



Learning to notice collaboration: examining the impact of professional development on mathematics teachers' enhanced awareness in CSCL settings

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Abstract

Acknowledging the pivotal role of noticing in teachers' professional work, it is noteworthy that its application in dialogic activities remains an area that has yet to be studied. In this study, we examine mathematics teachers' noticing of dialogue among peers working together on problem-solving tasks and investigate the impact of a professional development intervention focused on dialogue on teachers' noticing practices. Through think-aloud interviews, 14 teachers provided insights into their noticing practices by attending to and responding to video excerpts of dyads engaged in collaborative problem-solving in computer-supported learning environments. Their noticing practices were analyzed using a Bakhtinian-informed dialogic framework. Subsequently, the teachers participated in a professional development intervention centered around dialogue and were interviewed again using the same video excerpts. The second round of interviews was also analyzed using the same dialogic framework. The findings shed light on the initial state of teachers' noticing and indicate a discernible improvement in their ability to notice specific dialogic attributes. These findings offer valuable insights into how collaboration and dialogue between students can be effectively supported. Additionally, the study discusses how teachers envision dialogue and considers the capacity and limitations of incorporating a dialogic vision into the noticing paradigm.

Keywords Collaboration · CSCL · Noticing · Dialogic pedagogy · Mathematics teacher education · Professional development

Introduction

The ways teachers think about knowledge and learning, including the extent to which dialogue is perceived as a method of knowledge and verification, is linked to students' achievement in solving problems and learning complex subjects (Bråten et al., 2017). To

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understand the degree to which dialogue may be enacted in classrooms, it is important to study teachers' noticing. In this article, we present a study on mathematics teachers' noticing as a learned practice. A key question in our study focuses on whether teachers' engagement and responses to peer dialogue change following an intervention designed to enhance their awareness of student learning dialogues. More precisely, we inquire about the alterations in teachers' noticing, that is, attending and responding, to diverse dialogic attributes after the intervention. Additionally, we explore the individual trajectories of learning to notice by investigating how the attending and responding to dialogue evolved for different teachers. The intricate connections between "seeing" (attending) and "acting" (responding) are scrutinized by examining how these changes manifest in teachers' responses to various collaborative learning situations.

The concept of noticing sits at the intersection of three theoretical frameworks: collaborative learning, computer-supported collaborative learning (CSCL), and dialogic pedagogy. To better understand how noticing processes work, we combine their distinctive and complementary contributions. In the following theory section, we elaborate on the three. As an analytical tool, dialogue is used to identify collaboration.

The challenge of monitoring and regulating a collaborative learning dialogue

In contrast to individualistic or competitive pedagogical school cultures, collaborative learning proponents encourage the use of pedagogical designs such as cooperative classrooms and "positive interdependence." Cooperative learning improves how students learn (Kyndt et al., 2013; Gillies, 2016). The call for cooperative modes of pedagogical organization can be justified in three ways, the first two being cognitive, according to which learning together surpasses other modes of learning (Chi & Wylie, 2014), and socioemotional, according to which cooperation conveys the gist of social, emotional, and communicational skills needed for one's well-being (Johnson et al., 2010) and combats the climate of distortions of oppression, alienation, and violence of all kinds in schools (Aronson, 2021). Cooperative learning is also *political*, in the sense that the skills and dispositions nurtured by cooperation also contribute to the development of democratic citizens and a democratic society (Slakmon & Schwarz, 2019).

Fundamental cooperative learning elements include student inclination and behaviors such as positive interdependence, individual accountability, promotive interaction, social skills, and group processing (Johnson & Johnson, 2017). Their application into successful pedagogy depends on six instructional practices: pre-instructional decisions such as curricular planning and grouping, explanation of the task and the cooperative structure, monitoring, supporting, consolidating, and assessing. However, these accounts are not sufficiently finely tuned to guide monitoring. Monitoring implies "teacher's movement from group to group to monitor interaction carefully during the lesson" (Johnson & Johnson, 2017, p.490). Elsewhere, the practice of monitoring is divided into stages of pre-practice decision, observation, and interventions (Kaendler et al., 2015). There are no cognitive and sociocultural processes involved in these accounts of observing and intervening. The same is true for "promotive interaction," defined as moments "when individuals encourage and facilitate each other's efforts to complete tasks and achieve in order to reach the group's goals" (Johnson & Johnson, p.589). Next, we outline the three approaches addressing the elaborative gap in monitoring and promoting interaction: noticing, dialogic pedagogy, and CSCL.

Teachers' noticing

Noticing contributes to “monitoring” by delving into its challenges in greater detail, making sense of learning interactions (Sherin & Jacob, 2011). The noticing approach tries to understand how teachers attend to learning events, how they interpret what they see, what their interpretation is based on, and how the interpretations turn into responses (van Es & Sherin, 2021). Responding is determined by interpretations of the students' thinking. These interpretations are initially conditioned by allocating attendance to specific moves (Arcavi & Isoda, 2007; Rowland et al., 2015; Sherin & Jacobs, 2011).

