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Practicable method to identify ferromagnetic objects

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Ferromagnetic objects are ubiquitous in the world and the identification of ferromagnetic objects is important in many applications. Determining the position and magnetic parameters of the ferromagnetic object from its magnetic signature is challenging and can be used in several fields [1,2]. Many methods have been proposed for identifying ferromagnetic objects [3,4]. The identification accuracy of these methods is sensitive to the fluctuation of the geomagnetic field. Therefore, there is a need to eliminate the influence of geomagnetic field. In this regard, this paper proposes a practicable ferromagnetic object identification method based on a spherical harmonic function model. Using a magnetometer array containing four magnetic sensors in a stereoscopic distribution, it can efficiently identify the object and eliminate the interference of fluctuation of the geomagnetic field. It is worth noting that the size, shape, and intensity of the ferromagnetic object as well as its distance to the magnetometer array will affect the identification. In this study we assume that the size of the ferromagnetic object is small comparing with its distance to the magnetometer array, and its intensity is not significantly larger than the geomagnetic field of its location.

The measured magnetic field at any position, B_m, consists of the ambient geomagnetic field, B_0, and the ferromagnetic object magnetic field B_{fo}, and can be expressed as follows:

$$B_m = B_0 + B_{fo}. \tag{1}$$

The geomagnetic field B_0 is assumed to be identical for all types of magnetometers whereas the magnetic field of the ferromagnetic object differs in a sense array system. Then, B_{fo} can be expressed as the negative gradient of the scalar potential function, V [5]:

$$B_{fo} = -\nabla V. \tag{2}$$

The parameter V can be expanded using a spherical harmonic model, as shown in eq. (3) [5].

$$V(r,\theta,\varphi) = a\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}(g_n^m\cos m\varphi + h_n^m\sin m\varphi)P_n^m(\cos\theta), \tag{3}$$

where a is the reference radius of the ferromagnetic object and (r, θ, φ) represents the standard spherical coordinates (θ=0 coincides with the positive z-direction), $P_n^m(\cos\theta)$ is the associated Legendre function of degree n and order m, and N is the truncation degree, and g_n^m and h_n^m are the Gaussian coefficients.

Therefore, the magnetic field generated by the object can be expanded as shown in eq. (4):

$$B_r = -\frac{\partial V}{\partial r} = \sum_{n=1}^{N}(n+1)\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}(g_n^m\cos m\varphi + h_n^m\sin m\varphi)P_n^m(\cos\theta),$$

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$$
\begin{aligned}
B_\theta &= -\frac{1}{r}\frac{\partial V}{\partial \theta} \\
&= -\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}(g_n^m \cos m\varphi + h_n^m \sin m\varphi)\frac{\partial P_n^m(\cos\theta)}{\partial\theta}, \\
B_\varphi &= \frac{-1}{r\sin\theta}\frac{\partial V}{\partial\varphi} \\
&= \frac{-1}{\sin\theta}\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n} m(-g_n^m \sin m\varphi + h_n^m \cos m\varphi)P_n^m(\cos\theta),
\end{aligned} \tag{4}
$$

where B_r, B_θ, and B_φ are the spherical field vector components referenced to the local tangent plane.

When the distance between the target and the sensor is more than three times the largest dimension of the target, a ferromagnetic object is usually treated as a magnetic dipole [6]. In such a situation, the higher-order terms can be neglected because the magnetic field decays rapidly. In the following equations, the measured magnetic field of the ferromagnetic object is simplified to the 1st degree.

After setting N=1, we have $P_n^m(\cos\theta)$ as follows:

$$P_1^0 = \cos\theta,\ P_1^1 = \sin\theta.$$

Hence, parameter V can be simplified as

$$V(r,\theta,\varphi) = a\left(\frac{a}{r}\right)^2(g_1^0\cos\theta + g_1^1\cos\varphi\sin\theta + h_1^1\sin\varphi\sin\theta). \tag{5}$$

The field components B_r, B_θ, and B_φ are simplified as

$$
\begin{cases}
B_r = -\dfrac{\partial V}{\partial r} = 2\left(\dfrac{a}{r}\right)^3\left(g_1^0\cos\theta + g_1^1\cos\varphi\sin\theta + h_1^1\sin\varphi\sin\theta\right), \\
B_\theta = -\dfrac{1}{r}\dfrac{\partial V}{\partial\theta} = 2\left(\dfrac{a}{r}\right)^3\left(g_1^0\sin\theta - g_1^1\cos\varphi\cos\theta - h_1^1\sin\varphi\cos\theta\right), \\
B_\varphi = \dfrac{-1}{r\sin\theta}\dfrac{\partial V}{\partial\varphi} = 2\left(\dfrac{a}{r}\right)^3\left(g_1^1\sin\varphi - h_1^1\cos\varphi\right),
\end{cases} \tag{6}
$$

where g_1^0, g_1^1, and h_1^1 are expressed as in eq. (7) [7].

$$
\begin{cases}
g_1^0 = \dfrac{\mu_0 M\cos\theta_d}{4\pi a^3}, \\
g_1^1 = \dfrac{\mu_0 M\sin\theta_d\cos\varphi_d}{4\pi a^3}, \\
h_1^1 = \dfrac{\mu_0 M\sin\theta_d\sin\varphi_d}{4\pi a^3}.
\end{cases} \tag{7}
$$

μ_0=4π·10^{-7} H/m provides the free space permeability, M is the magnetic moment magnitude, (θ_d, φ_d) represent the dipole direction in spherical coordinates, and the other variables are defined as before.

