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Neutrino mass hierarchy and lepton flavor mixing

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In the standard model of particle physics there are three species of neutrinos whose masses were originally assumed to be zero. But the discovery of solar and atmospheric neutrino oscillations indicates that neutrinos are massive and lepton flavors are mixed. In this brief review we first give an overview of our current knowledge about the neutrino mass spectrum and lepton flavor mixing angles, and then comment on the seesaw mechanisms which allow us to understand the origin of tiny neutrino masses. We pay particular attention to the nearly tri-bi-maximal neutrino mixing pattern and the Friedberg-Lee symmetry to derive it. A relatively promising possibility of detecting hot and warm neutrino dark matter in the Universe will also be discussed.

neutrino mass, lepton flavor mixing, cosmic neutrino background, dark matter

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The standard model (SM) of particle physics consists of twelve matter particles: six quarks, three charged leptons and three neutrinos. The masses of three neutrinos were originally assumed to be zero in the SM, and thus there should be no lepton flavor mixing. This point of view turns out to be wrong, because the peculiar quantum phenomena of neutrino oscillations have convincingly been observed in a number of solar, atmospheric, reactor and accelerator neutrino experiments since 1998 [1]. There are two prerequisites for neutrino oscillations to happen: (1) neutrinos must have non-degenerate masses; (2) lepton flavor mixing angles must be non-vanishing. Both of them imply that one has to go beyond the SM. In this sense, the discovery of neutrino oscillations is the first great breakthrough of the SM since it was established in 1968.

In this brief review we shall first describe some salient features of the neutrino mass spectrum and lepton flavor mixing pattern extracted from current neutrino oscillation data, and then try to understand why neutrino masses are considerably smaller than quark masses but lepton flavor mixing angles are much larger than quark flavor mixing angles. A few typical seesaw mechanisms will be briefly introduced. Our main concern is the nearly tri-bi-maximal neutrino mixing pattern [2] and possible flavor symmetries behind it. We shall highlight such a physical picture: two relatively large neutrino mixing angles may result from a kind of flavor symmetry (e.g. the A_4 or Friedberg-Lee symmetry), while the smallest neutrino mixing angle θ_{13} and the Dirac-type CP-violating phase δ should arise from a mechanism of flavor symmetry breaking either at the tree level or via quantum (loop) corrections. Finally, the role of neutrinos in cosmology will be stressed and a relatively promising possibility of detecting hot and warm neutrino dark matter in the Universe will be discussed.

1 Neutrino masses and flavor mixing angles

In the standard three-flavor scheme there are totally twelve free parameters in the lepton sector: three charged lepton masses, three neutrino masses, three lepton flavor mixing angles and three CP-violating phases. Without loss of any generality, we assume neutrinos to be Majorana particles. The masses of three charged leptons (e, μ, τ) have been determined to an excellent degree of accuracy [1], but the absolute mass scale of three neutrinos remains unknown. Cur-

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rent cosmological data yield $m_1+m_2+m_3 \leqslant 1$ eV [1]; and current neutrino oscillation data allow us to determine two neutrino mass-squared differences [3]:

$$\begin{aligned} \Delta m_{21}^2 &\approx +7.6\times10^{-5}\ \text{eV}^2, \\ \Delta m_{31}^2 &\approx \pm2.4\times10^{-3}\ \text{eV}^2. \end{aligned} \tag{1}$$

Because the sign of Δm_{31}^2 is still unknown, we are left with two possible neutrino mass spectra: one is the normal mass hierarchy $m_1<m_2<m_3$ and the other is the inverted mass hierarchy $m_3<m_1<m_2$. One of the primary goals of the future neutrino oscillation experiments is just to determine the true neutrino mass spectrum. In the assumption of a normal mass hierarchy, Figure 1 illustrates the mass spectrum of leptons and quarks, where all the mass values have been renormalized to the electroweak scale [4]. One can see a big gap between neutrino masses and charged fermion masses. This "flavor desert" spans six orders of magnitude between O(0.5) eV to O(0.5) MeV, and it might be able to accommodate keV sterile neutrinos as a good candidate for warm dark matter [5].

