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von Neumann measurement-related matrices

HuaiXin Cao*

School of Mathematics and Information Science, Shaanxi Normal University, Xi'an 710119, China

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Quantum mechanics is the theory for small physical systems. It does provide a mathematical and conceptual framework for the development of laws that a physical system must obey. It has several basic postulates. The first is about state space, in which quantum mechanics takes place. The second is about the evolution of quantum system. The third is about quantum measurement, which describes what an experimentalist will obtain when he/she observes the system. These three postulates provide a connection between the physical world and the mathematical formalism of quantum mechanics [1].

The issue of quantum measurement lies at the heart of the problem of the interpretation of quantum mechanics, which can be interpreted as follows.

A measurement always causes the system to jump into an eigenstate of the dynamical variable that is being measured, the eigenvalue of this eigenstate belongs to being equal to the result of the measurement (P. A. M. Dirac, 1958).

Therefore, quantum measurement is an "irreversible" process. Once the measurement is completed and the information is extracted, the initial state is collapsed. It is also a "non-deterministic" process. Because the eigenstates of a quantum system are not unique and we do not know which one is collapsed when measurement is implemented.

In fact, for many applications of quantum computation and quantum information, scientists concern primarily with von Neumann measurements or projective measurements. When the other axioms of quantum mechanics are taken into account, von Neumann measurements augmented by unitary operations turn out to be completely equivalent to general measurements. In order to make quantum measurements clear, one should try to solve the problem with von Neumann measurements first. Moreover, a von Neumann measurement gives arise to an observable composed by the projectors. Conversely, an observable induces a von Neumann measurement with the projectors of spectrum projections of the observable.

Very recently, Zhao, Ma, Zhang, and Fei [2] discussed the action of von Neumann measurements on quantum systems. Since a quantum state in a quantum system is a positive operator of trace 1 mathematically, it can be expanded by the basis of orthonormal traceless Hermitian matrices, or the generators of special unitary group, which is referred as Bloch representation [1]. It turns out that the action of a von Neumann measurement on a quantum system is factually the action of a von Neumann measurement on the basis of orthonormal traceless Hermitian matrices.

With this motivation, the authors of ref. [2] associated with an important matrix to a give von Neumann measurement and a basis of orthonormal traceless Hermitian matrices, named the von Neumann measurement-related matrix. They proved that such a matrix is not only idempotent, basis-independent, but also singular. For an m-dimensional quantum system, the matrix corresponding to a von Neumann measurement is a square matrix of order $(m-1)^2$. In ref. [2], they also proved that this matrix is of rank $m-1$ by mathematical techniques, which makes the matrix related to a von Neumann measurement much clear. These results promote an understanding of von Neumann measurements mostly.

As one application, the authors considered the nullity of quantum correlation. As we know, if a bipartite state is invariant under some von Neumann measurements on a sub-

*Corresponding author (email: caohx@snnu.edu.cn)

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system, then it is classical at this subsystem, called a classical-quantum state, or a quantum-classical state. Or else it is quantum [3]. When a bipartite state is both classical-quantum and quantum-classical, we call it a classical-classical state.

Classicality corresponding to the maximal information about a subsystem can be obtained by some specific local measurements without altering correlations with the rest of the system. It has application to the theory of decoherence where they describe the classical correlation between the pointer states of some measurement apparatus and the internal quantum states [4]. The set of classical states can be used to define quantum correlation measures using a notion of minimum distance as relative entropy of discord [5] and geometric quantum discord [6]. Utilizing the property of the matrix related to a von Neumann measurement and based on the coefficient matrices of a quantum state in Bloch representation, ref. [2] presented three necessary conditions for a bipartite state to be classical-quantum, quantum-classical and classical-classical, respectively. These conditions are called the nullity conditions for quantum correlation in ref. [2] and are better than the necessary conditions given in ref. [6] in some cases. Additionally, the classicality, i.e. the nullity condition for quantum correlation of Bell-diagonal states is analyzed explicitly in ref. [7]. Recently, quantum correlations have been widely discussed [8] while entanglement transfer, entanglement concentration, entanglement dynamics, ghost imaging with entangled photons, entanglement swapping, and entanglement concentration with strong projective measurement have been extensively researched, for instance in refs. [9,10], respectively.

As far as the von Neumann measurement is applied in extensive fields, for example, quantum state discrimination, entanglement detection and some other quantum information process, I believe that the results in ref. [2] about the von Neumann measurement related matrix will be used in many aspects.

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