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Performance of cryogenic regenerator with ^{3}He as working fluid

HUANG YongHua*, FANG Lei & WANG RuZhu

Institute of Refrigeration and Cryogenics, Shanghai Jiao Tong University, Shanghai 200240, China

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Almost all experimental and theoretical studies on regenerative cryocoolers at temperatures below 20 K mention the use of ^{4}He as working fluid. A preliminary qualitative evaluation indicates that because of the superfluid phase transition, a working fluid of ^{3}He would overcome the cooling temperature limitation set by ^{4}He. Starting with a comparison of the thermophysical properties of ^{3}He and ^{4}He, cryogenic regenerator simulations applied to the third/last stage of a pulse tube refrigerator, with ^{3}He and ^{4}He separately, were implemented to quantitatively analyze performance differences of the regenerator with respect to regenerator loss, cooling power and COP. Results conclude that ^{3}He could significantly improve the performance of a regenerative cryocooler.

cryocooler, regenerator, ^{3}He, ^{4}He

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From such diverse fields as material science, medical science, meteorology and aerospace technology, the demands of ultra-low temperatures below 2 K and/or large cooling powers at temperatures below 20 K have served as incentives to develop more efficient refrigerators for devices such as magnetic resonance imaging and quantum communication devices. Regenerative cryocoolers are seen as a solution satisfying these demands. In the last 40 years, most cryocoolers used in scientific studies and even commercial products employ working substances of ^{4}He [1–3]. ^{4}He is an excellent refrigerant and easily available in the market. Its thermophysical properties including equation of state are well known. However, due to a small thermal expansion coefficient, the cooling capacity of ^{4}He regenerative cryocoolers diminishes quickly at liquid helium temperatures close to the lambda transition line of this fluid. The lowest non-load cooling temperatures achieved by commercial regenerative cryocoolers are of order 2.3 K, while the cooling power at 4.2 K can reach 1–1.5 W. Much of the research on cryocoolers has focused on such issues as 1) improving heat transfer at the cold end, 2) modifying the structures, and 3) adjusting the phase shifter. These efforts bring limited benefits to the coefficient of performance (COP) of modern cryocoolers. In light of experiences on regular refrigerators, the working substance is an important aspect. The stable isotope ^{3}He exhibits advantages with better integrated features in density, specific heat, enthalpy, thermal conductivity and viscosity for regenerative cryocoolers, which have been verified by preliminary experimental investigations. de Waele et al. [4] were the first to introduce ^{3}He in a three-stage pulse tube cryocooler and succeed in attaining 1.87 K at the cold end of the last stage. Satoh and Numazawa [5] then achieved a cooling temperature of 1.47 K using ^{3}He as working fluid in a G-M cryocooler with a new regenerative material. Jiang et al. [6] further lowered the cooling temperature down to 1.27 K in a separated-type two-stage pulse tube cryocooler. They also compared experimental results obtained by substituting ^{3}He for ^{4}He. More recently, a four-stage pulse tube cryocooler developed by Nast et al. [7] at Lockheed-Martin using ^{3}He in the last stage, reached a temperature of 3.8 K with an input power of 300 W and a frequency of 31 Hz. These experimental studies indicate that higher efficiencies/cooling powers or lower cooling temperatures can be realized using ^{3}He.

*Corresponding author (email: huangyh@sjtu.edu.cn)

In collaboration with our group, Radebaugh et al. [8,9] from NIST developed a new version of the regenerator modeling code-REGEN3.3, which incorporates the thermophysical properties of both ^{4}He and ^{3}He. Preliminary calculations of high frequency cryocoolers with ^{3}He and ^{4}He have been conducted to investigate how geometry and materials, among other aspects, affect the performance of the regenerator working at 4 K at the cold end and 20 K at the hot end. It was found that (a) a gadolinium oxysulfide ceramic (GOS+$Er_{0.5}Pr_{0.5}$) performed best out of nine different matrix materials for this application, (b) using ^{3}He contributes significantly in enhancing performance, and c) either a hot-end temperature above 35 K or an average pressure higher than 1.0 MPa has little effect on COP of the regenerator. This article reports on the theoretical analysis and simulations of the performance of regenerators looking particularly at variations in matrix porosity, aspect ratio, hot-end temperature, frequency, pressure ratio, and average pressure.

