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# Constrained portfolio strategies in a regime-switching economy

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## Abstract

We implement an allocation strategy through a regime-switching model using recursive utility preferences in an out-of-sample exercise accounting for transaction costs. We study portfolios turnover and leverage, proposing two procedures to constrain the allocation strategies: a low-turnover control (LoT) and a maximum leverage control (MaxLev). LoT sets a dynamic threshold to trim minor rebalancing, reducing portfolio turnover, mitigating costs. MaxLev calculates dynamic adjustments to the risk aversion parameter to constrain the portfolio leverage. The MaxLev adjustments depend on the risk aversion and permitted portfolio leverage, which enables optimal strategies considering the leverage constraints. The study uses US equity portfolios, and shows that, first, models with LoT result in superior return-to-risk measures than those without it when transaction costs increase. Second, considering transaction costs, the return-to-risk measures of the models using MaxLev closely match or exceed those from the corresponding unconstrained regime-switching benchmarks. Third, MaxLev returns have lower volatility and higher return-to-risk than conventional numerically constrained benchmarks. Fourth, the certainty equivalent returns indicate that models using MaxLev and LoT outperform both single-state models and unconstrained regime-switching models with statistical significance.

**Keywords** Regime switching models · Dynamic asset allocation · Stochastic differential recursive utility · Analytical solutions · Transaction costs · Leverage and turnover constraints

**JEL Classification** G11 · O16 · G24 · C02 · E44

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## 1 Introduction

According to Munk (2013), recursive utility functions, as discussed by Epstein and Zin (1989), became popular among practitioners due to a superior accuracy in characterizing the investor's preferences. Given that lifetime utility at time $t$ can be captured by a utility index referred to in the literature as felicity, previous dynamic functions like the power utility and the exponential utility considered that investor's felicity was time-additive, meaning that it would be merely incremental. Unlike them, recursive functions consider that tomorrow's felicity also depends on today's felicity, enabling investors to time the uncertainty resolution, which is reflected through the disentanglement of the attitudes toward atemporal risks (relative risk aversion) from the attitudes toward shifts in consumption over time (elasticity of intertemporal substitution). One function that captures such recursive preferences in continuous time is the stochastic differential utility introduced by Duffie and Epstein (1992).

To the best of our knowledge, the asset allocation model presented by Campani, Garcia, and Lewin (2021), herein denoted the CGL model, is the only model to apply the stochastic differential recursive utility function in a regime-switching framework. It consists of an approximate analytical method, which the authors demonstrate to be sufficiently accurate to solve the allocation problem. Before this model, the literature solved dynamic allocation under multiple regimes with the power utility function, via numeric methods such as the Monte Carlo simulation—Sass and Haussmann (2004), Guidolin and Timmermann (2007), and Liu (2011).

Lewin and Campani (2020a, b) test the CGL model in different settings, sharing the same finding: the CGL returns consistently outperform reference portfolios. So far, the CGL returns have been presented in the scope of unconstrained strategies and without accounting for transaction costs, which is not unusual in the literature. For example, Ang and Bekaert (2002) and Graflund and Nilsson (2003) present optimal solutions without accounting for costs.

Our study assesses the impacts of costs and the strategy's weights in the CGL model implementation. We endorse the literature that strategies might become unpractical if aspects such as portfolio leverage and rebalancing were not constrained. Thus, our objective is to propose filters to constrain the CGL portfolio's maximum leverage and to lower its turnover.

We propose a maximum leverage control, referred to as the MaxLev filter. Analogously to the drawdown control by Nystrup et al. (2019), MaxLev dynamically adjusts the risk aversion parameter ($\gamma$) to constrain the maximum leverage. First, it collects the optimal portfolio leverage for a theoretical 100% probability for each regime. Then, it calculates the adjustments over $\gamma$ to confine the optimal leverage inside the maximum leverage permitted for each regime, weighting them by the out-of-sample probabilities. Although the adjustments originate from the leverage limits, they remain dependent on $\gamma$. Thus, MaxLev presents a constraining method constantly proportional to $\gamma$. It distinguishes from other numerical methods that just trim excessive weights, in which the constrained leverage might become flattened at the level of the limits if the base case is often excessively leveraged. With MaxLev, leverage is constantly proportional to the regimes' probabilities, as expected from an unconstrained allocation. Therefore, the risk exposure is balanced by the regimes' expectations, enabling an optimal strategy that remains inside the leverage limits at each point in time.

We also propose a low-turnover control (LoT) to rebalance the portfolio if the turnover from $t$ to $t + 1$ is above a threshold. The rebalancing policy is dynamically recalculated, updating the threshold by the historic turnover during significant probability changes.

The study compares the CGL model with and without MaxLev and LoT in different levels of transaction costs using single-state and regime-switching benchmarks in an out-of-sample exercise. The empirical results show that MaxLev matches or improves the return-to-risk measures relative to the benchmarks, while LoT produces economic value when cost levels are higher. The certainty equivalent returns indicate that the CGL model using MaxLev and LoT outperform most benchmarks with statistical significance.

## 2 Literature review

Ang and Timmermann (2012) have reviewed the literature since the seminal paper by Hamilton (1989). The authors indicate the increasing importance of regime-switching models in asset allocation strategies, where the power utility functions have been widely used. For example, Ang and Bekaert (2002) and Guidolin and Timmermann (2007, 2008) numerically solved the problem using the constant relative risk aversion utility (CRRA). Later, Guidolin and Hyde (2012) applied the same procedure while Çanakoğlu and Özekici (2012) found the explicit solution for maximizing the expected utility of terminal wealth with a hyperbolic absolute risk aversion function (HARA), but suggest that further studies should involve other utility functions. In turn, Ang and Bekaert (2002) state that the recursive utility functions improve regime-switching effects in allocation problems.

Using the recursive utility, Kraft et al. (2017) and Xing (2017) solved the consumption-investment optimization with Epstein and Zin's (1989) function, but neither performed it observing regimes. For such a case, Campani, Garcia, and Lewin (2021) presented a regime-switching allocation strategy (CGL model) in continuous time based on Duffie and Epstein's (1992) stochastic differential recursive utility function. They offered an approximate analytical solution based on Campani and Garcia (2019), given a system of partial differential equations. Campani, Garcia, and Lewin (2021) and Lewin and Campani (2020a, b) present optimal allocations based on the CGL model. We advance the research based on such a model accounting for transaction costs and leverage and turnover constraints—a gap left open.

The literature presents several approaches to constrain portfolio leverage, indicating there is no consensus. One is to shrink the covariance matrix toward an identity matrix. Ledoit and Wolf (2004) and Fiecas, Franke, Von Sachs, and Tadjuidje (2017) apply it to constrain mean–variance portfolios. In another approach considering the quadratic utility, Clarke, De Silva, and Thorley (2002) and DeMiguel et al. (2009b) use a factor to shrink the weight vector. Later, Dal Pra, Guidolin, Pedio, and Vasile (2018) simply prevent the weights exceeding leverage limits using power utility preferences. These approaches, however, have significant limitations. In the first approaches, the problem of modifying high dimensional covariance matrices becomes more pronounced for regime-switching models (Nystrup et al. 2019). In the latter, if leverage is often excessive, the constrain flattens it at the boundaries level, unbalancing the risk exposure to the regimes' expectations.

Inspired by Nystrup et al. (2019) drawdown control, we overcome such limitations constraining leverage around the premise of a dynamically adjusted $\gamma$, the risk aversion parameter. The authors increase $\gamma$ in the domain of losses, as a reaction mechanism. However, in case of successive losses, it traps the investor in elevated risk settings.

Although we also calculate dynamic adjustments for $\gamma$, the investor's risk preferences do not disconnect from the base case ($\gamma$) in our method. Thus, $\gamma$ remains an unobservable behavioral parameter. Based on the maximum leverage permitted, we only infer adjustments to $\gamma$, assuming the leverage limits might be conditioned on the regimes' expectations. We emphasize that we do not set $\gamma$ according to the regimes; hence we do not shape the utility function—in a behavioral sense—as regime dependent. Instead, we consider that the leverage limits impact the utility function. Therefore, the method allows an optimal portfolio strategy that respects the leverage constraints at each point in time.

In addition, the literature mitigates transaction costs by imposing rebalancing policies that constrain portfolio turnover. For example, Lunde and Timmermann (2004) and Guidolin and Timmermann (2008) timed the regimes' duration to separate long and short-run movements. Instead, we follow Clarke, De Silva, and Thorley (2002), defining a rebalancing policy upon a minimum turnover threshold (limit). However, while they consider a static threshold, we dynamically compute it based on regimes' probabilities. Thus, we maintain an updated rebalancing policy.

## 3 Method

### 3.1 The regime-switching economy

We apply the strategy developed by Campani, Garcia, and Lewin (2021) with a continuous-time regime-switching model in which investors maximize their stochastic differential recursive utility functions through optimal portfolio strategies.

#### 3.1.1 State variable

We assume a regime-switching economy governed by the unobservable state variable $Y_t$ representing an independent right-continuous-time Markov chain process admitting only values in $R = \{1, 2, \ldots, m\}$, where $R$ is a finite set of $m$ possible regimes. Following Hamilton (1989), the state variable behavior is modeled using transition probabilities that rule if the economy will remain at the same regime or jump to a new one after an exponentially distributed length of time. If $i$ is the current regime and $\lambda_{ij}$ is the density of transition probabilities between regimes $i$ and $j$, the probability to jump to another regime $j$ at time $\Delta t$ is:

$$P_{ij,\Delta t} = \frac{\lambda_{ij}}{\sum_{k\neq i}\lambda_{ik}}\left(1 - e^{-\sum\limits_{k\neq i}\lambda_{ik}\Delta t}\right), \text{ with } j \neq i \in R > 0,\ \lambda_{ij} > 0, \tag{1}$$

where $\lambda_{ii} = -\sum_{k\neq i}\lambda_{ik} \leq 0$ such that $P_{ij,\Delta t} = \frac{\lambda_{ij}}{-\lambda_{ii}}\left(1 - e^{\lambda_{ii}\Delta t}\right)$. Hence, the probability of staying at the same regime $i$ over the next $\Delta t$ is given by $P_{ii,\Delta t} = e^{\lambda_{ii}\Delta t}$, with $\lambda_{ij}$ assumed constant.

