



# Kant's Categories of Quantity and Quality, Reconsidered: From the Point of View of the History of Logic and Natural Science

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## Abstract

According to Kant, the division of the categories “is not the result of a search after pure concepts undertaken at haphazard,” but is derived from the “complete” classification of judgments developed by traditional logic. However, the sorts of judgments that he enumerates in his table of judgments are not all ones that traditional logic has dealt with; consequently, we must say that he chose the sorts of judgments in question with a certain intention. Besides, we know that his choice of judgments and categories is strongly influenced by certain views of natural science that he fully accepts. For this reason, his argumentations are sometimes seriously inconsistent. As to Kant's argumentations of categories, many problems have already been pointed out, but in this paper, I take up the categories of quantity and quality once more, and make clear his argumentations' hidden logic and its distortion from the point of view of the history of logic and natural science. First, I confirm that there are non-negligible problems in his explanation to the effect that his derivation of the categories of quantity and quality is based on the quantity and quality of judgments. Next, I reconsider the meaning of his treating the categories of quantity and quality as pure concepts of the understanding. Finally, I conclude that by having recourse to the categories of quantity and quality Kant tried unjustly to apriorize the distinction between the “extensive magnitude” and “intensive magnitude” that has a long formational history since Aristotle.

**Keywords** Immanuel Kant · Categories of quantity · Categories of quality · Infinite judgments · Extensive magnitude · Intensive magnitude

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## 1 Introduction

In the *Critique of Pure Reason*, Kant gives us twelve pure concepts of the understanding, or categories, as the basic forms of our thinking. According to him, they were not collected in a rambling, arbitrary way. Rather, he says, the basic forms of our thinking are shown in the basic forms of our judgments, and he presents the latter in a table of judgments. Kant is aware that the division in the table differs on some points from that of the traditional logic. He says that “this division seems to depart in several points [...] from the customary technique of the logicians” (Kant, 1998b, A 70–71/B 96, p. 148; English translation is from: Kant, 1998a, A 70–71/B 96, p. 206), and adds some supplementary explanations (Kant, 1998b, A 71–76/B 96–101, pp. 148–153; 1998a, A 71–76/B 96–101, pp. 207–210). However, he notes, the difference is “not essential” (Kant, 1998b, A 70/B 96, p. 148; 1998a, A 70/B 96, p. 206). That is to say, when he derives the basic “functions” of human thinking, qua pure concepts of the understanding (categories), he is thinking that it is the forms of judgments that show us the functions, and that the grounds of their division lie in the logicians’ ordinary ways of dividing judgments. According to Kant, the traditional logic “seems to all appearance to be finished and complete” (Kant, 1998b, B VIII, p. 15; 1998a, B VIII, p. 106) and it is sufficient to be the firm basis of the derivation of categories. In such a context, Kant presents the table of categories that corresponds to that of judgments.<sup>1</sup>

In order to confirm whether such a way of arguing of Kant’s—in which, after presenting the table of judgments as the one showing the basic forms of the thinking of the understanding, he derives from them the pure concepts of the understanding (categories)—is valid or not, we must examine the following points: 1) whether the choice of the forms of judgments in the table of judgments is valid or not; and 2) whether the means of deriving each category from the division of judgments is valid or not. Thus, we must point out that in both points, Kant’s procedures have serious problems. It is the purpose of this paper to clarify them from the point of view of the history of logic and natural science.

When we regard Kant’s table of categories, we notice that those of “quantity” and “quality” account for half of all twelve. When, in the *Categories*, Aristotle enumerated his categories, those of quantity and quality constituted one out of ten respectively, thus occupying one-fifth of his total categories. From this, we must acknowledge that Kant’s 50% is, at a glance, abnormal. Of course, the real problem is not their proportion. When we compare Kant’s “categories of quality” with his “categories of quantity,” we know that his treatment of the former is based on a grave problem. Therefore, in this paper, I take up “quality of judgments” and the “categories of quality” that correspond to them, as well as “quantity of judgments” and the corresponding “categories of quantity,” and try to clarify the issues.

<sup>1</sup> For the relation between the table of judgments and the table of categories, see, e.g., Bennett (1966, pp. 71–83), Cleve (1999, pp. 87–90), Gardner (1999, pp. 131–135), Sgarbi (2016, pp. 135–150), Tomida (2017b, pp. 159–182), Wilkerson (1976, pp. 45–47), Wood (2005, pp. 41–44), and Young (1992, pp. 101–110).