1 Properties of the two helium isotopes

The inherent properties of ^{3}He and ^{4}He determine their applications in cryocoolers. The critical temperature and pressure of ^{4}He are 5.1953 K and 0.2275 MPa, respectively. The isobaric specific heat exhibits a sharp peak around 2.17 K, where superfluid phase transition takes place. As shown in Figure 1, the thermal expansion coefficient of ^{4}He under pressures of 0.1–1 MPa is close to zero at 2.1–2.2 K, which is thought to be the theoretical limit of the cooling temperature for any regenerative cryocoolers charged with ^{4}He. By contrast, the critical temperature of the isotope ^{3}He is 3.3157 K, while the superfluid phase transition temperature is only 2.6 mK, which is three orders of magnitude lower than that for ^{4}He. The zero expansion coefficient line for ^{3}He appears at much lower temperatures, suggesting that a cooling temperature below 1 K is possible for a ^{3}He cooler. Furthermore, the higher values of specific heat and compressibility for ^{3}He, as well as the better characteristics associated with enthalpy differences at equal temperature difference, thermal conductivity and viscosity, indicate a higher performance of such a regenerator compared with that using ^{4}He. The net refrigeration power and the second law efficiency will be specifically used in what follows as the two effectual criteria to evaluate the influence of thermophysical properties of these fluids.

In general, the amount of heat exchange in a regenerator is over ten times the rest in a refrigerator, so the relative loss indicates the effectiveness of the regenerator within the cryocooler. For a regenerator working with oscillating flow, the time-averaged acoustic power $\langle P\dot{V}\rangle_{\rm h}$ that drives the current stage enters at its hot end. Here P stands for pressure and $\dot{V}$ for specific volume. The subscript h indicates the

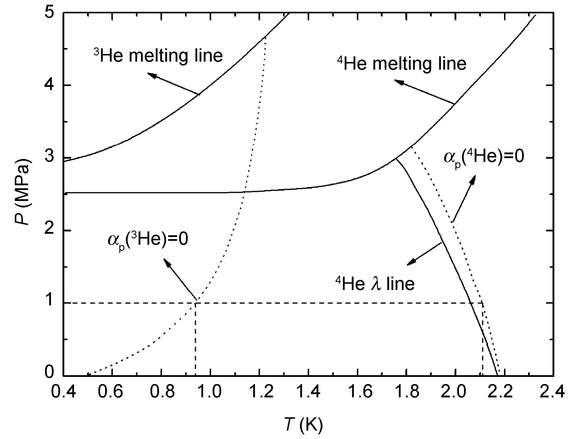

Figure 1 Zero expansion coefficients of helium isotopes.

parameter is to be evaluated at the hot-end. In our previous study on regenerator modeling and loss analysis [4], the net refrigeration power could be written as follows:

$$\dot{Q}_{\rm net}=\underbrace{\langle P\dot{V}\rangle_{\rm h}}_{<1>}\underbrace{\left[1-\frac{\langle \Delta P\dot{V}\rangle_{\rm h}}{\langle P\dot{V}\rangle_{\rm h}}\right]}_{<2>}\underbrace{\left[\frac{Z_{\rm c}T_{\rm c}}{Z_{\rm h}T_{\rm h}}\right]}_{<3>}\times\underbrace{\left[1-\frac{\langle \dot{H}\rangle_{\rm P}}{\langle P\dot{V}\rangle_{\rm c}}\right]}_{<4>}\underbrace{\left[1-\frac{\dot{Q}_{\rm reg}}{\dot{Q}_{\rm gross}}-\frac{\dot{Q}_{\rm cond}}{\dot{Q}_{\rm gross}}-\frac{\dot{Q}_{\rm pt}}{\dot{Q}_{\rm gross}}\right]}_{<5>},\qquad (1)$$

