Erratum to: On Galvin's theorem for compact Hausdorff right-topological semigroups with dense topological centers

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In the third paragraph on p. 2421 and in Theorem 1' of [3] it is wrongly assumed that “the multiplication on a topological semigroup T can be extended to a right continuous multiplication on βT ”; in other words, we wrongly assume that “ βT is a compact Hausdorff right-topological semigroup for any topological semigroup T ”. So using [3, Theorem 1] we can only obtain the following two theorems in place of [3, Theorem 1'], where X' denotes the limit-point set of a space X and $J(E)$ denotes the set of all idempotents in a compact Hausdorff right-topological semigroup E .

Firstly by [3, Theorem 1] and [6] we can obtain the following instead of [3, Theorem 1'].

Theorem 1' (See [1, Lemma 2.1] and [4, Theorem 3.3] for I and II). *Let T be a discrete semigroup. Then the following statements hold:*

I. *Let $v \in (\beta T)' \cap J(\beta T)$ and let $\{V_n\}_{n=1}^\infty$ be a sequence of neighborhoods of v in βT . Then there exists a 1-1 map $\tau_\cdot: \mathbb{N} \rightarrow T$ such that $FP(\langle \tau_n \rangle_{n=k}^\infty) \subseteq V_k$ for all $k \geq 1$. In particular, any neighborhood of $v \in (\beta T)' \cap J(\beta T)$ contains an infinite IP-set in T .*

IIa. *For any IP-set $FP(\langle t_n \rangle_{n=1}^\infty)$ in T , there exists a $v \in J(\beta T)$ such that $v \in \bigcap_{k=1}^\infty \text{cls}_{\beta T} FP(\langle t_n \rangle_{n=k}^\infty)$.*

IIb. *Let H be a subsemigroup of T such that H' is a non-empty subsemigroup of βT . Then for any infinite IP-set $A \subseteq H$, there exists an idempotent $v \in (\beta T)' \cap \text{cls}_{\beta T} A$.*

III. *If T is weakly left cancelable, then $(\beta T)'$ is T -invariant and so $(\beta T)'$ is a left ideal in βT .*

Let G be a Hausdorff topological group with continuous multiplication

$$\lambda: G \times G \xrightarrow{(t,x) \mapsto tx} G$$

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and with identity $\mathbb{1}$. Let $\mathcal{U}_{\mathbb{1}}$ be the neighborhood system of $\mathbb{1}$ in G . Let $I = [0, 1]$ be endowed with the usual topology. A function $\varphi: G \rightarrow I$ is *left uniformly continuous*, denoted by $F_{\text{luc}}(G)$, if and only if given $\varepsilon > 0$ there exists a $U \in \mathcal{U}_{\mathbb{1}}$ such that $|\varphi(x) - \varphi(tx)| < \varepsilon$ for all $x \in G$ and $t \in U$. Define the evaluation map

$$e_{\text{luc}}: G \xrightarrow{x \mapsto (\varphi(x))_{\varphi \in F_{\text{luc}}(G)}} I^{F_{\text{luc}}(G)},$$

and let $G^{\text{LUC}} = \text{cls}_{I^{F_{\text{luc}}(G)}} e_{\text{luc}}(G)$, which is a compact Hausdorff subspace of the compact product space $I^{F_{\text{luc}}(G)}$.

G^{LUC} is called the *LUC-compactification* of G , which is a compact Hausdorff right-topological semigroup with respect to the canonical multiplication (see [2, 5, 7]). Then by [3, Theorem 1] we can obtain the following Theorem 1''.

Theorem 1''. *Let G be any Hausdorff topological group. Then*

(I) *Let $v \in (G^{\text{LUC}})' \cap J(G^{\text{LUC}})$ and let $\{V_n\}_{n=1}^{\infty}$ be a sequence of neighborhoods of v in G^{LUC} . Then there exists a 1-1 map $\tau: \mathbb{N} \rightarrow G$ such that $FP(\langle \tau_n \rangle_{n=k}^{\infty}) \subseteq V_k$ for all $k \geq 1$. In particular, any neighborhood of $v \in (G^{\text{LUC}})' \cap J(G^{\text{LUC}})$ contains an infinite IP-set in G .*

(IIa) *For any IP-set $FP(\langle t_n \rangle_{n=1}^{\infty})$ in G , there exists a $v \in J(G^{\text{LUC}})$ such that*

$$v \in \bigcap_{k=1}^{\infty} \text{cls}_{G^{\text{LUC}}} FP(\langle t_n \rangle_{n=k}^{\infty}).$$

(IIb) *Let H be a subsemigroup of G such that H' is a non-empty subsemigroup of G^{LUC} . Then for any infinite IP-set $A \subseteq H$, there exists an idempotent $v \in (G^{\text{LUC}})' \cap \text{cls}_{G^{\text{LUC}}} A$.*

(III) *$(G^{\text{LUC}})'$ is G -invariant and so $(G^{\text{LUC}})'$ is a left ideal in G^{LUC} .*

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