Let us begin with “quantity of judgments.”

## 2 “Quantity of Judgments” and “Categories of Quantity”

Under the rubric of “quantity of judgments,” Kant enumerates the following three forms:

Universal judgments (*Allgemeine Urteile*)

Particular judgments (*Besondere Urteile*)

Singular judgments (*Einzelne Urteile*)

Although Kant lists them in this order in the table of judgments, under “categories of quantity,” he reverses the order.<sup>2</sup> Now, if we list the judgments in the order of the corresponding categories, they read: “singular judgments,” “particular judgments,” and “universal judgments.” These three correspond, respectively, to the three sorts of propositions in traditional logic: “propositio singularis” (singular proposition), “propositio particularis” (particular proposition), and “propositio universalis” (universal proposition).<sup>3</sup>

A singular proposition has, as a subject, a word or a phrase that stands for one determinate object (for example, “Socrates” or “that man”). A particular proposition and a universal one have, in their subjects, “some” and “all” (and the like), respectively. Thus, they assert that regarding one object, or some plural objects, or all of plural objects, something holds (or does not hold). This division of propositions was, in the traditional logic, one of the basic categories of propositions. It is, in its own way, persuasive that Kant derives from “singular judgments,” “particular judgments,” and “universal judgments,” the categories of “unity” (grasping an object as one thing), “plurality” (grasping objects as multiple things), and “totality” (plural objects that share identical states), respectively.

However, we must turn our attention to the fact that, although in the traditional logic, logicians usually list “propositio indefinita” (indefinite proposition), in addition to “propositio singularis,” “propositio particularis,” and “propositio universalis,” Kant ignores it. For example, in Peter of Spain’s *Tractatus (Summule logicales)*, one of the standard medieval textbooks of logic, propositions are classified from the point of view of “quantitas” (quantity) as follows:

Propositionum cathegoricarum alia universalis, alia particularis, alia indefinita, alia singularis.

Propositio universalis est illa in qua subicitur terminus communis determinatus signo universalis, ut “*omnis homo currit*” [...].

[...]

Propositio particularis est illa in qua subicitur terminus communis signo particulari determinatus, ut “*aliquis homo currit*.” [...]

<sup>2</sup> For this, see Frede & Krüger (1970) and Thompson (1989).

<sup>3</sup> For these phrases, see, for example, the passage that I will cite later from Hispanus (1972).

Indefinita est illa in qua subicitur terminus communis sine signo, ut “*homo currit*.”

Propositio singularis est illa in qua subicitur terminus singularis vel terminus communis iunctus cum pronomine demonstrativo, ut “*Sortes currit*” vel “*iste homo currit*.” (Hispanus, 1972, pp. 4–5)<sup>4</sup>

(Categoric[al] propositions are universal, particular, indefinite or singular.

A Universal Proposition is one in which a common term is [the] subject, determined by a universal marker [...], as in “every man runs” [...].

[...]

A Particular Proposition is one in which a common term is [the] subject, determined by a particular marker, as in “some man runs.” [...]

A[n Indefinite ...] Proposition is one in which a common term is [the] subject without a marker, as in “man runs.”

A Singular Proposition is one in which a singular term is [the] subject, or a common term constructed with a demonstrative pronoun is, as in “Socrates runs” or “that man runs.”)<sup>5</sup>

The “indefinite proposition” (*propositio indefinita*) is one that begins with a common term, like “man,” that is not accompanied by “all,” “some,” or the like.<sup>6</sup> Kant ignores this sort of judgment without offering any reason. Given the value Kant places on the classification of judgments in the traditional logic as grounds for the legitimacy of his own table of categories, his disregard for the indefinite proposition already shows that he does not blindly follow traditional logic, even if the proposition in question is de-emphasized in the theory of syllogisms of traditional logic. Rather, it seems that he chooses judgments with a certain intention of his own and likewise his categories.

Thus, here already is a symptom of his intentional choice. It is in his arguments concerning the “quality of judgments” and the corresponding “categories of quality” that the problem is more obvious.

