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Optimal Pricing and Return-Freight Insurance: Strategic Analysis of *E*-Sellers in the Presence of Reputation Differentiation*

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Abstract Motivated by the practice that e-sellers cooperate with insurance companies to offer consumers the return-freight insurance (RI), this paper aims to investigate the competing e-sellers' RI strategies. Regarding the information asymmetry in the online context, reputation system is widely applied by e-platforms. In an online market with two competing e-sellers that sell the same product but are differentiated in their reputation, this paper builds an analytical model to explore the e-sellers optimal pricing and RI strategies. Combined with sellers' reputation and their RI strategies, the equilibrium outcomes under four cases are discussed. This paper reveals the conditions that e-sellers are willing to offer RI. Specifically, the findings demonstrate that low reputation e-seller is more likely to offer RI. Moreover, when the sellers are more divergent, they are more likely to co-exist in the market. Insurance premium and RI compensation play critical roles in their decisions. RI introduction tends to increase the price, thus offsets the benefits of RI, but does not affect the total consumer surplus.

Keywords Game theory, product returns, reputation, return-freight insurance.

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1 Introduction

With the rapid development of information technology and the prevalent of Internet, consumers increasingly transfer from offline to online. According to the report of the United Nations[1], with the outbreak of COVID-19, the importance of online retail has become more prominent. In 2020, the global online retail sales reached 267,000 US dollars, accounting for 19% of the total retail sales all over the world. As the world's largest online retail market, in 2020, China's online sales increased by 10.9% from last year, reaching 11.76 trillion yuan, accounting for 24.9% of total retail sales, according to CNNIC[2].

Unlike traditional offline markets, consumers face more uncertainty and risk when shopping online because they cannot experience or touch products directly. Previous studies claimed that the reputation system is the main way to reduce the information asymmetry between consumers and sellers in the online marketplace, by indicating the performance of the sellers in their previous transactions[3]. Most e-platforms, e.g., Amazon, Taobao, eBay, adopted reputation system to help consumer to distinguish sellers. Besides, as one of the most popular way to to attract customers, in practice, the lenient customer return policy has been extensively provided by the online platforms, i.e., the money-back guarantee (MBG)[4]. For example, the e-sellers on Taobao have long offered a 7-day "no doubt" full refund return policy.

Although MBG allows consumers to return unsatisfactory products and obtain a full refund[5, 6], it cannot completely address potential consumer concerns in online shopping. Under the MBG policy, the return of goods due to product mismatch (rather than product defects) usually requires consumers to bear the return freight. According to the investigation of previous studies, the main reason for online product returns is that the product fails to meet the expectations and tastes of consumers[7], rather than product quality defects, especially for experience-based products. The average return rate of clothing and shoes is even as high as 40%[8]. Regarding the mismatch issue, the return shipping fee arising from the potential return has become an important obstacle for consumers when purchasing online[9]. In response to the problem that return freight costs still exist under the MBG, insurance companies in China provide a new type of Internet insurance, that is, online shopping return freight insurance, so-called return insurance (RI). Considering the potential risk in mismatch and the associated return freight costs, RI aims to reduce consumers return risk and increase consumers willingness to buy. Specifically, under RI, consumers can get refund of the return freight provided by the insurance company after the products are returned, i.e., return-freight compensation.

RI is an important strategy for e-sellers to increase their competitiveness in the online marketplace but providing RI does not necessarily bring benefits all the time. E-sellers providing RI will increase consumers' trust and willingness to buy, but it will also increase costs. In addition, the online marketplace is a competitive environment that different e-sellers selling the same products. Consumers make purchasing decisions by comparing the offers from different e-sellers on the same platform. Since the reputation is the cumulative of a seller's past performance, the existing seller with adequate reputation rating is more attractive to consumers, while a new seller who starts its business has a natural disadvantage of lacking reputation scores and difficult to survive in the e-platform. E-sellers with different reputations face an important question that whether they should provide a more generous return policy, i.e, RI. In other words, when the competitor provides RI, should the seller also provide it? The existing literature do not provide enough answers, so further investigation is necessary. Thus, to fill the research gap, this paper focuses on the potential basis for e-sellers to offer RI strategy. Specifically, this study aims to explore the following questions:

1) When should the differentiated e-sellers offer RI?

2) How does RI affect the pricing, demand, and profit?

3) How does RI adoption interact with the reputation mechanism?

To answer these questions, we develope an asymmetric duopoly three-stage game theory model in which two differentiated e-sellers sell the same product. Consumers' valuation of products is heterogeneous, and they make purchase decisions based on the information provided by the e-seller, that is, the seller's reputation level, price, and RI strategy. Assuming the reputation level is exogenous, the e-seller needs to decide whether to adopt RI and then set the corresponding price to maximize the profit. We assume that when consumers are dissatisfied with the purchased products, they can always return the products to the e-seller and get a full refund. Besides, we mainly consider the returns due to non-quality issues, i.e., product mismatching, in the case that the return shipping costs need to be borne by the consumers. When the seller provides the RI, the consumer can get the return-freight compensation, otherwise, the consumer should pay the return shipping fee themselves. We solve and compare the equilibrium results under different RI adoption configurations to explore the equilibrium RI adoption strategies of competing sellers under the background of the reputation mechanism in the e-commerce market.

This paper is organized as follows. The related literature review is provided in Section 2. Section 3 presents the model setup and the description of the problem. Section 4 presents the equilibrium results in different cases. Section 5 discusses the optimal RI strategy of sellers and the optimal pricing of the RI provider. Section 6 provides the conclusion and discussions.

2 Literature Review

In this study, we focus on the optimal pricing and RI strategies of the e-sellers with differentiated reputations. Then, this study naturally relates to three research streams: Consumer return, RI, and reputation system in the online context. We review each of these streams and demonstrate how our research contributes and joins them.

The reputation system is widely adopted by e-platforms to deal with information asymmetry issues in the online context. Previous studies claimed the signaling role of reputation, helps consumers distinguish trustworthy sellers[10–12]. Previous studies widely highlighted the importance of reputation in the online marketplace. For example, previous research indicated that sellers with high reputations tend to increase consumers perceived trust[10], purchase intention[13, 14] and willingness to pay[15, 16]. Therefore, reputation management has become one of the most critical parts of business management nowadays. However, previous research indicated the limitation of the reputation system, i.e., reputation system cannot diminish the information asymmetry between sellers and consumers[17]. The additional consumer protection programs need to be introduced in the e-marketplace to help increase trade efficiency and reduce the adverse selection[17].

