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Illiquidity Comovement and Market Crisis*

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Abstract This paper presents a rational expectation equilibrium model to explore how the financial contagion occurs between the unlinked markets that do not share common fundamentals. In the proposed model, the authors assume two of the three risky assets share no common fundamental factors, but are connected by one intermediate asset via cross fundamentals. Through this channel, investors transmit fundamental risk from one asset to another by dint of the cross fundamentals. This mechanism causes liquidity comovement and subsequently becomes a source of market crisis: Through the contagion mechanism, an initial liquidity shock in one asset can result in a drop tendency in liquidity and price informativeness for another asset. Such comovement in liquidity offers a new explanation for idiosyncratic assets in financial contagion.

Keywords Contagion, crisis, illiquidity, rational expectation equilibrium.

1 Introduction

The outbreak of COVID-19 has significantly shaken the global financial markets, triggering a spate of crises. On Mar 12th, The S&P and Dow Index suffered their biggest one-day falls, down by more than 9% since the financial crisis in 1987. Also, the European countries have experienced tremendous drops, with Britain's FTSE falling 9.81%, Germany's DAX off 11.42% and France's CAC dropping 12.28%. Inescapably, high market uncertainty superposition resulted in the market plummet in Latin American countries. Two days later, in Brazil, the

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Sao Paulo stock Exchange index plunged 14.78%, the biggest one-day drop in 21 years; Chile's Santiago stock market IPSA index dropped 6.33 percent on Monday, the biggest one-day drop in nine years. Interestingly, the crisis usually accompanies with financial contagion where prices experience downward movement. As shown in Yuan[1], large downward price movements are contagious. Undoubtedly, financial contagion can not only occur in local markets, but also spread to the global financial markets owning to its highly contagion effect. It seems that prices contagion in one market is plausible for explaining the contagion among closely linked markets since they share similar macroeconomic risks[2, 3], such as the markets in developed countries. However, these contributions can not completely account for the cross-regional contagion if markets are not closely linked[4], for example, the emerging countries like South East Asia and Latin America.

Although it is challenging to figure out how many detailed factors that affect the price movement, there are two widely acknowledged factors that influence the asset price undoubtedly, i.e., the fundamental and the noise[5]. The fundamentals represent the macroeconomic factors, such as international trade, foreign direct investment, or the value factors of one firm. While the noise traders usually function as the liquidity provider. An interesting question to ask is then, what was the driving force behind this transmission of negative shocks from one country to another especially for the countries that are not linked? Was it liquidity shock driven? Perhaps liquidity shock alone cannot completely account for the cross-sectional pattern of contagion. When hit with a liquidity shock, market participant would like to liquidate their positions in developed countries since highly liquid markets can attenuate the impact of sell orders on prices, not emerging markets[6]. Hence, the contagion mechanism between the developed countries and the emerging countries still needs to be further clarified.

In general, assets with related macroeconomic fundamentals may experience similar volatility tendency. Intuitively, it seems that assets from the unlinked countries cannot be directly affected from the developed countries via the fundamental since their fundamentals are different. However, the market crisis during COVID-19 turns out actually the contagion exists. In this paper, we try to derive this result by introducing a third intermediate that bridges the unlinked countries from the view of fundamental.

To construct the contagion model, it is necessary to seek one intermediate that connects the two types of unlinked countries. To this end, this intermediate must function as the bridge that interact with the two unlinked countries at the same time. Specifically, we specify one asset (for example, the market index) that represents one country and analyze the comovement of the three assets. In our model, asset A and asset C do not share any common fundamental factor, instead, asset A and asset B share an exposure to one fundamental risk, and asset B and asset C share another fundamental risk. We therefore refer to Grossman and Stiglitz[7]'s model and extend it to a three-risky-asset setting where traders can learn information from other asset prices. This mechanism prompts the information transmission from one asset to another because prices tend to aggregate all the differential information in equilibrium. In other words, cross-asset learning helps the investors know more about whole markets and reduces the uncertainty about fundamentals, however, it can also propagate the fundamental risk by means of prices.

We discover that, in this mechanism, liquidity displays positive comovement effect among the three assets. That is, an initial drop in liquidity for one asset will give rise to the same plight for the intermediate asset, and eventually, it can cause liquidity shortage for the third asset. Similarly, the comovement effect also happens for price informativeness. In other words, as the noise trading volume increases in one asset, the price informativeness tends to be low for that asset, which can, through cross-asset learning, induce augmented noise trading risk and thus results in low price informativeness for another asset.