The unit of analysis in noticing studies varies from microanalytic testing of eye-gaze to prolonged excerpts. Educational noticing studies typically involve (in-service and/or prospective) teachers observing and analyzing learning interactions together (Holstein et al., 2022; Sherin & van Es, 2005; Star & Strickland, 2008; Taylan, 2017; van Es & Sherin, 2002; Walkoe, 2015). Online and video-based professional development programs for noticing are often used (Ebby et al., 2023; Larison et al., 2022; Santagata et al. 2021).

Teachers' patterns of noticing change over time. However, studies that compare the dialogic and mathematical attributes in teachers' noticing indicate that teachers are more inclined to attend and respond to content-specific attributes of learners talk, whether in classroom discussion (Ayalon & Nama, 2023) or in cooperative learning (Calor et al., 2022). Stahnke and Blömeke (2021) examined differences in noticing between novice and expert teachers during teacher-led discussion and collaboration. While expert teachers attended and often responded to students' work, novice teachers offered fewer responses and mostly attended to instruction. Training improves teachers' noticing skills (Stockero, 2021; Walkoe, 2015). Teachers tend to notice events that resonate with their resources, educational goals, and pedagogical orientation (Schoenfeld, 2011). Teachers' resources are used in the act of noticing, especially at times when teachers face unpredictable learning situations for which they have established no solutions or schemes for action (Rowland et al., 2015).

Dialogic pedagogy: dialogue as a social construction of knowledge

By emphasizing the role of language and shared meaning-making in learning, dialogic education has made enormous strides in our understanding of what constitutes “promotive interaction” (Wegerif, 2019). The quality of students' dialogue and the quality of the dialogic mediation provided by the teacher determines the overall quality of the collaboration (Kobbe et al. 2007; Gillies et al. 2008). Without systematic mediation, students do not operate at a sufficiently high, interactive level of cognitive involvement when they talk and work together (Nystrand, 1997; King, 2002; Chi & Wylie, 2014). However, learning to learn together and operating at the interactive level of cognitive engagement enable students to reason, invent, ask questions that challenge the current level of understanding, build on the ideas of others, connect ideas, solidify these ideas, solve problems, and make decisions together.

Teachers can support productive collaboration by improving the dialogue between students. The dialogic teaching approach (Alexander, 2020) is one of the widely used approaches in the field, along with collaborative argumentation-based learning (Baker et al., 2019), dialogic instruction (Nystrand, 1997), dialogic inquiry (Wells, 1999), dialogical pedagogy (Skidmore, 2000), inquiry dialogue (Reznitskaya & Wilkinson, 2017), and accountable talk (Michaels & O'Connor, 2015). Dialogic teaching helps teachers mediate

dialogic skills with students and trains them in group thinking that uses the listening, organizing, and creative abilities of others to arrive at common solutions.

Studies on teachers' learning on dialogic pedagogy have yielded mixed results. Often using professional development courses as sites for intervention, many studies experimented with various ways of "training the trainers," usually through collaborative analysis of representations of instruction (Dudley & Vrikki, 2019; Hennessy & Davies, 2019). Changes in teachers' instructional practices have been not systematic and not connected to teachers' epistemology (Wilkinson et al., 2017). At this stage, Hofmann notes, "research needs to explain the generative mechanisms that can bring about change: we need to understand how professional learning interventions have their effect and why they may or may not lead to professional change" (Hofmann, 2019, p. 213). The current study focuses on attending and responding and stands at the intersection of epistemic cognition and instructional practices. We hope that this research will shed light on the (sociocognitive) generative processes that enable instructional change as well as the changes themselves.

We look at pedagogy from a Bakhtinian perspective, from which learning can be seen as voices that change within as a result of communication with other voices (Wegerif, 2011). A dialogue emerges when distinct voices share a space. Successful learning dialogues involve the creation of a dialogic space in which different voices emerge, are shared, and developed through discussion (Palmgren-Neuvonen et al., 2021). We present our working definitions to four interconnected Bakhtin-inspired themes—voices, expanding the dialogue, inter-animation, and relations—and the cooperative learner behaviors that are associated with them.

A voice in a dialogue is a perspective about an idea that is communicated to others, "the speaking personality, the speaking consciousness" (Bakhtin, 1981, p. 431). People generate meanings and ascribe them to reality through negotiation with communicative and technological means (Wegerif & Major, 2019). A monologue is a situation in which one voice is explicated by an individual or a group of people. A dialogic gap occurs when two equally valid and distinct voices interact, driving the dialogue forward (Abdu et al., 2022). Expansion of the dialogue is one mechanism through which a dialogue can move forward, by introduction of new ideas into the conversation and the associated acts of evaluating them. Dialogue can be expanded by widening it or by deepening it. Widening occurs when new voices are introduced, such as when students consider alternative solutions to a problem. Deepening the dialogue involves engaging in a critical inquiry of existing voices through exploration, elaboration, and refutation of assumptions and conclusions (Ludvigsen et al., 2019; Wegerif, 2011).