### 3 “Quality of Judgments” and “Categories of Quality”

Under the name of “Quality of judgments,” Kant lists the following items:

Affirmative judgments (*Bejahende Urteile*)

Negative judgments (*Verneinende Urteile*)

Infinite judgments (*Unendliche Urteile*)

<sup>4</sup> We can find in Aristotle the prototype of the classification, which is composed of “καθόλου” (universal), “ἐν μέρει” (particular), “ἀδιόριστος” (indefinite), and “καθ’ ἕκαστον” (singular). For “καθόλου” and “καθ’ ἕκαστον,” see Aristotle (1949b, 17a38f., p. 52f.); for “ἐν μέρει” and “ἀδιόριστος,” see Aristotle (1964, 24a17f., p. 3).

<sup>5</sup> English translation is from: Hispanus (1990, p. 4). I replaced the word “nonfinite” (which is for the Latin “indefinita”) in the original English translation with “indefinite.”

<sup>6</sup> For the “indefinite proposition,” see also Ockham (1954, pp. 229–232).

Traditionally propositions have been classified from the point of view of the “quality” (*qualitas*) into “affirmative propositions” and “negative propositions” alone. If we take a further example from Peter of Spain:

Propositionum categoricarum alia affirmativa, alia negativa. Affirmativa est illa in qua predicatum affirmatur de subiecto, ut “*homo currit.*” Negativa est illa in qua predicatum removetur a subiecto, ut “*homo non currit.*” (Hispanus, 1972, p. 5)

(Categor[ical] Propositions are affirmative or negative. An Affirmative is one in which the predicate is affirmed of the subject, as in “man runs.” A Negative is one in which the predicate is removed from the subject, as in “man [does not run].”) (Hispanus, 1990, p. 4)<sup>7</sup>

Kant’s “affirmative” and “negative” judgments correspond to these “affirmative” and “negative” propositions. Kant further lists, as the categories correspondent to them, “reality” (*Realität*) and “negation” (*Negation*).

As to his style of arguing in which he derives the category of “reality” from the property of affirmative judgments, and the category of “negation” from the property of the negative judgments, if we glance at the procedures alone, they may seem justified. But when we note the treatment that he gives to the third sort of judgments, called “infinite judgments” (*unendliche Urteile*) and the correspondent category of “limitation” (*Limitation*), it becomes clear that they are not at all justified.<sup>8</sup>

When Kant adds “infinite judgments” to the quality of judgments, and equates them with “affirmative” and “negative” judgments, his firm intention is operating there. In the traditional logic, as shown in the citation from Peter of Spain’s *Tractatus* above, the qualities of propositions were “affirmative” and “negative” alone. However, since Aristotle, propositions that have a negative subject or a negative predicate have been considered secondarily and have been argued under the name of “infinite enunciation” (*enuntiatio infinita*) or “infinite proposition” (*propositio infinita*).

In the *De interpretatione* Aristotle distinguishes between “name” (*ὄνομα*) and “verb” (*ῥῆμα*) as basic ingredients that compose a statement (Aristotle, 1949b, 16a1, p. 49), and taking “not man” (*οὐκ ἄνθρωπος*) as an example, he says:

“Not man” is not a name, nor is there any correct name for it. It is neither a phrase nor a negation. Let us call it an indefinite name (*ὄνομα ἀόριστον*). (Aristotle, 1949b, 16a29–32, p. 50. English translation is from: Aristotle, 1963b, 16a29–32, p. 44.)

<sup>7</sup> I replaced the word “Categorematic” (which is for the Latin “categoricarum”) in the original English translation with “Categorical” and also replaced the sentence “man runs not” (which is for the Latin “*homo non currit*”) with “man does not run.”

<sup>8</sup> Needless to say, there is already much literature concerning Kant’s “infinite judgment.” See, e.g., Bennett (1966, pp. 77–78 & p. 89), Gardner (1999, p. 134), Hegel (1986, pp. 324ff.), Höffe (2003, pp. 126–127), Longuenesse (1998, pp. 294–298), Menne (1982, 1989), Serck-Hanssen (2013), Siebel (2020), Stang (2012), Tonelli (1966), Wolff (2017), and Young (1992, pp. 107–108).