Furthermore, our model shows that there exists a negative interaction between illiquidity and price informativeness associated with cross-asset learning. That is, an increase in illiquidity lowers price informativeness and vice versa. In reality, assets values may be affected by a series of common fundamentals. This implies an initial liquidity shock of one asset can propagate to other assets and be amplified through mutual interaction. Hence, accompanying with the financial contagion across the assets, the whole market may be in low price informativeness and high liquidity risk, leaving the market in crisis.

Our paper is related to the burgeoning rational expectation equilibrium (REE) models about multiple assets, in papers that feature such models[2, 3, 8], rational traders can use the price information to update their expectations. Goldstein, et al.[2] extended Grossman and Stiglitz[7]'s model to segmented markets and assume price information can flow across markets. Cespa and Foucoult[3] supposed that price information is available to dealers across the markets. While in our model, we retain the assumption that price information is accessible among investors, and distinctively we focus on the process of price contagion from one asset to another and its potent insights for market crisis.

Numerous theoretical and empirical literature has greatly contributed to the study about the price comovement effect within markets. Papers about such research findings mainly focus on the contagion through a correlated information or liquidity shock[1, 3, 9]. Our paper is also related to the extensive works about contagion among financial markets[4, 10–12]. Allen and Gale[10] provided contagion caused by linkages among financial intermediaries. Kodres and Pritsker[4] also developed a multiple asset REE model to explain the financial contagion. We follow Kodres and Pritsker[4]'s model and try to explain the financial contagion from the way of illiquidity contagion. Recent research about financial contagion about multiple assets is Zeng, et al.[12] who constructed the contagion model from the view of common sentiment that bridges the connection between prices learned by rational traders. However, in our model, we innovatively adopt the shared fundamental risk factors to describe how the contagion occurs between unlinked assets and our model is the complement of Zeng, et al.[12]. And the shared fundamental factors represent the systematic macroeconomic risk.

The rest of this paper is structured as follows. In Section 2, we propose the model, as well as constructing the trading environment and solve for traders' strategies. In Section 3, we define the equilibrium and discusses its properties. Finally, we make the conclusion in Section 4.

2 The Model

2.1 Environment

To illustrate the contagion among three countries, we consider three segmented markets in which there are four assets traded including one riskless asset (with elastic supply) of unitary return and three risky assets A, B and C in each market with the corresponding payoffs:

$$\begin{cases} v_A = \theta_1 + \varepsilon_A, \\ v_B = \theta_1 + \theta_2 + \varepsilon_B, \\ v_C = \theta_2 + \varepsilon_C, \end{cases}$$

where ε_A, ε_B and ε_C are residual uncertainty terms conditional on the fundamental values θ_1 and θ_2, and $\theta_i \sim N\left(0, \tau_{\theta_i}^{-1}\right)$, $\varepsilon_k \sim N\left(0, \tau_{\varepsilon_k}^{-1}\right)$ $(\tau_{\theta_i}, \tau_{\varepsilon_k} > 0)$, $i = 1, 2$, $k = A, B, C$. The most important feature here is that A and C share no common fundamental risk, while A and B, B and C share common fundamental risk factors θ_1, θ_2, respectively. Here is an example, asset A can be regarded as one company which produces plastic toy, and asset B can be regarded as one company which produces plastic medical equipment, while asset C can be regarded as one company which manufacture medicine. Intuitively, the fundamentals of asset A and asset B are relevant because they are affected by the price of plastic. Similarly, the fundamentals of asset B and asset C are relevant since they belong to same industry. However, the fundamentals of asset A and asset C are irrelevant since they belong to different industries.

There are three classes of rational traders in the economy corresponding to the three assets A, B, C: A-informed traders† (of mass α) endowed with the private information θ_1. B-informed traders (of mass β) with the private information θ_2. C-informed traders (of mass λ) with the private information θ_2 and C-uninformed traders (of mass $1-\lambda$). As argued by Paul[13], one of the most important functions of the price system is the decentralized aggregation of information since no single person or institution can process all information relevant to pricing, here we leave out B-informed traders with θ_1 informed. The riskless asset is traded at price 1, and the risky assets are traded at prices p_A, p_B and p_C, respectively. Furthermore, similar with Cespa and Foucault[3], we assume there is no cross-asset trading so that effects can only arise from cross-asset learning rather than from hedging effects.

