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A mean-field Bak-Sneppen model with varying interaction strength

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We propose a mean-field Bak-Sneppen (MFBS) model with varying interaction strength. The interaction strength, here denoted by α, specifies the degree of interaction, and varies smoothly between 0 for no interaction and 1 for full interaction (restoring the original BS model). Our simulations of the MFBS model reveal some interesting features. When α is non-zero, the MFBS model can evolve to a self-organized critical (SOC) state. The critical exponent of the avalanche size distribution, τ, is insensitive to changes in α. The critical exponent of average avalanche size, γ, and the avalanche dimension exponent, D, both increase slightly with $\alpha < 0.5$ but remain constant if $\alpha > 0.5$. The critical threshold f_c decreases almost linearly with α.

Bak-Sneppen model, self-organized criticality, critical exponents

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Self-organized criticality (SOC) proposed by Bak et al. [1] can be used to explain the origin of power laws displayed by many different systems, such as forest fires [2–4] and power systems [5,6].

The Bak-Sneppen (BS) evolution model [7] is a prototype displaying SOC. The BS model mimics the biological evolution in a very simple but most characterized way: L^d species are located on a d-dimensional lattice of linear size L. Initially, L^d random numbers $P(f)$, chosen from a uniform distribution between 0 and 1, are assigned independently to each species that reflects the fitness of that species. At each time step, the global extremal site, i.e. the species with the smallest fitness within the system, and its $2d$ nearest neighbors are assigned new random numbers also chosen from the same $P(f)$. The updates continue until a stationary state is reached where the density of the fitness is uniform above f_c and vanishes under this critical threshold.

The BS model has received much attention since its introduction [8,9]. It features behaviors such as avalanches created when there is a cascade of fitness changes below the threshold. Li and Cai [10] introduced the so-called LC-avalanche based on the average fitness as variable. Some exact results have been found for the LC-avalanche [11,12]. Following Li and Cai, another avalanche has been defined [13]. The application of LC-avalanche has also been extended to the analysis of stock markets [14,15].

We focus on the failing of the original BS model in not explicitly treating the strength of the interactions, namely, when the extremal site and its nearest neighbors are updated at the same time. This is not true in real situations; that is, it is necessary to distinguish the strength of interactions. We have already considered such situations in [16] by relating the interaction strength to the respective fitness of each neighbor. Here, we introduce an even simpler modified BS model by introducing a form of interaction strength; we call this the mean-field Bak-Sneppen (MFBS) model with interaction strength. We will see that this model may facilitate our simulations. Our ultimate aim is that this modified version enables theoretical aspects of the model to be studied in more detail.

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1 A mean-field Bak-Sneppen model

Our mean-field Bak-Sneppen model is defined as follows:

(1) L^d species are located on a d-dimensional lattice of linear size L. Initially, L^d random numbers, chosen from a uniform distribution between 0 and 1, $P(f)$, are assigned independently to each species as fitness.

(2) At each iteration, the smallest fitness within the system will be updated by a random number from $P(f)$. The fitness of its $2d$ nearest neighbors will be updated with random numbers also taken from $P(f)$, but with probability α fixed in the range $0<\alpha<1$.

(3) Repeat (2).

As stated above, we interpret α as interaction strength. Since α is fixed during iterations and is independent of fitness, the model has characteristics of a mean-field approach. If α is set to zero, there is no interaction and all fitness values will eventually become unity where no SOC can be observed. If α is set to unity, the original BS model is restored. In between this range, we have an infinite number of MFBS models, the features of which we study extensively here.

2 Simulation results

We shall first generate data from the MFBS model under various conditions. Our basic set-up is: d=1, L=10000, α is incremented between 0 and 1 in steps of 0.1. For each α, we ran 100 simulations with different initial conditions; most results obtained (mainly critical exponents and critical threshold f_c) are averaged over 100 different realizations.

2.1 Avalanche size distribution

Our simulations reveal that, as long as α is greater than 0, the MFBS model can display SOC characteristics, namely, a power-law distribution for the avalanche sizes. Normally, we need to observe the avalanches based on f_c. However, one knows that the critical point cannot be precisely identified. An alternative way is to choose an auxiliary parameter f_0 close to f_c (we will give details below how to obtain the critical value numerically) and observe the so-called f_0-avalanches near the critical point. For a certain value of f_0, an f_0-avalanche of size S is defined as a sequence of S successive events for which the smallest fitness $f_{min}<f_0$ is confined between two events when $f_{min}>f_0$. This definition ensures that the so-called mutation events during an avalanche are spatially and temporally correlated. This also guarantees the hierarchical structure of avalanches: larger avalanches consist of smaller ones.

Therefore, the avalanche size distribution can be described by

$$P(S) \sim S^{-\tau}, \tag{1}$$

where τ is the critical exponent for the f_0-avalanche as f_0 approaches f_c.