Substituting g_1^0, g_1^1, and h_1^1 from eq. (7) into eq. (6), the following equations can be obtained:

$$
\begin{cases}
B_r = \dfrac{\mu_0 M}{2\pi r^3}(\cos\theta_d\cos\theta + \sin\theta_d\sin\theta\cos(\varphi-\varphi_d)), \\
B_\theta = \dfrac{\mu_0 M}{4\pi r^3}(\cos\theta_d\sin\theta \\
\qquad - \sin\theta_d(\cos\varphi_d\cos\theta\cos\varphi + \sin\varphi_d\sin\theta\sin\varphi)), \\
B_\varphi = \dfrac{\mu_0 M}{4\pi r^3}(\sin\theta_d\sin(\varphi-\varphi_d)).
\end{cases} \tag{8}
$$

The values of B_{fox}, B_{foy}, and B_{foz} in a rectangular coordinate system can be obtained using eq. (9):

$$
\begin{pmatrix} B_{fox} \\ B_{foy} \\ B_{foz} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{pmatrix} \times \begin{pmatrix} B_r \\ B_\theta \\ B_\varphi \end{pmatrix}. \tag{9}
$$

Then, we have

$$
\begin{aligned}
&B_{fox} \\
&= \frac{\mu_0 M}{4\pi r^3}\big(2\sin\theta\cos\varphi(\cos\theta\cos\theta_d + \sin\theta_d\sin\theta\cos(\varphi-\varphi_d)) \\
&+\sin\theta\sin\varphi(\cos\theta_d\sin\theta - \sin\theta_d(\cos\phi_d\cos\theta\cos\varphi + \sin\varphi_d\sin\theta\sin\varphi)) \\
&+\cos\theta\sin\theta_d\sin(\varphi-\varphi_d)\big), \\
&B_{foy} \\
&= \frac{\mu_0 M}{4\pi r^3}\big(2\cos\theta\cos\varphi(\cos\theta\cos\theta_d + \sin\theta_d\sin\theta\cos(\varphi-\varphi_d)) \\
&+\cos\theta\sin\varphi(\cos\theta_d\sin\theta - \sin\theta_d(\cos\phi_d\cos\theta\cos\varphi + \sin\varphi_d\sin\theta\sin\varphi)) \\
&-\sin\theta\sin\theta_d\sin(\varphi-\varphi_d)\big), \\
&B_{foz} \\
&= \frac{\mu_0 M}{4\pi r^3}\big(-2\sin\varphi(\cos\theta\cos\theta_d + \sin\theta_d\sin\theta\cos(\varphi-\varphi_d)) \\
&+\cos\varphi(\cos\theta_d\sin\theta - \sin\theta_d(\cos\phi_d\cos\theta\cos\varphi + \sin\varphi_d\sin\theta\sin\varphi))\big).
\end{aligned} \tag{10}
$$

Eq. (1) can be divided into three components in a rectangular coordinate system as follows:

$$
\begin{cases}
B_{mx} = B_{0x} + B_{fox}, \\
B_{my} = B_{0y} + B_{foy}, \\
B_{mz} = B_{0z} + B_{foz}.
\end{cases} \tag{11}
$$

Substituting B_{fox}, B_{foy}, and B_{foz} from eq. (10) into eq. (11), the following equation set can be obtained:

$$
\begin{aligned}
B_{mx} &= B_{0x} + \frac{\mu_0 M}{4\pi\left((x_m-x_{fo})^2 + (y_m-y_{fo})^2 + (z_m-z_{fo})^2\right)^{\frac{3}{2}}} \\
&\times\big(2\sin\theta\cos\varphi(\cos\theta\cos\theta_d + \sin\theta_d\sin\theta\cos(\varphi-\varphi_d)) \\
&+\sin\theta\sin\varphi(\cos\theta_d\sin\theta - \sin\theta_d(\cos\phi_d\cos\theta\cos\varphi + \sin\varphi_d\sin\theta\sin\varphi)) \\
&+\cos\theta\sin\theta_d\sin(\varphi-\varphi_d)\big), \\
B_{my} &= B_{0y} + \frac{\mu_0 M}{4\pi\left((x_m-x_{fo})^2 + (y_m-y_{fo})^2 + (z_m-z_{fo})^2\right)^{\frac{3}{2}}}
\end{aligned}
$$