Regarding the verb, taking examples containing negation, he says:

“Does not recover” (*οὐχ ὑγιαίνει*) and “does not ail” (*οὐ κάμνει*) I do not call verbs. For though they additionally signify time and always hold of something, yet there is a difference—for which there is no name. Let us call them indefinite verbs (*ἀόριστον ῥῆμα*) [...]. (Aristotle, 1949b, 16b11–15, p. 50; 1963b, 16b11–15, pp. 44–45)

In this manner, Aristotle calls the name and verb that contain negation “indefinite name” and “indefinite verb,” respectively, and considers the statements that contain them. For example, he says:

Now an affirmation signifies something about something, this last being either a name or a “non-name” [...]. [E]very affirmation will contain either a name and a verb or an indefinite name and a verb. (Aristotle, 1949b, 19b5f., pp. 58–59; 1963b, 19b5f., pp. 53–54)

Moreover, he says:

“every not-man is not-just” signifies the same as “no not-man is just.” (Aristotle, 1949b, 20a39–40, p. 61; 1963b, 20a39–40, p. 56)

In response to Aristotle’s mention of “indefinite name,” “indefinite verb,” and the statements that contain them, Boethius translates “ὄνομα ἀόριστον” (indefinite name) into Latin as “nomen infinitum” and “ἀόριστον ῥῆμα” (indefinite verb) as “verbum infinitum” (Boethius, 1894a, p. 341f.; 1894b, p. 424ff.), and he discusses the statements that contain them under the name of “enuntiatio infinita” (Boethius, 1894a, p. 341f.; 1894b, p. 520f.). (Hereafter, following Boethius’s wording, I use the word “infinite” instead of “indefinite” in such cases.)

Boethius’s wording of “enuntiatio infinita” is adopted by seventeenth-century logic books. For example, Johannes Rodolphus Faber mentions it in his *Totius logicae peripateticae corpus* (1623) (Faber, 1623, p. 279ff.), and in the *Manuale logicum* (1650) Johannes Scharff says that “enuntiationes fiunt vel finitæ, vel infinitæ” (Scharffius, 1650, p. 96).<sup>9</sup>

Thus, what Kant calls “infinite judgments” (*Unendliche Urteile*) traces back to Aristotle. Boethius translated Aristotle’s “ἀόριστον” into Latin by “infinitum” and called a statement containing “nomen infinitum” or “verbum infinitum” “enuntiatio infinita.” Therefore, Boethius’s “infinitum” and “infinita” do not mean “without end” or “endless” or “infinite” in this sense. As Ackrill’s translation of Aristotle’s *De interpretatione* shows, Ackrill’s use of “indefinite” is proper. When we use the English adjective “infinite” in this context, we must keep this fact in mind and understand it as the word that means “indefinite.”

If that is so, why is a judgment containing a negative name or verb called “infinite”? For example, in the case of “not man,” the name denotes something not human but nor does it offer a concrete and determinate meaning. Therefore, the

<sup>9</sup> For this, see also Menne (1989, p. 319).

statement that contains such a name or verb is called “enuntiatio infinita” (infinite enunciation). Later, it is also called “propositio infinita” (infinite proposition), or “judicium infinitum” (infinite judgment).

We can find an example in which the German phrase “unendliches Urtheil” is used in the context of “judicium infinitum,” in Meier’s *Auszug aus der Vernunftlehre* (1752), which Kant long used as a textbook of logic. Meier says in German (and Latin) as follows:

In einem logischen Urtheile stellen wir uns entweder vor, daß das Prädicate dem Subjecte zukomme, oder nicht zukomme [...]. Jenes ist **ein bejahendes Urtheil** (iudicium affirmans, affirmatiuum), dieses **ein verneinendes** (iudicium negans, negatiuum). Z. E. die Seele kan denken, die Materie kan nicht denken. In einem verneinenden Urtheile ist die Verneinung des Verbindungsbegrifs [...]. Und wenn in einem Urtheile entweder in dem Subjecte oder Prädicate, oder in beyden zugleich eine Verneinung ist, wenn nur der Verbindungsbegriff nicht verneinet wird, so ist es ein bejahendes Urtheil, welches **ein unendliches Urtheil** genennet wird (iudicium infinitum). Man kan also alle verneinende Urtheile in bejahende verwandeln, wenn man die Verneinung von dem Verbindungsbegriffe weg zum Prädicate leßt. Z. E. die Seele ist nicht sterblich, die Seele ist unsterblich. **Die Beschaffenheit der Urtheile** (qualitas iudicii) besteht in ihrer Bejahung und Verneinung. (Meier, 1752, § 294, pp. 81–82)

In this passage Meier says that in the case of the judgment “Die Seele ist nicht sterblich” (The soul is not mortal), since the copula “ist” (is) is denied by “nicht” (not), the judgment is a negative one. If, however, by moving the negation to the predicate, we make a new predicate “unsterblich” (immortal) and make a judgment “Die Seele ist unsterblich” (The soul is immortal), then it becomes affirmative. Meier’s view here echoes Aristotle’s, who, as we have already seen above, thinks that “‘every not-man is not-just’ signifies the same as ‘no not-man is just.’”