In Figure 1, we display a log-log plot of the avalanche size distribution of MFBS setting α=0.5. To reduce the statistical errors we smooth the data points using bins of exponential width. Here τ is measured to be 0.890±0.002 in this case. We also measured τ for different α's; these are plotted in Figure 2 which shows that τ is insensitive to changes in α. This means that the nature of the model is nearly independent of the interaction details. The value of τ for α=1 is consistent with that measured in [7].

2.2 Avalanche dimension

It can be inferred that, an avalanche is a compact cluster which relates spatial and temporal dimensions. Analogous to fractals, one can define the number of sites covered by an avalanche:

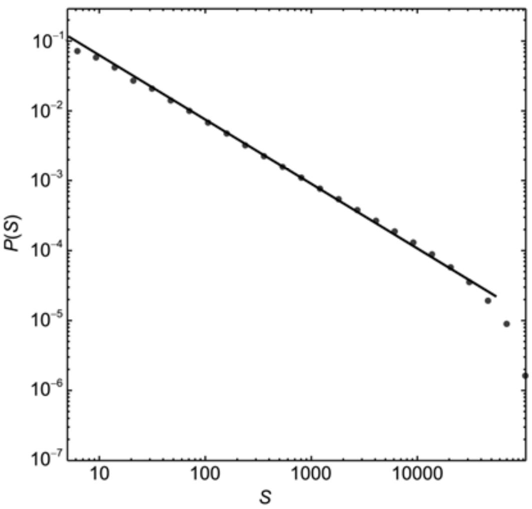

Figure 1 Avalanche size distribution for MFBS with α = 0.5.

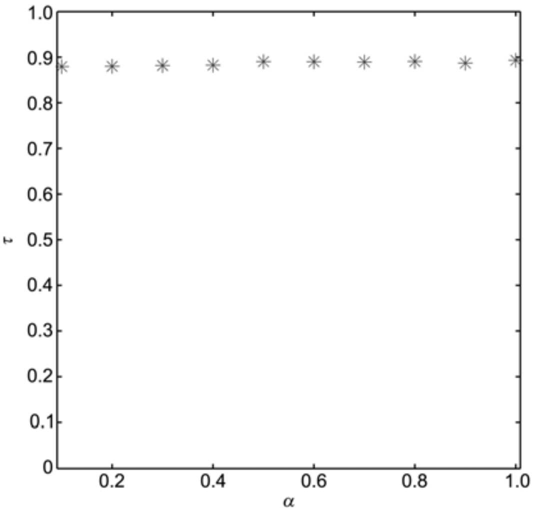

Figure 2 The dependence of critical exponent τ on interaction strength α.

$$n_{\rm cov} \sim S^{D/d}, \tag{2}$$

where d is the dimension, S the avalanche size, and D is called the avalanche dimension.

Figure 3 shows in a log-log plot the variation of $n_{\rm cov}$ with S for α=0.7, from which we obtain D=2.349±0.002. We also measured the values of D for different α, which is graphed in Figure 4. Clearly D increases slightly with α if the latter is less than 0.5; if α is greater than 0.5, D is nearly constant with α. Our measurement of D at α=1 is consistent with that from [8].

2.3 Average avalanche size and critical fitness

According to the scaling requirement of critical phenomena, the average size of f_0-avalanches should diverge as the critical point is approached. Therefore, for avalanche size, we assume the following scaling ansatz:

$$< S >_{f_0} \sim \left(f_{\rm c} - f_0\right)^{-\gamma}, \tag{3}$$

where γ is the critical exponent for the average size, and $f_{\rm c}$ is the critical threshold. According to [3], an exact equation can be derived which relates γ and $f_{\rm c}$:

$$\gamma =< n_{\rm cov} >_{f_0} \left(f_{\rm c} - f_0\right) / \left(1 - f_0\right), \tag{4}$$

where f_0 is chosen close to $f_{\rm c}$.

Based on eq. (4), we have calculated γ and $f_{\rm c}$ for different α. Figure 5 displays the plot f_0 vs.$(1-f_0)/<n_{\rm cov}>$ for α=0.7 from which we have γ=2.720±0.001 and $f_{\rm c}$=0.828±0.001.

Figures 6 and 7 show the dependence of γ and $f_{\rm c}$ on α, respectively. As can be seen, γ rises slightly with a decreasing rate before peaking at about α=0.5 and remaining constant thereafter, whereas $f_{\rm c}$ falls nearly linearly with α. γ and $f_{\rm c}$ for α=1 are in agreement with previous results [3].

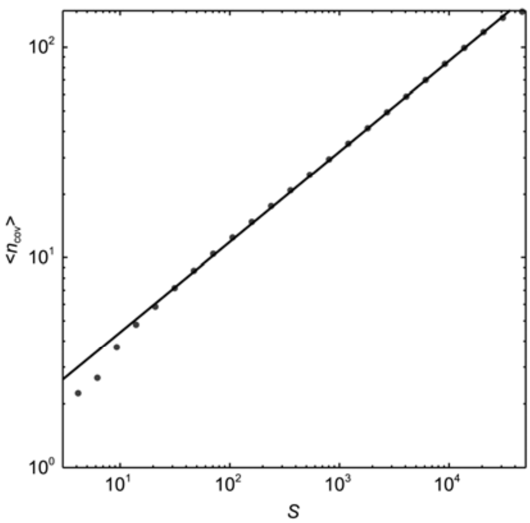

Figure 3 Avalanche dimension for MFBS with α=0.7.