In addition, the return problem is one of the most critical parts of business management[18]. Extant studies made lots of efforts on the optimal design of return policy under different conditions. The optimal return strategy between the retailer and the manufacturer has attracted the most attention in the offline context. From the perspective of supply chain management, previous research explored the parameter settings of the optimal return policy, e.g., coverage duration[19], restocking fee[20], hassle fee[6] as the decision variable. Besides, the impacts of the return policy on the operation strategy were widely explored, for example, pricing[21, 22], inventory[23, 24], sales performance[25] and supply chain coordination[26, 27].

In the online context, the return problem between the e-seller and the consumer attracted increasing attention from scholars in recent years[18]. As a kind of after-sale service, return policy plays an important role in consumers' purchasing decisions[28]. The generous return policies, e.g., MBG, increase consumers' willingness to buy[29]. Regarding information asymmetry issues in the online context, previous studies investigated the optimal design of return policy by assuming consumers have partial knowledge of their true valuation[30], consumer uncertainty about product attributes[20], the quality risk that some consumers value an appropriate product more than others[6]. In a quality uncertain context, the lenient return policy can signal the quality[31]. Besides, the impact of a lenient return policy on sellers performance has been widely explored. For example, a low-quality retailer has suggested benefits from a lenient return policy more than a high-quality retailer[32]. Based on the research on the conditions and the optimal setting of return policy, in practice, the e-platform generally drafts a standardized return policy for all e-sellers , e.g., a 7-day return guarantee in Taobao.com, and a 30-day return policy in Amazon.

Return insurance (RI) is a new type of Internet insurance that is generated for the return shipping costs of online returns[33]. Since RI emerged in recent years along with the serious return issues in online shopping, this important research topic is still largely unexplored. For example, most of the research is limited to the scenario of monopoly or homogeneous sellers. Previous studies explored the reasonable insurance premium and compensation in the context of consumers directly purchasing RI, considering the uncertainty of product fitting[34], and the impact of different RI strategies and compensation levels on retailers profits[35]. With the consideration of the uncertainty of online product fit and its impact on returns and insurance demand, the optimal pricing strategy including premium and compensation under online reviews was explored[34]. The optimal RI strategy for the e-seller under different sales formats, i.e., the reselling format and the agency selling format was investigated[36]. Regarding the RI pricing, previous research indicated the optimal policy of an insurance company is not affected whether the policy holder is an online retailer or a consumer[37]. In a market consisting of a manufacturer and an e-retailer, previous work indicated the introduction of RI raises the price level but not necessarily beneficial[38].

To sum up, previous studies mainly focused on the RI strategy in a monopolistic context without the consideration of asymmetric sellers' competition. In practice, a more critical and general problem is whether the competitive sellers should offer RI or not. Based on the literature mentioned above, our research combines RI and reputation mechanisms in the online context. Considering the information asymmetry in the e-commerce market and the signaling role of seller's reputation[3], it is meaningful to investigate the RI adoption strategies of the e-sellers differentiated in their reputation. In the competitive environment, the reputation level of e-seller will affect their pricing strategy as well as the RI adoption strategy. In other words, RI may have different effects on different types of e-sellers. The symmetric RI strategy of e-sellers discussed by previous studies may not be suitable for the competition of e-sellers under the reputation system in practice. It is of practical and theoretical significance to explore the relevant conditions of the RI strategies of the differentiated e-sellers and discuss the implications.

3 Model

In this paper, we consider the case that multiple e-sellers selling homogeneous products in an online market (e.g., Taobao.com). Specifically, we assume two e-sellers are differentiated in their reputation, one of them with a relatively high reputation ($i = H$), the other with a low reputation ($i = L$). Due to the information asymmetry, consumers tend to evaluate the product based on the seller's reputation level. As a positive signal, the high reputation has a positive role for consumers in inferring product quality[10−12]. Therefore, in our model, we assume that there is a value reduction for consumers if purchasing from a low reputation e-seller, represented by θ, $\theta \in (0, 1)$. Consumers have unit demand for the product and exhibit heterogeneous valuations for the product, i.e., from 0 to 1. Let v denote the consumer's valuation, which is uniformly distributed on [0, 1]. Thus, the consumers WTP for seller L is θv.

The baseline mismatch probability of the homogeneous product is represented by $1 - \alpha$. In other words, the baseline probability that the product satisfies the consumers is α, which is related to the feature of the product but irrelevant to the sellers. Besides, e-sellers selling the same product may have different satisfying rates due to the difference in the service management in practice. Thus, we suggest a high reputation seller with a satisfying rate α_H, and a low reputation seller with a satisfying rate α_L.

In the case of returning the mismatched product, the salvage value for the returned product is s, and each returned product generates a unit return handling cost h for the e-seller. The hassle cost is represented by f, including the return shipping cost. Besides, consumers are heterogeneous in their costs of time value and efforts when returning the product, without loss of generality, these costs normalized to zero. It worth noting that the consumer protection policies require e-sellers to cover all the expenses if the return is due to product quality issues. In other words, whether there is RI or not, if there are any quality issues, consumers can return the product with no cost. Therefore, we restrict our analysis context to the non-quality issues, i.e., product mismatching, that is, the situations that consumer needs to bear the return shipping cost (i.e., f).

Specifically, under the RI strategy, consumers can get a full refund, as well as the return-freight compensation m when returning the mismatched product. The insurance company set the RI price r to maximize the profit. The two sellers each decide the RI strategy (k_i), either offer RI strategy ($k_i = R$) or not ($k_i = N$). The RI strategy set of the high reputation seller and low reputation seller represent as $K = k_H k_L$, where there are four combinations, $K = \{NN, RR, RN, NR\}$.