If that is the case, how did the Latin word “infinitum” (indefinite) shift to the German word “unendlich” (infinite in the sense of “endless”)? Perhaps Crusius’s view of “negative concept” (*negativer Begriff*) is one of those that prompt this change. In the *Weg zur Gewißheit und Zuverlässigkeit der menschlichen Erkenntniß* (1747), Crusius presents the view that whereas a positive concept strictly determines what it itself represents, a negative concept can represent an infinite number of objects. In absorbing this view, Albert Menne says, “this seems the origin of the false translation of ‘propositio infinita’ as ‘unendliches Urtheil’; its correct translation is ‘unbestimmtes Urtheil’” (Menne, 1989, p. 319; English translation is mine). Indeed, in the book, as to the “negatively determinate concept” (*negative bestimmter Begriff*), Crusius says:

in any negatively determinate concept one thinks something positive, that is, something indefinite. For if not so, one does not know what one is talking about. [...] In this manner we can replace it with all the properties [...] as they occur to our minds [...]. Such random determinations can be made infinitely. (Crusius, 1747, § 122, pp. 212–213. English translation is mine.)

The “indefinite name” and “indefinite verb” are nothing but linguistic items that signify what Crusius calls “negatively determinate concepts.” Therefore, as Crusius says, when we try to think concretely what a particular concept represents, we might consider an infinite number of items. For this reason, it is said, judgments that contain such names or verbs become understood as “unendliche Urteile.” Incidentally, Crusius says that “an infinite proposition (*Propositio infinita*) can be either affirmative or negative” (Crusius, 1747, § 226, p. 426) and he does not intend to take it up as an equal of affirmative and negative propositions.

In any case, at the time of Kant, “unendliche Urteile” (infinite judgements) are taken up as a term in the textbooks of logic. However, according to Tonelli, the textbooks that take them up are in the minority (Tonelli, 1966, p. 151). As to the type of infinite judgments with infinite predicates, they are customarily treated as affirmative judgments. This demonstrates that Kant’s way of listing “affirmative judgments,” “negative judgments,” and “infinite judgments” as equal items is never general or self-evident. Thus, he does not merely follow traditional logic that, as he says, “seems to all appearance to be finished and complete.”

#### 4 Kant’s Distorted Logic

Even if Kant’s procedure, which lists “affirmative judgments,” “negative judgments,” and “infinite judgments” as equal items, is valid, we must greatly doubt the validity of his method of deriving from the infinite judgments the category of “limitation.”

First, let us review the record of his lecture on logic—called *The Vienna Logic* that is estimated to have been delivered in the early 1780s. The following long citation is the entire relevant passage. He says:

as to their *quality*, they are divided into affirmative, negative, and infinite judgments. Even if the *logici* say that infinite judgments can be used as affirmative ones, that is a proposition which can be expounded in a special note. Basically, however, it is something different as to form, and in the beginning one must divide just as the distinction of the action[s] of our understanding is. [...]

*Quality* is the relation of concepts insofar as they stand in the relation of unity with one another[;] in accordance with this they are divided into affirmative judgments, if I combine one concept with the other positively, into negative judgments, if I separate one concept from the other, into infinite judgments, if I restrict one concept by the other. E.g., men are mortal[;] here I affirm mortality of men, or I think men as they stand under the concept of mortality. No man is mortal[;] here I deny mortality of man. If I think man, I think him as he is distinct from all that which is mortal. *Anima non est mortalis*, is a negative proposition. On the other hand, *Anima est non mortalis* is an infinite proposition. —All affirmative propositions show their affirmation through the *copula<sub>L</sub> est*, which *copula<sub>L</sub>* indicates the relation of two concepts. When the *copula<sub>L</sub> est* occurs *simpliciter*, it means the connection of two concepts—when the *copula<sub>L</sub> est* is affected with the *non*, it means the opposition of the two



concepts and indicates that the one concept does not belong to the other, or is not contained in the *sphaera* of the other. E.g., *anima non est mortalis*[:] here I represent that mortality does not include the soul. If I say, however, *anima est non mortalis*, then I say not merely that the soul contains nothing mortal, but also that it is contained in the *sphaera* of everything that is not mortal. In this case something special is said, then, namely, that I do not merely exclude one concept from the *sphaera* of another concept, but also think the concept under the whole remaining *sphaera*, which does not belong under the concept that is excluded. I do not actually say, *est immortalis*, but instead I say that the soul can be counted among all the concepts in general that may be thought outside the concept of mortality. And this actually constitutes infinite judgments. —

