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Universalities of earthquake-network characteristics

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The scale-free and small-world properties are studied in detail for the complex earthquake networks constructed from the seismic data sets taken from California (USA), Japan, Iran and Chile. It is found that, in all these geographical regions, both the exponent γ of the power-law connectivity distribution and the clustering coefficient C take the universal invariant values $\gamma \approx 1$ and $C \approx 0.85$, respectively, as the cell size, which is the scale of coarse graining needed for construction of network, becomes larger than a certain value. A possible physical interpretation is given to the emergence of such remarkable invariance.

earthquake networks, scale-freeness, small-worldness, universalities of characteristic quantities

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Dynamics governing seismicity is yet largely unknown, and accordingly the relevant research areas are necessarily empirical, owing their developments to case studies. This seems to be true even in the processes of establishing two celebrated classical laws known as the Omori law for temporal pattern of aftershocks and the Gutenberg-Richter law for the relation between frequency and magnitude.

In the absence of detailed knowledge about underlying dynamics, it is of central importance to clarify the pattern of correlations in order to extract information on features of the dynamics as a time series. Here, our primary interest is in the event-event correlations between successive earthquakes. As reported in the literature, both the spatial distance [1] and time interval [2,3] between two successive events strongly deviate from Poissonian statistics, supporting rationality of the concept of the event-event correlations. Accordingly, the following working hypothesis may be framed: two successive events are indivisibly correlated at least at the statistical level, no matter how distant they are. In this context, it is also noted that an earthquake can in fact trigger the next one which is more than 1000 km away [4]. Thus, the correlation length can indeed be divergently large, indicating a similarity to critical phenomena.

Although seismicity is generically assumed to be a complex phenomenon, it is actually a nontrivial issue to characterize its complexity in a unified way. The network approach offers a powerful tool to analyzing kinematical and dynamical structures of complex systems in a holistic manner (see [5] as a general reading). In a series of recent works [6–9], we have introduced and developed the concept of earthquake network in order to reveal the complexity of seismicity. In this approach, the event-event correlations are replaced by edges connecting vertices (for the details of constructing an earthquake network, see Section 1). It was found that the networks constructed in some geographical regions are complex ones, which are scale-free [10] and small-world [11]. Close studies have shown [12] that they are hierarchically organized [13] and possess the property of assortative mixing [14]. It has also turned out [15] that time evolution of network characterizes main shock in a peculiar manner. In addition, it has been found [16] that the scaling relation holds for the exponents of the power-law connec-

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tivity distribution and network spectral density.

Thus, the network approach may offer a novel possibility to deeper physical understandings of seismicity. Since the approach is inherently empirical in the sense that it is based on analyses of real data, it is essential to examine how universal the properties observed are.

In this paper, first we summarize the recent discovery [17] about the universal behaviors of the characteristics of earthquake networks found through analysis of the data sets taken from California, Japan and Iran, as well as Chilean one included here anew. The earthquake networks in these four geographical regions are found to be of the scale-free and small-world type. We carefully study the dependencies of the exponent γ of the power-law connectivity distribution and the clustering coefficient C on a single parameter that is the cell size of division needed for constructing a network (see Section 1). We show a remarkable fact that both γ and C come to take the universal invariant values:

$$\gamma \approx 1, \tag{1}$$

$$C \approx 0.85, \tag{2}$$

respectively, as the cell size becomes larger than a certain value. Then, we shall present a possible physical explanation about the emergence of such invariance.

1 Construction of earthquake network

The method of constructing an earthquake network proposed in [6] is as follows.

A geographical region under consideration is divided into cubic cells. A cell is regarded as a vertex of a network if earthquakes with any values of magnitude (above a certain detection threshold) occurred therein. Two successive events define an edge between two vertices, but if they occur in the same cell, then a tadpole (i.e. a self-loop) is attached to that vertex. These edges and tadpoles represent the event-event correlations (recall the working hypothesis mentioned earlier).