$$\times\left(2\cos\theta\cos\varphi\left(\cos\theta\cos\theta_d+\sin\theta_d\sin\theta\cos(\varphi-\varphi_d)\right)\right.$$
$$+\cos\theta\sin\varphi\left(\cos\theta_d\sin\theta-\sin\theta_d\left(\cos\phi_d\cos\theta\cos\varphi+\sin\varphi_d\sin\theta\sin\varphi\right)\right)$$
$$\left.-\sin\theta\sin\theta_d\sin(\varphi-\varphi_d)\right),$$
$$B_{mz}=B_{0z}+\frac{\mu_0 M}{4\pi\left((x_m-x_{fo})^2+(y_m-y_{fo})^2+(z_m-z_{fo})^2\right)^{\frac{3}{2}}}$$
$$\times\left(-2\sin\varphi\left(\cos\theta\cos\theta_d+\sin\theta_d\sin\theta\cos(\varphi-\varphi_d)\right)\right.$$
$$\left.+\cos\varphi\left(\cos\theta_d\sin\theta-\sin\theta_d\left(\cos\phi_d\cos\theta\cos\varphi+\sin\varphi_d\sin\theta\sin\varphi\right)\right)\right),\tag{12}$$

where x_{fo}, y_{fo}, and z_{fo} are the coordinates of the ferromagnetic object; x_m, y_m, and z_m are the coordinates of the magnetometer; and the other variables are defined as before.

For a certain magnetic sensor array, B_{mx}, B_{my}, and B_{mz} can be measured using the magnetometer, and the coordinates of the magnetometers x_m, y_m, and z_m can also be measured. Therefore, there are nine unknowns in eq. (12), namely, B_{0x}, B_{0y}, B_{0z}: the ambient geomagnetic field; x_{fo}, y_{fo}, and z_{fo}: the position of the ferromagnetic object; and, M, θ_d, φ_d : the ferromagnetic object magnetic moment.

A vector magnetometer can be used to obtain one set of magnetic field measurements for three components at its location; therefore, at least three independent vector magnetometers are needed to solve eq. (12), which has nine variables. In practice, multiple measurements and the least-square method are required to reduce errors. This study used four independent vector magnetometers. The method can also obtain the ambient geomagnetic field (B_{0x}, B_{0y}, B_{0z}) and hence eliminate the interference of fluctuation of the geomagnetic field.

We evaluated the performance of the proposed method by applying it to an actual magnetic dipole target in a real-world setting. A standard magnetic source 2 A m^2, magnetic sensors, and a fluxgate magnetometer Mag690 with an intrinsic noise less than 0.02 nT/Hz@1 Hz were used. In the experiment, the shape of the magnetic source is cuboid and its size is 36 mm×7 mm×7 mm. The distance between the magnetic source and the sensor is more than 1000 mm, which is more than three times the largest dimension of the magnetic source. In that situation, the magnetic source can be treated as a magnetic dipole.

The four fluxgate magnetometers were set up in a stereoscopic distribution and their distance are about 50 cm. We took the readings from the four independent vector magnetometers. The properties of the ferromagnetic object were calculated from the magnetic field measurements by solving the nonlinear equation set (12). The relative error vs. the distance between the ferromagnetic object and the origin for the estimated location and magnetic moment are shown in Figure 1(a) and (b), respectively. The relative error for the RMS location was 0.4106% and that for the RMS magnetic moment was 9.43%.

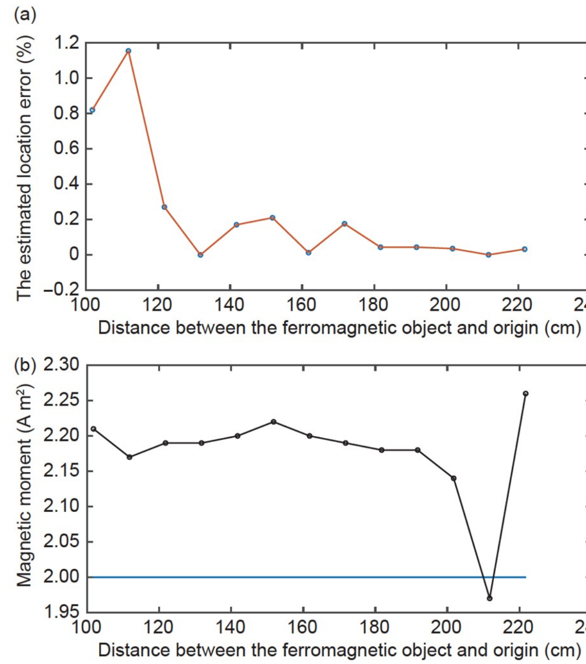

Figure 1 (a) Estimated location relative error vs. the distance. The RMS error is 0.4106%. (b) Estimated magnetic moment relative error vs. the distance. The RMS error is 9.43%.

From Figure 1, it is observed that the estimated location and magnetic moment are approximately the same as those of the defined values. Based on this, the effectiveness of the method can be confirmed. It can identify ferromagnetic objects in a three-dimensional space and meanwhile eliminate the interference of fluctuation of the geomagnetic field. Therefore, the method has a wide range of potential applications.

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