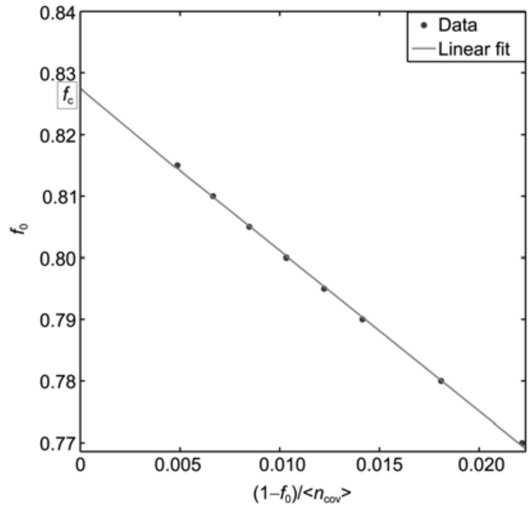

Figure 5 Measurement of critical exponent γ and critical threshold $f_{\rm c}$ for MFBS with α=0.4.

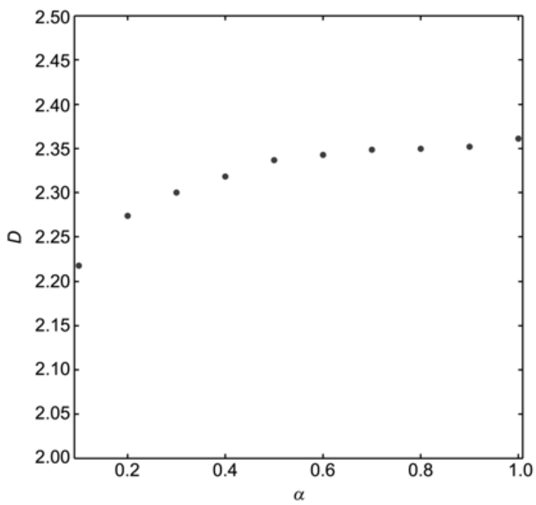

Figure 4 Dependence of avalanche dimension on α.

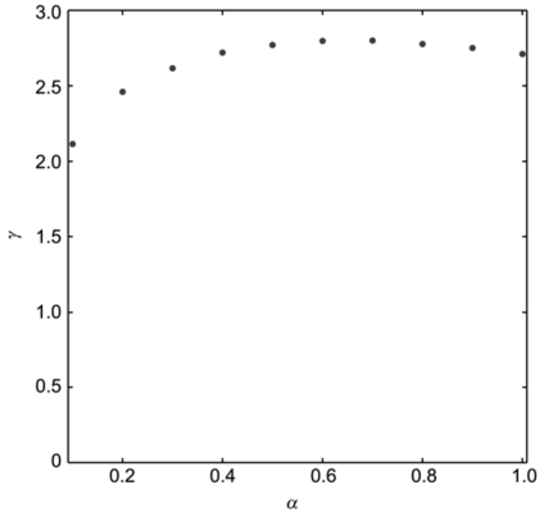

Figure 6 Dependence of γ on α.

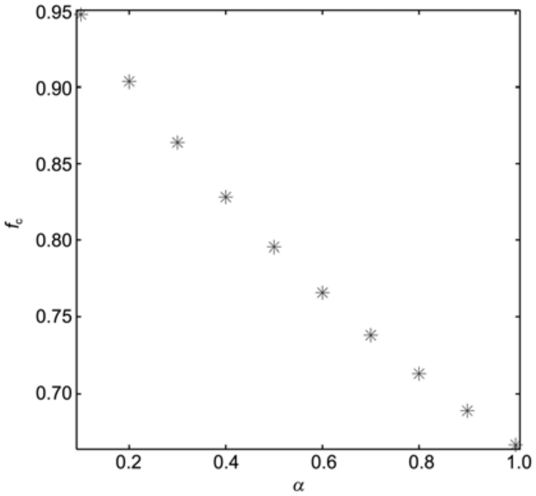

Figure 7 Dependence of critical threshold f_c on α.

3 Discussion and conclusions

We presented a MFBS model by introducing a parameter α to describe the interaction strength among nearest neighbors. We find that as long as the interaction strength is non-zero, our model can self-organize into a critical state where the size distribution of avalanches follows a power law. The critical exponent τ for the avalanche size distribution is insensitive to the variation of α. The critical exponent γ for average avalanche size, and the avalanche dimension D increases slowly when α is smaller than 0.5 but saturates thereafter. f_c decreased nearly linearly over the entire range of α.

It would be worth checking, both theoretically and numerically, if α=0.5 is indeed a turning point for MFBS. We would also need to understand the relationship between f_c and α. For instance, can an exact equation be derived similar to that obtained for the original BS model?

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