The irrational traders named as noise traders provide market liquidity and their trading is exogenously given as u_A, u_B and u_C, respectively, where $u_A \sim N\left(0, \tau_{u_A}^{-1}\right)$, $u_B \sim N\left(0, \tau_{u_B}^{-1}\right)$, $u_C \sim N\left(0, \tau_{u_C}^{-1}\right)$, and $\tau_{u_A} > 0$, $\tau_{u_B} > 0$, $\tau_{u_C} > 0$.

When trading occurs, p_A will be incorporated into the strategies of B-traders and therefore influence the formation of p_B which can further affect p_C via C-traders' free riding. Due to the fact that asset A and asset C share no common fundamentals, the price contagion cannot happen directly between these two assets. In our model, the contagion mechanism bridges the connections between assets A and C through the intermediate B, which ultimately results in the price information about asset A to be revealed in asset C. A parsimony version of such a

†For convenience, we omit A-uninformed traders and B-uninformed traders since they have no impact on price informativeness and our results do not depend on them.

view could be depicted in Figure 1. Since asset A and asset C are symmetric, so the contagion path can also be from asset C to asset B, and then from asset B to asset A.

$$\boxed{\text{Asset A}} \xrightarrow{P_A} \boxed{\text{Asset B}} \xrightarrow{P_B} \boxed{\text{Asset C}}$$

Figure 1 The contagion mechanism

To sum up, $(\theta_1, \theta_2, \varepsilon_A, \varepsilon_B, \varepsilon_C, u_A, u_B, u_C)$ are underlying random variables which describe the economy. They are all independent with each other. In the economy just characterized, each investor has CARA preference with risk-averse coefficient 1 and only trade one asset.

2.2 Traders' Strategies

Trading of A-informed traders is affected by their information set $\{\theta_1, p_A, p_B, p_C\}$, and the expectation operator conditional on the information set can be given as $E\left[\cdot\left|\theta_1, p_A, p_B, p_C\right.\right]$. They choose the riskless asset D_A and risky asset X_A to maximize their expected utility:

$$E\left[-\mathrm{e}^{-W_A}\left|\theta_1, p_A, p_B, p_C\right.\right],$$

where $W_A = (v_A - p_A) X_A + D_A$.

The CARA-normal setup implies the optimal demand of one A-informed trader is:

$$X_A(\theta_1, p_A, p_B, p_C) = \frac{E\left[\theta_1\left|\theta_1, p_A, p_B, p_C\right.\right] - p_A}{\mathrm{Var}\left[\theta_1\left|\theta_1, p_A, p_B, p_C\right.\right] + \tau_{\varepsilon_A}^{-1}}. \tag{1}$$

Further calculation, we have $x_A = \tau_{\varepsilon_A}(\theta_1 - p_A)$.

Similarly, B-informed traders have the information set $\{\theta_2, p_A, p_B, p_C\}$, and they maximize:

$$E\left[-\mathrm{e}^{-W_B}\left|\theta_2, p_A, p_B, p_C\right.\right],$$

subject to $W_B = (v_B - p_B) X_B + D_B$ with the riskless asset D_B . So their optimal holdings of asset B are:

$$X_B(\theta_2, p_A, p_B, p_C) = \frac{\theta_2 + E\left[\theta_1\left|\theta_2, p_A, p_B, p_C\right.\right] - p_B}{\mathrm{Var}\left[\theta_1\left|\theta_2, p_A, p_B, p_C\right.\right] + \tau_{\varepsilon_B}^{-1}}. \tag{2}$$

C-informed traders own information set $\{\theta_2, p_A, p_B, p_C\}$. Maximizing their expected utility in the same way delivers:

$$X_C(\theta_2, p_A, p_B, p_C) = \tau_{\varepsilon_C}(\theta_2 - p_C). \tag{3}$$

Solving the optimal demands for C-uninformed traders with the information set $\{p_A, p_B, p_C\}$ likewise gives:

$$Y_C(p_A, p_B, p_C) = \frac{E\left[v_C\left|p_A, p_B, p_C\right.\right] - p_C}{\mathrm{Var}\left[v_C\left|p_A, p_B, p_C\right.\right]}. \tag{4}$$

Hence, according to normal-distributed setup, the price of A-asset is a linear function of $\{\theta_1, p_B, p_C, u_A\}$, and similarly, the prices of B-asset and C-asset are linear functions of $\{\theta_2, p_A, p_C, u_B\}$ and $\{\theta_2, p_A, p_B, u_C\}$, respectively. We then define the rational expectation equilibrium and solve for the prices.