### 3.1.2 Assets dynamics

As Lewin and Campani (2020a,b), we estimated the model for excess returns $(\widehat{r})$ over the riskless asset $(r_f)$. We consider that the risk-premia from the $n$ risky assets are defined through the following multidimensional stochastic process:

$$\begin{bmatrix} d\hat{r}_{1,t} \\ d\hat{r}_{2,t} \\ \dots \\ d\hat{r}_{n,t} \end{bmatrix} = \boldsymbol{\mu}_{s,t}dt + \boldsymbol{\sigma}_{s,t}\boldsymbol{dZ_t}$$
$$= \begin{bmatrix} \mu_{1,t} \\ \mu_{2,t} \\ \dots \\ \mu_{n,t} \end{bmatrix} dt + \begin{bmatrix} \sigma_{11,t} & 0 & \dots & 0 \\ \sigma_{21,t} & \sigma_{22,t} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \sigma_{n1,t} & \sigma_{n2,t} & \dots & \sigma_{nn,t} \end{bmatrix} \begin{bmatrix} dZ_{1,t} \\ dZ_{2,t} \\ \dots \\ dZ_{n,t} \end{bmatrix}, \tag{2}$$

where $\boldsymbol{\mu}_{s,t}$ is a $n \times 1$ vector of the instantaneous expected risk-premia (drifts), $\boldsymbol{\sigma}_{s,t}$ is an $n \times n$ lower triangular volatility matrix, and $\boldsymbol{dZ_t}$ is a column vector with $n$ increments of independent standard Wiener processes. Note that both $\boldsymbol{\mu}_{s,t}$ and $\boldsymbol{\sigma}_{s,t}$ are time-varying and conditioned by the state variable $Y_t$. In turn, with $Y_t = i, i \in R$, we find:

$$\boldsymbol{\mu}_{s,t} = \boldsymbol{\mu}_{s,i} = \begin{bmatrix} \boldsymbol{\mu}_{1,i} \\ \boldsymbol{\mu}_{2,i} \\ \dots \\ \boldsymbol{\mu}_{n,i} \end{bmatrix} \text{ and } \boldsymbol{\sigma}_{s,t} = \boldsymbol{\sigma}_{s,i} = \begin{bmatrix} \sigma_{11,i} & 0 & \dots & 0 \\ \sigma_{21,i} & \sigma_{22,i} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \sigma_{n1,i} & \sigma_{n2,i} & \dots & \sigma_{nn,i} \end{bmatrix}, \tag{3}$$

in which, $\boldsymbol{\mu}_{j,i}$ coefficients and $\boldsymbol{\sigma}_{s,i}$ matrices are constant for each $j = \{1, 2, \dots, n\}$. We underline that $\boldsymbol{\sigma}_{s,i}$ elements are defined as partial volatilities, *e.g.*, $\sigma_{21,i}$ denotes partial volatility of asset 2 in relation to the first Wiener process $(dZ_{1,t})$ in regime $i$, and also that $\boldsymbol{\sigma}_{s,i}\boldsymbol{\sigma}_{s,i}^T$ represent the regime-dependent variance–covariance matrix. Still, as the drifts are regime-dependent and simultaneously time-dependent, it means that they can vary in time even if the regime remains unchanged. Such drifts are stored in an $n \times m$ drift matrix:

$$\boldsymbol{D}_{s,t} = \begin{bmatrix} \boldsymbol{\mu}_{1,1} & \boldsymbol{\mu}_{1,2} & \cdots & \boldsymbol{\mu}_{1,m} \\ \boldsymbol{\mu}_{2,1} & \boldsymbol{\mu}_{2,2} & \cdots & \boldsymbol{\mu}_{2,m} \\ \cdots & \cdots & \cdots & \cdots \\ \boldsymbol{\mu}_{n,1} & \boldsymbol{\mu}_{n,2} & \cdots & \boldsymbol{\mu}_{n,m} \end{bmatrix}. \tag{4}$$

Provided such assets and state variable processes, we obtain the regime-parameters through the maximum likelihood (ML) estimation methodology. These parameters are: the drift matrix $\boldsymbol{D}_{s,t}$, the volatility matrix $\boldsymbol{\sigma}_{s,i}$ and the transition probabilities $P_{ij,\Delta t}$ (where $j \neq i \in R$). In fact, the number of parameters to be estimated is $[mn + mn(n+1) \div 2 + m(m-1)]$. Given the unobservable nature of the regimes, we assume that investors can infer the regimes' occurrences through filtered probabilities observing the assets' past returns. We detail the probabilities estimation according to Hamilton's (1989) procedure in the supplementary file.

### 3.2 The portfolio strategy

After configuring the regime-switching economy, we address the portfolio strategy. First, considering $W_t$ as the wealth in $t$ and $\boldsymbol{\alpha}_t$ as the $1 \times n$ vector of portfolio shares of the risky assets, and $(1-\boldsymbol{\alpha}_t 1)$ as the riskless asset share, wealth dynamics can be expressed as:

$$dW_t = (1 - \boldsymbol{\alpha}_t 1) W_t r_f dt + W_t \boldsymbol{\alpha}_t \frac{dS_t}{S_t} = W_t r_f dt + W_t \boldsymbol{\alpha}_t \left[ \boldsymbol{D}_{s,t} \boldsymbol{\pi}_t dt + (\boldsymbol{V} \boldsymbol{\pi}_t) \boldsymbol{dZ}_t \right], \tag{5}$$

where 1 is a column vector of $n$ ones, $\frac{dS_t}{S_t}$ is the column vector with $n$ infinitesimal risky asset returns, $\boldsymbol{\pi}_t$ is a column vector with the $m$ filtered probabilities in $t$ and $\boldsymbol{V}$ is an $1 \times m$ row vector containing the regime-dependent covariance matrices ($\boldsymbol{\sigma}_{s,i}$).

#### 3.2.1 Utility function

In the CGL model, the investor's preferences are characterized as continuous-time and modeled by the stochastic utility function from Duffie and Epstein (1992):

$$J_t = E_t \left[ \int_{u=t}^{T} f(C_u, J_u) du + \frac{W_T^{1-\gamma}}{1-\gamma} \right], \tag{6}$$

where $E_t$ is the expected value in the current moment ($t$); $T$ is the investment horizon; $f$ is the recursive aggregator of the utility function $J_t$ in function of consumption rate $C_u$ (in moment $u$) and $J_u$, the continued utility in $u$. In turn, $W_T$ is the investor's terminal wealth while $\gamma$ is the risk aversion coefficient. The following function details the utility function aggregator:

$$f(C, J) = \frac{\beta}{1-\frac{1}{\psi}}(1-\gamma)J\left\{\left[\frac{C}{\left[(1-\gamma)J\right]^{\frac{1}{1-\gamma}}}\right]^{1-\frac{1}{\psi}} - 1\right\}, \tag{7}$$

where $\beta$ is the time preference rate of the investor's utility (felicity) and $\psi$ is the elasticity of intertemporal substitution, i.e., consumption choices over time. Thus, we must set $\psi$, $\beta$ and $\gamma$ to configure the strategy using recursive utility.

Campani and Garcia (2019) analyze the sensitivity of consumption and portfolio choices over the value of both preference parameters $\gamma$ and $\psi$ for a problem similar to Campani, Garcia, and Lewin (2021) but in a single-state model. They indicate that the value of $\psi$ affects consumption preferences but barely affects the allocation strategy. Later, Campani, Garcia, and Lewin (2021), considering regime-switching models, conclude that the impact of consumption-to-wealth ratio variations is minimal in the allocation strategy. Thus, we simplify the current application disregarding intermediary consumption. Considering that $\psi > 1$ is this case where substitution effects dominate and the investor is willing to postpone consumption, we define $\psi = \infty$ to represent the investor that waits for the terminal horizon to consume their wealth. Then, given a problem without intermediary consumption, the value of $\beta$ does not significantly affect the allocation strategy. Campani, Garcia, and Lewin (2021) and Guidolin and Timmermann (2007) also show that the investment horizon has a negligible impact on the allocation strategy considering frequent rebalancing, like in our application. Following them, we consider $\gamma = 5$ our base case.

Campani, Garcia, and Lewin (2021) demonstrate that the general solution quantifying the investor total optimal utility in $t$ ($V_t = \sup J_t$) admits the wealth-separable solution:

$$V(W_t, \boldsymbol{\pi_t}, \tau) = H(\boldsymbol{\pi_t}, \tau)\frac{W_t^{1-\gamma}}{1-\gamma}, \tag{8}$$

where $\tau = T - t$ is the time until the final horizon and $H(\boldsymbol{\pi_t}, \tau)$ is a function in terms of the time to horizon and the regimes' probabilities vector. But, as an exact analytical expression for $H(\boldsymbol{\pi_t}, \tau)$ is not yet available in the literature, Campani, Garcia, and Lewin (2021), based on the Bellman equation solve the problem by offering the following approximate analytical expression:

$$H(\boldsymbol{\pi_t}, \tau) = exp\left[A_0(\tau) + \sum_{i=1}^{m} A_i(\tau)\pi_{i,t} + \sum_{i=1}^{m} B_i(\tau)\pi_{i,t}^2 + \sum_{j<i} C_{ij}(\tau)\pi_{i,t}\pi_{j,t}\right], \tag{9}$$

where $\pi_{i,t}$ is regime $i$ probability at time $t$. Meanwhile, $A_0$, $A_i$,$B_i$, and $C_{ij}$ are time-varying coefficients obtained from the solution of the Bellman equation under a system of partial differential equations (PDE). The supplementary file provides details for solving the Bellman equation, while Campani, Garcia, and Lewin (2021) demonstrate the PDE.

### 3.2.2 Portfolio weights

Given the approximate solution for $V(W_t, \boldsymbol{\pi_t}, \tau)$ and the coefficients $A_0$, $A_i$, $B_i$, and $C_{ij}$ from function $H(\boldsymbol{\pi_t}, \tau)$, the CGL model presents the optimal weights for the regime-switching allocation using the recursive utility function given by the following form:

$$\begin{aligned}\boldsymbol{\alpha_t} &= \frac{1}{\gamma}\left(\boldsymbol{D}_{s,t}\boldsymbol{\pi_t}\right)^T\left[(\boldsymbol{V\pi_t})(\boldsymbol{V\pi_t})^T\right]^{-1} \\ &+\frac{1}{\gamma}\sum_{i=1}^{m}\left[A_i(\tau)+2B_i(\tau)\pi_{i,t}+\sum_{j\neq i}C_{ij}(\tau)\pi_{j,t}\right]\boldsymbol{\sigma_{i,\pi}}(\boldsymbol{V\pi_t})^{-1}\end{aligned}, \tag{10}$$

where $\boldsymbol{\alpha_t} = \left[\boldsymbol{\alpha}_{1,t} \ldots \boldsymbol{\alpha}_{n,t}\right]$, $\boldsymbol{\sigma_{i,\pi}} = \left[\sigma_{i1,\pi}\ \sigma_{i2,\pi}\ \ldots\ \sigma_{in,\pi}\right]$, $i \in R$ and $j = \{1, 2, \ldots, n\}$.

### 3.2.3 Maximum leverage control (MaxLev)

After determining the unconstrained optimal weights, we must ensure that the strategy is confined within the maximum leverage permitted. To constrain it by the maximum leverage levels, we propose dynamically adjusting the risk parameter $\gamma$. We emphasize that $\gamma$ is not set according to the regimes (as the parameter is unobservable). We limit leverage by calculating adjustments to the base case $\gamma$, obtaining an optimal portfolio strategy that preserves such a maximum leverage policy.

It contrasts to conventional numerically constrained (NC) methods that prevent the weights from exceeding leverage limits by trimming excessive weights. For example, NC procedures might flatten the portfolio leverage at the policy limits if the base case frequently leads to excessive leverage. Instead, using MaxLev, leverage is constantly proportional to the regimes' probabilities and the risk exposure, balanced to the regimes' expectations.