To understand why neutrino masses are so small as compared with the masses of other fundamental fermions, one has explored various theoretical or phenomenological models [6]. Among them, the most popular mechanisms are the so-called seesaw mechanisms, which attribute the smallness of the masses of three known neutrinos to the existence of some unknown heavy degrees of freedom and lepton number violation above the electroweak scale [7]. There are three typical seesaw mechanisms on the market: (a) type-I seesaw — heavy right-handed neutrinos are added into the SM and the lepton number is violated by their Majorana mass term [8,9]; (b) type-II seesaw — one heavy Higgs triplet is added into the SM and the lepton number is violated by its interactions with both the lepton doublet and the Higgs doublet [10–13]; (c) type-III seesaw — heavy triplet fermions are added into the SM and the lepton number is violated by their Majorana mass term [14]. In each case the light neutrino masses are inversely proportional to the heavy particle masses which are assumed to be much larger than the Fermi scale. That is why three known neutrinos are really light. A number of variations of such seesaw pictures, in particular those TeV seesaw mechanisms which can be tested at the Large Hadron Collider [7], have also been discussed in the literature.

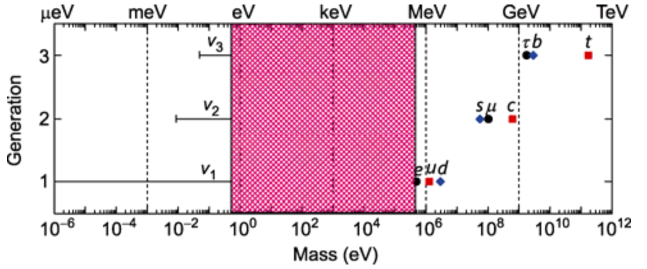

Figure 1 The mass spectrum of six leptons and six quarks at the electroweak scale [4] in the assumption of a normal neutrino mass hierarchy [5].

The phenomenon of lepton flavor mixing reflects the fact that there is a mismatch between the lepton mass and flavor eigenstates. In the basis where the mass eigenstates of charged leptons are identified with their flavor eigenstates, the lepton flavor mixing matrix V links the flavor eigenstates of three neutrinos to their mass eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu1} & V_{\mu2} & V_{\mu3} \\ V_{\tau1} & V_{\tau2} & V_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{2}$$

In the neglect of two Majorana-type CP-violating phases which are irrelevant to neutrino oscillations, one usually takes $V_{e2}=\cos\theta_{13}\sin\theta_{12}$ and $V_{e3}=\sin\theta_{13}e^{-i\delta}$ together with $V_{\mu3}=\cos\theta_{13}\sin\theta_{23}$ to parametrize this neutrino mixing matrix. Then the flavor mixing angles θ_{12} and θ_{23} are respectively associated with solar and atmospheric neutrino oscillations, and they have been determined to a good degree of accuracy from a global analysis of current neutrino oscillation data [3]: $\theta_{12}\approx34°$ and $\theta_{23}\approx45°$. An upper bound on the smallest neutrino mixing angle is found to be $\theta_{13}\leqslant10°$. The ongoing Daya Bay reactor antineutrino oscillation experiment in China aims to probe θ_{13} up to the level of about 3° [15]. Note that the size of θ_{13} remains unknown, although current experimental data give some preliminary hints that it might be around 6° (at the 1σ to 2σ confidence level [16–18]). Because this angle and the unknown Dirac-type CP-violating phase δ control the effect of CP violation in neutrino oscillations, it is extremely important to measure it in those running and upcoming reactor and long-baseline neutrino oscillation experiments. Besides the Daya Bay experiment, the Double Chooz and RENO reactor antineutrino oscillation experiments and the T2K and NOνA accelerator neutrino oscillation experiments are also underway to probe θ_{13} (for a recent review, see [19]).

In the same parametrization of the quark flavor mixing matrix, one may see an obvious difference between lepton and quark flavor mixing patterns. Three quark mixing angles are $\theta_{12}\approx13°$, $\theta_{23}\approx2°$ and $\theta_{13}\approx0.2°$ [1], which are much smaller than the corresponding lepton mixing angles. One naively expects that the smallness of quark flavor mixing is intrinsically related to the fact that up- and down-type quarks both have a very strong mass hierarchy [6] (see also Figure 1): $m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$. Because a mixing angle generally depends on the mass ratios of each quark sector, it will not be a surprise if its magnitude is naturally small. In this sense, the largeness of two lepton flavor mixing angles implies that three neutrino masses should not have a strong hierarchy, although three charged lepton masses have a very strong hierarchy as shown in Figure 1 (namely, $m_e \ll m_\mu \ll m_\tau$). That is why some attention has been paid to the nearly degenerate neutrino mass spectrum in model building, in order to obtain sufficiently large lepton flavor mixing angles (see, for example [20–22]).