Inter-animation involves attempting to incorporate other voices by repeating them in one's own words (not necessarily agreeing, Matusov, 1996). This is a two-sided process, because the addressee is one's only path toward recognition. A voice being heard in a dialogue enables speakers to understand their utterances more fully by transforming and contextualizing them in response to others' responses (Blommaert, 2005). Effective dialogue also requires students to engage in persuasive interaction by listening to each other, entering into the realm of others' words, and attempting to change them from within (Wegerif, 2011). To be open to the possibility of changing one's perspective, it is necessary to develop an ideological dialogic stance (O'Connor & Michaels, 2007; Boyd & Markarian, 2015) and the willingness and the capacity for perspective taking and changing (Wegerif & Major, 2019).

The dialogic conditions of mutuality and relationality exist in social relations. When learners' voices have equal status on matters of thinking, speaking, and persuasion, they can be heard. These voices may change as a result of the dialogue, so none of the voices

present are externally authoritative regarding what is, and what is not, correct (Scott et al., 2006). Interaction with authoritative voices, dialogically defined, is a monologue in which only one voice is present and is imposed on others, while other voices are silenced or not explored to their full potential. This can happen in collaborative learning when one takes the lead and the other is led.

Computer-supported collaborative learning and dialogue

The third approach to nurturing collaborative learning is CSCL. The first difference between cooperative learning and CSCL is the presence of computers and digital tools. In this approach, computation serves many functions, from simply being the setting of the content-learning environment, to roles of scaffolding, prompting, documenting, and assessing the collaborative aspects of the learning assignments (Cress et al., 2021). The second, more subtle difference between collaborative learning and CSCL is the shift toward distributed and expansive views of thinking and learning. The CSCL community is interested in the new, that is, the additional knowledge, whatever its form and representation that none of the earlier collaborators “possessed.” Instead, this additional knowledge has to be jointly created (Dillenbourg, 1999). Collaborative learning designs strive to refrain from fragmentation of the work and favor less rigid, predesigned divisions of labor. In addition, these designs express a greater depth of analysis regarding the processes of collaboration (Kaendler et al., 2015).

Designing assignments for students to work at the highest level of cognitive engagement (the dialogic level) proves to be overly complicated for teachers (Chi et al., 2018; Hennessy & Davies, 2019). Tools for supporting dialogue in CSCL settings and collaboration include macro and micro scripting (Hämäläinen & Häkkinen, 2010; Kollar et al., 2018), technology for grouping (Maqtary et al., 2019), designated communication and information sharing (Major et al., 2018; Strauß & Rummel, 2020), dashboards for teachers, providing them with online status of participation and dynamics (Buder et al., 2021), clustered (conceptual, spatial) organizations of the problem space (Tabach & Schwarz, 2018), and support for teachers in assessing learners’ solutions (Olsher & Lavie, 2021)

The meeting between the three paradigms focuses on teachers’ noticing during the educational dialogue held by groups of learners in the computer-mediated collaborative learning environment. The lack of attention to dialogue and relationships between group members can have negative consequences (Barron, 2003). The opposite is also true: supporting dialogue may empower people and improve group performance sometimes even more than a content-focused response (Calor et al., 2022; Dekker & Elshout-Mohr, 2004). Mathematics learning can happen on several modalities (speech, movement, visual attention), rooted by a variety of representations (graphic or algebraic), and levels of discourse (personal, interpersonal). These together paint a complex picture of mathematical discourse and the way it can be analyzed. Among the studies of teachers’ attunement to dialogue, Calor and colleagues’ research (Calor et al., 2022) follows four teachers who apply a scheme of instructional actions to foster small group learning on top of content-specific instruction. The scheme includes what we would call attending (e.g., “diagnosing the group’s maximum level of understanding”) and responding (providing process support). They identify three noticing challenges: (1) deciding whether to provide support in mathematics, in dialogue, or in both, (2) adjusting the degree of control over the mathematical discourse in relation to the student’s work, and (3) recognizing the right time to return control to the group. When mathematical discussions stop, and

the group fails to progress, a targeted intervention to support the dialogue is insufficient, and teachers must elicit mathematical discourse. However, even under these circumstances, teachers can refer to dialogue.

Overall, the literature research suggests a link between teachers' views on knowledge and learning, including their perceptions of dialogue, and student achievement. Learning is also related to levels of interaction and cognitive engagement. Studying dialogue in classrooms through the lens of teacher noticing, however, is lacking. This study aims to address this gap. It draws upon cooperative learning, CSCL, and dialogic pedagogy to posit that the intricate connection between "seeing" (attending) and "acting" (responding) in teachers' noticing can be illuminated by studying the interplay of these frameworks. In this study, we explore how their engagement with and responses to student dialogues change after they are exposed to diverse dialogic features through an intervention. We specifically examine how teachers' collaboration noticing, in terms of attending to and responding to various dialogue features, evolves after the intervention. Furthermore, we examine how different teachers' learning trajectories of noticing develop differently in response to diverse dialogue attributes.