This simple procedure enables one to map a given seismic time series to a growing stochastic network, which is an earthquake network that we have been referring to.

Several comments on the construction are in order. Firstly, it contains a single parameter: the cell size L, which is the scale of coarse graining. That is, all earthquakes occurred in a given cell are identified and represented by the relevant vertex. Once a set of cells is fixed, then an earthquake network is unambiguously defined. Secondly, an earthquake network is a directed one in its nature. However, directedness does not bring any difficulties to statistical analysis of connectivity (or degree, i.e. the number of edges attached to the vertex under consideration) needed for examining the scale-free property, since by construction the in-degree and out-degree are identical for each vertex except the initial and final vertices in analysis, so they need not be distinguished. Therefore, vertices except the initial and final ones have the even-number values of connectivity. Thirdly, a full directed earthquake network should be reduced to a simple undirected network, when its small-world property is examined (see Figure 1). That is, tadpoles are removed and each multiple edge is replaced by a single edge.

To practically set up cells and identify a cell for each earthquake, here we employ the following procedure. Let θ_0 and $\theta_{\max}$ be the minimal and maximal values of latitude covered by a data set, respectively. Similarly, let ϕ_0 and $\phi_{\max}$ be the minimal and maximal values of longitude. Define θ_{av} as the sum of the values of latitude of all the events divided by the number of events contained in the analysis. The hypocenter of the ith event is denoted by (θ_i, ϕ_i, z_i), where θ_i, ϕ_i, and z_i are the values of latitude, longitude and depth, respectively. The north-south distance between (θ_0, ϕ_0) and (θ_i, ϕ_i) reads $d_i^{\mathrm{NS}} = R\cdot(\theta_i - \theta_0)$, where R ($\cong$ 6370 km) is the radius of the Earth. On the other hand, the east-west distance is given by $d_i^{\mathrm{EW}} = R\cdot(\phi_i - \phi_0)\cdot\cos\theta_{\mathrm{av}}$. In these expressions, all the angles should be described in the unit of radian. The depth is simply $d_i^{D}=z_i$. Now, starting from the point $(\theta_0,$

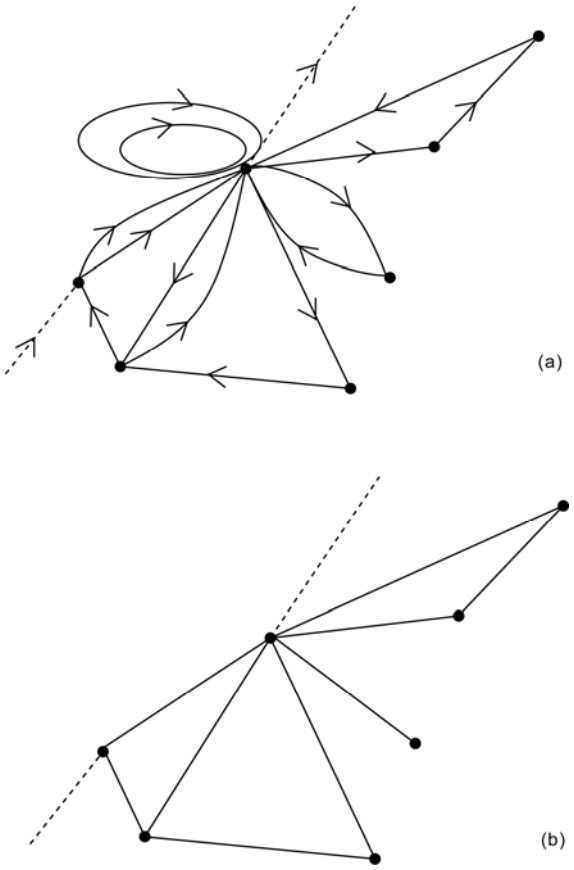

Figure 1 Schematic descriptions of earthquake network: (a) full directed network and (b) reduced undirected simple network. The vertices with the dotted lines indicate the initial and final events in analysis.