2 Nearly tri-bi-maximal neutrino mixing

In 1996 Fritzsch and the present author first pointed out that the lepton flavor mixing matrix might consist of two terms: the leading term is a constant matrix containing only two large mixing angles relevant to the solar and atmospheric neutrino oscillations, and the sub-leading term represents a small perturbation to the leading term and thus gives rise to the smallest mixing angle and CP-violating effects [20]. We obtained the constant flavor mixing matrix based on the $S(3)_L \times S(3)_R$ flavor symmetry for charged leptons and the $S(3)$ flavor symmetry for Majorana neutrinos [20–22]:

$$U_0 = \begin{pmatrix} \sqrt{1/2} & \sqrt{1/2} & 0 \\ -\sqrt{1/6} & \sqrt{1/6} & \sqrt{2/3} \\ \sqrt{1/3} & -\sqrt{1/3} & \sqrt{1/3} \end{pmatrix}, \tag{3}$$

the so-called democratic mixing pattern. In 2002 a twisted form of U_0 was proposed by Harrison, Perkins and Scott in order to get a better fit of neutrino oscillation data [23]:

$$V_0 = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ \sqrt{1/6} & -\sqrt{1/3} & \sqrt{1/2} \end{pmatrix}, \tag{4}$$

the so-called tri-bi-maximal mixing pattern. This scenario predicts $\theta_{12} \approx 35.3°$, $\theta_{23}=45°$ and $\theta_{13}=0°$ in the standard parametrization, and thus it is essentially consistent with current experimental data on neutrino oscillations.

A new and more reasonable picture of lepton flavor mixing turns out to be [2]

$$V = V_0 + \Delta V, \tag{5}$$

where the leading term V_0 may be a natural consequence of a certain flavor symmetry and the perturbation ΔV results from the flavor symmetry breaking and (or) finite quantum corrections. In particular, ΔV is responsible for the generation of θ_{13} and δ together with slight corrections to the original values of θ_{12} and θ_{23} given by V_0. The resultant lepton flavor mixing matrix V is therefore called the nearly tri-bi-maximal mixing pattern. It is currently the most popular pattern of lepton flavor mixing.

It is possible to derive V_0 based on a number of flavor symmetries [24,25]. Among them, the most interesting one should be the A_4 flavor symmetry [26–28] which has attracted a lot of attention. The so-called Friedberg-Lee symmetry [29] is also an interesting organizing principle for the texture of the Dirac neutrino mass matrix which can lead to the tri-bi-maximal neutrino mixing pattern if the μ-τ symmetry is simultaneously imposed. To see this point, let us consider the following neutrino mass term in the flavor basis where the charged lepton mass matrix is diagonal:

$$\begin{aligned} L_\nu = {} & a\left(\overline{\nu_\tau} - \overline{\nu_\mu}\right)\left(\nu_\tau - \nu_\mu\right) + b\left(\overline{\nu_\mu} - \overline{\nu_e}\right)\left(\nu_\mu - \nu_e\right) \\ & + c\left(\overline{\nu_e} - \overline{\nu_\tau}\right)\left(\nu_e - \nu_\tau\right) + m_0\left(\overline{\nu_e}\,\nu_e + \overline{\nu_\mu}\,\nu_\mu + \overline{\nu_\tau}\,\nu_\tau\right). \end{aligned} \tag{6}$$

with a, b, c and m_0 being real [29,30]. Switching off the term proportional to m_0, one finds that the above effective neutrino mass term is invariant under the transformation $\nu_\alpha \rightarrow \nu_\alpha + z$ (for $\alpha = e, \mu, \tau$), where z is a space-time independent constant element of the Grassmann algebra. This invariance is referred to as the Friedberg-Lee symmetry, and it is broken by $m_0 \neq 0$. The neutrino mass matrix identified by eq. (6) can be diagonalized by a unitary transformation, which is just the neutrino mixing matrix [29,30]:

$$V = V_0 \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}, \tag{7}$$

where $\tan 2\theta = \sqrt{3}(c-b)/(b+c-2a)$. It becomes obvious that $V=V_0$ holds if the effective neutrino mass term in eq. (6) has the μ-τ permutation symmetry (i.e. $b=c$). Some applications of the Friedberg-Lee symmetry and its breaking to the case of Majorana neutrinos, either with a seesaw mechanism or without it, have been discussed by a number of authors in the literature [31–34].