Materials & method

Using a snowball method, we recruited 14 mathematics teachers experienced in teaching parabola to high school students to participate in a two-staged experiment (8 women and 6 men). The first stage involved teachers watching CSCL mathematical tasks and having think-aloud interviews on their noticing. Following this, they participated in a lesson with various activities focused on enhancing dialogic awareness. This was followed by a reflective writing task and a second interview on their noticing. Importantly, both stages built upon the same mathematical tasks to assess changes in teachers' noticing and responses to the dialogic and collaborative attributes of the task.

The teacher watched students working on two mathematical tasks with GeoGebra widgets (Tomaschko et al., 2018; Yerushalmy & Olsher, 2020). The students solved task 1 alone and then solved task 2 in pairs. Both tasks required students to create three examples of parabola functions that pass through two randomized points. Learners clicked on a "new points" button to generate new randomized points (x and y range was $[-10, 10]$). In task 1 (see Table 1), the widget was designed so that one point was always on the X -axis, and the other on the Y -axis. In task 2, the widget was designed so that the two points had the same y -value and different x -values (Abdu et al., 2022). The students were asked to repeat the task three times, each time with a pair of new points, and to submit examples as different as possible. Students were then asked to record how these three examples were different (Fig. 1).

Data collection

The experiment had three stages. The first stage of the experiment had two parts. In the first part, (30 min), the teachers solved the two mathematical tasks. They then (second part, around 50 min) participated in a think-aloud interview (Van Someren et al., 1994). The interview was made up of ten episodes curated for their dialogic learning opportunities (Table 1). In every episode, a student-dyad collaborates to solve the second

Table 1 Learning opportunities (dialogic attributes) identified in the episodes

Episode	Student-Dyad	Dialogue expanding		Inter- animation			Relations				Voices	
		1	2	3	4	5	6	7	8	9	10	11
1	Lia and Sara (a)		V		V		V			V	V	
2	Amnon and Tamar (a)					V	V	V	V			V
3	Tsila and Gila (a)				V		V			V		V
4	Orit and Ayalla				V	V				V		V
5	Rona and Sean				V		V		V		V	V
6	Gal and Ella	V		V	V	V			V		V	
7	Natalia and Matilda				V	V	V			V	V	V
8	Amnon and Tamar (b)				V		V		V	V		V
9	Tsila and Gila (b)				V	V	V			V	V	V
10	Lia and Sara (b)		V		V	V	V			V		V

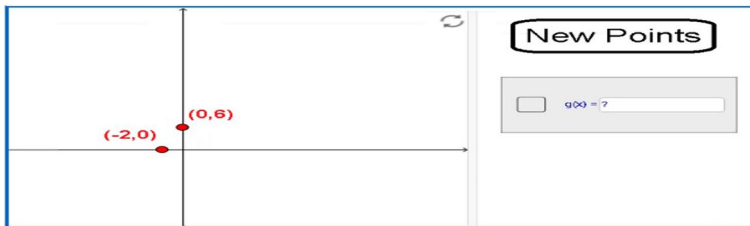


Fig. 1 A GeoGebra-based applet for the first mathematical task. The points’ locations were random integers between (-10) and (10) that changes upon clicking the “new points” button

mathematical task remotely. In every episode, a student -dyad collaborates to solve the second mathematical task remotely. Prior to every episode was a presentation of the student’s solutions to task 1 (e.g., Fig. 2), followed by short video excerpts of the student -dyad’s collaboration¹. At the end of each episode the teachers were asked to elaborate orally on what they saw. They were also asked whether what happened at that moment merited responding, and if so, what type of response. The experimenter’s role at this stage was to ensure that the teacher would refer to three questions: (1) Describe what you saw. (2) Would you intervene in the students’ work? Why? (3) If you would, how would you do that? The interviewer asked for elaboration at times and controlled the transition from an episode on the basis of completing the account for the currently viewed episode.

Table 1 presents the initial analysis regarding the dialogic attributes in the episodes: (1) deepening the dialogue, (2) widening the dialogue, (3) perspective changing, (4) persuasive interaction, (5) repeating the other’s voice, (6) agreement and collective actions, (7) authoritative interaction, (8) one leads the other is led, (9) students have equal status, (10) dialogic gap, (11) monologue. The first three student-dyads appeared again by the end (denoted with a and b).

¹ The film is available at <https://youtu.be/cCj4rsbilzE>.