3.1 Consumers

Given sellers RI strategies K and seller is price p_i^K, the utility of a consumer purchasing from seller i with valuation (v) is:

$$U_i^K = \alpha_i(\theta_i v - p_i^K) - (1 - \alpha_i)\ (f - m^{k_i}), \tag{1}$$

where

$$\theta_i = \begin{cases} 1, & i = H, \\ \theta, & i = L, \end{cases} \qquad m^{k_i} = \begin{cases} 0, & k_i = N, \\ m, & k_i = R. \end{cases}$$

The consumer purchases if the utility is non-negative. Besides, the consumer makes the purchase decision to maximize the utility, i.e., purchase from seller i who offers the higher utility. In our model, the price, reputation, and RI strategy are the factors that influence the consumers purchasing decisions. The first term in Equation (1) captures consumers expected utility if the purchase is satisfied and kept, i.e., $\alpha_i(\theta_i v - p_i^K)$. The second term, $(1 - \alpha_i)\ (f - m^{k_i})$, is consumer's expected disutility if the purchase is unsatisfied and returned. If a consumer purchase from the seller does not offer RI, the expected disutility is $(1 - \alpha_i)f$, when the seller offers RI, the expected disutility of a returned product will get return-freight compensation m, denote as $(1 - \alpha_i)\ (f - m)$.

Specifically, a consumer with valuation v will purchase from seller L if and only if $U_L^K \geq 0$ and $U_L^K \geq U_H^K$. We can derive the indifference values v_1^K and v_3^K by setting $U_L^K = 0$ and $U_L^K = U_H^K$, respectively. Similarly, the consumer will purchase from the seller H if and only if $U_H^K \geq 0$ and $U_H^K \geq U_L^K$, which gives the indifference values v_2^K and v_3^K by setting $U_H^K = 0$ and $U_H^K = U_L^K$, respectively. As consumers valuation, v is uniformly distributed from 0 to 1 and the market size is 1. Therefore, consumers with evaluation $v > v_3^K$ will purchase from seller H. The total demand of seller L and H are $D_L^K = v_3^K - v_1^K$ and $D_L^K = 1 - v_3^K$. If $v_3^K \leq v_1^K$, seller L is driven out of the market, if $v_3^K \geq 1$, seller H is driven out of the market. The two sellers would both exist in the market if $v_1^K \leq v_3^K < 1$. In our paper, we mainly focus on the case that two sellers coexist in the market. Figure 1 illustrates the consumer segments for the two e-sellers.

3.2 Sellers

Given the RI strategy set K, the two sellers make the pricing decisions to maximize their profits. The profit of seller i can be generalized as

$$\pi_i^K = (\alpha_i\ p_i^K + (1 - \alpha_i)(s - h) - c - r^{k_i})\ D_i^K, \tag{2}$$

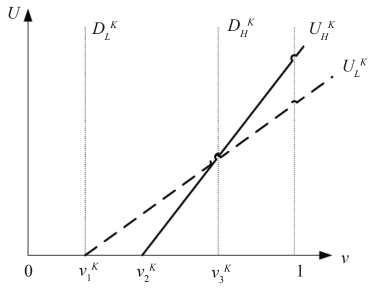

Figure 1 Consumers' segments

where

$$r^{k_i} = \begin{cases} 0, & k_i = N, \\ r, & k_i = R. \end{cases}$$

The term $\alpha_i\ p_i^K + (1-\alpha_i)(s-h) - c - r^{k_i}$ captures the margin profit of seller i in the case of K. If the seller selects to offer RI, the margin profit of the seller would decrease r. We define $Z_i^{k_i} = \alpha_i\theta_i + (1-\alpha_i)(s-h-f+m^{k_i}) - c - r^{k_i}$ as the maximum net product value shared by the seller i and the consumer with the highest net valuation, and $w_i^{k_i} = \alpha_i\theta_i - (1-\alpha_i)(f - m^{k_i})$ as the highest net valuation of the consumer ($v = 1$). That is to say, $Z_i^{k_i}$ is the summary of $w_i^{k_i}$ and seller i's margin profit, reflecting the efficiency of seller i in selling the product. $Z_i^{k_i}$ is referred to as the maximum net shared value of seller i[5]. In this paper, we present an extension of previous work by discussing RI strategies of sellers differentiated in their reputation. Besides, we define $\tau_i = Z_i^R - Z_i^N$ as the net value of offering RI for consumers. For both sellers, we have $\tau_i = (1-\alpha_i)\, m - r$. $C_i^{k_i} = c + r^{k_i} - (1-\alpha_i)\left(s - h - f + m^{k_i}\right)$ represents the expected cost of seller i selling a product with RI. We present the equilibrium results in different cases in the following subsections.

3.3 Insurance Provider

Given the RI strategy set of the duopoly K, the insurance company charges the RI price r for each insurance policy. The profit of the insurance company is denoted as

$$\pi_{provider}^K = (r - (1-\alpha_i)\, m - c_0)\left(\sum_{i=H,L} {M_i}^{k_i}\right), \tag{3}$$

where

$${M_i}^{k_i} = \begin{cases} 0, & k_i = N, \\ D_i^{k_i}, & k_i = R. \end{cases}$$

In our setting, insurance providers face uncertain demand. The above formula shows that the demand of the insurance provider is demonstrated by the equilibrium RI strategy of the duopoly. In the case that the equilibrium strategy of the two sellers is not offering, the demand of the insurance company is 0. Otherwise, if both sellers adopt RI, the demand of the insurance company is equal to the overall demand of the market. At this time, the overall demand level of the market is affected by many factors, such as the prices of the two sellers, the degree of difference in the reputation of the two sellers. Specifically, in our paper, the factors including e-commerce factors, i.e., product-fit uncertainty of online shopping as well. In the insurance companys profit formula, it can be observed that the insurance companys marginal revenue is the first part on the right, $r-(1-\alpha_i)\,m$, where $(1-\alpha_i)\,m$ is the product of the product benchmark return rate and return-freight compensation, which represents the expected cost of insurance provider. Besides, without loss of generality, we assume $c_0=0$.

3.4 Game Sequence

The game sequence is as follows. In the first stage, the insurance provider offers RI and decides the RI price. Then, the sellers decide whether to offer RI simultaneously in the second stage. After observing each others RI strategy, sellers simultaneously make their pricing decisions in the third stage. In the fourth stage, consumers make purchasing and possibly return decisions.