where $\dot{Q}_{\rm net}$ stands for the net refrigeration power, $\dot{Q}_{\rm gross}$ the gross refrigeration power, $\dot{Q}_{\rm reg}$ the thermal loss associated with enthalpy flow caused by imperfect heat transfer and limited heat capacity in the regenerator, $\dot{Q}_{\rm cond}$ the conduction heat leak through the regenerator, $\dot{Q}_{\rm pt}$ the loss associated with an imperfect pulse tube or any irreversible expansion process at the cold end, and $\langle \dot{H}\rangle_{\rm P}$ the time averaged enthalpy flow. The subscripts c and h stands for the cold end and hot end, respectively. By writing the net refrigeration power in this manner we have separated out the terms that are functions of only the gas properties from those that also are dependent on the hardware. The first term on the right side of the equation is the acoustic power input at the hot end of the regenerator. The second term represents the effect of pressure drop in the regenerator and is both hardware and gas dependent. The third term represents the reduction in acoustic power due to temperature change and real gas behavior associated with compressibility. The fourth term represents the effect of real gas enthalpy flow. The terms in the last set of brackets are responsible for the various losses in the regenerator and the cold head, and are both hardware and gas dependent. Therefore, the COP of the last stage of the regenerator can be written as

$$\mathrm{COP}=\frac{\dot{Q}_{\mathrm{net}}}{\langle P\dot{V}\rangle_{\mathrm{h}}}. \tag{2}$$

For an ideal gas and a perfect regenerator, the COP equals $T_c/(T_h-T_c)$; it reduces to (T_c/T_h) if we assume that the reversible expansion work at the cold end is not being fed back to the hot end of this regenerator. Thus, the thermodynamic second-law efficiency of the last stage is given by

$$\eta=\left(T_{\mathrm{h}}/T_{\mathrm{c}}\right)\mathrm{COP}. \tag{3}$$

Combining eqs. (1)–(3) gives an expression for the second law efficiency of the last stage in a form that separates the variables

$$\eta=\left[1-\frac{\langle\Delta P\dot{V}\rangle_{\mathrm{h}}}{\langle P\dot{V}\rangle_{\mathrm{h}}}\right]\left[\frac{Z_{\mathrm{c}}}{Z_{\mathrm{h}}}\right]\left[1-\frac{\langle\dot{H}\rangle_{\mathrm{P}}}{\langle P\dot{V}\rangle_{\mathrm{c}}}\right]\times\left[1-\frac{\dot{Q}_{\mathrm{reg}}}{\dot{Q}_{\mathrm{gross}}}-\frac{\dot{Q}_{\mathrm{cond}}}{\dot{Q}_{\mathrm{gross}}}-\frac{\dot{Q}_{\mathrm{pt}}}{\dot{Q}_{\mathrm{gross}}}\right]. \tag{4}$$

Assuming the pulse tube loss is zero, the efficiency η calculated from eq. (4) refers only to the regenerator, which then helps in understanding the effect of the working substance on the regenerator. In addition, if only real gas effects are taken into account, net refrigeration power equals gross refrigeration power. As a consequence, the acoustic power loss for a perfect regenerator due to the pressure drop is zero. Following these considerations, eq. (4) becomes

$$\eta=\frac{Z_{\mathrm{c}}}{Z_{\mathrm{h}}}\left[1-\frac{\langle\dot{H}\rangle_{\mathrm{P}}}{\langle P\dot{V}\rangle_{\mathrm{c}}}\right], \tag{5}$$

by which we find that the efficiency depends only on the thermophysical properties of the working fluid. These properties for ^{3}He and ^{4}He are taken from Huang et al. [10,11] and McCarty [12], respectively. The compressibility curves for both ^{3}He and ^{4}He with respect to temperature are plotted in Figure 2. The compressibility of both gases is seen to be close to 1 at 20 K. On the lower temperature side, however, the value for ^{3}He is significantly greater than that for ^{4}He at fixed temperatures and pressures. Therefore, the value of Z_c/Z_h for ^{3}He will be greater than that for ^{4}He. Figure 3 shows the efficiency factor $\left[1-\langle\dot{H}\rangle_{\mathrm{P}}/\langle P\dot{V}\rangle_{\mathrm{c}}\right]$ associated with the enthalpy flow. Clearly, the efficiency factor for ^{3}He is also much higher than that for ^{4}He. Based upon the above qualitative analysis, one may deduce that a ^{3}He cryocooler has a higher efficiency than a ^{4}He cryocooler. A quantitative analysis based on numerical simulations of the regenerator will be presented below.