3 The Equilibrium

Definition Given the fractions (α, β, λ) of the corresponding three types of informed traders, an REE is composed of the price functions p_A, p_B and p_C, and traders' optimal demand functions contingent on their information such that market clearing condition satisfies:

$$\alpha X_A + u_A = 0, \tag{5}$$

$$\beta X_B + u_B = 0, \tag{6}$$

$$\lambda X_C + (1-\lambda) Y_C + u_C = 0. \tag{7}$$

The equations (5)–(7) mean that all the buy orders equal sell orders. In other words, the total trading positions equal zero. According to the definition of the equilibrium and traders' strategies, prices of the three assets can roughly be assumed to the following formation:

$$p_A = A_1\theta_1 + A_2 u_A + A_3 p_B + A_4 p_C, \tag{8}$$

$$p_B = B_1\theta_2 + B_2 u_B + B_3 p_A + B_4 p_C, \tag{9}$$

$$p_C = C_1\theta_2 + C_2 u_C + C_3 p_B + C_4 p_A. \tag{10}$$

We then derive the sufficient statistic $\{z_A, z_B, z_C\}$ of the corresponding prices $\{p_A, p_B, p_C\}$. Specifically, let

$$z_A = \frac{p_A - A_3 p_B - A_4 p_C}{A_1} = \theta_1 + \frac{A_2}{A_1} u_A, \tag{11}$$

$$z_B = \frac{p_B - B_3 p_A - B_4 p_C}{B_1} = \theta_2 + \frac{B_2}{B_1} u_B, \tag{12}$$

$$z_C = \frac{p_C - C_3 p_B - C_4 p_A}{C_1} = \theta_2 + \frac{C_2}{C_1} u_C. \tag{13}$$

It is obvious that $\{z_A, z_B, z_C\}$ is informationally equivalent to $\{p_A, p_B, p_C\}$. Hence the expression $E[\theta_1 | \theta_2, p_A, p_B, p_C]$ in (2) can be rewritten as $E[\theta_1 | \theta_2, z_A, z_B, z_C]$ which is equal to solve $E[\theta_1 | z_A]$. And the conditional expectation $E[v_C | p_A, p_B, p_C]$ in (4) is equal to solving $E[v_C | z_A, z_B, z_C] = E[\theta_2 | z_B, z_C]$.

By computation, plugging traders' strategies into market clearing condition, we immediately derive the following proposition.

Proposition 1 *There exists a unique linear REE in which prices are given as follows:*

$$p_A = A_1\theta_1 + A_2 u_A,$$

$$p_B = B_1\theta_2 + B_2 u_B + B_3 z_A,$$

$$p_C = C_1\theta_2 + C_2 u_C + C_3 z_B,$$

where the coefficients are endogenously determined and given as follows:

$$A_1 = 1, \quad A_2 = \frac{1}{\alpha\tau_{\varepsilon_A}}, \quad A_3 = 0, \quad A_4 = 0, \quad B_1 = 1, \quad B_2 = \frac{1}{\beta M}, \quad B_3 = \frac{(\alpha\tau_{\varepsilon_A})^2 \tau_{u_A}}{M},$$

$$B_4 = 0, \quad M = \tau_{\theta_1} + (\alpha\tau_{\varepsilon_A})^2 \tau_{u_A}, \quad C_1 = \frac{\lambda\tau_{\varepsilon_C}(\tau_{\varepsilon_C} + \tau) + (1-\lambda)\tau_{\varepsilon_C}(\lambda\tau_{\varepsilon_C})^2 \tau_{u_C}}{N(\tau_{\varepsilon_C} + \tau)},$$

$$C_2 = \frac{(\tau_{\varepsilon_C} + \tau) + (1-\lambda)\lambda\tau_{\varepsilon_C}^2 \tau_{u_C}}{N(\tau_{\varepsilon_C} + \tau)}, \quad C_3 = \frac{(1-\lambda)\tau_{\varepsilon_C}(\beta M)^2 \tau_{u_B}}{N(\tau_{\varepsilon_C} + \tau)}, \quad C_4 = 0,$$
$$N = \lambda\tau_{\varepsilon_C} + (1-\lambda)\frac{\tau_{\varepsilon_C}\tau}{\tau_{\varepsilon_C} + \tau}, \quad N = \lambda\tau_{\varepsilon_C} + (1-\lambda)\frac{\tau_{\varepsilon_C}\tau}{\tau_{\varepsilon_C} + \tau},$$
$$\tau = \tau_{\theta_2} + (\beta M)^2 \tau_{u_B} + (\lambda\tau_{\varepsilon_C})^2 \tau_{u_C}.$$