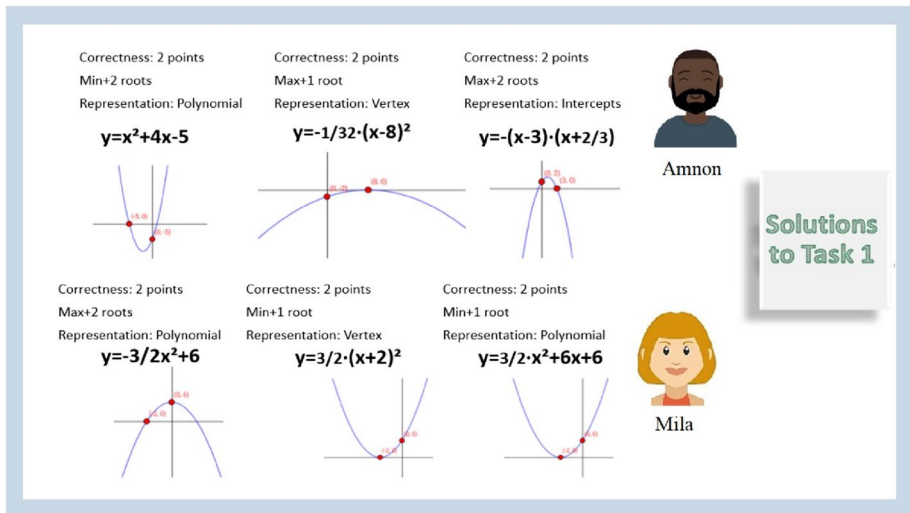


Fig. 2 An example of the information presented in the experimental video to inform teachers regarding students' personal example spaces in previous tasks

In the second stage, the teachers participated in an 80-min lesson. The instructor explained the research and the purpose of the intervention. Six activities followed. First, the participants wrote their definitions of dialogue on a shared digital writing board. Second, the participants were exposed to dialogic concepts and ideas (encapsulated in the theory section of this paper), the notions of voices, dialogic gap, and mutuality. Third, in the plenary led by the authors, they discussed instructional practices that could promote dialogue. Among these practices were allocating individual thinking time prior to and during group learning, encouraging student involvement, and the cultivation of the quality of interaction between learners. Fourth, the teachers were asked to write their personal instructional ideas on how to support dialogue. For example, embracing and celebrating mistakes in the classroom provides valuable opportunities for learning and growth. In the fifth part of the intervention, the teachers shared their ideas and engaged with each other's practices. In the last activity, the teachers were asked to list what they would pay attention to in collaborative learning the next time they see the film of the experiment and hopefully, in their classroom.

The third stage happened approximately 2 weeks after the second stage. The teachers were interviewed again following the same protocol as in the first stage (i.e., post-intervention), with one change. Before the first interview, teachers solved the two mathematical tasks. This time, teachers read their list of things they would pay attention to to elicit dialogue.

Analytical procedures

The 28 video recordings were transcribed and put into MS Excel spreadsheets. We omitted the interviewer's inferences and off-topic utterances of teachers and had 1685 and 1817 units of meaning (hereafter, and "utterance") in the pre and post, respectively. Every utterance was inserted into one row on a spreadsheet, ranging in length from one word to more complex sentences (Derry et al., 2010). We used the following criteria to separate the utterances: (1) changes in the speaker, (2) changes in the object of speech,

Table 2 Noticing dialogic utterances per dialogic attributes, before and after the intervention

Dialogic category	Dialogic attributes	Attend		Respond		Total noticing (attend and respond)			
		Pre	Post	Pre	Post	Pre	Post	Sig.	<i>t</i>
Dialogue Expanding	Deepening the dialogue	1	0	0	2	1	2	0.34	-1
	Widening the dialogue	1	0	1	1	2	1	0.34	1
Inter-animation	Perspective changing	4	0	2	4	6	4	0.61	0.52
	Persuasive interaction	3	6	0	3	3	9	< 0.05	-2.48
	Repeating other's voice	0	0	5	0	5	0	0.05	2.11
Relations	Agreement and collective attention	25	56	5	15	30	71	< 0.05	-3.56
	Authoritative interaction	2	8	0	2	2	10	0.12	-1.67
	One leads and another is led	35	49	4	8	39	57	< 0.05	-2.26
	Students have equal status	2	8	3	7	5	15	< 0.05	-2.51
Voices	Dialogic gap	9	20	3	24	12	44	< 0.05	-4.02
	Monologue	1	23	1	5	2	28	< 0.05	-4.45
Total		83	170	24	71	107	241		

and (3) complex arguments in which the object of speech did not change, but the learner developed, disclaimed, or refuted the previous part of the argument.

We then coded two aspects of the utterances: the stage of noticing (Schoenfeld, 2011) and dialogic activity (Trausan-Matu et al., n.d.). We did not distinguish between what teachers attended to and the interpretations they made—in think-aloud protocols it is impossible sometimes to distinguish between the two. Two trained coders independently coded the interviews, discussed the results, and refined the coding scheme. They then randomly selected three more interviews to code and compared their results. Inter-rater reliability between the coders at this stage was (89%) for noticing and (96%) for dialogic activities. The coding was then finalized by coder 1.

To simplify the analysis of the dataset and reduce its granularity, we eliminated duplicate combinations of the four variables: teachers, episodes, noticing stage, and dialogic activity. When the values of these variables were the same, they were considered a single combination, allowing to focus on the presence of a particular combination rather than the frequency with which it occurred within a single episode. We arrived at 577 and 550 utterances of coded data in the pre and post, respectively. These utterances were allocated into four categories: expanding the dialogue, inter-animation, interpersonal relationship, and voices (Table 2). The utterances and the coding were further analyzed using SPSS.