Affirmation and negation are qualities in judgment, accordingly. A negative judgment is not just any judgment that is negative, but a negative judgment where the negation affects the *copula*<sub>L</sub>. A judgment is an affirmative judgment, accordingly, where it does not affect the *copula*<sub>L</sub> but rather the predicate, as occurs in an infinite judgment, and where the *copula*<sub>L</sub> is without any negation[:] consequently, all infinite judgments are affirmative, because the negation affects only the predicate. But although every infinite judgment has the nature of the affirmative, nonetheless, there is always a negation there, not of the judgment, i.e., of the relation of the concepts, but of the predicate.

The relation is the same, to be sure, as in an affirmative judgment, but the negation is still always there, and consequently infinite judgments are distinct from the affirmative judgment. In logic, this matter seems to be a subtlety. But in metaphysics it will be a matter of importance not to have passed over it here. For there the distinction between reality, negation, and limitation is greater. In the case of limitations I think something positive, but not merely positive, but rather negative, too, and it is something positive that is restricted. —They are called *judicia infinita* because they are unlimited. They only say what is not, and I can make uncountably many such predicates, for the *sphaera* of the predicates which, affected by *non*, can be said of the subject, is infinite. (Kant, 1966, pp. 929–931. English translation is from: Kant, 1992, pp. 369–371.)

In this passage, too, as in the *Critique of Pure Reason*, judgments are divided, from the point of view of quality, into “affirmative” (*bejahende*), “negative” (*verneinende*), and “infinite” (*unendliche*). Regarding infinite judgments, as far as Kant’s assertion—“In this case something special is said, then, namely, that I do not merely exclude one concept from the *sphaera* of another concept, but also think the concept under the whole remaining *sphaera*, which does not belong under the concept that is excluded”—is concerned, he is right. It is certain that when regarding A, we say that A is something that is not B, we think of A as what exists outside the sphere of the concept of B and “think the concept under the whole remaining *sphaera*” that exists outside the sphere of the concept of B.

Further, in the last paragraph of the citation above he says: “In the case of limitations I think something positive, but not merely positive, but rather negative, too, and it is something positive that is restricted.” If the phrase “something positive that is restricted” means that something that is restricted, as in “something that is not B,” is affirmed by

the *copula*<sub>L</sub>, in this case, too, he is right. However, as Kant's statement in the citation above—that “in metaphysics [...] the distinction between reality, negation, and limitation is greater”—suggests, if he here, too, bears the derivation of the category of “limitation” (*Limitation*) from “infinite judgments” (*unendliche Urteile*) in mind, that is another story.

As we see later, what Kant thinks about the category of “limitation” is that B is affirmed in a certain limited way. Therefore, if Kant's statement that in the “infinite judgments” one thinks “something positive that is restricted” means that a certain property is affirmed in a limited way, then we must call it a sort of *ignoratio elenchi*. In fact, what he does in the *Critique of Pure Reason* is such a type of distortion of logic.

In the *Critique of Pure Reason*, after presenting the table of judgments, Kant makes several comments about it. In the second comment he brings up the “infinite judgments” and says:

Likewise, in a transcendental logic **infinite judgments** must also be distinguished from **affirmative** ones, even though in general logic they are rightly included with the latter and do not constitute a special member of the classification. General logic abstracts from all content of the predicate (even if it is negative), and considers only whether it is attributed to the subject or opposed to it. Transcendental logic, however, also considers the value or content of the logical affirmation made in a judgment by means of a merely negative predicate, and what sort of gain this yields for the whole of cognition. If I had said of the soul that it is not mortal, then I would at least have avoided an error by means of a negative judgment. Now by means of the proposition “The soul is non-mortal” (*die Seele ist nichtsterblich*) I have certainly made an actual affirmation as far as logical form is concerned, for I have placed the soul within the unlimited domain of undying beings. Now since that which is mortal contains one part of the whole domain of possible beings, but that which is undying the other, nothing is said by my proposition but that the soul is one of the infinite multitude of things that remain if I take away everything that is mortal. But the infinite sphere of the possible is thereby limited only to the extent that that which is mortal is separated from it, and the soul is placed in the remaining space of its domain. But even with this exception this space still remains infinite, and more parts could be taken away from it without the concept of the soul growing in the least and being affirmatively determined. In regard to logical domain, therefore, this infinite judgment is merely limiting with regard to the content of cognition in general, and to this extent it must not be omitted from the transcendental table of all moments of thinking in judgments, since the function of understanding that is hereby exercised may perhaps be important in the field of its pure *a priori* cognition. (Kant, 1998b, A 71–73/B 97–98, pp. 149–150; 1998a, A 71–73/B 97–98, pp. 207–208)<sup>10</sup>