To examine how the sellers can use RI to gain an advantage, we analyze the duopoly case which starts with the benchmark that both sellers do not offer RI, i.e., $K=NN$. As the game involves multiple rounds of strategic interactions, we use sub-game perfect Nash equilibrium as our solution concept. We summarize our notations in Table 1.

Table 1 Summary of notations

Notations	Explanation
i	Index of sellers with different reputation, $i\in H,L$
k_i	Seller i's RI strategy, not offering RI, $k_i=N$; offering RI, $k_i=R$
K	Duopoly's RI strategy combination, where $K=k_Hk_L$
v	Consumer's valuation on the product
θ	The perceived value reduction of a low reputation seller
f	The return hassle cost, i.e., the return shipping fee
c	Unit cost of the product
r	The insurance premium for per unit purchase
m	The compensation for per unit return
α_i	The perceived matching probability when purchase from seller i
U_i^K	The utility of consumer purchasing from seller i
P_i^K	Price of seller i
D_i^K	Demand of seller i
π_i^K	Profit of seller i
$\pi_{provider}^K$	Profit of the insurance provider

4 Duopoly

4.1 Case NN

We start from the case $K=NN$ where neither seller offers RI strategy as the benchmark. According to the principle of profit and utility maximization, we applied backward induction to provide the following results.

Lemma 4.1 *When neither seller offers RI, the equilibrium retail prices are* $p_H^{NN*} = w_H^N/\alpha_H - (2Z_H^N + Z_L^N)/(4\alpha_H - \theta\alpha_L)$, $p_L^{NN*} = w_L^N/\theta\alpha_L - (2\alpha_H Z_L^N + \theta\ \alpha_L Z_H^N)/\theta\alpha_L(4\alpha_H - \theta\alpha_L)$*; the equilibrium market share are* $D_H^{NN*} = (2\alpha_H - \theta\alpha_L)(Z_H^N - t_1\ Z_L^N)/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$, $D_L^{NN*} = \alpha_H(t_2 Z_L^N - Z_H^N)/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$*; the equilibrium profits are* $\pi_H^{NN*} = (2\alpha_H - \theta\alpha_L)^2\left(Z_H^N - t_1\ Z_L^N\right)^2/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)^2$, $\pi_L^{NN*} = \theta\alpha_H\alpha_L\left(t_2 Z_L^N - \ Z_H^N\right)^2/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)^2$*, where* $t_1 = \alpha_H/(2\alpha_H - \theta\alpha_L)$, $t_2 = (2\alpha_H - \theta\alpha_L)/\theta\alpha_L$.

Lemma 4.1 clarifies the equilibrium results of seller H and L in the case NN. Note that guaranteeing two sellers coexist in the market, i.e., $D_H^{NN*} > 0$ and $D_L^{NN*} > 0$, requires the assumption that $t_1 < Z_H^N/Z_L^N < t_2$. Lemma 4.1 shows that the difference between Z_H^N/Z_L^N and t_1 is positively related to the profit of seller H, meanwhile, the difference between Z_H^N/Z_L^N and t_2 is positively associated with the profit of seller L. Moreover, with the increase of θ, the range between t_1 and t_2 shrinks.

4.2 Case RR

Next, we consider the case where both sellers offer RI to consumers (i.e., $K = RR$). Similarly, we obtain the consumer segmentation under this case by the maximizing utility of consumers and the optimal pricing of sellers. Thus, we have the equilibrium results present in Lemma 4.2.

Lemma 4.2 *When both sellers offer* RI*, the equilibrium prices are* $p_H^{RR*} = w_H^R\ /\alpha_H - (2Z_H^R + Z_L^R)/(4\alpha_H - \theta\alpha_L)$, $p_L^{RR*} = w_L^R/\theta\alpha_L - (2\alpha_H Z_L^R + \theta\ \alpha_L Z_H^R)/\theta\alpha_L(4\alpha_H - \theta\alpha_L)$*; the equilibrium market share are* $D_H^{RR*} = (2\alpha_H - \theta\alpha_L)(Z_H^R - t_1\ Z_L^R)/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$, $D_L^{RR*} = \alpha_H(t_2 Z_L^R - Z_H^R)/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$*; the equilibrium profits are* $\pi_H^{RR*} = (2\alpha_H - \theta\alpha_L)^2(Z_H^R - t_1\ Z_L^R)^2/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)^2$, $\pi_L^{RR*} = \theta\alpha_H\alpha_L(t_2 Z_L^R - \ Z_H^R)^2/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)^2$.

Similarly, we find guaranteeing two sellers coexist in the market requires the assumption that $t_1 < Z_H^R/Z_L^R < t_2$. In the case RR, the sales price p_i^{RR*} increases with the insurance premium r and the compensation m. Since the insurance premium increases sellers operating costs, sellers who offer RI tend to increase the price. Besides, the higher compensation that consumers can get if they return the mismatched product, the higher the price consumers are willing to pay. Therefore, the optimal price of sellers increases with insurance premium r and the compensation m. Comparing with the benchmark that neither seller offers RI strategy, in the case RR, we have Proposition 4.3 as follows.

Proposition 4.3 *Compared with the benchmark NN where neither seller offers* RI*, in the case RR,* (i) *seller i charges a higher price,* $p_i^{RR*} > p_i^{NN*}$*;* (ii) *the market share increases, i.e.,* $D_i^{RR*} \geq D_i^{NN*}$*, if* $\tau_i \geq 0$*, and vice versa.*

Proposition 4.3 represents the comparison result between the equilibrium when both sellers provide RI and the equilibrium when neither. We see that both sellers would charge a higher price when offering RI. The risen price can be explained by two aspects, the increase in the cost and the competitiveness. Compared with the case that neither sellers offer RI, in the case of RR, the market competition becomes more intensive. With the price increasing, consumers with a relatively lower evaluation of the product will not purchase, which makes the total market share decline.

4.3 Case RN

Consider the asymmetric case where the high reputation seller H offers RI while the low reputation seller L does not. The equilibrium results are derived and presented in Lemma 4.4.