Results

We conducted a series of two-tailed *t*-tests for paired samples ($n = 14$) to answer the overarching question regarding the change in teachers' noticing attending after the intervention. We counted attending and responding utterances per episode—before and after the intervention (Table 4). After the intervention, teachers noticed the various dialogic attributes significantly ($p < 0.05$) more than in the pre-test (107 to 241 cases). Specifically, they attended to dialogue significantly ($p < 0.05$) more than in the pre-test (83 to 170 cases) and responded to dialogue significantly ($p < 0.05$) more than in the pre-test (24 to 71 cases).

Teachers' learning: noticing outcomes of the intervention

Next, we assessed the change in teachers' attending to and responding to different dialogic attributes after the intervention. We conducted a series of two-tailed *t*-tests for paired samples to test for changes in teachers' noticing per all 11 dialogic attributes (Table 2). We counted attending and responding utterances per dialogic attribute—before and after the intervention. Per dialogic attribute (as in the case of per episodes), after the intervention, the teachers noticed dialogue, attended to dialogue, and responded to dialogue significantly more than in the pre-test ($p < 0.05$). Beyond these significant changes in listening to dialogue, responding to dialogue, and total utterances after the intervention, we observed significant changes after the intervention in persuasive interaction, one leads and another is led, agreement and joint action, dialogic gap, monologue, and students with equal status (Table 2).

Individual noticing practices

To account for individual changes in teachers' noticing of dialogue we counted attending and responding utterances per teacher before and after the intervention (Table 3). As in the case of per dialogic attribute, after the intervention the participants noticed dialogue, attended to dialogue, and responded to dialogue significantly more than in the pre-test ($p < 0.05$). In particular, 9 of the 14 participants showed a change in at least one more dialogic utterance per episode after the intervention. Three participants showed milder increases (2, 6, and 13), and two participants showed a mild decrease (5 and 9).

Table 3 Dialogic utterances per participant in the pre- and post-tests

Participant	Attend		Respond		Total noticing (attend and respond)	
	Pre	Post	Pre	Post	Pre	Post
1	3	8	1	11	4	19
2	16	21	2	5	18	26
3	2	15	0	7	2	22
4	2	17	4	7	6	24
5	8	6	4	2	12	8
6	7	7	2	7	9	14
7	4	15	1	3	5	18
8	11	17	1	7	12	24
9	5	1	1	0	6	1
10	0	9	0	3	0	12
11	3	12	1	2	4	14
12	0	11	1	2	1	13
13	10	11	4	10	14	21
14	12	20	2	5	14	25
Total	83	170	24	71	107	241

The “agentivity” of specific episodes to enhance noticing

To identify the change per episode, we conducted a series of two-tailed *t*-tests for paired samples to test for changes in teachers’ noticing per every episode (Table 4). We counted attending and responding utterances per episode—before and after the intervention. Per episode (as in the case of per participant and per dialogic attribute), after the intervention, the participants noticed dialogue, attended to dialogue, and responded to dialogue significantly more than in the pre-test ($p < 0.05$). In particular, we observed increases in dialogic utterances per all episodes but one (episode 6). In particular, we observed significant increases ($p < 0.05$) in teachers’ noticing of dialogue in episodes 2,3,5,7, and 9 (Table 4).

Teachers’ responses to dialogue after the intervention

Of the 241 utterances coded as noticing dialogue, 170 were coded as attending to and 71 as responding to dialogue. To characterize the types of teachers’ responses in light of what participants attended, we analyzed the 71 responding utterances along the four dialogic categories—dialogue expanding, inter-animation, relations, and voices.

Throughout the experiment, we observed only a few utterances related to dialogue expansion, with no significant changes between the pre- and post-tests among related dialogic attributes. We therefore focused on attributes within these categories in which we observed a significant change after the intervention: persuasive interaction, one leads and another is led, agreement and joint action, dialogic gap, monologue, and students with equal status.

Participants’ responses that aimed to foster inter-animation focused on eliciting persuasive interaction among students. The teachers instructed students to explicate their thoughts in ways that could increase inter-animation by their peers (four utterances). For example, episode two contains an interaction reflecting an imbalance between two students, Amnon and Tamar, characterized by Amnon carrying the situation forward and Tamar following and praising his solution (Table 2). Participant 2 attended to this imbalance (“the boy took over the activity”) and offered a response that elicited Amnon to state, “This is what I think” and asked Tamar, “What do you see in it?”

Table 4 Dialogic utterances per episode in the pre- and post-tests, and its distribution per the noticing stages

Episode	Attend		Respond		Total noticing (attend and respond)			
	Pre	Post	Pre	Post	Pre	Post	Sig.	<i>t</i>
1	8	13	0	1	8	14	0.34	-1.00
2	14	24	9	25	23	49	< 0.05	-2.63
3	4	21	0	7	4	28	< 0.05	-2.67
4	4	12	1	1	5	13	0.12	-1.69
5	6	20	1	8	7	28	< 0.05	-2.43
6	18	21	6	2	24	23	0.83	0.22
7	6	15	0	12	6	27	< 0.05	-3.02
8	9	16	5	4	14	20	0.37	-0.94
9	0	7	0	5	0	12	< 0.05	-2.45
10	14	21	2	6	16	27	0.18	-1.45
Average	8.3	17	2.4	7.1	10.7	24.1		

