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Exponential speedup of quantum newton optimization algorithm for general polynomials

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The gradient-based method plays a central role in optimization problems, and is widely applied in engineering control, financial analysis, weather forecast, and training of machine learning models.

One important gradient-based algorithm is the estimation of the gradient vector of loss function. In classical algorithms, the gradient cannot be accomplished with less than $O(d)$ basic operations, where d is the number of variables in the problem. As data size of information processing tasks is usually at a level of billions or more, computation resources are pushed to their limits in classical computing.

Taking advantage of state superposition and entanglement, quantum computing speeds up the solutions of various important problems, such as factoring, searching, the linear system of equations, and the like. The study of quantum optimization is of interest, and several quantum gradient-based methods have been proposed. In 2005, Jordan [1] proposed a quantum gradient estimation algorithm, which exponentially reduces the query complexity to a constant, but takes up $O(\text{poly}(d))$ quantum memory space. In 2019, Rebentrost et al. [2] presented a quantum gradient descent algorithm for even order and homogeneous polynomials. With a complexity $O(\text{polylog}(d))$, it achieves exponential speedup compared with classical methods. Together with linear combinations of unitaries (LCU) [3], those quantum gradient methods are expected to find broader application in more general cases. LCU has proved its capability to exponentially improve the dependence on the solution precision of the linear system of differential equations [4].

Against this background, Gao et al. [5] recently proposed a new quantum optimization algorithm for general polynomials. Compared with the results in ref. [2], two new techniques are introduced in this work, i.e., the modified amplitude encoding method and the loss function's second order deviation. The new algorithm is an iterative one, where each iteration preserves the logarithmic $O(\text{polylog}(d))$ scaling advantage, but it converges much faster than the quantum first order methods. As demonstrated in their work, the new algorithm reduces the full iteration times from thousands to less than ten in solving a non-linear equation, representing a significant speedup in quantum optimization. Given the wide application of optimization, this new quantum algorithm will benefit varieties of optimization-related problems.

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