Lemma 4.4 *When the strategy set is $K = RN$, the equilibrium retail prices are* $p_H^{RN*} = w_H^R/\alpha_H - (2Z_H^R + Z_L^N)/(4\alpha_H - \theta\alpha_L)$, $p_L^{RN*} = w_L^N/\theta\alpha_L - (2\alpha_H Z_L^N + \theta\alpha_L Z_H^R)/\theta\alpha_L(4\alpha_H - \theta\alpha_L)$*; the equilibrium market share are* $D_H^{RN*} = (2\alpha_H - \theta\alpha_L)(Z_H^R - t_1 Z_L^N)/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$, $D_L^{RN*} = \alpha_H(t_2 Z_L^N - Z_H^R)/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$*; the equilibrium profits are* $\pi_H^{RN*} = (2\alpha_H - \theta\alpha_L)^2(Z_H^R - t_1 Z_L^N)^2/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)^2$, $\pi_L^{RN*} = \theta\alpha_H\alpha_L(t_2 Z_L^N - Z_H^R)^2/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)^2$.

In this case, we find two sellers coexist in the market requires the assumption that $t_1 < Z_H^R/Z_L^N < t_2$. The optimal price of seller H who offers the RI increases with insurance premium r and compensation m. Meanwhile, the optimal price of seller L who does not offer the RI increases with insurance premium r but decreases with compensation m. Compared with the benchmark that neither seller offers RI strategy, in the case RN, seller H offers RI while seller L does not, we have

Proposition 4.5 *Compared with the benchmark NN where neither seller offers* RI*, in the case RN,* (i) *seller H charges a higher price,* $p_H^{RN*} > p_H^{NN*}$*, seller L charges a lower price,* $p_L^{RN*} \le p_L^{NN*}$*;* (ii) *the market share of seller H increases, i.e.,* $D_H^{RN*} \ge D_L^{NN*}$*, the market share of seller L decreases,* $D_L^{RN*} \le D_L^{NN*}$ *if* $\tau_H \ge 0$.

Proposition 4.5 shows that seller H offers RI tend to charge a higher price to obtain the maximum profit, it's worth mention that its operating costs increased as well. Meanwhile, seller H enjoys more demands if $Z_H^R \ge Z_H^N$, since RI decreases consumers perceived risk and attracts more consumers. On the other hand, seller L does not offer RI tends to decrease its price and enjoys fewer demands. It's worth noting that the profit of seller H may not be better off since the increased cost tends to lower the margin profits.

4.4 Case NR

Finally, we consider the asymmetric case where only the low reputation seller L offers RI to consumers. The equilibrium results are derived and presented in Lemma 4.6.

Lemma 4.6 *When the strategy set is NR, the equilibrium retail prices are* $p_H^{NR*} = w_H^N/\alpha_H - (2Z_H^N + Z_L^R)/(4\alpha_H - \theta\alpha_L)$, $p_L^{NR*} = w_L^R/\theta\alpha_L - (2\alpha_H Z_L^R + \theta\ \alpha_L Z_H^N)/\theta\alpha_L(4\alpha_H - \theta\alpha_L)$*; the equilibrium market share are* $D_H^{NR*} = (2\alpha_H - \theta\alpha_L)(Z_H^N - t_1\ Z_L^R)/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$, $D_L^{NR*} = \alpha_H(t_2 Z_L^R - Z_H^N)/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$*; the equilibrium profits are* $\pi_H^{NR*} = (2\alpha_H - \theta\alpha_L)^2\left(Z_H^N - t_1\ Z_L^R\right)^2/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)^2$, $\pi_L^{NR*} = \theta\alpha_H\alpha_L\left(t_2 Z_L^R - Z_H^N\right)^2/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)^2$.

In the case $K = NR$, the equilibrium results as shown in Lemma 4.6, two sellers coexist in the market requires the assumption that $t_1 < Z_H^N/Z_L^R < t_2$. Similarly, the optimal price of seller L increases with the insurance premium r. Comparing with the benchmark that neither seller offers RI strategy, in the case NR, we have

Proposition 4.7 *Compared with the benchmark NN where neither seller offers* RI*, in the case NR,* (i) *seller L charges a higher price,* $p_L^{RR*} > p_L^{NN*}$*, seller H charges a lower price,* $p_H^{RR*} \leq p_H^{NN*}$ (ii) *the market share of seller L increases, i.e.,* $D_L^{NR*} \geq D_L^{NN*}$*, the market share of seller H decreases,* $D_H^{NR*} \leq D_H^{NN*}$ *if* $\tau_L \geq 0$.

Proposition 4.7 shows that seller L offers RI tends to price higher and enjoys more demands, while seller H tends to price lower and obtain fewer demands. Since the high reputation seller tends to enjoy a price premium, regardless of RI. In the case of NR, seller L price higher and seller H price lower, which reduces the differentiation between sellers caused by their reputation. For all the cases, we find a seller who offers RI tends to charge a higher price.

5 Comparison Analysis

5.1 Optimal RI Strategy

In this section, we discuss the equilibrium of RI adoption strategies of two sellers. We define $e^K = Z_H^{k_H}/Z_L^{k_L}$ as seller H's efficiency of selling the product relative to seller L. Calculating the conditions that both sellers exist in the market, we have the following result.

Proposition 5.1 *For any given* RI *strategy combination K, the two differentiated sellers coexist in the market if and only if* $t_1 \leq \mathrm{e}^K \leq t_2$.

Proposition 5.1 presents the condition for the two differentiated sellers to coexist, i.e., $D_H^{K*} \geq 0$ and $D_L^{K*} \geq 0$. e^K depends on the differentiation in the efficiency in selling the product between two sellers. Lemma 4.1 implies that the two differentiated sellers can coexist if the efficiency of seller H selling the product relative to that of seller L selling the product is comparable. Besides, seller H will be driven out of the market if $e^K < t_1$; seller L will be driven out of the market if $e^K > t_2$. Since the boundary values t_1 and t_2 depend on the feature difference between two sellers, the difference in sellers reputation θ and their return rate, α_L, and α_H. In this work, we focus on the case when two sellers are in sustained competition, that is, they can both exist in the market. It is interesting for us to find that, with the increase of θ, t_1 increases while t_2 decreases, that is, the increase in sellers difference in their reputation would narrow the boundary. Similarly, the difference in sellers return rate would narrow the scope. Under different cases, we can category the equilibrium as follow. Comparing the profit of sellers in different cases, we obtain the proposition as follows.