In their responses to the relations between students, teachers often sought to balance the status (15 utterances). One way of doing so was to encourage students to reach an agreement and collective attention. Participant 1 also identified the imbalance in episode 2 (“I see that he is the one that mainly participated. Tamar was less active”) and proposed to tell the students, “It should be done in collaboration” and that “Participant should make their own individual contributions.” Another common way of responding is to balance the relations included negating situations in which one leads and another is led (eight utterances). For example, other participants responded to the case of imbalance in Amnon and Tamar’s solution (episode 2) by suggesting that Amnon, by offering that “he should not be allowed to take over the activity” (teacher 2) and Tamar should be asked “to lead the next move” (teacher 13). In one case, it was recommended that they should “change partners because Amnon had completely taken over the discussion” (teacher 4). A third way to promote the relations between learners was through attempts to balance the status of the students (seven utterances). For example, in episode 5, Sean listened to Rona’s solution to the task. Rona did not apply algebraic consideration to her solution and solved the task by moving a parabola and guessing a proper placement for it. In her noticing, participant 14 attended the imbalance in their relations (“she is very confident,” but he did not ask any challenging questions) and proposed to ask Sean to think of other parabolas that can be created to “make the dialogue to a level that is [...] more balanced.”

Teachers’ responses also focused on enhancing students’ voices. Many of the responses after the intervention were aimed at facilitating the emergence of a dialogic gap between learners (23 utterances). Participant 7 addressed the incorrectness in Rona’s solution to episode 5 (“The problem is that she did not reach an accurate solution, because the parabola cannot pass next [to the two red points]”). Participant 8 proposed a response that included a refutation of Rona’s solution and asked her to think of another way to solve the task. He suggested that Rona will “ask him if he has a better solution, and then I would let him talk”. At other times, teachers’ responses aimed at eliciting students’ voices and were focused on promoting a monologic narrative (five utterances). For example, participant 8 attended to the imbalance in episode 2 between Amnon and Tamar (“Amnon really led everything, and she hardly spoke”). The response offered included finding the correct moment in which she (the teacher) would ask Tamar to “continue from here to the end of the solution”. Tamar, in this case, tried to focus on repeating what Amnon said (his voice), thus maintaining a monologic interaction.

Interestingly, the data reveal pedagogical tensions when teachers pondered their responses between adhering to the correctness of the group’s mathematical discourse and eliciting individual student voices. Although some participants thought this was a useful method to help guide students facing difficulties in reaching a correct solution, other teachers saw it as a constraint that might hinder some students’ ability to express or explain themselves. Instead, they suggested a dual approach of maintaining both natural discourse and mathematical, methodical discourse.

Discussion

We operationalized dialogue using Bakhtinian definitions and used them to train the participants and to analyze their pre- and post-intervention noticing. This approach is used to identify the merits of noticing education as part of teachers’ professional development. Overall, our findings support those of Stockero on mathematics noticing training (Stockero, 2021) and expand them to include in-service teachers and the noticing of collaboration. Teachers’ dialogue noticing skills changed as a result of training. A total of 9 of the 14 participants showed

an increase in noticing per episode after the intervention. Teachers scoring low in the use of dialogic noticing in the pre-test experienced greater changes after the intervention. These teachers learned to notice dialogue in places where they had not previously paid attention. Teachers with an initial strong focus on dialogic noticing showed moderate changes after the intervention. Three participants showed milder increases, and two participants showed slight decreases in dialogic utterances. To us, these decreases are a natural outcome of an intervention aimed to elicit teachers' orientations toward dialogue, as it highlights the limitations of this nondirective approach. Some teachers, we hypothesize, are not sensitive to such an instructional approach (i.e., intervention in stage 2 of the experiment); teachers 5 and 9 are two possible examples of the matter. Significant changes were found in participants' attending and responses to six main dialogic attributes within three categories: inter-animation, relations, and voices. No major changes were recorded regarding extending the dialogue category.

The greatest changes were observed in the relations category, where increases were recorded in agreement and collective attention, one leads and another is led, and students have equal status. Participants attended to, and even more importantly, responded to potential imbalances that may hinder effective collaboration. Accordingly, the participants became even more attuned to how students work together toward a common goal and the strategies they use. Our findings suggest that teachers may be able to learn all measures of effective collaboration quite quickly: attend to agreement and collective action, equal status, and respond to situations in which none of the voices present is externally authoritative. In their responses, teachers used two main balancing strategies: first, by encouraging both students to work together and reach an agreement (e.g., "each participant should make their own contribution"), and second, by addressing only one student, either limiting the role of leader or elevating the one being led ("asking her to lead the next move"). Thus, noticing the relational status between students can help teachers understand the extent to which all students are able to contribute to the learning process and to identify barriers that prevent students from fully participating.