It should be noted that the efficiency of a regenerator is an aggregated consequence of the thermodynamics, heat transfer and oscillating flow dynamics in the porous medium. The numerical model REGEN3.3 was developed upon these considerations and is able to calculate not only the energy balance outputs but also the distributions of

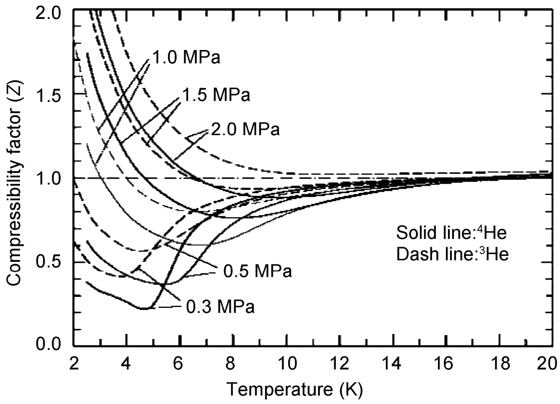

Figure 2 Compressibility factor of fluids ^{4}He and ^{3}He.

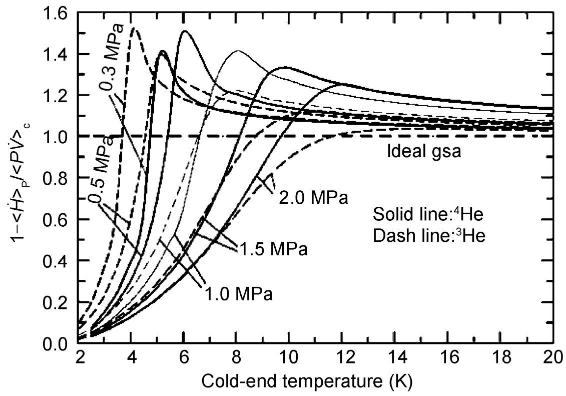

Figure 3 Contribution of real-gas enthalpy flow to the efficiency of a stage with the hot end at 20 K.

temperature, pressure and flow velocity, among others, inside the regenerator. The thermodynamic second-law efficiency was chosen as the overall indicator to investigate the performance of the regenerator under different working substances.

2 Comparison of regenerator performance between working fluids ^{3}He and ^{4}He

A previous study showed that in quite wide ranges of pressure, pressure ratio, frequency and regenerator length, the optimum value of the ratio V_{rg}/V_E is between 7 and 10 [13], where V_{rg} is the volume of gas in the regenerator, and V_E is the swept volume of gas at the regenerator cold end. We also found that for a 4 K regenerator the best matrix material is a mixture of GOS and $Er_{0.5}Pr_{0.5}$ prepared in a spherical geometry. Our numeral work adopts this GOS+$Er_{0.5}Pr_{0.5}$ configuration with a diameter of 100 μm. The porosity n_g, aspect ratio A_g/m_c, cold-end pressure ratio P_r, phase angle between mass flow and pressure at the cold end ϕ_c, hot-end temperature T_h, average pressure P_0, and frequency f are all considered as factors influencing cooler performance and discussed in turn in the following section.

2.1 Effect of matrix porosity and aspect ratio

The effect of matrix porosity on the regenerator performance is quite significant. As the porosity increases, the flow resistance of the working substance decreases. However, the effective heat exchange area decreases at the same time. This contradiction indicates that there must be an optimum value for the porosity. Figure 4 shows the effect of porosity n_g varying from 0.35 to 0.7 on the second law efficiency and relative loss. For $n_g \approx 0.4$, the second law efficiency increased by about 120% for ^{3}He compared with that for ^{4}He, while the relative loss decreased by only about 20%. This result indicates the tremendous potential of ^{3}He to improve regenerator performance. However, this enhancement vanishes with increments in porosity. In terms of efficiency and loss curves, the COP rise and loss reduction are not concomitant. This is probably a consequence of the rapid growth of conduction loss in the regenerator due to the increase in porosity.

The irregularity of the oscillating gas flow could be omitted for a regenerator with small diameter. Thus, regenerators with the same length and matrix material theoretically have identical efficiency. Ones with larger cross-sectional areas possess the ability to transfer larger mass flows and hence produce higher refrigeration powers. In fact, a large-area regenerator can be regarded as a cluster of small-area regenerators. That is to say, the COP remains constant under given conditions such as constant frequency, average pressure, pressure ratio, and phase angle, if the ratio of cross-sectional area to mass flow rate is fixed. This concept can be seen to be confirmed in Figure 5, and signifies that the cross area and the mass flow rate at the cold end are not necessarily independent parameters during optimization.