Thus, we have:

$$\begin{aligned}
&E\left[v_B \left|\theta_2, p_A, p_B, p_C\right.\right] = \theta_2 + \frac{(\alpha\tau_{\varepsilon_A})^2 \tau_{u_A} z_A}{\tau_{\theta_1} + (\alpha\tau_{\varepsilon_A})^2 \tau_{u_A}},\\
&\mathrm{Var}\left[v_B \left|\theta_2, p_A, p_B, p_C\right.\right] = \left(\tau_{\theta_1} + (\alpha\tau_{\varepsilon_A})^2 \tau_{u_A}\right)^{-1} + (\tau_{\varepsilon_B})^{-1},\\
&E\left[v_C \left|p_A, p_B, p_C\right.\right] = \frac{\left[\beta\left(\tau_{\theta_1} + (\alpha\tau_{\varepsilon_A})^2 \tau_{u_A}\right)\right]^2 \tau_{u_B} z_B + (\lambda\tau_{\varepsilon_C})^2 \tau_{u_C} z_C}{\tau},\\
&\mathrm{Var}\left[v_C \left|p_A, p_B, p_C\right.\right] = \tau^{-1} + \tau_{\varepsilon_C}^{-1}.
\end{aligned} \tag{14}$$

Proof Note that the information set $\{\theta_1, p_A, p_B, p_C\}$ and $\{\theta_1\}$ are equal when solving for the conditional expectation $E[\theta_1 |\bullet]$, according to the lemma in the Appendix, it is not difficult to compute the strategies of A-informed traders, that is, $X_A = \tau_{\varepsilon_A}(\theta_1 - p_A)$.

We further plug the above conditional expectation and variance into Equations (2) and (4), and obtain the following results:

$$X_B = \frac{\tau_{\varepsilon_B}\left(M\theta_2 + \tau_{\varepsilon_B}(\alpha\tau_{\varepsilon_A})^2 \tau_{u_A} z_A\right) - \tau_{\varepsilon_B} M p_B}{\tau_{\varepsilon_B} + M},$$
$$Y_C = \frac{\left((\beta M)^2 \tau_{u_B} z_B + (\lambda\tau_{\varepsilon_C})^2 \tau_{u_C} z_C\right)\tau_{\varepsilon_C} - \tau_{\varepsilon_C}\tau p_C}{\tau_{\varepsilon_C} + \tau}.$$

Plugging traders' strategies into the market clearing conditions (5), (6) and (7), we immediately derive the equilibrium prices. ∎

Proposition 1 shows that the equilibrium price will reveal the fundamental value aggregated by the informed traders as well as the noise trading. It is obvious that in our model the statistic of p_A (z_A) will be integrated into the price of asset B (z_B) which will be reflected in the price of asset C by observing the other assets' prices. The price integration process is in line with the contagion mechanism in Figure 1.

As is defined (see Cespa and Foucault[3]), we measure the illiquidity of asset k ($k = A, B, C$) by the sensitivity of price p_k to liquidity demand u_k. Thus, we have

$$L_A = \frac{\partial p_A}{\partial u_A} = \frac{1}{\alpha\tau_{\varepsilon_A}}, \tag{15}$$

$$L_B = \frac{\partial p_B}{\partial u_B} = \frac{1}{\beta\left(\tau_{\theta_1} + (\alpha\tau_{\varepsilon_A})^2 \tau_{u_A}\right)} = \frac{1}{\beta\left(\tau_{\theta_1} + (1/L_A)^2 \tau_{u_A}\right)}, \tag{16}$$

$$L_C = \frac{\partial p_C}{\partial u_C} = \frac{(\tau_{\varepsilon_C} + \tau) + (1-\lambda)\lambda\tau_{\varepsilon_C}^2\tau_{u_C}}{N(\tau_{\varepsilon_C} + \tau)} = \frac{1}{\tau_{\varepsilon_C}} + \frac{(1-\lambda)(1+\lambda\tau_{\varepsilon_C}\tau_{u_C})}{\lambda\tau_{\varepsilon_C} + \tau_{\theta_2} + (1/L_B)^2\tau_{u_B} + (\lambda\tau_{\varepsilon_C})^2\tau_{u_C}}. \tag{17}$$

Above equations describe the expression of illiquidity with respect to the three assets, respectively. In the following part, we will make some comparative statics of the illiquidity as for the agent masses.