The increase within the inter-animation category is mainly associated with persuasive interaction, by which students enter the realm of their peers' words and attempt to change them from within (Wegerif, 2011). This increase possibly emerged because persuasive interaction focuses on how students try to convince each other rather than take over and express authority. Attending to this persuasive interpersonal stance for interaction provides teachers with insights into the ways students engage with each other and the extent to which they are open to considering alternative viewpoints. Accordingly, in their responses, the participants aimed to elicit interanimation, by asking students to re-explicate their peers' thoughts (e.g., "what do you see in it?").

The participants developed a greater sensitivity to the fundamental dialogic idea of multivoicedness, manifested in an increased awareness of the number of voices present in a learning dialogue. In particular, participants identified (1) the dialogic gap representing the difference between voices and (2) the monologue referring to a situation with one voice expressed. Teachers' responses after the intervention were primarily focused on facilitating the emergence of a dialogic gap between learners ("[I would] ask him whether he has a better solution, and I would let him talk"). However, even after the intervention, some of the responses aimed to elicit students' voices still promoted monologic narratives that enhance the existing voice, which they find to be promising or productive (Tamar should continue Amnon's idea "from here to the end of the solution"). Possibly, this is because mathematics teachers tend to refer to the mathematical aspects of a learning situation, such as correctness and appropriateness (Calor et al., 2022). A response to a monologic interaction in line with the dialogic view of collaborative learning can focus on eliciting more voices in the interaction, in addition to those already voiced.

Noticing is the pragmatic site in which knowledge, instructional intentions, and values converge into practices of attending and responding. It is a dialogic space in which different voices dwell in tension and that determines teachers' political and moral dimensions of resource allocation (what's worth attending, what's the meaning of the seen, how to regulate the acted). Teachers' repertoire is meant to serve two masters: the dialogic tension in noticing "ways of seeing math" and "additional voices" is productive. It must be maintained, without one strand of noticing eliminating the other. Students need the openness of the dialogue to pursue ideas. Such exploration can turn meaningless without correctness. Attending and responding are rich practices that require in-depth considerations by the teacher. It is impossible to reduce noticing dialogue to "best practice" behaviors. Learning to attend to dialogic moves in students' work can be a steppingstone for teachers to nurture collaborative learning skills among students. Our findings can be seen as a call for developing teacher training programs in which teachers can learn to attend to content-specific and dialogical attributes of a collaborative learning situation.

Because of its condensed practice-based nature, noticing is a promising starting point for reflecting and expanding teachers' instructional repertoire. Ayalon and Nama (2023) conclude that instructional practices integrating both dialogic and content aspects of learning are critical to supporting student participation. The ability to notice dialogue can be an important resource for teachers in nurturing collaborative mathematics learning. Noticing practices are influenced by resources and approaches at the teachers' disposal (Schoenfeld 2011). Our study confirms this. The fact that the teachers knew how to translate dialogic into noticing after the intervention without directly dealing with instruction perhaps demonstrates that teachers have a sense of noticing practices but may require additional intermediate level theories as a base for support.

Limitations and future research

Studies on iterative learning processes can be sensitive to practice effect bias. Although our study design did not include a separate control group, we took several steps to minimize and account for practice effects. First, the professional development intervention was grounded in a robust theoretical framework that provided participants with novel lenses through which to view collaboration and dialogue. They did not know these lenses (e.g., concepts, language) before. This theoretical grounding is critical in distinguishing the learning outcomes attributable to the intervention from those that might arise from repeated exposure to the material (e.g., practice). Second, the inclusion of a reflective writing task as part of the intervention served as an additional measure to deepen participants' engagement with the dialogic concepts. This task required teachers to articulate their observations and insights, which likely contributed to a more profound learning experience than mere repeated exposure to data. Third, we analyzed changes in noticing practices quantitatively to assess the change and qualitatively to understand the changes' characteristics. If mere practice or familiarity were the only factors, we would expect a general increase in noticing but not necessarily a deeper or more nuanced understanding of dialogic interactions, which was observed.

The research design did not include the study of causal relations between what teachers attend to and their responses. Additional research is necessary to claim causality and to arrive at a more complete generalization of the results. One way to further pursue our line of research would be to examine specific groups of teachers, experts, and novices with different mathematical activities and other dialogic frameworks. Although we were pleased with the reported increase in noticing dialogue, which shows that noticing is a learned practice

related to the intervention, the increase in noticing dialogue was accompanied by an observed decrease in noticing mathematical activity, a phenomenon also known as attentional tunneling (Wickens & Alexander, 2009). Further studies should focus on noticing mathematics and dialogue to understand when these facets complement or exclude each other.

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Declarations

Declarations of interest The authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements) or non-financial interest (such as personal or professional relationships, affiliations, knowledge, or beliefs) in the subject matter or materials discussed in this manuscript.

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January 17, 2022

Dr. Ben-Zion Slakmon
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This is to inform you that your research proposal no. 0004368-1 entitled:
**"Teachers' noticing of dialogue in computer-supported
collaborative mathematics learning"**

meets the requirements of the Ethics Committee of Tel-Aviv University.

The investigator's information:
The document is for ethical purposes only and valid until 16-01-2023.

* Please note: Applications for the extension of a certificate will be
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Sincerely,


Prof. Mair Lahav
Chairman, The Ethics Committee
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