ϕ_0, $z_0 \equiv 0$), divide the region into cubic cells with a given value of the cell size L. Then, the cell of the ith event can be identified by making use of d_i^{NS}, d_i^{EW} and d_i^D.

Closing this section, we wish to emphasize the following point ascertained by our examinations. Although numerical values of characteristics of a network generically depend on how cells are set up, gross properties of a network do not change.

2 Scale-free and small-world properties of earthquake network

The four independent seismic data sets we analyze here are from (i) California, http://www.data.scec.org, (ii) Japan, http://www.hinet.bosai.go.jp, (iii) Iran, http://irsc.ut.ac.ir and (iv) Chile (by the time when the present work was completed, the Chilean data was not made open to public yet). The periods and the geographical regions covered are as follows: (i) between 00:25:8.58 on January 1, 1984 and 23:15:43.75 on December 31, 2006, 28.00°N–39.41°N latitude, 112.10°W–123.62°W longitude with the maximal depth 175.99 km, (ii) between 00:02:29.62 on June 3, 2002 and 23:54:36.21 on August 15, 2007, 17.96°N–49.31°N latitude, 120.12°E–156.05°E longitude with the maximal depth 681.00 km, (iii) between 03:08:11.10 on January 1, 2006 and 18:26:21.90 on December 31, 2008, 23.89°N–43.51°N latitude, 41.32°E–68.93°E longitude with the maximal depth 36.00 km and (iv) between 04:21:57.0 on October 2, 2000 and 18:31:57.3 on March 29, 2007, 29.01°S–35.50°S latitude, 69.51°W–73.95°W longitude with the maximal depth 293.30 km, respectively. The total numbers of events in these periods are (i) 404106, (ii) 681546, (iii) 22845 and (iv) 17004.

We have constructed the earthquake networks form these data sets following the procedure explained in Section 1 and analyzed their properties. In particular, we focus our attention to its scale-free and small-world properties.

First, let us look at scale-freeness of earthquake network. In Figure 2, we present plots of the connectivity distribution (or, degree distribution) $P(k)$, which gives the probability of finding a vertex with k edges, for those four geographical regions. As can be seen there, it decays as a power law:

$$P(k) \sim \frac{1}{k^{\gamma}}, \tag{3}$$

showing that the full earthquake networks are in fact scale-free.

The scale-free nature indicates that there are quite a few hubs with large values of connectivity, which make the network heterogeneous. We have taken a close look at the vertices of some main shocks in each data set and found that they correspond to hubs. An important point is that, aftershocks following each main shock tend to return to the locus of the main shock. This empirical fact explains why main shocks make their vertices hubs of a network.