Proposition 2 *The masses of the informed traders have a negative effect on illiquidity, that is,* $\frac{\partial L_A}{\partial \alpha} < 0$, $\frac{\partial L_B}{\partial \alpha} < 0$, $\frac{\partial L_B}{\partial \beta} < 0$, $\frac{\partial L_C}{\partial \alpha} < 0$, $\frac{\partial L_C}{\partial \beta} < 0$.

The effect of the masses of informed traders on the illiquidity through two ways. The first is the direct effect, as shown in the literature (Grossman and Stiglitz[7]; Vives[14]), illiquidity (the reciprocal of market depth) is negatively related to the masses of the informed traders or the information precision of informed traders. Hence, increasing the masses of A-informed traders (B-informed traders) will decrease the illiquidity of asset A (asset B). The second is the indirect effect, increasing the masses of A-informed (B-informed) traders has a negative effect on the illiquidity of asset B (asset C). This can occur because increasing the masses of A-informed traders has a positive effect on price informativeness of asset A, which will give rise to high information precision of B-informed traders when learning from p_A and further lead to low illiquidity of asset B. Same explanation for the variation of the illiquidity of asset C.

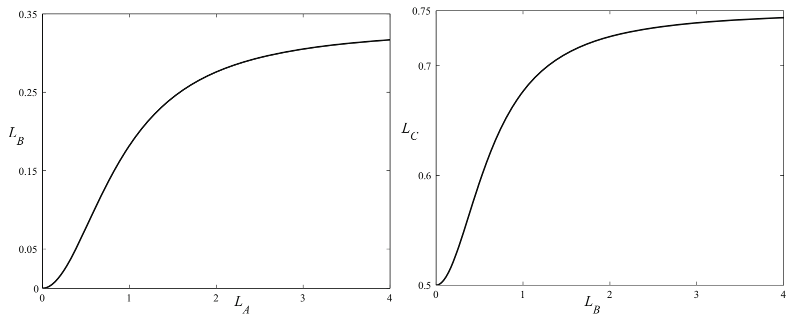

Figure 2 The comovement of illiquidity when $\beta = 0.5, \tau_{\theta_1} = \tau_{\theta_2} = 6, \tau_{u_A} = \tau_{u_B} = \tau_{u_C} = 5, \lambda = 0.5, \tau_{\varepsilon_C} = 2$

Proposition 3 (Comovement effect) $\frac{\partial L_B}{\partial L_A} > 0, \frac{\partial L_C}{\partial L_B} > 0$.

This proof is obvious in above equations. Among the three assets, an increase in the illiquidity of asset A will make asset B more illiquid, which can eventually trigger an increase in illiquidity for asset C. Proposition 3 shows that illiquidity exhibits positive interaction among the three risky assets no matter whether asset fundamentals are relevant or not if cross-asset learning prices is available. The following examples in Figure 2 describe this comovement effect as for the illiquidity.

Proposition 4 $\frac{\partial L_B}{\partial \tau_{u_A}} < 0, \frac{\partial L_C}{\partial \tau_{u_A}} < 0$.

Proposition 4 shows that the initial high volatility of noise trading (τ_{u_A} low) shock can engender a final liquidity decline (L_B, L_C increase) via the intermediate asset B. Through cross-asset learning, noise trading in asset A will be infused into the price of asset B which can be further learned by C-traders. This negative impact is an explanation for the financial contagion of the COVID-19 shock, when market is impacted by the bad news, traders need to liquidate their assets to obtain cash and avoid loss from the downward price, leading to the pessimistic sentiment and high volatility of noise trading (τ_{u_A} decreases). Through our contagion mechanism, the initial noise trading shock will result in a negative effect on liquidity.

Remark 1 Increasing the number of A-informed traders augments the trading intensity of B-informed traders.

According to (2), the trading intensity in term of B-informed traders towards their private information θ_2 can be given as:

$$\frac{1}{\mathrm{Var}\left[\theta_1 \left|\theta_2, p_A, p_B, p_C\right.\right] + \tau_{\varepsilon_B}^{-1}} = \left[\left(\tau_{\theta_1} + (\alpha\tau_{\varepsilon_A})^2\tau_{u_A}\right)^{-1} + \tau_{\varepsilon_B}^{-1}\right]^{-1}.$$

When α increases, B-informed traders face less fundamental risk about θ_1, thus taking more aggressively strategies towards their information.