We firstly collect the unconstrained weights at 100% probability for each regime through Eq. (10) with $\boldsymbol{\widehat{\pi}_i} = i^{th}$ column of an $m$-order identity matrix. We then compute leverage as the sum of positive weights that exceeds 100% of the portfolio shares, storing in a $1 \times m$ vector the unconstrained leverages conditioned by each regime ($\boldsymbol{UncLev}$), whose elements are:

$$UncLev_i = \left[\sum_{k=1}^{n+1}\max\left(\hat{\boldsymbol{\alpha}}_{\hat{\boldsymbol{\pi}}_i,k}, 0\right)\right] - 1, \text{with}\, i \in R = \{1, 2, \ldots, m\}, \tag{11}$$

where $\widehat{\boldsymbol{\alpha}}_{\widehat{\pi}_i,k}$ is the $k$th element of the row vector $\widehat{\boldsymbol{\alpha}}_{\widehat{\pi}_i} = \left[\boldsymbol{\alpha}_{\widehat{\pi}_i}\ \left(1 - \boldsymbol{\alpha}_{\widehat{\pi}_i}1\right)\right]$ formed by the risky asset weight vector along with the riskless asset weight, all conditioned by $\boldsymbol{\widehat{\pi}_i}$. So, to build the leverage policies, we assume that investors' risk appetite adapts according to the economic states, *e.g.*, in a shift from crash to rally states, pessimistic investors may become more optimistic and prone to set leverage at a different level. Therefore, we define maximum leverage values ($L_{b,i}$) conditioned by regime $i \in R$, where we also consider the possibility to build $z$ leverage policies, with $z$ as a positive integer representing the number of policies investigated at the research and $b = \{1, 2, \ldots, z\}$. The policies are then collected in a $z \times m$ matrix as follows:

$$\boldsymbol{MaxLev} = \begin{bmatrix} L_{11} & \cdots & L_{1m} \\ \vdots & \ddots & \vdots \\ L_{z1} & \cdots & L_{zm} \end{bmatrix}. \tag{12a}$$

The method enables investors to assign regime-conditioned leverage policies to meet their market expectations, although we do not explore this idea. The $z$ leverage policies (limits) adopted in the research are as follows (this range of values is justified in Sect. 3.4):

$$\boldsymbol{MaxLev} = \begin{bmatrix} 300\% & 300\% & 300\% & 300\% \\ 200\% & 200\% & 200\% & 200\% \\ 100\% & 100\% & 100\% & 100\% \\ 0\% & 0\% & 0\% & 0\% \end{bmatrix}. \tag{12b}$$

We advance by calculating adjustments for $\gamma$ to confine $UncLev_i$ within limits stated in $\boldsymbol{MaxLev}$, with which we infer the base case risk parameter ($\gamma = 5$) conditionally to the limits and regimes. The procedure emerges on a new $z \times m$ matrix whose elements are:

$$\gamma_{b,i} = \gamma \times max[(1 + UncLev_i) \div (1 + L_{b,i}), 1], \tag{13}$$

where the maximum operator preserves the original value of $\gamma$ when the element $UncLev_i$ is already below the limit imposed by $L_{b,i}$. Multiplying the new matrix $b$th row (expressed by $\boldsymbol{\gamma_b} = \left[\gamma_{b,1} \ldots \gamma_{b,m}\right]$) by the column vector of regimes probabilities at time $t$ ($\boldsymbol{\pi_t}$), we obtain a dynamic value as our risk parameter, which is an average of $\boldsymbol{\gamma_b}$ (dynamically) weighted by $\boldsymbol{\pi_t}$. Plugging it on Eq. (10), we then find the weights constrained by policy $b$ at time $t$:

$$\begin{aligned} \boldsymbol{\alpha_{b,t}} = & \frac{1}{(\boldsymbol{\gamma_b \pi_t})} \left(\boldsymbol{D_{s,t} \pi_t}\right)^T \left[(\boldsymbol{V\pi_t})(\boldsymbol{V\pi_t})^T\right]^{-1} \\ & + \frac{1}{\gamma} \sum_{i=1}^{m} \left[ A_i(\tau) + 2B_i(\tau)\pi_{i,t} + \sum_{j \neq i} C_{ij}(\tau)\pi_{j,t} \right] \boldsymbol{\sigma_{i,\pi}} (\boldsymbol{V\pi_t})^{-1} \end{aligned}. \tag{14}$$

### 3.3 The application of the CGL model

Regime-switching models may offer more hedged positions than single-state models, given the regimes' expectations. Still, such protection might lead to peaks of excessive leverage in particular conditions, such as high correlations in less uncertain states. Thus, we study a single class portfolio to increase the chances of observing this effect for controlling it.

#### 3.3.1 Data set

We allocate $n = 3$ risky assets along with the risk-free asset. The risky assets are the first, third, and fifth quintiles from the value-weighted daily returns of the portfolios formed on size by Kenneth French. We will denote them as small, mid, and large caps, respectively. Along with such a US equity portfolio, the risk-free asset is the 3-month Treasury bill secondary market rate extracted from the Board of Governors of the Federal Reserve System. We converted the data series to weekly observations as Lewin and Campani (2020a, b), given that a daily frequency could defy the application feasibility, and a monthly frequency would limit the amplitude of the out-of-sample exercise. Thus, the data set encompasses 3552 weekly observations, starting on January 8th, 1954, and ending on November 30th, 2021.

#### 3.3.2 The out-of-sample exercise

We organized the weekly observations in 174 rolling windows (three widows per year). Following DeMiguel et al. (2009a) and Bulla et al. (2011), we set them with approximately ten years of data. Then, the regime parameters were (re)estimated via ML with observations prior to the first window date and held during the four subsequent months. Meanwhile, at every new week, we define the strategy from the filtered probabilities estimated in $t$ for $t + 1$. Hence, replicating only the available information at the investment decision moment, this procedure delivers out-of-sample returns. Thus, the exercise extends from January 3rd, 1964, to November 30th, 2021.

#### 3.3.3 Number of regimes

To define the number of regimes ($m$), we consider $m = 1$ as the single regime model presented in the results section. Meanwhile, $m = 4$ is the highest number of states often observed in the literature. For example, Guidolin and Timmermann (2008) identify four regimes in a study with US equity indices. In addition, Guidolin and Ono (2006) indicate a saturation ratio between the number of estimated parameters and the data series length. They work with ratio values of around 30. Given the rolling window length, in our application, only models with $m \leq 4$ present ratios above 30. Hence, we tested models with $m = 2, 3, 4$. Table 1 shows their information criteria (IC), in which we present 3 out of the 174 (re)estimations obtained using rolling windows: 1, 87, and 174. The differences between the models' IC are relatively stable during the (re)estimations. Therefore, as Table 1 indicates, $m = 4$ dominates the other models; thus, we set the application with four regimes.

**Table 1** Information criteria

| m | AIC | BIC | H-Q |
|---|---|---|---|
| *Oldest rolling window* | | | |
| 2 | – 10.373 | – 10.359 | – 10.396 |
| 3 | – 10.416 | – 10.393 | – 10.453 |
| 4 | – 10.457 | – 10.424 | – 10.512 |
| *Intermediate rolling window* | | | |
| 2 | – 9.631 | – 9.617 | – 9.654 |
| 3 | – 9.694 | – 9.671 | – 9.732 |
| 4 | – 9.757 | – 9.723 | – 9.811 |
| *Most recent rolling window* | | | |
| 2 | – 9.000 | – 8.986 | – 9.023 |
| 3 | – 9.027 | – 9.004 | – 9.065 |
| 4 | – 9.043 | – 9.010 | – 9.098 |

The table indicates the information criteria for the models with n = 3 risky assets under 2, 3, and 4 regimes. The columns present Akaike (AIC), Bayes-Schwartz (BIC), and Hannan-Quinn (H-Q) for three rolling windows from the out-of-sample exercise: the oldest (May/1954–Dec/1963), the intermediary (May/1983–Dec/1992), and the most recent (Jan/2012–Aug/2021). The full out-of-sample exercise was performed with 174 rolling windows

### 3.3.4 Low-turnover control (LoT)

We define turnover as the percentage of the total portfolio value to be exchanged for new positions. Guidolin and Timmermann (2008) affirm that weights of single regime strategies differ from multi-regimes as the latter capture the short-run market timing effects, while slower long-run movements drive the former. Hence, rapid state shifts are expected to create higher turnover than single regimes strategies, underlining the urgency of restricting turnover to manage costs. Our rebalancing policy consists of two parts. First, we filter out total portfolio turnovers below a given threshold. We then eliminate portfolio changes that occur due to minor updates of the probabilities. Second, we assure that the weights remain close to the most updated strategy imposing a minimum of one monthly rebalancing.

The LoT filter informs whether the best decision is to rebalance at every moment in which the investor needs to decide on the optimal allocation strategy (i.e., weekly). LoT recommends rebalance if the optimal strategy generated by the model at the decision-making moment is significantly different from the current position (otherwise, it recommends holding the current position). We are not computing the turnover between optimal weights from the previous and the current weeks, but the previous week's optimal weights impacted by the market movements (from the current week) and the current week optimal weights.

The rebalancing decision is made given a threshold that defines a minimum turnover limit. Clarke, De Silva, and Thorley (2002) applied a process with a threshold, but they consider it static. In its turn, LoT dynamically adapts it—thus constantly updating the

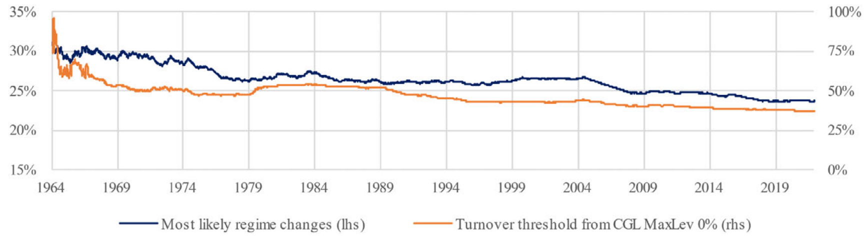


**Fig. 1** Most likely regime changes dynamically adapting the threshold. We show the most likely regime changes and their effects over the turnover threshold, illustrated by CGL MaxLev 0%. The most likely regime is given by the highest filtered probability at each moment (hence, it is independent from the allocation strategy). We accrue the changes when the most likely regime alters from $t-1$ to $t$. The percentage of most likely regime changes ($c$) is the historic number of changes relative to the number of observations. We started these observations in 1959 to present the data of the out-of-sample exercise, from January 3rd, 1964 to November 30th, 2021, discarding the first five years to avoid initial noise

rebalancing policy. The process is as follows. At each rebalancing decision, we observe the percentage of most likely regime changes ($c$) as given by the filtered probabilities, and the optimal total portfolio turnovers considering past historical data. We assume that $c$ is a proxy for the number of probabilities updates that shall be considered. We now define $u = (1 - c)$ and calculate the $u^{\text{th}}$ percentile at the historical dataset of optimal portfolio turnovers: this will be the minimum accepted turnover to define whether there will be rebalancing. As such, the threshold will recommend rebalancing only upon significant turnovers and, on average, at the same rate as the historically most likely regime changes. Figure 1 illustrates the effects of LoT filtering.