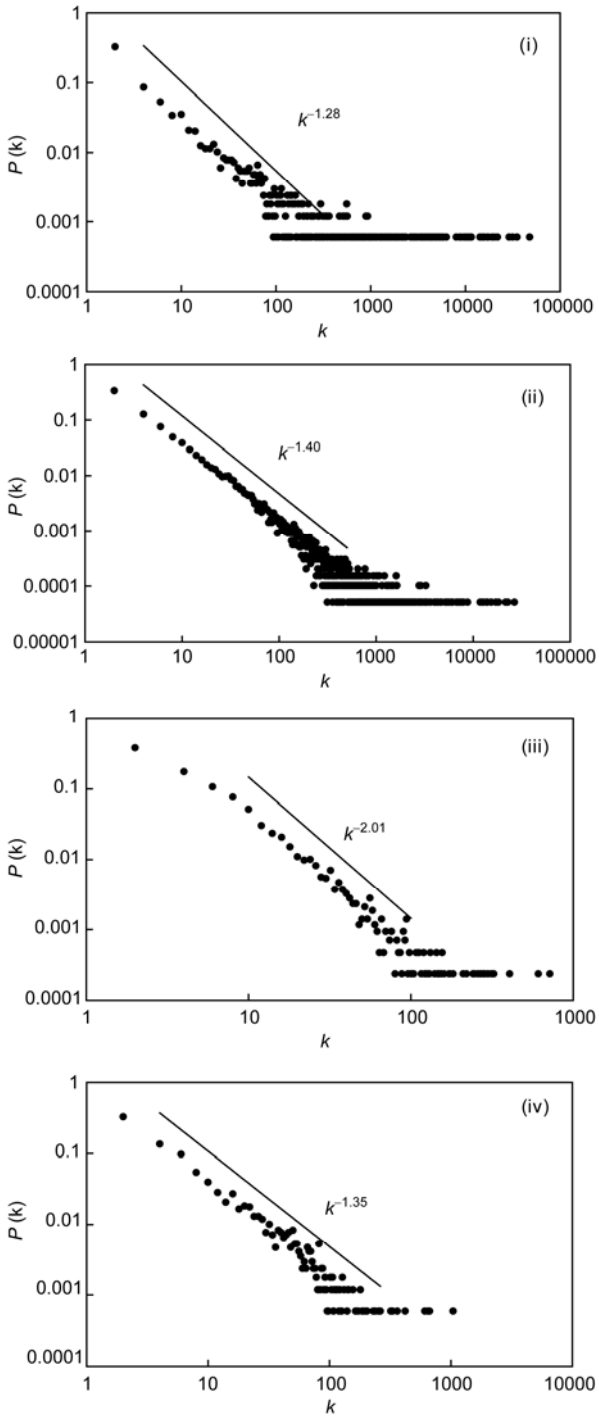

Figure 2 The log-log plots of the connectivity distributions of the earthquake networks in (i) California, (ii) Japan, (iii) Iran and (iv) Chile, for the cell size 20 km × 20 km × 20 km. The numbers of vertices are (i) 1674, (ii) 19662, (iii) 4274 and (iv) 1692, respectively. All quantities are dimensionless.

Second, we discuss small-worldness of the reduced simple earthquake. Here, the following two characteristic quantities are of primary relevance. One is the clustering coefficient and the other is the average path length [11], both of

which are explained below.

The clustering coefficient C of a simple network with N vertices is defined by

$$C = \frac{1}{N}\sum_{i=1}^{N} c_i. \tag{4}$$

c_i appearing on the right-hand side is given by $c_i \equiv$ (number of edges of the ith vertex and its neighbors)/$[k_i(k_i-1)/2]$ with k_i being the connectivity of the ith vertex. Equivalently, it is calculated also as follows. Let A be the symmetric adjacency matrix of the reduced simple network. Its element $(A)_{ij}$ is 1 (0), if the ith and jth vertices are linked (unlinked). Then, c_i is also written as follows:

$$c_i = \frac{(A^3)_{ii}}{k_i(k_i-1)/2}. \tag{5}$$

c_i tells about the tendency that two neighboring vertices of the ith vertex are linked together. By definition, C takes a value between 0 and 1. An important point is that C of a small-world network is much larger than that of the corresponding random network (i.e. classical Erdös-Rényi random graph) given by

$$C_{\text{random}} = \frac{< k >}{N} << 1, \tag{6}$$

where $<k>$ is the average value of connectivity of the random network. That is,

$$C >> C_{\text{random}}. \tag{7}$$

The other important characteristic quantity is the average path length $\bar{l}$, which quantifies that, for a pair of vertices, how many steps the shortest path linking them contains. A small-world network has a small value for it.

In Table 1, we present the results about these two characteristic quantities. It shows that the reduced simple earthquake networks in California, Japan, Iran and Chile are in fact of the small-world type.

Table 1 The values of the clustering coefficient C and average path length $\bar{l}$ of the networks in (i) California, (ii) Japan, (iii) Iran and (iv) Chile[a)]

	C	C_{random}	$\bar{l}$
(i)	0.7886	0.030	2.3640
(ii)	0.4002	0.0021	2.7640
(iii)	0.0641	0.0022	3.4438
(iv)	0.1629	0.0096	2.8783

a) The cell size employed is 20 km×20 km×20 km. The total numbers of vertices are as in Figure 2. For comparison, the values of corresponding random networks, C_{random}, are also presented.