How to introduce ΔV remains an open question, but there are a few general ways. The simplest way is to explicitly break the flavor symmetry at the tree level, such that ΔV may arise either from the charged lepton sector or from the neutrino sector, or from both sectors. The first example of this kind was given in [2], where non-vanishing θ_{13} and CP violation can both result from a simple pattern of ΔV. At the loop level ΔV may stand for a summary of finite quantum corrections to V_0 [35]. If a flavor symmetry model is built at a super-high energy scale above the electroweak scale, the quantum effect due to renormalization-group evolution may also correct the constant mixing matrix V_0. In this case ΔV is also the source of non-zero θ_{13} and CP violation [36,37]. In each case the smallness of θ_{13} is understandable: it is a consequence of flavor symmetry breaking or an effect of quantum corrections, so it must be smaller than the other two flavor mixing angles. Because CP violation is in general a peculiar effect of three (or more) flavor families, the Dirac-type CP-violating phase δ is naturally associated with the generation of θ_{13} in the above-mentioned scenarios.

The upcoming neutrino oscillation experiments will test the $V=V_0+\Delta V$ picture and provide us with more hints about model building. Note that this general approach can always be combined with the seesaw mechanisms. Note also that an alternative, such as $V=U_0+\Delta U$, is also likely to interpret the observed lepton flavor structure although a slightly larger perturbation ΔU is needed [38]. On the other hand, it is worthwhile to point out that the Majorana-type CP-violating phases may play an important role in quantum corrections to the constant neutrino mixing matrix (e.g. V_0 or U_0). In general the Dirac- and Majorana-type phases are entangled with each other, so they are intrinsically of the Majorana type and contribute to the neutrinoless double beta decay [39–42].

3 Hot and warm neutrino dark matter

Although we have known quite a lot about three species of neutrinos, there are many open questions in neutrino physics and neutrino cosmology. For example, are there extra species of light or heavy neutrinos? Are neutrinos the Dirac or Majorana particles? Can neutrinos play an important role in dark matter? Do neutrinos have something to do with the cosmological matter-antimatter asymmetry? Here we only focus on the problems of hot and warm neutrino dark matter and how to directly detect them in the laboratory.

Thanks to recent developments in cosmology, the existence of dark matter in the Universe has been established. But what it is made of remains a fundamental puzzle. Within the SM three kinds of neutrinos could constitute hot dark matter (i.e. the cosmic neutrino background) after they were decoupled from matter in the early Universe when the temperature was around 1 MeV. But it is known that hot dark matter can only have a tiny contribution to the total matter density of the Universe [1]. A careful analysis of the structure formation indicates that most dark matter should be cold (non-relativistic) or warm (semi-relativistic) at the onset of the galaxy formation, when the temperature of the Universe was about 1 keV [1]. A lot of attention has so far been paid to possible candidates for cold dark matter. In comparison, warm dark matter is also an intriguing possibility of accounting for the observed non-luminous and non-baryonic matter content in the Universe. A very good candidate for warm dark matter should be sterile neutrinos, if their masses are in the keV range and their lifetimes are much longer than the age of the Universe [43,44]. They could be produced in the early Universe in several ways, and their existence may allow us to solve or soften several problems that we have encountered in current dark matter simulations (e.g. to damp the inhomogeneities on small scales by reducing the number of dwarf galaxies or to smooth the cusps in the dark matter halos). Some preliminary observational hints of keV sterile neutrinos as warm dark matter have recently been discussed [45–47].