Remark 2 Increasing the mass of A-informed traders weakens the uncertainty of θ_2 in p_B, that is $\frac{\partial(\mathrm{Var}[\theta_2|p_B])}{\partial\alpha} < 0$. As shown in Remark 1, α increases, B-informed traders will trade more aggressively to θ_2, as a result, more information about θ_2 will be revealed in p_B.

As standard in the literature (e.g., Vives[5]; Mondria, et al.[15]), we can use the precision $\frac{1}{\mathrm{Var}(v|prices)}$ of stock payoff conditional on its prices to measure "price informativeness" (or "market efficiency", "information efficiency","price efficiency"). We define price informativeness of assets A and C as $I_A = \left[\mathrm{Var}\left(v_A \left|p_A, p_B, p_C\right.\right)\right]^{-1}$ and $I_C = \left[\mathrm{Var}\left(v_C \left|p_A, p_B, p_C\right.\right)\right]^{-1}$. As shown in (12), we have the following results.

Proposition 5 (a) $\frac{\partial I_C}{\partial\alpha} > 0$, $\frac{\partial I_C}{\partial\beta} > 0$; (b) $\frac{\partial I_C}{\partial\tau_{u_A}} > 0$, $\frac{\partial I_C}{\partial\tau_{u_B}} > 0$.

Proof According to (12), we obtain

$$I_C = \left(\left(\tau_{\theta_2} + \left[\beta\left(\tau_{\theta_1} + (\alpha\tau_{\varepsilon_A})^2\tau_{u_A}\right)\right]^2\tau_{u_B} + (\lambda\tau_{\varepsilon_C})^2\tau_{u_C}\right)^{-1} + \tau_{\varepsilon_C}^{-1}\right)^{-1} > 0.$$

By derivation, the results hold. ∎

The above results are intuitive. According to Remark 1, a high proportion of A-informed traders will give rise to high trading intensity of θ_2, and thus more information about θ_2 will be injected into p_B , which therefore leads to high price informativeness of asset C. It is then not difficult to find increasing the mass of B-informed traders can also result in high price informativeness of asset C since asset B and C share common fundamental factor. Hence, through our contagion mechanism, raising the informed trading of asset A will generate high price informativeness of asset B, and eventually induce high price informativeness of asset C.

When the noise trading of asset A is in high volatility, through this contagion mechanism, price informativeness about asset B tends to be low, ultimately resulting in low price informativeness about asset C.

Proposition 6 $\frac{\partial I_C}{\partial L_A} < 0$, $\frac{\partial I_C}{\partial L_B} < 0$, $\frac{\partial I_C}{\partial L_C} < 0$.

Proof Combing (12) with (14) and (15), we have

$$\begin{aligned} I_C &= \left(\left(\tau_{\theta_2} + \left(\beta\left(\tau_{\theta_1} + (1/L_A)^2\tau_{u_A}\right)\right)^2\tau_{u_B} + (\lambda\tau_{\varepsilon_C})^2\tau_{u_C}\right)^{-1} + \tau_{\varepsilon_C}^{-1}\right)^{-1} \\ &= \left(\left(\tau_{\theta_2} + (L_B)^{-2}\tau_{u_B} + (\lambda\tau_{\varepsilon_C})^2\tau_{u_C}\right)^{-1} + \tau_{\varepsilon_C}^{-1}\right)^{-1} \\ &= \left(\left(\frac{(1-\lambda)(1+\lambda\tau_{\varepsilon_C}\tau_{u_C})}{L_C - \tau_{\varepsilon_C}^{-1}} - \lambda\tau_{\varepsilon_C}\right)^{-1} + \tau_{\varepsilon_C}^{-1}\right)^{-1}. \end{aligned}$$

By derivation, we obtain the above results. ∎

Intuitively, high illiquidity will make the trading harder and therefore prevent new information from being incorporated into price, causing low price informativeness. Figure 3 offers an intuitive description of such relationship.

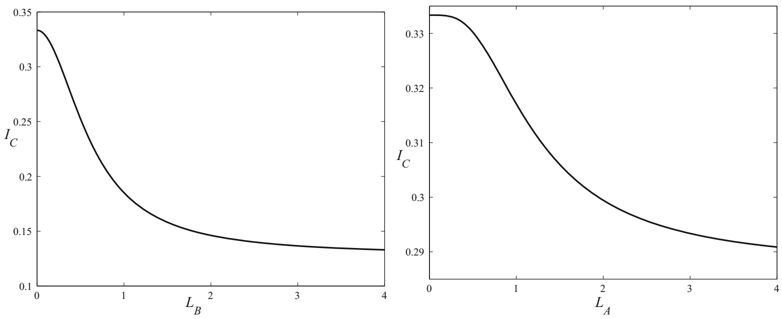

Figure 3 The effect of illiquidity on price informativeness when $\beta = 0.5, \tau_{\theta_1} = \tau_{\theta_2} = 6, \tau_{u_A} = \tau_{u_B} = \tau_{u_C} = 5$,$\lambda = 0.5, \tau_{\varepsilon_C} = 2$