3 Disappearance of cell-size dependence and universalities of network characteristics

Quantities characterizing a network are usually dimensionless, as the connectivity distribution, clustering coefficient and average path length are. On the other hand, in the case of earthquake network, their numerical values depend on the cell size L, in general. This issue is discussed in this section.

First of all, it is reasonable to make the cell size dimensionless. We rescale L using the dimension of a geographical region under consideration. Let L_{LAT}, L_{LON} and L_{DEP} be the dimensions of the whole region in the directions of latitude, longitude and depth, respectively. From these, we construct the following dimensionless quantity:

$$l_3 \equiv L\Big/\left(L_{\text{LAT}}L_{\text{LON}}L_{\text{DEP}}\right)^{1/3}. \tag{8}$$

The values of the denominator in those four geographical regions are respectively as follows: (i) 617.80 km, (ii) 1973.78 km, (iii) 583.45 km and (iv) 444.91 km.

Secondly, let us discuss the cell-size dependence of the exponent γ in eq. (3). In Figure 3, we present plots of γ of the scale-free earthquake networks with respect to l_3. To calculate γ from the data, we have employed the method of maximum-likelihood estimation for a power-law distribution. As can be seen in Figure 3, the result is remarkable: in each case, the dependence of γ on the cell size disappears if the size becomes larger than each certain value. In addition, in all the four geographical regions, γ takes the universal invariant value $\gamma \approx 1$.

The above result may be interpreted as follows. According to the method of network construction, there are two competitive factors. As the cell size increases, vertices get merged, yielding vertices with larger values of connectivity, while at the same time geographically neighboring vertices are absorbed each other, loosing the roles of hubs. The former decreases the value of γ, whereas the latter increases it. Disappearance of the cell-size dependence of γ may be due to the balance between these two competitive effects.

Finally, let us discuss the cell-size dependence of the clustering coefficient C. In Figure 4, we present its plots with respect to l_3 for the earthquake networks in those four geographical regions. Again, quite remarkably, C also takes the universal invariant value $C \approx 0.85$ as the cell size becomes larger than a certain value.

This result may be explained as follows. As the cell size increases, the number of vertices decreases, and the network approaches a complete graph (i.e. a fully linked network) having the maximum value of the clustering coefficient (C=1), but at the same time cells swallow triangular loops (recall the A^3-nature of C in eq. (5)) attached to them as they become larger. The former effect increases the value of C, whereas the latter decreases it. Disappearance of the cell-size dependence of C may be due to the balance between these two competitive mechanisms.

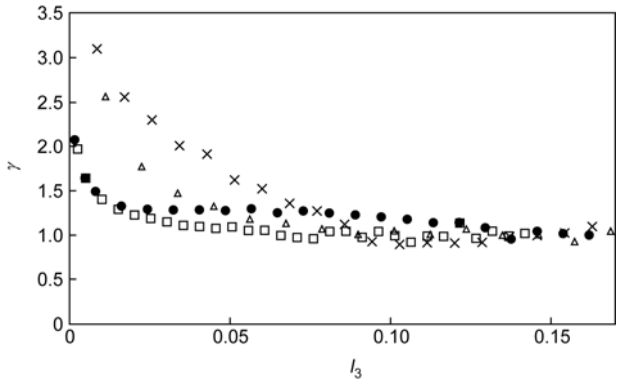

Figure 3 Dependence of the exponent γ on the dimensionless cell size l_3: California (●), Japan (□), Iran (×) and Chile (△). All quantities are dimensionless.