Figure 1 shows the most likely regime changes ($c$) representing the dynamic rebalancing policy, and indicates that significant probabilities updates occur 24–31% of the time. In 1979–2004 and 2009–2021, there are plateaus of $c$ at approximately 26 and 24%, respectively. In 2004–2009, $c$ reduces given a long period under only one regime, as the results section will show. We exemplify these effects over CGL MaxLev 0%. In 1979–2004 and 2009–2021, its turnover threshold spans from 42 to 54% and 37–41%, respectively. Meanwhile, it extends from 27 to 96% in the full sample. Thus, it dynamically adapts as time passes.

### 3.3.5 Transaction costs

We assume the investor accounts for transaction costs. We studied Bulla et al. (2011), Gârleanu and Pedersen (2013), and Nystrup et al. (2019), who fixed transaction costs at 10 basis points (0.10%) for dynamic asset allocation for daily trading. The last authors additionally propose to account for holding costs, charged at the risk-free rate over the short sales. In our application, holding costs are underlying the model, as we assume that the investor borrows at the risk-free rate for short selling. Nevertheless, to be conservative and account for potential inefficiencies, like illiquidity costs, we present the returns for higher costs. Therefore, we will present the exercise considering the following levels of transaction costs: 0.10, 0.20, and 0.40%.

### 3.4 Benchmarks

The out-of-sample exercise presents the net returns from the proposed model (CGL MaxLev) to constrain leverage at zero, 100, 200, and 300% levels. At these leverage levels, the CGL MaxLev volatilities are (approximately) restricted by the volatility of the investigated assets, as indicated in Sect. 4.3. CGL MaxLev is compared to four models as follows. First, two models without regimes: the equal-weights and the single regime model. Then, two regime-switching models: the unconstrained CGL model and the numerically constrained CGL model. The regime-switching models are presented with and without the LoT filter. Every case considers the four levels of transaction costs to compare different cost perspectives.

#### 3.4.1 Equal-weights portfolio ($1/n$)

DeMiguel et al. (2009a) demonstrate that a $1/n$ portfolio, despite being a naïve strategy, outperforms a number of dynamic models based on optimal rules. Thus, it represents a common benchmark.

#### 3.4.2 Single regime model (SR)

The SR model corresponds to the recursive utility preferences of an investor who does not account for a multiple regime economy. We use it to assess the impact of the regime-switching on the overall performance.

#### 3.4.3 Unconstrained CGL model (UNC)

We compute the unconstrained CGL model as in the original CGL model from Campani, Garcia, and Lewin (2021), without any adjustments on $\gamma$ (i.e., fixed preferences). Later, we empirically observe that the maximum leverage under such a setup is not distant from 100, 200, and 300% when $\gamma$ is 75, 100, and 200, respectively. This comparison reveals the impacts from the proposed filtering process (MaxLev) versus the original application, in which we solve the problem without constraints.

#### 3.4.4 Numerically constrained CGL model (NC)

An alternative constraining approach is simply preventing the weights from exceeding leverage limits recalculating them proportionally to the constraint when the optimal leverage exceeds the maximum permitted. We apply the NC procedure at the leverage levels: zero, 100, 200, and 300%. This model benchmarks CGL MaxLev relative to a constrained portfolio using fixed preferences.

#### 3.4.5 Other benchmarks

We calculated the exercise for the power utility, as Guidolin and Hyde (2012). It is a particular case from our codes, obtained when $\psi = 1/\gamma$. However, the results from the power utility are very close to those of the recursive utility, given $\psi = \infty$, as expected. It occurs because the value of $\psi$ impacts consumption, but it minimally impacts the allocation strategy (Campani and Garcia, 2019). In addition, Campani, Garcia, and Lewin (2021) show that the impact of consumption-to-wealth ratio variations is also minimal in the allocation model. Thus, both results converge as the investor postpones consumption to the final horizon by setting $\psi = \infty$. The power utility results are indicated in the supplementary file.

### 3.5 Results tests

Following Fugazza et al. (2015) and Campani, Garcia, and Lewin (2021), we use the annualized certainty equivalent returns ($CER$) to compare and rank different strategies. The authors provide the derivation of the following expression used to compute $CER$:

$$CER_i(\gamma, t) \equiv \frac{F}{T}\left\{\frac{1}{W_t}\left[\frac{1}{K-T}\sum_{\tau=1}^{K-T}\left[W_{\tau+T}\left(\hat{\omega}_{i,t}(\gamma, T)\right)\right]^{\frac{1}{1-\gamma}}\right] - 1\right\}, \tag{15}$$

where $F$ is the data frequency (52 weeks per year), $T$ is the horizon (520 weeks), $K$ is the number of out-of-sample returns, $\widehat{\omega}_{i,t}$ are the proportions of the wealth invested in asset $i$, and $W_t$ is the initial wealth (set to 1).

The next section presents the $CER$ differences to compare two portfolios. Below such figures, it will also report the 95% bootstrap confidence intervals drawn from 1,000,000 samples with replacement, using the bias-corrected and accelerated percentile method due to non-normalities in the out-of-sample returns.

## 4 Results

### 4.1 The four-regime model

We analyze the estimated parameters in Table 2. Panel A indicates the single-state parameters as a reference of estimates that do not account for regime-switching. Panels B.1 and B.2 show the regimes' parameters with the characteristics of four economic states: crash, bear market, bull market, and rally. The regimes are interpreted as follows.

In the crash state, expected returns are highly negative and volatile, thus uncertainty is exceedingly high. However, after a crash regime, the transition probabilities reveal similar chances of shifting to any of the three less uncertain states, indicating that recovery does not necessarily follow it. On average, the crash and the bear market's duration are 10 weeks, while the bull market lasts 28 weeks. Therefore, volatilities in the bull market converge with those from the single-state, although the bull market's returns are slightly higher than the single-state. In turn, the rally state presents the highest returns and the lowest volatilities and correlations.

The crash state, bear, and bull markets show high correlations, from 78 to 95%, close to the single-state. The only regime with relatively lower correlations is the rally,

**Table 2** Estimated parameters (%)

| Panel A: Single State Model | | Small caps | Mid-caps | Large caps |
|---|---|---|---|---|
| Expected returns | | | | |
| | | 9.57 | 9.67 | 7.45 |
| Volatility and correlation matrix | | | | |
| | Small caps | 18.40 | | |
| | Mid-caps | 92.57 | 17.91 | |
| | Large caps | 76.10 | 88.30 | 15.45 |

| Panel B.1: Four State Model | | | Small caps | Mid-caps | Large caps |
|---|---|---|---|---|---|
| Expected returns | | | | | |
| | Regime 1 | | − 11.71 | − 11.16 | − 8.58 |
| | Regime 2 | | − 12.52 | − 5.04 | 0.58 |
| | Regime 3 | | 12.15 | 13.94 | 11.99 |
| | Regime 4 | | 80.54 | 56.19 | 31.06 |
| Volatility and correlation matrix | | | | | |
| | Regime 1 | Small caps | 32.12 | | |
| | | Mid− caps | 93.02 | 31.66 | |
| | | Large caps | 78.78 | 90.09 | 26.86 |
| | Regime 2 | Small caps | 8.61 | | |
| | | Mid− caps | 92.29 | 10.87 | |
| | | Large caps | 78.39 | 89.18 | 11.68 |
| | Regime 3 | Small caps | 17.27 | | |
| | | Mid− caps | 94.36 | 15.11 | |
| | | Large caps | 82.55 | 90.42 | 10.92 |
| | Regime 4 | Small caps | 7.41 | | |
| | | Mid− caps | 83.32 | 8.22 | |
| | | Large caps | 54.92 | 75.41 | 9.81 |

| Panel B.2: Four State Model | | Regime 1 | Regime 2 | Regime 3 | Regime 4 |
|---|---|---|---|---|---|
| Transition probabilities | | | | | |
| | Regime 1 | 89.73 | 3.71 | 3.41 | 3.15 |
| | Regime 2 | 3.53 | 89.54 | 0.12 | 6.80 |
| | Regime 3 | 2.32 | 0.21 | 96.43 | 1.04 |
| | Regime 4 | 0.29 | 12.32 | 1.95 | 85.44 |
| Steady state | | | | | |
| | Ergodic Prob | 18.11 | 31.46 | 29.69 | 20.74 |
| | Duration (weeks) | 10 | 10 | 28 | 7 |

This table was computed with excess returns over the risk-free rate. Correlation matrices present the volatilities in their diagonals. Weekly returns and volatilities are annualized for presentation. The indicated parameters were estimated considering the complete data set

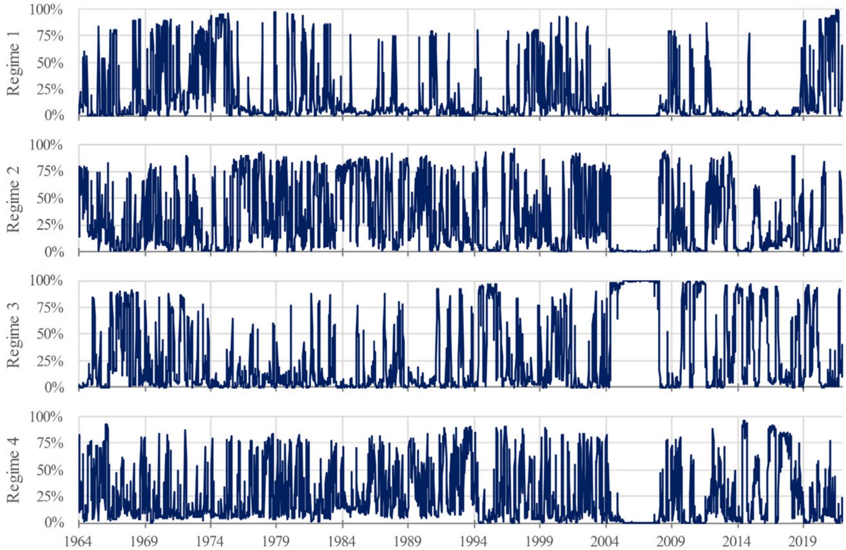


**Fig. 2** Out-of-sample probabilities. The out-of-sample probabilities are the filtered probabilities from $t + 1$ in $t$, estimated considering the rolling windows described in Sect. 3.3

but the ergodic probabilities reveal its chances are only 21%. At the same time, Eq. (10) shows that the CGL model inverts the var-cov matrix to compute the weights. Then, high correlations lead to extremely low numbers, more sensitive to return variations. Thus, more pronounced under less uncertain (volatile) states, high correlations might oversize portfolio hedges relative to the investor's risk aversion. Therefore, we propose to control leverage to contain it.