Proposition 5.2 *For sellers H and L, the* RI *strategy represents as*: $k_i = R$ *if* $\tau_i \geq 0$; $k_i = N$ *if* $\tau_i < 0$. *Specifically, the equilibrium of the duopoly is* (i) $K = RR$*, if* $\tau_H \geq 0$*;* (ii) $K = NN$ *if* $\tau_L < 0$*;* (iii) $K = NR$ *if* $\tau_L \geq 0$ *and* $\tau_H < 0$.

Proposition 5.2 shows that in a duopoly market, offering RI is an optimal strategy for both sellers, if the net value of offering RI is non-negative, i.e., $\tau_i \geq 0$, no matter what strategy is offered by the competitor. If $\tau_H \geq 0$, i.e., $r^* \leq (1 - \alpha_H)m$, it is beneficial for both sellers to offer the RI to consumers, otherwise, if $\tau_L < 0$, it is optimal for them to not adopt RI. Besides, if $\tau_L \geq 0$ and $\tau_H < 0$, it is optimal for the low reputation seller L to offer RI not the high reputation seller H, which will lead to the equilibrium $K = NR$, both sellers will not offer RI. Therefore, if it is beneficial for seller H to offer RI, then seller L could always obtain more profit to adopt RI. Moreover, if it is not beneficial for seller L to offer RI, then seller H should not offer RI as well. Considering the case for both sellers to offer RI, it is easy for us to reveal Corollary 5.3 that seller H is less likely to offer RI compared with seller L.

Corollary 5.3 *Low reputation seller L is more likely to offer* RI.

5.2 Optimal Pricing of RI Provider

Now, we take the insurance company into account. When the market equilibrium is RR, both sellers provide RI, and the insurance companys sales are $D_H^{RR*} + D_L^{RR*}$; and when the market equilibrium is NR, only low-reputation seller L provides RI, the sales of the insurance company are D_L^{NR*}; the insurance company sets insurance premium r to maximize its profit, as we mentioned before. Therefore, the results are present as follows.

Proposition 5.4 *For $K = RR$, the optimal price of* RI *is* $r^{RR*} = m(1 - \frac{(2\alpha_H+\theta\alpha_H)\alpha_L}{2\alpha_H+\theta\alpha_L}) + \frac{3\alpha_H\alpha_L\theta(s-f-h)+\theta\alpha_L Z_H^N+2\alpha_H Z_L^N}{4\alpha_H+2\theta\alpha_L}$*; for $K = NR$, the optimal* RI *price is* $r^{NR*} = m(1 - \alpha_L) + \frac{(s-f-h)(\theta\alpha_L(\alpha_L-\alpha_H)-2\alpha_H(1-\alpha_L))+2\alpha_H Z_L^N-2\theta\alpha_L Z_H^N}{4\alpha_H-2\theta\alpha_L}$.

Proposition 5.4 shows the optimal pricing of the insurance provider under different cases. According to Proposition 5.2, there are three possible outcomes, i.e., RR, NN, and NR. Besides, when the equilibrium is NN, there is no pricing problem for the insurance provider since neither offer RI. Therefore, insurance provider makes their pricing decisions under two cases, i.e., RR and NR. In addition, the insurance company provides RI only when their marginal profit is non-negative, that is, $r^* \geq (1-\alpha_i)\,m$ needs to be met. As we mentioned before, the condition that sellers would adopt RI is $\tau_i \geq 0$, that is, $(1-\alpha_H)\,m - r^* \geq 0$ needs to be met. Therefore, we have the highest price of an insurance provider which should be $r^* = (1-\alpha_i)\,m$. Specifically, when the price is $r^* = (1-\alpha_H)\,m$, both sellers will choose to provide RI. However, the expected profit margin of the insurance provider is negative. When the price is $r^* = (1-\alpha_L)\,m$, only low-reputation sellers will choose to provide RI which is not provided by high-reputation sellers, the expected profit margin of the insurance company is 0. Therefore, when the insurance company charges the same price for the two differentiated sellers, the equilibrium result of the insurance premium is represented as $r^* = (1-\alpha_L)\,m$. In this case, it is optimal for the low-reputation seller to provide RI, while the high-reputation seller does not. In addition, it is worth mentioning that the insurance provider can charge the seller separately. The optimal pricing of the insurance company is $r_i = (1-\alpha_i)\,m$. At this time, the insurance company's profit is 0, and the consumer surplus value does not increase either. In practice, the insurance provider charges e-sellers individually according to the return rates.

Considering a more general situation, for the homogeneous product, there is a benchmark return rate α. Sellers differentiated in their reputation may result in different return rates. For example, the expectation of the product of a high reputation may be higher than that of a low reputation seller. We assume that $\alpha_H = \alpha + \sigma$ and $\alpha_L = \alpha - \sigma$, i.e., $\alpha_L < \alpha < \alpha_H$.

According to our analysis, the insurance premium is thus $r^* = (1-\alpha)\,m$. Compared with the benchmark case, when both sellers choose to offer RI, the profit of the high-reputation seller decreases, while the low-reputation seller's profit increases. In the long term, seller H will not offer RI. In addition, when $t1 < e^K < \frac{2\alpha_H^2}{\theta\alpha_L(3\alpha_H-\theta\alpha_L)}$, the RI provider's profit is positive, when $\frac{2\alpha_H^2}{\theta\alpha_L(3\alpha_H-\theta\alpha_L)} < e^K < t2$, the profit of the RI provider is negative.

It's worth noting that when the insurance provider's price is $r^* = (1-\alpha)\,m$, seller i chooses to offer RI will not increase its market share and profit. This may be because when seller i provides RI, the cost increases, which requires an increase in sales prices to prevent some consumers with a lower perceived value from buying. Although RI reduces the perceived risk of consumers which tends to increase the demand, the increase in selling prices reduced the market share in the meanwhile. Thus, if the insurance premium equals the expected insurance compensation, sellers profit remains the same regardless of RI adoption or not. In addition, when the insurance company's pricing is $r^* = (1-\alpha)\,m$, the total market share remains the same.