Figure 6 illustrates how the ratio of cross-sectional area and mass flow rate at the cold end A_g/m_c in the range of 0.15–0.6 cm^2 s g^{-1} affects the second law efficiency and relative loss of the regenerator at different charging pressures. The efficiency falls after rising as A_g/m_c increases, while the relative loss behaves in the reverse manner. The interpretation is that there must be an optimum value for the area-mass flow ratio to maximize the performance of the regenerator. At pressure P_0 = 0.5 MPa, the second law efficiency of a regenerator with ^{3}He is about 130% higher than that with ^{4}He, at the A_g/m_c ratio of 0.3 cm^2 s g^{-1}. In practice, we could adjust the mass flow rate at the cold end to optimize A_g/m_c.

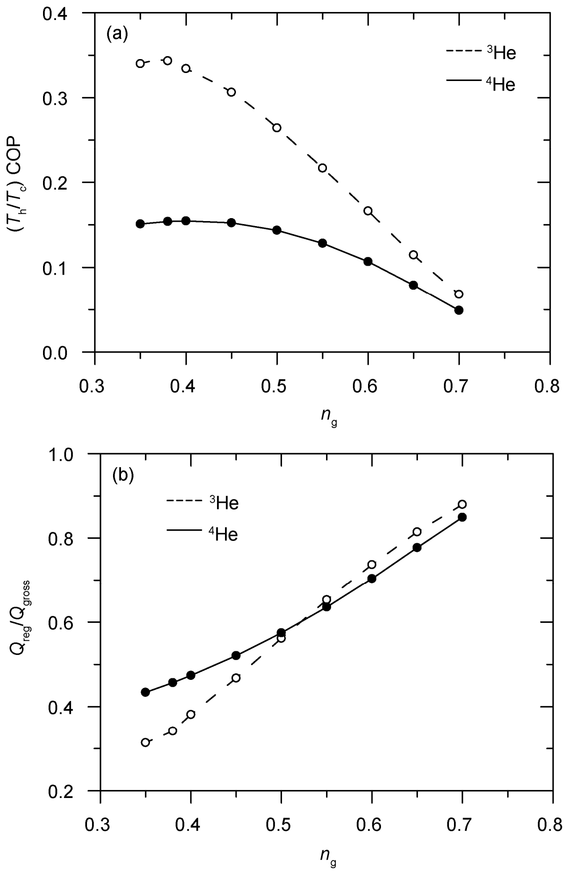

Figure 4 Effect of matrix porosity on regenerator performance (T_c=4 K, T_h=20 K, P_r=1.5, P_0=0.5 MPa, f=30 Hz, ϕ_c= −30°).

2.2 Effect of hot-end temperature

The influences of the hot-end temperature T_h on the regenerator efficiency and relative loss are presented in Figure 7 for pressures 0.5 and 1.0 MPa in the temperature range from 10 to 45 K. As T_h increases, the efficiency decreases with an accelerating rate. At pressure 1.0 MPa, the influence of T_h is relatively imperceptible. The two curves for either ^{3}He or ^{4}He intersect at a temperature around 30 K. Below this temperature, the regenerator working at lower gas pressures achieves higher efficiencies, while the reverse occurs at higher temperatures. At T_h = 20 K, the efficiency is enhanced

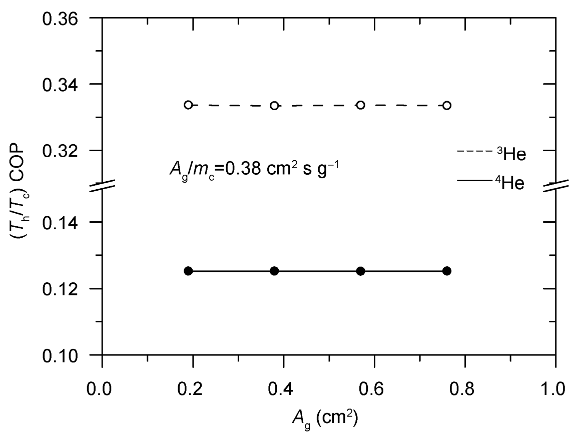

Figure 5 Relationship between efficiency and cross-sectional area at fixed A_g/m_c.