<sup>10</sup> I replaced the sentence “The soul is not mortal” (which is for the German sentence “die Seele ist nichtsterblich”) in the original English translation with the one “The soul is non-mortal.” In this passage Kant presents an example of infinite judgment, and therefore, his example is not “die Seele ist nichtsterblich” but “die Seele ist nichtsterblich.” If we translate his example into “The soul is not mortal,” it is nothing but a negative judgment, not an infinite one.

As Kant says with regard to the proposition, “The soul is non-mortal,” thereby “the soul is placed in the remaining space of its domain,” that is, it is placed in the space that is formed by “tak[ing] away everything that is mortal” from “the whole domain of possible beings.” Thus, the soul is treated as an entity outside the sphere of the mortal, that is, it is treated as an entity in the sphere of the non-mortal. Since it is supposed that the sphere of the non-mortal contains “the infinite multitude of things,” Kant states that “the soul is one of the infinite multitude of things that remain if I take away everything that is mortal.” If we use the word “limited” here, his statement means that the soul is one of an infinite number of things in a sphere whose members are “limited” to the non-mortal. Of course, it never means that the soul is mortal to a “limited” degree.

However, when we take up the arguments that Kant offers later in the “Analytic of Principles” and consider what he says about the category of “limitation,” we find that he uses the term “limitation” in the sense that something has a certain property to a “limited” degree. That is to say, although he pretends to derive the category of “limitation” from the judgment of the form “A is not-B,” he uses the category in the sense that although A is B, it is not 100% B. In order to make this point clear, let us turn to Kant’s arguments in the “Analytic of Principles.”

## 5 Kant’s Explanation in the Theory of “Schemata”

In the second comment about the table of judgments cited above, Kant uses the verb “limit” (*beschränken*). As he says, “the infinite sphere of the possible is [...] limited only to the extent that that which is mortal is separated from it, and the soul is placed in the remaining space of its domain.” The limitation that is discussed here is only a limitation in the sense of reducing A’s ways of being in the manner of “not being B.” Kant makes no further explanation. However, when in the first chapter of the “Analytic of Principles” that treats “transcendental schemata,” he argues the schemata of the three categories—“reality,” “negation,” and “limitation”—the discussion moves at a stroke in a different direction. He says:

Reality is in the pure concept of the understanding that to which a sensation in general corresponds, that, therefore, the concept of which in itself indicates a being (in time). Negation is that the concept of which represents a non-being (in time). The opposition of the two thus takes place in the distinction of one and the same time as either a filled or an empty time. Since time is only the form of intuition, thus of objects as appearances, that which corresponds to the sensation in these is the transcendental matter of all objects, as things in themselves (thinghood, reality). Now every sensation has a degree or magnitude, through which it can more or less fill the same time, i.e., the inner sense in regard to the same representation of an object, until it ceases in nothingness ( $= 0 = \textit{negatio}$ ). Hence there is a relation and connection between, or rather a transition from reality to negation, that makes every reality representable as a quantum, and the schema of a reality, as the quantity of something insofar as it fills time, is just this continuous and uniform generation of that quantity in

time, as one descends in time from the sensation that has a certain degree to its disappearance or gradually ascends from negation to its magnitude. (Kant, 1998b, A 143/B 182–183, p. 244; 1998a, A 143/B 182–183, pp. 274–275)

Here, Kant restricts his discussion of “categories of quality” to “degrees” of “sensation.” Although in this passage the word “limitation” does not explicitly appear, we can perceive that the category of “limitation” is treated as having a certain degree of a sensation. In fact, in the *Prolegomena* (1783), regarding this, he says:

in those [categories] for quantity (*Größe*) and quality (*Qualität*) there is merely a progression from Unity to Totality, or from something to nothing (for this purpose the categories of quality must stand thus: Reality, Limitation, full Negation) [...]. (Kant, 1968b, § 39, n., p. 325. English translation is from: Kant, 2002, § 39, n., p. 117.)

