid. So the velocity field of the two-dimensional simulation cannot reflect the actual movement. Besides, the velocity field is determined by the movement of the glacial bottom to a large extent. However, it is still unclear how the sliding velocity at the glacial bottom changes with increasing glacier temperature. Further study and observation are therefore needed. Future work should be focused on the solution of the three-dimensional Stokes equation coupled with Heat Transfer equation and the Mass Balance model in order to better simulate the glacier change and its response to climate change.

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