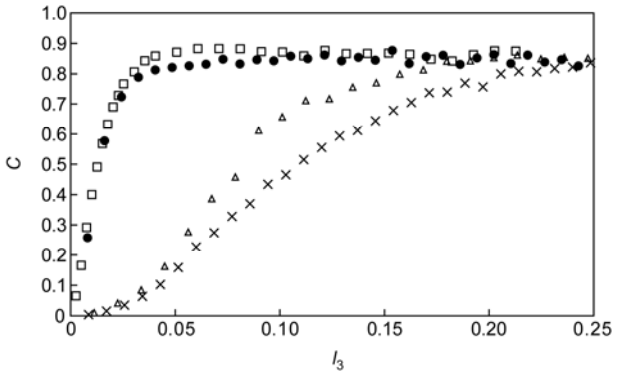

Figure 4 Dependence of the clustering coefficient C on the dimensionless cell size l_3: California (●), Japan (□), Iran (×) and Chile (△). All quantities are dimensionless.

4 Concluding remarks

We have studied the scale-free and small-world properties of the earthquake networks in California, Japan, Iran and Chile. We have observed that these networks are of the scale-free and small-world type and their cell-size dependence qualitatively coincides with each other. We are firmly convinced that the discovered invariant values in eqs. (1) and (2) are universal and intrinsic in the seismicity of the Earth.

An additional remark is about incompleteness of a seismic data set. One might wonder if the incompleteness leads to change of the present results. Regarding this point, one should recall an important feature of a complex network: that is, it has a high degree of tolerance against "random attacks", i.e. random removals of vertices [18]. In fact, earthquake networks do not have a centrality with a small value of connectivity. Since the incompleteness of a data set is not biased (i.e. not due to "intelligent attacks"), we can confidently assume the robustness of the results presented here.

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1 Abe S, Suzuki N. Law for the distance between successive earthquakes. J Geophys Res-Solid Earth, 2003, 108: ESE 19-1
2 Corral Á. Long-term clustering, scaling, and universality in the temporal occurrence of earthquakes. Phys Rev Lett, 2004, 92: 108501
3 Abe S, Suzuki N. Scale-free statistics of time interval between successive earthquakes. Physica A, 2005, 350: 588–596
4 Steeples D W, Steeples D D. Far-field aftershocks of the 1906 earthquake. Bull Seismol Soc Am, 1996, 86: 921–924
5 Buchanan M. Nexus: Small worlds and groundbreaking science of networks: New York: W W Norton & Company, Inc., 2002
6 Abe S, Suzuki N. Scale-free network of earthquakes. Europhys Lett, 2004, 65: 581–586
7 Abe S, Suzuki N. Small-world structure of earthquake network. Physica A, 2004, 337: 357–362
8 Abe S, Suzuki N. Scale-invariant statistics of period in directed earthquake network. Eur Phys J B, 2005, 44: 115–117
9 Abe S, Suzuki N. Complex-network description of seismicity. Nonlinear Proc Geophys, 2006, 13: 145–150
10 Barabási A L, Albert R. Emergence of scaling in random networks. Science, 1999, 286: 509–512
11 Watts D J, Strogatz S H. Collective dynamics of 'small-world' networks. Nature, 1998, 393: 440–442
12 Abe S, Suzuki N. Complex earthquake networks: Hierarchical organization and assortative mixing. Phys Rev E, 2006, 74: 026113
13 Ravasz E, Barabási A L. Hierarchical organization in complex networks. Phys Rev E, 2003, 67: 026112
14 Newman M E J. Assortative mixing in networks. Phys Rev Lett, 2002, 89: 208701
15 Abe S, Suzuki N. Dynamical evolution of clustering in complex network of earthquakes. Eur Phys J B, 2007, 59: 93–97
16 Abe S, Suzuki N. Scaling relation for earthquake networks. Physica A, 2009, 388: 2511–2514
17 Abe S, Suzuki N. Determination of the scale of coarse graining in earthquake networks. Europhys Lett, 2009, 87: 48008
18 Albert R, Jeong H, Barabási A L. Error and attack tolerance of complex networks. Nature, 2000, 406: 378–382