6 Discussion and Conclusion

To attract customers and dispel their purchasing concerns, e-sellers have innovative introduced return freight insurance policies in practice. Specifically, e-sellers and the insurance company cooperate to provide insurance against reverse logistics, i.e., RI. Considering the competition of selling the same product in the e-platform, should RI be adopted is a critical problem faced by e-sellers in practice. As a competitive strategy, our article explores the conditions of RI adoption for e-sellers differentiated in their reputation. We explore competing sellers RI adoption equilibrium strategies and the pricing strategy of the insurance provider.

This paper contributes to the literature from several aspects. First, by extracting the key parameters of each seller in the asymmetric duopoly structure, we were able for the first time to obtain simple conditions for the two e-sellers to coexist under all combinations of pricing and service strategies: Coexistence only depends on the two sellers relative maximum net share value. Under different pricing and RI strategy selection combinations, we can determine the conditions for the coexistence of the two sellers based on the key parameters of the merger of the two sellers, that is, the boundary value formed by the difference in the reputation of the seller and the difference in the return rate. As far as we know, the existing literature only focuses on the purchase of RI by single or symmetrical e-sellers, and without the consideration of the differences in the level of reputation of e-sellers. This study provides new insights on how the two sellers choose the RI strategy equilibrium and the RI provider's best pricing to compete in pricing and customer return strategies. The results indicate that the low-reputation seller is more likely to offer RI. We find that the total consumer surplus does not increase when the seller offers RI. Through exploration in practice, it can be observed that many sellers have given up on adopting this policy, while some sellers have been offering it. Recently, in response to many sellers giving up on providing RI, Taobao has adjusted the pricing of RI. Our results are consistent with practice and provide new insights on the pricing strategy of the RI provider.

The optimal strategy for insurance providers depends on the difference in seller return rate as well as their reputation. This study extends the understanding of the RI strategy of sellers when competing in the same platform selling the homogeneous product.

Our study can be extended in several ways. First, future work can examine the relationship between RI strategy and online reviews, i.e., how the RI approach dynamically changes sellers' reputation ratings. Second, this research can be expanded to look into the case where consumers can obtain RI on their own. Third, future research could look into extensions that take multiple sellers' competition into account in a hypercompetitive context. Moreover, empirical studies can be employed to investigate how the RI strategy affects consumers' purchase decisions in practice.

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Appendix

Proof of Lemma 4.1

Since consumers purchase if the utility is non-negative, given $U_L^{NN}=0$, we have $v_1^{NN}=\frac{f-f\alpha_L+p_L^{NN}\alpha_L}{\theta\alpha_L}$; given $U_H^{NN}=0$, we have $v_2^{NN}=\frac{f-f\alpha_H+p_H^{NN}\alpha_H}{\alpha_H}$; given $U_L^{NN}=U_H^{NN}$, we have $v_3^{NN}=\frac{-f\alpha_H+p_H^{NN}\alpha_H+f\alpha_L-p_L^{NN}\alpha_L}{\alpha_H-\theta\alpha_L}$. Accordingly, we have $\pi_H^{NN}=(1-v_3^{NN})((\alpha_H-k)p_H^{NN}-c+(1-\alpha_H)(s-h))$ and $\pi_L^{NN}=(v_3^{NN}-v_1^{NN})((\alpha_L-k)p_L^{NN}-c+(1-\alpha_L)(s-h))$. Calculating $\frac{\partial\pi_H^{NN}}{\partial p_H^{NN}}=0$ and $\frac{\partial\pi_L^{NN}}{\partial p_L^{NN}}=0$, we can easily have the optimal pricing strategies and the equilibrium results.

Proof of Lemma 4.2

Given $U_L^{RR}=0$, $U_H^{RR}=0$ and $U_L^{RR}=U_H^{RR}$, we have $v_1=\frac{f-m-f\alpha_L+m\alpha_L+p_L^{RR}\alpha_L}{\theta\alpha_L}$, $v_2=\frac{f-m-f\alpha_H+m\alpha_H+p_H^{RR}\alpha_H}{\alpha_H}$ and $v_3=\frac{-f\alpha_H+m\alpha_H+p_H^{RR}\alpha_H+f\alpha_L-m\alpha_L-p_L^{RR}\alpha_L}{\alpha_H-\theta\alpha_L}$. Accordingly, $\pi_H^{RR}=(1-v3)((\alpha_H-k)p_H^{RR}-c-r+(1-\alpha_H)(s-h))$, $\pi_L^{RR}=(v3-v1)((\alpha_L-k)p_L^{RR}-c-r+(1-\alpha_L)(s-h))$. Letting $\frac{\partial\pi_H^{RR}}{\partial p_H^{RR}}=0$ and $\frac{\partial\pi_L^{RR}}{\partial p_L^{RR}}=0$, we can easily have the optimal pricing strategies and the equilibrium results.

Proof of Proposition 4.3

Comparing p_i^{RR*} with p_i^{NN*}, we can have $p_H^{RR*}-p_H^{NN*}=(w_H^R-w_H^N)/\alpha_H-(2(Z_H^R-Z_H^N)+(Z_L^R-Z_L^N))/(4\alpha_H-\theta\alpha_L)$, and $p_L^{RR*}-p_L^{NN*}=(w_L^R-w_L^N)/\theta\alpha_L-(2\alpha_H(Z_L^R-Z_L^N)+\theta\,\alpha_L(Z_H^R-Z_H^N))/\theta\alpha_L(4\alpha_H-\theta\alpha_L)$, therefore, we can easily get $p_i^{RR*}>p_i^{NN*}$. Similarly, comparing D_i^{RR*} with D_i^{NN*}, we have $D_H^{RR*}-D_H^{NN*}=(2\alpha_H-\theta\alpha_L)((Z_H^R-Z_H^N)-t_1(Z_L^R-Z_L^N))/(\alpha_H-\theta\alpha_L)(4\alpha_H-\theta\alpha_L)$ and $D_L^{RR*}-D_L^{NN*}=\alpha_H(t_2(Z_L^R-Z_L^N)-(Z_H^R-Z_H^N))/(\alpha_H-\theta\alpha_L)(4\alpha_H-\theta\alpha_L)$. Thus, we have $D_i^{RR*}\geq D_i^{NN*}$ if $Z_i^R\geq Z_i^N$.