Conversely, price informativeness also has a negative impact on illiquidity, i.e., $\partial L_B/\partial I_A < 0$, $\partial L_C/\partial I_A < 0$. The negative interaction between illiquidity and price informativeness is a key driver of market crisis. In fact, assets value may be determined by various fundamental factors[16] which may generate a series of cross interactions as the above description. Hence, once the initial asset crises are triggered, the downward price may feed the panicky mind-set, which can further create a shock for illiquidity and price informativeness. Under our contagion mechanism, this inter-feedback from illiquidity and price informativeness is therefore the source of liquidity crisis, prompting the occurrence of financial crisis across the whole market (see Figure 4).

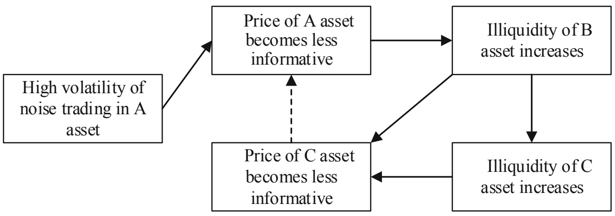

Figure 4 Explanation for market crisis

4 Some Extensions

If v_A and v_B are correlated with a correlation coefficient ρ, the format of the asset fundamentals will change to:

$$\begin{cases} v_A = \rho\theta_1 + \varepsilon_A, \\ v_B = \theta_1 + \theta_2 + \varepsilon_B, \\ v_C = \theta_2 + \varepsilon_C. \end{cases}$$

Follow the calculation process above, the main difference is that the strategies of A-informed traders change to $x_A = \tau_{\varepsilon_A}(\rho\theta_1 - p_A)$, and the price statistic z_A changes to $z_A = \theta_1 + \frac{1}{\alpha\rho\tau_{\varepsilon_A}}u_A$. So we only need to add the coefficient ρ in front of τ_{ε_A}. In this case, the propositions in our paper still hold. What's deserved to be considered is the role of the coefficient ρ in our contagion model, the following Proposition 7 will describe the effect.

Proposition 7 *For exogenous ρ, we have $\frac{\partial I_C}{\partial \rho} > 0$, $\frac{\partial L_C}{\partial \rho} < 0$.*

Proof According to (15), (16), (17), it is easy to obtain $\frac{\partial I_C}{\partial \rho} > 0$. The coefficient ρ increase implies that more informed information θ_1 will be incorporated into p_A. Therefore, the price informativeness of asset A will increase, and this will further enhance the price informativeness of asset C based on Proposition 5. As for the illiquidity, since L_A is negatively related to ρ, it is obvious that $\frac{\partial L_C}{\partial \rho} < 0$ according to the illiquidity comovement effect (Proposition 3). ∎

5 Conclusion

Advanced information technology and close inter-market linkage enable traders to learn information contained in the prices of assets. Our model shows that rational traders can update their expectations using the available private information and price information which bridges multiple assets via cross fundamentals. By this means, prices provide a way that transmits fundamental risk and noise trading risk. This makes the liquidity of various securities more mutually connected: 1) An initial shock in the illiquidity can have a positive effect on the illiquidity of the assets; 2) The drop in liquidity occurs jointly with the decrease in price informativeness.

As described above, our results suggest a potential explanations for market crisis. As a matter of fact, when extreme volatility happens in stock prices, market participants who provide liquidity may widen their quotes or stop providing liquidity. If this happens, prices of other assets would not be reliable as the information, but be a pessimistic noisy signal, finally triggering a chain reaction and leading to large liquidity crash across the whole market.

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Appendix

Lemma *Suppose that x and y are normally distributed, then according to projection theorem, we have*

$$E(x|y) = E(x) + \frac{\mathrm{Cov}(x,y)}{\mathrm{Var}(y)}(y - E(y)), \tag{18}$$

$$\mathrm{Var}(x|y) = \mathrm{Var}(x) - \frac{\mathrm{Cov}^2(x,y)}{\mathrm{Var}(y)}. \tag{19}$$