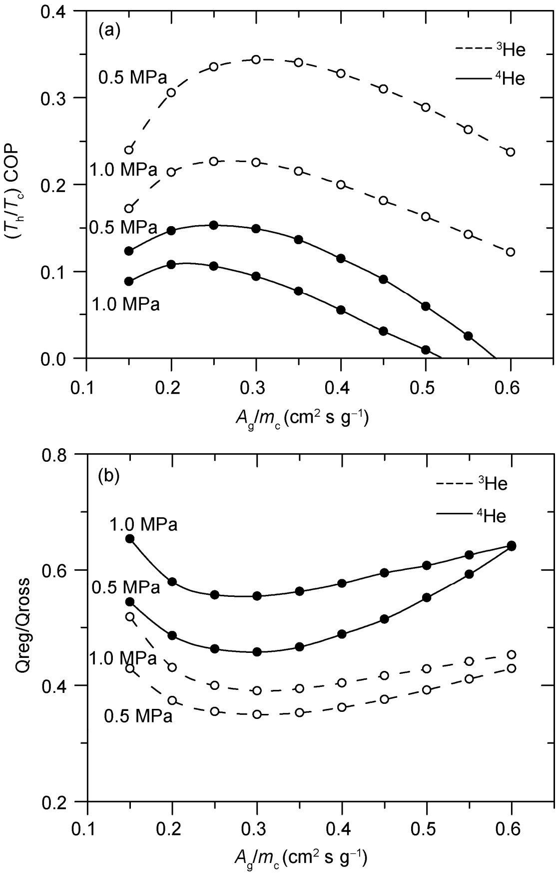

Figure 6 Effect of aspect ratio on regenerator performance (T_c=4 K, T_h=20 K, P_r=1.5, f=30 Hz, ϕ_c=–30°).

by about 120% and 110% for ^{3}He compared with ^{4}He at pressures 0.5 MPa and 1.0 MPa, respectively. We also find that the cryocooler with ^{4}He could not attain a cooling temperature at about 4 K if the hot-end temperature is above 32 K. However, this T_h limit for ^{3}He is 40 K or even higher, although the hot-end temperature should be lower than 30 K in practice due to considerations in efficiency.

2.3 Effect of cold-end pressure ratio

Increments at the cold end in the pressure ratio P_r, one of the dominant operating parameters of a cryocooler, will lead to a proportionate increase in the mass flow rate there. Figure 8 shows how the efficiency and loss varies with P_r from 1.3 up to 2.0. At a given frequency, the efficiency improves as the pressure ratio P_r rises. For small P_r, the major part of the gross refrigeration power is used in compensating losses, resulting in a small net refrigeration power. As the pressure ratio goes up, gross power and losses grow together and finally reach equilibrium where the efficiency nears a maximum at P_r =1.6. Although a larger value of P_r leads generally to a better performance from the regenerator, a high pressure ratio needs assistance from the compressor. In addition, the efficiency will not always rise with increasing pressure ratios; that is, to say declines may occur in some instances. As shown in the figure, the efficiency is improved by 140% when using ^{3}He compared with ^{4}He at f = 20 Hz and P_r = 1.5. Figure 8 also indicates that the higher the frequency is, the more moderate is the dependence of efficiency on the pressure ratio. To design and/or operate a practical cryocooler, an adequate frequency and pressure ratio should be determined under the twin considerations of overall economical and technical efficiencies.

2.4 Effect of phase angle

The phase angle between the leading mass flow and pressure at the cold end is a critical parameter affecting cryocooler performance. The second law efficiency and relative loss related to the phase angle ϕ_c variation from –45° to 30° at different pressures are plotted in Figure 9. The curves indicate that the performance of the regenerator with ^{3}He is much better than that with ^{4}He. Taking the optimum ϕ_c = –30° at P_0 = 0.5 MPa for example, the efficiency of a ^{3}He cryocooler is 120% higher than that for ^{4}He. Phase shifter devices such as the double inlet valve or the inertance tube could be used to adjust the phase angle and hence to maximize the refrigeration power output or COP.