Figure 2 presents the out-of-sample probabilities, filtered from $t + 1$ in $t$. For example, they show that crash state probabilities hit the highest level since 1964 during the COVID-19 pandemic, differently from the previous economic crisis. In addition, the probabilities reveal that before the subprime crisis, between 2004 and 2009, it was an unusually long bull market. Again, it contrasts with the other periods, where regimes frequently shift. The regime-switching models generate value by quantitatively tracking such shifts between economic cycles (states).

## 4.2 The regime-switching benchmarks

### 4.2.1 The unconstrained CGL model

It is the original configuration from Campani, Garcia, and Lewin (2021), i.e., it is the CGL model using fixed values of $\gamma$. Figure 3 shows that increasing $\gamma$ conserves the significant distance between the leverage peaks and their median levels, despite mitigating the overall portfolio leverage. For example: to hold the maximum leverage below 100% in such a sample, the unconstrained CGL model requires $\gamma > 250$. Consequently, the median leverage drops below 10%, and the correlation between

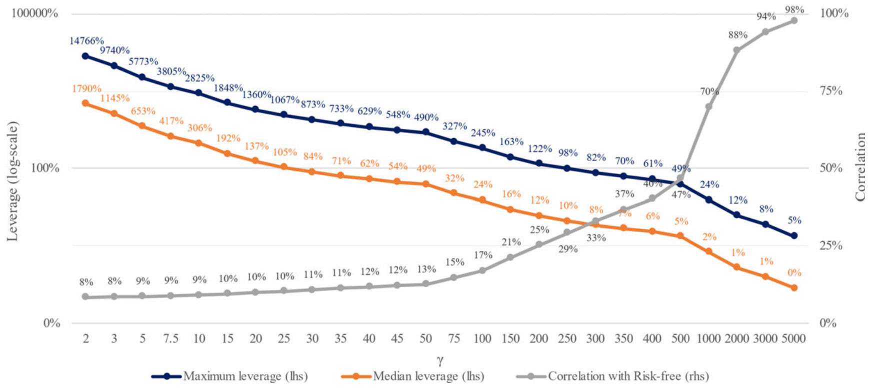


**Fig. 3** The unconstrained CGL model. The unconstrained CGL model is the original model presented by Campani, Garcia, and Lewin (2021). The figure indicates the maximum and median portfolio leverage from the portfolios obtained considering such a model, and the correlation between the portfolios' returns and the risk-free returns, calculated in the out-of-sample exercise

the portfolio and the risk-free returns rises above 29%. The correlations in Fig. 3 indicate an increasing overweight of the risk-free asset when $\gamma$ increases. In contrast, the literature usually represents the investor's risk preferences with $\gamma$ closer to 5, conducting the portfolio returns toward much lower correlations with the risk-free returns. For example, models with $\gamma \leq 25$ show correlations not greater than 10%, regardless of the leverage peaks greater than tenfold the investor capital—which pose limitations either due to implementation or compliance rules.

To assess the tradeoff between the leverage levels and the risky assets allocation, at the out-of-sample exercise, we pair the unconstrained CGL with $\gamma = 75, 100, 200$ to the constrained portfolios at 100, 200, and 300%, respectively. However, Fig. 3 reveals that it is inviable to obtain an unleveraged portfolio (leverage $= 0\%$) only increasing the fixed value of $\gamma$, without achieving almost 100% correlation to the risk-free returns. Consequently, the comparison of this particular case will not carry an unconstrained regime-switching benchmark.

#### 4.2.2 The numerically constrained CGL model

As presented in Sect. 3, our base case is $\gamma = 5$; and as Fig. 3 indicates, the median leverage of the unconstrained CGL of the base case is 653%. The left panel from Fig. 4 additionally shows that such a portfolio leverages 25th and 75th percentiles are 306 and 1327%, respectively. This wide range results from the states dynamically shifting and resetting leverage according to the states' expectations. Figure 4 (left panel) also reveals that the leverage obtained by the numerically constrained CGL is flat at the boundary level, except for outliers. In other words, up to the leverage limit of 300%, the leverage from the numerically constrained CGL model virtually does not detach from the limit.

The right panel of Fig. 4 also indicates that numerical constraints level the leverage of portfolios. Nevertheless, in cutting off such dynamics, they overlook the opportunity

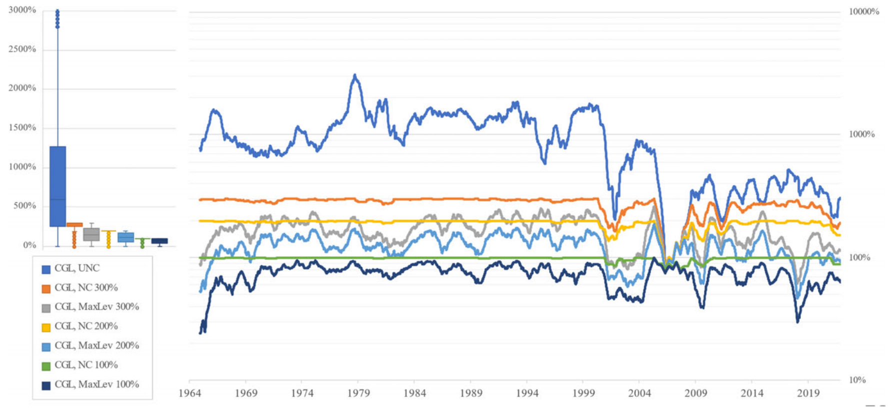


**Fig. 4** Portfolio leverage. The left panel shows the leverage distribution (scale limited to 3000%), while the right panel presents the 52-week moving average from the portfolios leverage calculated in in the out-of-sample exercise. Both panels present the unconstrained CGL model (UNC), the numerically leverage CGL model (NC), and CGL MaxLev. The color key at the bottom fits both panels. All models were computed with $\gamma = 5$

to mitigate risks by calibrating leverage according to regimes. In contrast, Fig. 4, in both panels, shows that the CGL MaxLev model maintains the dynamic adjustments of the portfolio leverage while constraining the maximum leverage. Thus, the numerically constrained CGL model is expected to offer higher but more volatile returns than the CGL MaxLev.

### 4.3 The out-of-sample performance

This section presents the results from the out-of-sample exercise performed from 1964 to 2021. As an additional robustness check, the supplementary file presents the results from its most recent subset, 2000 to 2021. Despite natural differences among the samples, the conclusions drawn from comparing the model's results are not significantly different.

Table 3 indicates that an investment in the risky assets admits volatilities up to 19% p-a. We will refer to it as an admissible market volatility. The risk of extreme events in these returns is indicated the by maximum drawdowns between 53 and 71%.

The risky assets results are an intuitive benchmark, particularly for strategies without transaction costs. Additionally, we set the leverage constraints from the exercise to align the proposed strategies (volatility) with the market volatility. Therefore, we present constrained portfolios up to 300% leverage, as CGL MaxLev 300% volatility approximates to such a level.

Table 4 shows the out-of-sample results of the $1/n$ portfolio, SR model, and CGL models. The CGL models are presented unconstrained (UNC), numerically constrained (NC), and MaxLev. Below Table 4, we discuss the models indicated on its panels.

**Table 3** Risky assets and the risk-free rate

| Assets (%) | Small caps | Mid-caps | Large-caps | Risk-free |
|---|---|---|---|---|
| Returns (p-a) | 11.9 | 12.2 | 10.3 | 4.4 |
| Volatility (p-a) | 18.8 | 18.4 | 15.8 | 0.4 |
| Sharpe ratio | 40.2 | 42.8 | 37.5 | – |
| Skewness | − 0.7 | − 0.5 | − 0.4 | 0.6 |
| Kurtosis | 6.8 | 6.1 | 5.4 | 0.5 |
| Maximum drawdown | 71.3 | 53.6 | 53.0 | 0.0 |

This table presents the annualized results obtained from weekly observations from January 3rd, 1964 to November 30th, 2021

#### 4.3.1 Reference models (panel A)

The $1/n$ portfolio is an unleveraged portfolio with the lowest turnover. It presents a Sharpe ratio of 36%, in line with the risky assets. Compared to them, the $1/n$ allocation lowers the maximum drawdown to approximately 46%, establishing an important benchmark. In turn, The SR model presents maximum leverage of 305%, while its turnover's 75th percentile is below 3%. Such a low turnover makes the costs in SR returns not as critical as in regime-switching models. Compared to the $1/n$ portfolio, the SR model has a slightly higher Sharpe ratio, but its maximum drawdown is almost two-fold. Despite the severe drawdown, highly leveraged portfolios eventually recover in long samples, as the case of the unconstrained CGL model with $\gamma = 5$ described below.

The unconstrained CGL model with $\gamma = 5$ is our base case, as it is in Campani, Garcia, and Lewin (2021). Notwithstanding its extreme leverage (analyzed in previous sections), the base case exposes the favorable impact of the LoT filter. The median turnover of 32% from the base case without LoT (optimal model) falls to 0% with LoT. The returns-to-risk measure (Sharpe ratio) reveals that LoT has a positive impact in this model when transaction costs are 0.40%. Moreover, the base case dominates $1/n$ portfolio and SR model in terms of return-to-risk, indicating the relevance of the regime-switching in allocation models considering the stochastic recursive utility function. However, due to the regime shifting process, the base case returns are more leptokurtic (thus, riskier) than the single-state benchmarks.

#### 4.3.2 Unconstrained models (panel B)

If, on the one hand, the CGL model with $\gamma = 10, 25, 50$ mitigates the leverage levels relative to the base case ($\gamma = 5$), on the other, these unconstrained configurations still lead to almost impractical leverage peaks: 2825, 1067, and 490%, respectively. The supplementary file presents the unconstrained CGL for more risk aversion settings. Still, one could also argue the outcome of applying MaxLev with $\gamma > 5$. We answer it by pointing out that the return-to-risk measures from the base case dominate those from the unconstrained models with $\gamma > 5$. As we learned from Fig. 3, increasing