How to directly detect hot neutrino dark matter is a great challenge to the present experimental techniques, simply because its average temperature is extremely low today ($T_\nu \approx 1.945$ K). Among several possibilities for the direct detection of the cosmic neutrino background [48], the most promising one is the relic neutrino capture experiment by means of radioactive beta-decaying nuclei [49–52]. The central idea is that a generic neutrino capture reaction $\nu_e + N \to N' + e^-$ will take place with no threshold on the incident neutrino energy, if N can undergo the beta decay $N \to N' + e^- + \overline{\nu}_e$ with an energy $Q_\beta = m_N - m_{N'} - m_e$ in the limit of vanishing neutrino masses (i.e. $m_i \to 0$ for each mass eigenvalue). The signal of such a neutrino capture process is characterized by the monoenergetic electron's kinetic energy $Q_\beta + E_i \approx Q_\beta + m_i$ for a given neutrino mass eigenstate that is cold enough today, as compared with the non- monoenergetic electron's endpoint energy $Q_\beta - m_i$ for the same neutrino mass eigenstate in the beta decay [50–52]. Hence there is a gap equal to $2m_i$ between the signal and the background as illustrated in Figure 2. A measurement of this gap will directly probe relic neutrinos of the Big Bang and determine their masses.

Taking the reaction $\nu_e + {}^3\mathrm{H} \to {}^3\mathrm{He} + e^-$ for example, one may estimate the relic neutrino capture rate as follows:

$$N_{\mathrm{C\nu B}} \approx 6.5 \sum_i \varsigma_i \,|V_{ei}|^2\, R\left(T_e,\ T_e^i\right)\ \mathrm{yr}^{-1}\ \mathrm{MCi}^{-1}, \qquad (8)$$

where ς_i denotes the ratio of the number density of relic ν_i neutrinos around the Earth and its average value in the Universe, and $R(T_e,\ T_e^i)$ is a Gaussian energy resolution function [53]. We see that this capture rate is sensitive to the neutrino mixing angles via $|V_{ei}|$. On the other hand, the beta-decay background is sensitive to both the neutrino mass spectrum and the values of $|V_{ei}|$. A detailed analysis of such flavor effects can be found in [53,54].

In the presence of one or more sub-eV sterile neutrinos, which might be cosmologically friendly [55,56], one may do a similar analysis of the above relic neutrino capture

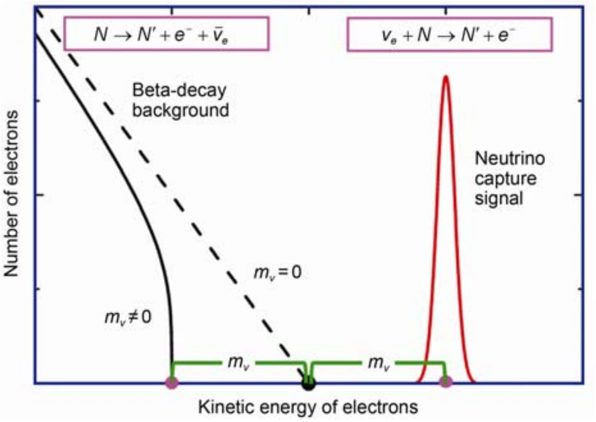

Figure 2 A schematic plot for the electron energy spectrum of capturing relic electron neutrinos on radioactive beta-decaying nuclei against the corresponding beta-decay background, where m_ν represents an overall neutrino mass for illustration.

process on radioactive beta-decaying nuclei and the corresponding background [53]. When the cosmic antineutrino background (i.e., hot antineutrino dark matter) is concerned, we have to consider some radioactive nuclei which can decay via electron capture (EC) [57]. A typical example is the isotope ^{163}Ho [58]. So a thresholdless capture of relic electron antineutrinos on EC-decaying ^{163}Ho nuclei may happen via

$$\overline{\nu_e} + {}^{163}\mathrm{Ho} + e^-_{i(\mathrm{shell})} \to {}^{163}\mathrm{Dy}^*_i \to {}^{163}\mathrm{Dy} + E_i , \qquad (9)$$

where $e^-_{i(\mathrm{shell})}$ is an orbital electron from the i-th shell of ^{163}Ho, and E_i denotes the corresponding binding energy of the electron hole in ^{163}Dy. A detailed analysis of active and sterile antineutrino flavor effects on this reaction and its EC-decay background can be found in [59] and [60].

The above ideas have also been used to capture keV sterile neutrino dark matter on radioactive beta-decaying ^{3}H and ^{106}Ru nuclei [5,61], and to capture keV sterile antineutrino dark matter on EC-decaying ^{163}Ho nuclei [60]. It is shown that the signatures of warm dark matter in the form of keV sterile neutrinos and antineutrinos should in principle be observable, provided the target is big enough, the energy resolution is good enough and the local gravitational clustering effect on dark matter is significant enough.

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