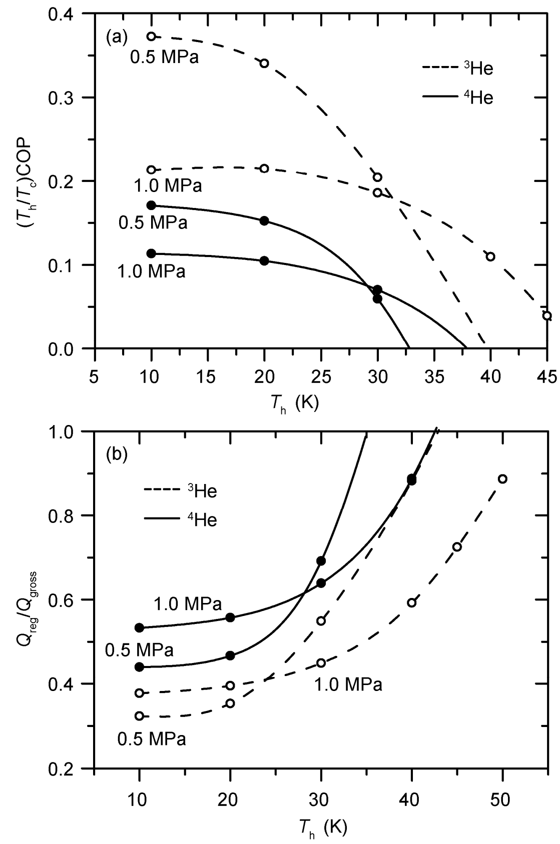

Figure 7 Effect of hot-end temperature on regenerator performance (T_c=4 K, P_r=1.5, f=30 Hz, ϕ_c=–30°, n_g=0.38).

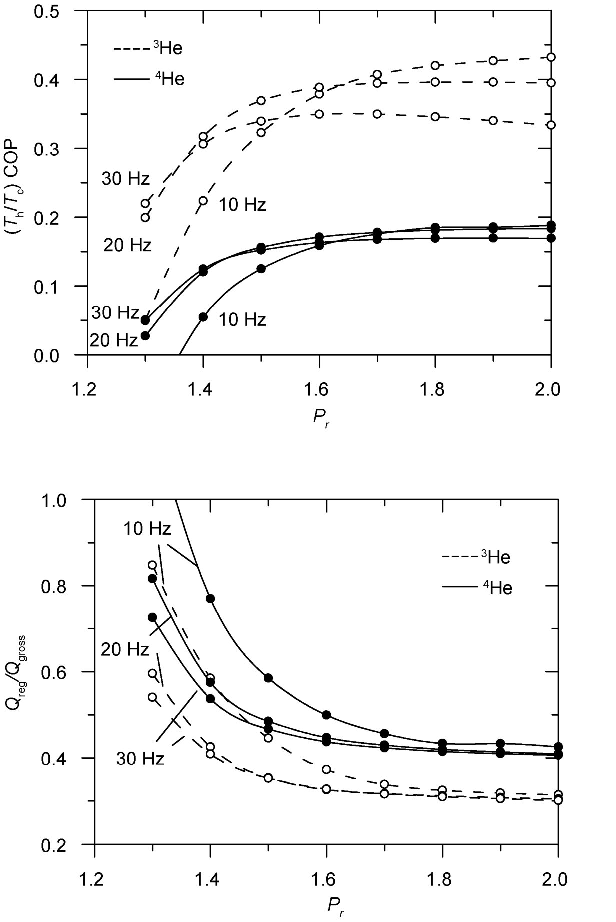

Figure 8 Effect of pressure ratio on regenerator performance (T_c=4 K, T_h=20 K, P_0=0.5 MPa, ϕ_c=–30°, n_g=0.38).

2.5 Effect of average pressure

The average pressure of the oscillating flow in a running cryocooler is close to the initial charging pressure, which directly determines the extensive properties of the working substance. Figure 10 shows the dependency of the efficiency and loss on the average pressure P_0 varying from 0.3 to 1.5 MPa. At a pressure between 0.5 MPa and 0.6 MPa, the second law efficiency of the regenerator with ^{3}He is 110% higher than that with ^{4}He. As the pressure increases further, both the flow resistance and temperature fluctuation intensify, and the efficiency decreases as a consequence. Meanwhile, high gas pressures require high strengths in the structures of both the compressor and the cold head of the cryocooler. It is interesting to find that for ^{3}He, pressure has almost no effect on the second law efficiency when it exceeds the optimum value.