**Table 4** The out-of-sample results (%)

| Panel A | | Equal weights (1/$n$) | | | | Single regime (SR) | | | | CGL, $\gamma = 5$, UNC | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A.I. Percentiles | | 25 | 50 | 75 | 100 | 25 | 50 | 75 | 100 | 25 | 50 | 75 | 100 |
| Leverage | | 0.00 | 0.00 | 0.00 | 0.00 | 0.68 | 29.22 | 69.56 | 305.23 | 305.57 | 652.77 | 1326.50 | 5772.60 |
| Weekly turnover | ~ LoT | 0.21 | 0.34 | 0.51 | 3.70 | 0.91 | 1.56 | 2.82 | 25.23 | 9.85 | 31.70 | 77.06 | 1156.71 |
| | LoT | – | – | – | – | – | – | – | – | 0.00 | 0.00 | 63.27 | 941.99 |
| A.II. Transaction Costs | | 0.00 | 0.10 | 0.20 | 0.40 | 0.00 | 0.10 | 0.20 | 0.40 | 0.00 | 0.10 | 0.20 | 0.40 |
| Returns | ~ LoT | 10.10 | 8.98 | 8.96 | 8.91 | 14.90 | 13.54 | 13.32 | 12.89 | 546.76 | 385.15 | 267.01 | 109.05 |
| | LoT | – | – | – | – | – | – | – | – | 445.36 | 342.45 | 262.27 | 142.30 |
| Volatility | ~ LoT | 12.59 | 12.59 | 12.59 | 12.59 | 19.09 | 19.08 | 19.08 | 19.08 | 99.91 | 99.50 | 99.39 | 100.03 |
| | LoT | – | – | – | – | – | – | – | – | 98.79 | 99.21 | 99.97 | 102.41 |
| Sharpe ratio | ~ LoT | 45.35 | 36.51 | 36.32 | 35.94 | 55.07 | 47.97 | 46.82 | 44.53 | 542.86 | 382.69 | 264.24 | 104.63 |
| | LoT | – | – | – | – | – | – | – | – | 446.39 | 340.75 | 257.96 | 134.66 |
| Skewness | ~ LoT | − 0.64 | − 0.64 | − 0.64 | − 0.64 | − 0.68 | − 0.69 | − 0.70 | − 0.72 | 2.48 | 2.22 | 1.94 | 1.32 |
| | LoT | – | – | – | – | – | – | – | – | 1.95 | 1.71 | 1.45 | 0.93 |
| Kurtosis | ~ LoT | 6.33 | 6.33 | 6.34 | 6.34 | 7.22 | 7.25 | 7.28 | 7.33 | 27.54 | 25.82 | 24.03 | 20.51 |
| | LoT | – | – | – | – | – | – | – | – | 33.04 | 30.78 | 28.46 | 23.92 |
| Max. drawdown | ~ LoT | 45.58 | 46.50 | 46.53 | 46.60 | 81.96 | 83.03 | 83.39 | 84.10 | 99.99 | 99.99 | 99.99 | 99.99 |
| | LoT | – | – | – | – | – | – | – | – | 99.99 | 99.99 | 99.99 | 99.99 |

**Table 4** (continued)

| Panel B | | CGL, $\gamma = 10$, UNC | | | | CGL, $\gamma = 25$, UNC | | | | CGL, $\gamma = 50$, UNC | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| B.I. Percentiles | | 25 | 50 | 75 | 100 | 25 | 50 | 75 | 100 | 25 | 50 | 75 | 100 |
| Leverage | | 129.36 | 306.27 | 633.86 | 2824.96 | 46.84 | 104.95 | 227.48 | 1067.35 | 21.97 | 48.87 | 103.67 | 490.23 |
| Weekly turnover | ~ LoT | 7.74 | 25.56 | 66.75 | 801.01 | 5.10 | 18.24 | 51.06 | 543.61 | 3.30 | 13.23 | 35.56 | 336.68 |
| | LoT | 0.00 | 0.00 | 48.87 | 801.01 | 0.00 | 0.00 | 33.79 | 543.61 | 0.00 | 0.00 | 22.87 | 336.68 |
| B.II. Transaction Costs | | 0.00 | 0.10 | 0.20 | 0.40 | 0.00 | 0.10 | 0.20 | 0.40 | 0.00 | 0.10 | 0.20 | 0.40 |
| Returns | ~ LoT | 131.05 | 100.39 | 75.35 | 33.80 | 46.96 | 38.36 | 31.55 | 18.85 | 24.38 | 20.13 | 17.18 | 11.48 |
| | LoT | 121.40 | 99.16 | 80.77 | 48.47 | 44.91 | 38.19 | 33.08 | 23.36 | 23.66 | 20.14 | 17.88 | 13.48 |
| Volatility | ~ LoT | 49.16 | 49.46 | 49.88 | 51.09 | 19.51 | 19.72 | 19.97 | 20.61 | 9.75 | 9.86 | 10.00 | 10.34 |
| | LoT | 47.42 | 48.00 | 48.73 | 50.55 | 19.07 | 19.36 | 19.70 | 20.51 | 9.71 | 9.85 | 10.02 | 10.43 |
| Sharpe ratio | ~ LoT | 257.62 | 194.10 | 142.25 | 57.57 | 218.23 | 172.32 | 135.98 | 70.16 | 205.02 | 159.56 | 127.87 | 68.59 |
| | LoT | 246.77 | 197.42 | 156.74 | 87.21 | 212.43 | 174.60 | 145.65 | 92.48 | 198.43 | 159.85 | 134.63 | 87.16 |
| Skewness | ~ LoT | 2.44 | 2.20 | 1.95 | 1.41 | 2.42 | 2.22 | 2.00 | 1.52 | 2.41 | 2.22 | 2.01 | 1.56 |
| | LoT | 2.66 | 2.47 | 2.26 | 1.79 | 2.45 | 2.27 | 2.07 | 1.62 | 2.35 | 2.16 | 1.96 | 1.51 |
| Kurtosis | ~ LoT | 26.83 | 25.72 | 24.53 | 22.05 | 26.36 | 25.54 | 24.61 | 22.61 | 26.03 | 25.32 | 24.50 | 22.68 |
| | LoT | 27.42 | 26.38 | 25.19 | 22.61 | 27.90 | 26.84 | 25.64 | 23.01 | 28.58 | 27.63 | 26.53 | 24.07 |
| Max. drawdown | ~ LoT | 58.31 | 71.41 | 81.03 | 94.77 | 26.77 | 31.79 | 41.40 | 64.56 | 13.95 | 15.28 | 20.28 | 38.59 |
| | LoT | 46.58 | 60.51 | 72.22 | 86.49 | 24.03 | 26.27 | 35.50 | 50.75 | 13.85 | 14.15 | 17.02 | 27.78 |

**Table 4** (continued)

| Panel C | | CGL, $\gamma = 75$, UNC | | | | CGL, $\gamma = 5$, NC 300% | | | | CGL, $\gamma = 5$, MaxLev 300% | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C.I. Percentiles | | 25 | 50 | 75 | 100 | 25 | 50 | 75 | 100 | 25 | 50 | 75 | 100 |
| Leverage | | 14.46 | 32.28 | 66.82 | 326.60 | 300.00 | 300.00 | 300.00 | 300.00 | 92.37 | 161.47 | 236.96 | 300.00 |
| Weekly turnover | ~ LoT | 2.50 | 9.98 | 27.26 | 231.55 | 4.07 | 11.60 | 43.69 | 335.83 | 6.40 | 20.54 | 50.65 | 342.65 |
| | LoT | 0.00 | 0.00 | 17.90 | 231.55 | 0.00 | 0.00 | 31.47 | 326.07 | 0.00 | 0.00 | 39.61 | 342.65 |
| C.II. Transaction Costs | | 0.00 | 0.10 | 0.20 | 0.40 | 0.00 | 0.10 | 0.20 | 0.40 | 0.00 | 0.10 | 0.20 | 0.40 |
| Returns | ~ LoT | 17.43 | 14.37 | 12.50 | 8.84 | 59.37 | 47.47 | 37.80 | 20.21 | 40.83 | 33.38 | 27.58 | 16.69 |
| | LoT | 16.94 | 14.31 | 12.87 | 10.02 | 57.93 | 47.95 | 39.95 | 25.13 | 38.59 | 32.76 | 28.44 | 20.18 |
| Volatility | ~ LoT | 6.51 | 6.59 | 6.68 | 6.91 | 28.70 | 29.10 | 29.56 | 30.62 | 18.57 | 18.77 | 19.00 | 19.52 |
| | LoT | 6.53 | 6.63 | 6.74 | 7.01 | 28.04 | 28.48 | 28.97 | 30.12 | 18.34 | 18.58 | 18.86 | 19.49 |
| Sharpe ratio | ~ LoT | 200.28 | 151.48 | 121.37 | 64.44 | 191.56 | 148.05 | 113.02 | 51.67 | 196.30 | 154.48 | 122.07 | 63.01 |
| | LoT | 192.09 | 149.80 | 125.83 | 80.37 | 190.90 | 152.98 | 122.76 | 68.87 | 186.43 | 152.67 | 127.55 | 81.05 |
| Skewness | ~ LoT | 2.40 | 2.22 | 2.02 | 1.58 | 0.30 | 0.23 | 0.16 | 0.00 | 0.99 | 0.92 | 0.84 | 0.68 |
| | LoT | 2.25 | 2.07 | 1.86 | 1.43 | 0.15 | 0.09 | 0.02 | − 0.13 | 0.86 | 0.79 | 0.71 | 0.54 |
| Kurtosis | ~ LoT | 25.74 | 25.08 | 24.32 | 22.59 | 5.65 | 5.45 | 5.24 | 4.80 | 10.82 | 10.46 | 10.08 | 9.31 |
| | LoT | 28.84 | 27.95 | 26.91 | 24.54 | 5.40 | 5.23 | 5.04 | 4.65 | 10.27 | 9.95 | 9.60 | 8.86 |
| Max. drawdown | ~ LoT | 9.39 | 11.40 | 15.73 | 26.88 | 53.59 | 61.51 | 67.66 | 87.58 | 34.26 | 39.74 | 45.74 | 69.37 |

**Table 4** (continued)

| Panel C | | CGL, $\gamma = 75$, UNC | | | | CGL, $\gamma = 5$, NC 300% | | | | CGL, $\gamma = 5$, MaxLev 300% | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | LoT | 9.71 | 10.51 | 13.16 | 18.55 | 54.64 | 59.85 | 65.67 | 76.66 | 31.94 | 35.31 | 37.28 | 46.01 |
| Panel E | | CGL, $\gamma = 200$, UNC | | | | CGL, $\gamma = 5$, NC 100% | | | | CGL, $\gamma = 5$, MaxLev 100% | | | |
| E.I. Percentiles | | 25 | 50 | 75 | 100 | 25 | 50 | 75 | 100 | 25 | 50 | 75 | 100 |
| Leverage | | 5.32 | 12.01 | 24.97 | 122.38 | 100.00 | 100.00 | 100.00 | 100.00 | 49.90 | 83.84 | 100.00 | 100.00 |
| Weekly turnover | ~ LoT | 1.13 | 4.60 | 12.52 | 106.83 | 2.56 | 8.79 | 34.35 | 175.36 | 3.32 | 12.70 | 35.06 | 188.69 |
| | LoT | 0.00 | 0.00 | 8.33 | 106.83 | 0.00 | 0.00 | 20.95 | 175.36 | 0.00 | 0.00 | 24.80 | 188.69 |
| E.II. Transaction Costs | | 0.00 | 0.10 | 0.20 | 0.40 | 0.00 | 0.10 | 0.20 | 0.40 | 0.00 | 0.10 | 0.20 | 0.40 |
| Returns | ~ LoT | 9.15 | 7.41 | 6.75 | 5.44 | 30.61 | 25.52 | 21.84 | 14.76 | 22.78 | 19.00 | 16.49 | 11.61 |
| | LoT | 8.99 | 7.39 | 6.88 | 5.86 | 30.51 | 26.09 | 23.04 | 17.14 | 22.48 | 19.31 | 17.37 | 13.59 |
| Volatility | ~ LoT | 2.49 | 2.52 | 2.55 | 2.63 | 17.63 | 17.77 | 17.94 | 18.32 | 10.18 | 10.29 | 10.40 | 10.67 |
| | LoT | 2.52 | 2.55 | 2.59 | 2.68 | 17.36 | 17.51 | 17.68 | 18.09 | 10.04 | 10.15 | 10.28 | 10.59 |
| Sharpe ratio | ~ LoT | 190.98 | 119.80 | 92.46 | 39.93 | 148.76 | 118.93 | 97.26 | 56.60 | 180.63 | 142.05 | 116.29 | 67.66 |
| | LoT | 182.57 | 117.80 | 96.24 | 54.86 | 150.41 | 123.94 | 105.47 | 70.49 | 180.22 | 146.90 | 126.23 | 86.88 |
| Skewness | ~ LoT | 2.33 | 2.17 | 1.99 | 1.60 | − 0.21 | − 0.23 | − 0.26 | − 0.31 | 0.59 | 0.55 | 0.50 | 0.40 |
| | LoT | 2.08 | 1.91 | 1.73 | 1.33 | − 0.18 | − 0.21 | − 0.24 | − 0.30 | 0.33 | 0.29 | 0.24 | 0.14 |
| Kurtosis | ~ LoT | 23.77 | 23.32 | 22.75 | 21.39 | 4.29 | 4.21 | 4.12 | 3.92 | 9.40 | 9.11 | 8.80 | 8.16 |
| | LoT | 27.72 | 27.11 | 26.33 | 24.44 | 4.25 | 4.19 | 4.12 | 3.95 | 9.00 | 8.76 | 8.49 | 7.91 |