Proof of Lemma 4.4

Given $U_L^{RN}=0$, $U_H^{RN}=0$ and $U_L^{RN}=U_H^{RN}$, we have $v_1^{RN}=\frac{f-f\alpha_L+p_L^{RN}\alpha_L}{\theta\alpha_L}$, $v_2^{RN}=\frac{f-m-f\alpha_H+m\alpha_H+p_H^{RN}\alpha_H}{\alpha_H}$, and $v_3^{RN}=\frac{-m-f\alpha_H+m\alpha_H+p_H^{RN}\alpha_H+f\alpha_L-p_L^{RN}\alpha_L}{\alpha_H-\theta\alpha_L}$, respectively. Then, we have $\pi_H^{RN} = (1 - v_3^{RN})((\alpha_H - k)p_H^{RN} - c - r + (1 - \alpha_H)(s - h))$ and $\pi_L^{RN} = (v_3^{RN} - v_1^{RN})((\alpha_L - k)p_L^{RN} - c + (1 - \alpha_L)(s - h))$. Calculating $\frac{\partial \pi_H^{RN}}{\partial p_H^{RN}} = 0$ and $\frac{\partial \pi_L^{RN}}{\partial p_L^{RN}} = 0$, we can easily have the optimal pricing strategies and the equilibrium results in the case RN.

Proof of Proposition 4.5

Comparing p_i^{RN*} with p_i^{NN*}, we can have $p_H^{RN*} - p_H^{NN*} = (w_H^R - w_H^N)/\alpha_H - 2(Z_H^R - Z_H^N)/(4\alpha_H - \theta\alpha_L) > 0$, $p_L^{RN*} - p_L^{NN*} = -(\theta\alpha_L(Z_H^R - Z_H^N))/\theta\alpha_L(4\alpha_H - \theta\alpha_L)$, therefore, we can easily get $p_H^{RN*} > p_H^{NN*}$, $p_L^{RN*} < p_L^{NN*}$. Similarly, comparing D_i^{RR*} with D_i^{NN*}, we have $D_H^{RN*} - D_H^{NN*} = (2\alpha_H - \theta\alpha_L)(Z_H^R - Z_H^N)/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$ and $D_L^{RN*} - D_L^{NN*} = \alpha_H(Z_H^N - Z_H^R)/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$, thus, we have $D_H^{RN*} \geq D_L^{NN*}$ and $D_L^{RN*} \leq D_L^{NN*}$ if $Z_H^R \geq Z_H^N$.

Proof of Lemma 4.6

Given $U_L^{NR} = 0$, $U_H^{NR} = 0$ and $U_L^{NR} = U_H^{NR}$, we have $v_1^{NR} = \frac{f - m - f\alpha_L + m\alpha_L + p_L^{NR}\alpha_L}{\theta\alpha_L}$, $v_2^{NR} = \frac{f - f\alpha_H + p_H^{NR}\alpha_H}{\alpha_H}$, and $v_3^{NR} = \frac{m - f\alpha_H + p_H^{NR}\alpha_H + f\alpha_L - m\alpha_L - p_L^{NR}\alpha_L}{\alpha_H - \theta\alpha_L}$. Thus, $\pi_H^{NR} = (1 - v_3^{NR})((\alpha_H - k)p_{HNU} - c + (1 - \alpha_H)(s - h))$ and $\pi_L^{NR} = (v_3^{NR} - v_1^{NR})((\alpha_L - k)p_{LNU} - c - r + (1 - \alpha_L)(s - h))$.Calculating $\frac{\partial \pi_H^{NR}}{\partial p_H^{NR}} = 0$ and $\frac{\partial \pi_L^{NR}}{\partial p_L^{NR}} = 0$, we can easily have the optimal pricing strategies and the equilibrium results in the case NR.

Proof of Proposition 4.7

Comparing p_i^{NR*} with p_i^{NN*}, we have $p_H^{NR*} - p_H^{NN*} = (Z_L^N - Z_L^R)/(4\alpha_H - \theta\alpha_L)$, and $p_L^{NR*} - p_L^{NN*} = (w_L^R - w_L^N)/\theta\alpha_L - (2\alpha_H(Z_L^R - Z_L^N))/\theta\alpha_L(4\alpha_H - \theta\alpha_L)$; comparing D_i^{NR*} with D_i^{NN*}, we have $D_H^{NR*} - D_H^{NN*} = (2\alpha_H - \theta\alpha_L)t_1(Z_L^N - Z_L^R)/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$ and $D_L^{NR*} - D_L^{NN*} = \alpha_H(t_2(Z_L^R - Z_L^N))/(\alpha_H - \theta\alpha_L)(4\alpha_H - \theta\alpha_L)$, thus, we have $p_L^{NR*} > p_L^{NN*}$, $p_H^{NR*} \leq p_H^{NN*}$, $D_L^{NR*} \geq D_L^{NN*}$, and $D_H^{NR*} \leq D_H^{NN*}$ if $Z_L^R \geq Z_L^N$.

Proof of Proposition 5.1

The e-sellers would coexist in the market if $D_H^{K*} > 0$ and $D_L^{K*} > 0$. For any given K, by calculating D_H^{K*} and D_L^{K*}, we can easily obtain the conditions.

Proof of Proposition 5.2

Compared the profit of seller H and seller L under different cases, Proposition 5.2 can easily be proved by matching the conditions between Proposition 4.3, Proposition 4.5 and Proposition 4.7. The detail is omitted.

Proof of Corollary 5.3

Corollary 5.3 can be easily be proved by Proposition 5.2. The detail is omitted.

Proof of Proposition 5.4

Given the optimal strategies of seller H and seller L, the RI provider set price to maximum the profit. If $K = NR$, we have $\pi_{provider}^{NR} = (r - (1 - \alpha_L)\, m)\, D_L^{NR}$, if $K = RR$, we have $\pi_{provider}^{RR} = (r - (1 - \alpha_L)\, m)\, (D_H^{RR} + D_L^{RR})$. By calculating $\frac{\partial \pi_p rovider^{NR}}{\partial r^{NR}} = 0$ and $\frac{\partial \pi_p rovider^{RR}}{\partial r^{RR}} = 0$, we can easily get the results.