3 Conclusion

The thermophysical properties of isotopes ^{3}He and ^{4}He are contrasted to elicit any benefits to be gained in substituting one for the other in a regenerator model, which is based on

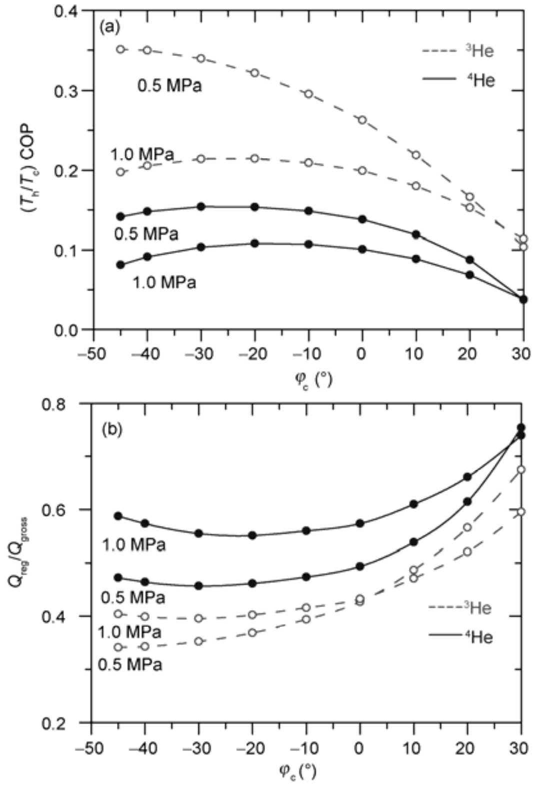

Figure 9 Effect of phase angle on regenerator performance (T_c=4 K, T_h=20 K, P_r=1.5, f=30 Hz, n_g=0.38).

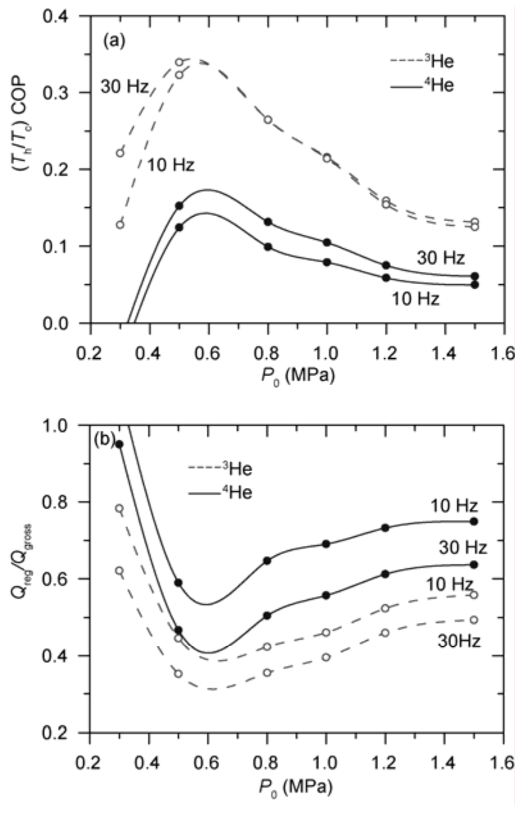

Figure 10 Effect of average pressure on regenerator performance (T_c=4 K, T_h=20 K, P_r=1.5, ϕ_c=–30°, n_g=0.38).

enthalpy flow theory. The qualitative analysis indicated that an advantage can be gained in using a working fluid of ^{3}He in cryocoolers. This was confirmed by a set of comparative numerical simulations of practical regenerators with ^{3}He and ^{4}He separately. This quantitative analysis helped to ascertain optimum geometric and operating parameters for the last stage of the regenerator. We believe that the efficiency of a regenerator with ^{3}He could be 100% higher than that with ^{4}He for a cryocooler operating at liquid helium temperatures, provided the geometric and operating parameters have been optimized. The results presented in this article are expected to be of useful guidance in the design and operation of the regenerator component and even the entire cryocooler with ^{3}He charged, that will be a potential competitive solution for the proposed applications.

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