**Table 4** (continued)

| Panel E | | CGL, $\gamma = 200$, UNC | | | CGL, $\gamma = 5$, NC 100% | | | | CGL, $\gamma = 5$, MaxLev 100% | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Max. drawdown | ~ LoT | 3.46 | 8.17 | 9.99 | 13.86 | 42.51 | 45.44 | 47.86 | 52.75 | 25.56 | 25.79 | 26.87 | 32.12 |
| | LoT | 3.84 | 7.27 | 8.21 | 10.31 | 43.91 | 46.67 | 48.60 | 52.26 | 25.49 | 25.71 | 25.99 | 28.38 |
| **Panel F** | | **n/a** | | | | **CGL, $\gamma = 5$, NC 0%** | | | | **CGL, $\gamma = 5$, MaxLev 0%** | | | |
| F.I. Percentiles | | – | – | – | – | 25 | 50 | 75 | 100 | 25 | 50 | 75 | 100 |
| Leverage | | – | – | – | – | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Weekly turnover | ~ LoT | – | – | – | – | 0.63 | 4.42 | 20.23 | 100.00 | 1.31 | 5.38 | 16.93 | 100.00 |
| | LoT | – | – | – | – | 0.00 | 0.00 | 8.97 | 100.00 | 0.00 | 0.00 | 10.08 | 100.00 |
| F.II. Transaction Costs | | – | – | – | – | 0.00 | 0.10 | 0.20 | 0.40 | 0.00 | 0.10 | 0.20 | 0.40 |
| Returns | ~ LoT | – | – | – | – | 17.06 | 14.81 | 13.73 | 11.59 | 13.52 | 11.62 | 10.85 | 9.33 |
| | LoT | – | – | – | – | 17.14 | 15.07 | 14.16 | 12.36 | 13.20 | 11.47 | 10.87 | 9.68 |
| Volatility | ~ LoT | – | – | – | – | 14.33 | 14.35 | 14.38 | 14.45 | 8.01 | 8.02 | 8.04 | 8.09 |
| | LoT | – | – | – | – | 14.17 | 14.20 | 14.23 | 14.30 | 7.87 | 7.89 | 7.92 | 7.97 |
| Sharpe ratio | ~ LoT | – | – | – | – | 88.45 | 72.61 | 64.92 | 49.80 | 114.01 | 90.13 | 80.34 | 61.06 |
| | LoT | – | – | – | – | 90.00 | 75.23 | 68.68 | 55.78 | 111.83 | 89.73 | 81.90 | 66.42 |
| Skewness | ~ LoT | – | – | – | – | − 0.61 | − 0.62 | − 0.62 | − 0.62 | − 0.10 | − 0.11 | − 0.12 | − 0.14 |
| | LoT | – | – | – | – | − 0.52 | − 0.53 | − 0.53 | − 0.54 | 0.01 | 0.00 | − 0.01 | − 0.03 |
| Kurtosis | ~ LoT | – | – | – | – | 4.56 | 4.54 | 4.52 | 4.46 | 7.14 | 7.11 | 7.08 | 7.01 |
| | LoT | – | – | – | – | 3.87 | 3.86 | 3.85 | 3.82 | 6.33 | 6.32 | 6.31 | 6.26 |

**Table 4** (continued)

| Panel F | | n/a | | | | CGL, $\gamma = 5$, NC 0% | | | | CGL, $\gamma = 5$, MaxLev 0% | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Max. drawdown | ~ LoT | – | – | – | – | 40.22 | 42.28 | 43.28 | 46.15 | 19.94 | 21.60 | 22.18 | 23.31 |
| | LoT | – | – | – | – | 40.25 | 42.13 | 42.97 | 44.89 | 21.29 | 23.17 | 23.76 | 24.92 |

The table presents the out-of-sample results from the equal weights portfolio ($1/n$), the single regime (SR), and the CGL model. The SR model represents the allocation under the recursive utility function without multiple regimes (single state) with $\gamma = 5$. The CGL model is presented considering alternative configurations: unconstrained (UNC), numerically constrained (NC), and MaxLev. We present the unconstrained CGL with $\gamma$ varying from 5 to 200. Meanwhile, the leverage constrains in NC and MaxLev are 0, 100, 200, and 300%. The table is horizontally organized in panels, each with sections I and II. Section I presents the percentiles of distributions, as leverage and turnover are independent from the transaction costs. Section II is organized according to the transaction costs: 0.00, 0.10, 0.20, and 0.40%. Except for the leverage data, the measures are segmented between ~ LoT and LoT in both sections, indicating the results without and with LoT, respectively. Returns and volatility were annualized for presentation. The out-of-sample exercise was conducted with weekly observations from January 3rd, 1964 to November 30th, 2021

$\gamma$ amplifies the overweight in the risk-free asset. Thus, the dominating return-to-risk measures reveal that the CGL strategy has more chances of successful performance than failure, and those chances are unchanged using MaxLev. Hence, the following panels apply NC and MaxLev with $\gamma = 5$.

Moreover, panel B shows that LoT eliminates the turnover up to the 50th percentile. The return-to-risk measures from three models reveal that LoT is favorable in any case of transaction costs. Contrasting to the base case, it indicates that under less extreme leverage levels, LoT generates value. For example, take the unconstrained CGL model with $\gamma = 25$. The Sharpe ratios for transaction costs of 0.20% are 134 and 146% for the cases without and with LoT filter, respectively. Naturally, these ratios decrease while increasing costs. Nevertheless, for transaction costs of 0.40%, the Sharpe ratio without LoT is 70%, while with LoT it is 92%. This pattern repeats over the other models, evidencing that LoT creates value when allocating for transaction costs, most notably when costs increase.

### 4.3.3 Maximum leverage at 300% (panel C)

The maximum leverage of the unconstrained CGL model with $\gamma = 75$ is 327%, yet we use it to benchmark the cases where leverage is limited to 300%. The leverage from the unconstrained CGL model with $\gamma = 100$ is 245%. Although a fine-tuning between $75 < \gamma < 100$ would result in maximum leverage closer to 300%, it is not critical to the current comparison. On the other hand, it is relevant to emphasize that these leverage levels correspond to the entire sample (1964–2021). The supplementary file shows the subset 2000–2021, where the maximum leverage for the same risk preference is significantly different. The difference between the maximum leverage magnitudes results from estimating more (or less) uncertain economic states in the samples. Hence, only increasing $\gamma$ is insufficient to effectively control (extreme) leverage at discretionary levels.

Sill, given an unconstrained case, $\gamma = 75$ benchmarks the return-to-risk measures relative to the limit of 300% leverage. Panel C shows that those measures from CGL MaxLev 300% are aligned with the unconstrained case. Meanwhile, we cannot lose sight that these unconstrained returns are more exposed to the risk-free rate (Fig. 3). Thus, matching Sharpe ratios reflect that MaxLev manages risks more effectively than the unconstrained model.

At the same time, considering transaction costs, the Sharpe ratios from CGL MaxLev 300% dominate those from CGL NC 300%. It results from an overly leveraged base case, where the NC procedure flattens leverage at the limits (Fig. 4); and while higher leverage levels increase returns, they also increase volatility. The outperformance from CGL MaxLev 300% relative to NC 300% is even magnified when considering higher transaction costs, underlining that it is valuable to dynamically adjust the leverage levels upon uncertainty. So, at the level of 300% leverage, the higher the costs, the more positive it is to constrain leverage using MaxLev.

#### 4.3.4 Maximum leverage at 200% (panel D)

The conclusions from comparing returns-to-risk are analogous to those from panel C. First, the alignment between Sharpe ratios with CGL unconstrained model ($\gamma = 100$) indicates that CGL MaxLev 200% offers superior risky assets management than such a benchmark, given a lower risk-free allocation. Then, at the same cost level, MaxLev's return-to-risk dominating those from NC evidence that MaxLev is a more effective constraining procedure at 200% leverage.

Another characteristic from MaxLev is that it skews the distribution slightly to the right side relative to the NC model. It results from MaxLev mitigating leverage proportionally between regimes. Then, when MaxLev adjusts leverage upon regime expectations, it mitigates the risk of negative events. In contrast, when leveling leverage by its limit, the NC procedure does not balance leverage between regimes. Thus, compared to NC returns, MaxLev's are consistently slightly more skewed to the right, pointing to a greater number of positive returns.

#### 4.3.5 Maximum leverage at 100% (panel E)

In contrast with panels C and D, panel E shows that CGL MaxLev 100% considering transaction costs outperforms its benchmarks' Sharpe ratio more pronouncedly. Therefore, it suggests that MaxLev generates even greater value for lower leverage limits. In addition, as in previous panels, MaxLev significantly lowers maximum drawdowns relative to NC, suggesting that it mitigates left-tail events in comparison with NC procedures. These findings are extensible to the comparison with $1/n$ portfolio and SR model.

#### 4.3.6 Unleveraged cases (panel F)

Constraining the portfolios at a 0% leverage level represents the unleveraged condition. It is a special case, as some investors cannot hold any leveraged positions. Hence, we suppress an unconstrained model from panel F, as it would require extremely high values of $\gamma$ for the unconstrained CGL to achieve the unleveraged condition. Furthermore, such a case would present negative risk premia considering transaction costs (the supplementary file shows unconstrained models approximating to 0% leverage). In contrast, the unleveraged condition is achieved by the $1/n$ portfolio along with both models from panel F. Comparing them using the same costs, the return-to-risk measures from the CGL MaxLev 0% dominate the benchmarks. At the same time, it also presents the lowest maximum drawdown among them. Such results evidence that MaxLev effectively generates unconstrained portfolios without eroding the return-to-risk measures.

### 4.4 Robustness checks

Table 5 shows the annualized certainty equivalent returns differences ($\Delta CER$) between CGL MaxLev with LoT and the benchmarks. Below we analyze them by the benchmarks.

**Table 5** Certainty equivalent returns differences ($\Delta CER$, %)

| TC (%) | Single regime (SR) | Equal weights ($1/n$) | CGL MaxLev ~ LoT | CGL UNC ~ LoT | CGL UNC LoT | CGL NC ~ LoT | CGL NC LoT |
|---|---|---|---|---|---|---|---|
| *MaxLev 300%, LoT* | | | | | | | |
| 0.00 | 2.10 | 2.63 | − 0.11 | 1.21 | 1.32 | − 0.13 | − 0.20 |
| | [1.26–3.17] | [1.81–3.62] | [− 0.22–0.01] | [0.94–1.49] | [1.05–1.58] | [− 0.41–0.16] | [− 0.46–0.09] |
| 0.10 | 1.92 | 2.50 | − 0.02 | 1.39 | 1.44 | − 0.01 | − 0.19 |
| | [1.01–3.08] | [1.62–3.62] | [− 0.15–0.12] | [1.08–1.68] | [1.14–1.73] | [− 0.33–0.34] | [− 0.48–0.14] |
| 0.20 | 1.61 | 2.15 | 0.11 | 1.35 | 1.33 | 1.08 | 0.52 |
| | [0.66–2.8] | [1.23–3.28] | [− 0.05–0.26] | [1.00–1.67] | [0.98–1.65] | [0.53–1.88] | [0.09–1.09] |
| 0.40 | 0.82 | 1.28 | 0.52 | 1.24 | 1.02 | 0.19 | − 0.37 |
| | [− 0.26–2.11] | [0.23–2.47] | [0.27–0.84] | [0.73–1.66] | [0.50–1.44] | [− 0.28–0.86] | [− 0.76–0.10] |
| *MaxLev 200%, LoT* | | | | | | | |
| 0.00 | 1.89 | 2.41 | − 0.07 | 1.02 | 1.13 | − 0.01 | − 0.07 |
| | [1.05–2.94] | [1.60–3.39] | [− 0.19–0.04] | [0.72–1.33] | [0.82–1.41] | [− 0.32–0.32] | [− 0.37–0.24] |
| 0.10 | 1.70 | 2.28 | 0.01 | 1.23 | 1.30 | 0.06 | − 0.10 |
| | [0.79–2.86] | [1.40–3.4] | [− 0.13–0.14] | [0.89–1.55] | [0.96–1.6] | [− 0.29–0.45] | [− 0.42–0.26] |
| 0.20 | 1.40 | 1.95 | 0.12 | 1.14 | 1.16 | 1.02 | 0.53 |
| | [0.47–2.6] | [1.04–3.07] | [− 0.03–0.27] | [0.76–1.49] | [0.78–1.49] | [0.48–1.80] | [0.08–1.09] |
| 0.40 | 0.68 | 1.13 | 0.49 | 0.90 | 0.78 | 0.20 | − 0.30 |
| | [− 0.37–1.94] | [0.12–2.32] | [0.26–0.76] | [0.35–1.34] | [0.24–1.22] | [− 0.3–0.87] | [− 0.72–0.19] |
| *MaxLev 100%, LoT* | | | | | | | |
| 0.00 | 1.59 | 2.11 | − 0.01 | 0.48 | 0.56 | 0.19 | 0.11 |
| | [0.78–2.62] | [1.34–3.07] | [− 0.12–0.10] | [0.15–0.81] | [0.23–0.87] | [− 0.16–0.59] | [− 0.22–0.48] |
| 0.10 | 1.40 | 1.99 | 0.07 | 0.74 | 0.80 | 0.18 | 0.01 |
| | [0.54–2.53] | [1.15–3.07] | [− 0.06–0.18] | [0.38–1.10] | [0.44–1.13] | [− 0.21–0.63] | [− 0.34–0.42] |

**Table 5** (continued)

| TC (%) | Single regime (SR) | Equal weights ($1/n$) | CGL MaxLev ~ LoT | CGL UNC ~ LoT | CGL UNC LoT | CGL NC ~ LoT | CGL NC LoT |
|---|---|---|---|---|---|---|---|
| 0.20 | 1.16 | 1.70 | 0.16 | 0.62 | 0.65 | 0.88 | 0.47 |
| | [0.27–2.31] | [0.85–2.79] | [0.03–0.30] | [0.21–1.00] | [0.25–1.01] | [0.36–1.61] | [0.03–1.03] |
| 0.40 | 0.57 | 1.03 | 0.45 | 0.30 | 0.28 | 0.20 | − 0.21 |
| | [− 0.39–1.78] | [0.11–2.14] | [0.27–0.67] | [0.24–0.76] | [0.26–0.70] | [− 0.30–0.85] | [− 0.65–0.30] |
| *MaxLev 0%, LoT* | | | | | | | |
| 0.00 | 0.66 | 1.18 | − 0.01 | | | 0.33 | 0.24 |
| | [0.03–1.48] | [0.67–1.83] | [− 0.09–0.17] | | | [− 0.03–0.73] | [− 0.10–0.61] |
| 0.10 | 0.51 | 1.09 | 0.02 | | | 0.15 | 0.03 |
| | [0.15–1.4] | [0.55–1.82] | [− 0.07–0.20] | | | [− 0.21–0.56] | [− 0.31–0.40] |
| 0.20 | 0.41 | 0.95 | 0.06 | | | 0.44 | 0.23 |
| | [0.26–1.31] | [0.41–1.67] | [0.03–0.25] | | | [0.02–0.97] | [− 0.15–0.68] |
| 0.40 | 0.18 | 0.64 | 0.16 | | | 0.12 | − 0.09 |
| | [− 0.5–1.11] | [0.12–1.34] | [0.05–0.36] | | | [− 0.27–0.60] | [− 0.44–0.31] |

The table indicates the differences between annualized certainty equivalent returns ($\Delta CER$). The differences correspond to the CER from the models indicated in the horizontal panels, minus the CER from the benchmarks in the columns—computed according to Sect. 3.5. The horizontal panels present CGL MaxLev 300, 200, 100, and 0% with LoT, under the transaction costs (TC): 0.00, 0.10, 0.20, and 0.40%. The benchmarks in the columns are the SR model, the $1/n$ portfolio, the CGL MaxLev model without LoT (~ LoT), the unconstrains CGL model with and without LoT, and the numerically constrained CGL model (NC) with and without LoT. As in Sect. 4.3, the models with $\gamma = 75, 100, 200$ are the benchmarks for CGL MaxLev 300, 200, and 100%, respectively. Section 4.3 also demonstrates that unconstrained CGL models do not offer unleveraged models. Furthermore, CGL MaxLev ~ LoT, CGL NC ~ LoT, and CGL NC LoT are paired with the horizontal panels at the correspondent leverage levels (300, 200, 100, 0%). Below the $\Delta CER$, we report the 95% bootstrap confidence intervals drawn from 1,000,000 samples with replacement, with the bias-corrected and accelerated percentile method. The out-of-sample exercise was conducted with weekly observations from January 3rd, 1964 to November 30th, 2021

#### 4.4.1 Single regime

Positive $\Delta CER$ and positive confidence intervals indicate that CGL MaxLev with LoT statistically outperforms the SR model, considering transaction costs up to 0.20%. Yet, for the highest transaction cost (0.40%), although $\Delta CER$ is positive, confidence intervals with different algebraic signals indicate that they are not statistically significant.

#### 4.4.2 Equal weights

Positive $\Delta CER$ and positive confidence intervals indicate that CGL MaxLev with LoT surpass $1/n$ portfolios at any cost level with statistical significance.

#### 4.4.3 CGL MaxLev without LoT

Table 5 confirms a conclusion from Sect. 4.3: LoT generates value considering higher transaction costs and low leverage limits. With 200 and 300% leverage constraints, CGL MaxLev with LoT outperforms the correspondent model without LoT considering the cost of 0.40% with statistical significance. Yet, lowering the maximum leverage limit, LoT also statistically outperforms the benchmark at the cost of 0.20%.

#### 4.4.4 Unconstrained models

Positive $\Delta CER$ and positive confidence intervals indicate that CGL MaxLev with LoT statistically outperforms the unconstrained CGL models at any level of transaction costs. However, to unleverage the unconstrained CGL model, it would be necessary to set $\gamma$ at uncommonly high values, resulting in an overweighted risk-free allocation (Fig. 3). Thus, as Sect. 4.3, we suppress such a benchmark for the unleveraged condition.

#### 4.4.5 Numeric constrained models

At the leveraged portfolios, CGL MaxLev with LoT statistically outperforms the NC models, considering transaction costs of 0.20%. In the remaining pairs, the values from $\Delta CER$ are not statistically different from zero, thus we cannot demonstrate that one model outperforms the other. However, given we already demonstrated that MaxLev outperforms the unconstrained model, the similarity to the NC models is not uninteresting. It indicates that MaxLev's lower volatility compensates NC models high returns, in the certainty equivalence—suggesting that MaxLev is a competitive constraining procedure.

## 5 Conclusion

We addressed the issue of constraining dynamic models for multiple regimes economies for recursive utility preferences applying the CGL model to solve an allocation strategy accounting for transaction costs—a gap left by Campani, Garcia, and

Lewin (2021) and Lewin and Campani (2020a, b). We studied a single asset class portfolio, given by equities, to observe the effects of highly correlated assets. In this setting, we identified four unobservable regimes with the characteristics of crash, bear market, bull market, and rally states.

There is a body of literature indicating that regime-switching portfolio strategies are challenged by elevated leverage and turnover, but solutions by Bulla et al. (2011), Fiecas, Franke, Von Sachs, and Tadjuidje (2017) or Dal Pra, Guidolin, Pedio, and Vasile (2018) are ineffective for our application. Thus, we propose filters to control the portfolio maximum leverage (MaxLev) and low-turnover (LoT), using the CGL model.

We conducted an out-of-sample exercise that indicated that the CGL model with MaxLev and LoT report competitive return-to-risk measures relative to both single-state and regime switching benchmarks. For example, comparing models with and without LoT, such a turnover control generates value considering transaction costs (and most notably when costs increase), as expected. Meanwhile, the annualized certainty equivalent returns indicate that CGL MaxLev outperforms the unconstrained CGL model (i.e., the original model from Campani, Garcia, and Lewin 2021) with statistical significance. Furthermore, compared to a conventional numerically constraining procedure, MaxLev mitigates the returns' volatility.

The scope of our study was to evaluate regime-switching portfolio strategies for recursive preferences for constrained applications. Future studies shall find research opportunities investigating the proposed filters for new settings such as portfolios with different correlations in the assets menu, and a more complex cost structure, like dynamically observed illiquidity costs, and/or additional types of filters as maximum drawdown control and stop-loss triggers.

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