Thus, Kant adds “infinite judgments” to the affirmative and negative ones, and from their characteristic of denying while affirming, he derives the category of “limitation,” without any substantive explanation. Originally, the limitation that infinite judgments make should be the limitation in the sense of denying a certain property, but Kant restricts the property to sensation and tries to make us understand the category of having a sensory property in a limited way.<sup>11</sup>

Originally, the viewpoints of “quantity” (*quantitas*) and “quality” (*qualitas*) that the traditional logic uses to classify propositions (judgments) were devices to apply Aristotle’s two categories (*ποσόν* and *ποιόν*) to propositions.<sup>12</sup> As previously described, with regard to the quality, “affirmative” and “negative” are the basis of its division. If a proposition affirms something by “is” (and the like), it is an “affirmative proposition” (affirmative judgment), and if it denies something by “is not” (and the like), it is a “negative proposition” (negative judgment). In this case, the thing affirmed and the thing denied are not restricted to the qualitative properties that are distinguished from the quantitative ones. Take, for example, a quantitative property like “being one meter long” or a qualitative property like “being white.” If we affirm it and say, “It is one meter long” or “It is white,” then the proposition is affirmative. If we deny it and say, “It is not one meter long” or “It is not white,” then the proposition is negative. In other words, the traditional viewpoint of the “quality” of

<sup>11</sup> For the fact that the “reality,” “negation,” and “limitation” that Kant lists as categories of quality are meant to have a full sensory property, not to have it at all, and to have it to some extent, see Maier (1930, pp. 37–38) and Warren (2001, p. 21). Warren says: “We are now in a position to characterize the relation between the third category of quality, i.e., limitation, and the other two. In traditional rationalist thought, the notion of limitation signified a *finite* degree of reality, a degree of reality insofar as it was not the greatest possible; reality, without this qualification, was regarded as in this sense unlimited, and negation was simply the absence of the reality.” (Warren, 2001, p. 21)

<sup>12</sup> It is said that Lucius Apuleius Madaurensis introduced the Latin expressions “*quantitas*” and “*qualitas*”—which correspond to Greek “*ποσότης*” and “*ποιότης*”—to represent two points of view for the classification of propositions. For his view, see his *Peri hermeneias* (Apuleius, 1621, p. 29f.). Incidentally, since Gustav Friedrich Hildebrand expressed doubts as to whether the *Peri hermeneias* was really written by Apuleius, the issue has been discussed among scholars.

propositions (judgments) is simply related to the affirmation and negation of propositions. It is never the one that restricts what is handled to the qualitative properties.

Nevertheless, Kant derives the “categories of quality” from this “quality of judgments” and regards them as exclusively related to the “sensations” qua qualitative properties. Thus, he restricts their range of application in an abnormal way, treating them as categories of “intensive magnitude.”

In this manner, by piling up the intentional choice of judgments, a sort of *ignoratio elenchi*, and the arbitrary restriction of the topic, Kant forces us to understand that from the three sorts of judgments (“affirmative judgments,” “negative judgments,” and “infinite judgments”) we can derive the three categories—“reality” as having a full sensory property, “negation” as not having it at all, and “limitation,” as having it to a certain limited degree—in a way that creates an illusion that some deep thought must be hidden in it.

This direction in Kant’s argument, in which the topic is restricted to the degree of the subject’s sensory property, is clearer in the “principle” called “anticipations of perception,” which is the basic principle to apply those categories to the intuition in a wider sense.<sup>13</sup> In order to better discuss it, we need to compare and contrast the “categories of quality” and the “anticipations of perception” with the “categories of quantity,” which Kant derives from the “quantity of judgments,” and the “axioms of intuition,” which is the principle of “categories of quantity.”

## 6 “Axioms of Intuition”

In the “Analytic of Principles,” Kant calls the “rules of the objective use of the categories” of quantity and quality, “axioms of intuition” (*Axiomen der Anschauung*)<sup>14</sup> and “anticipations of perception” (*Antizipationen der Wahrnehmung*), respectively.

In the first edition of the *Critique of Pure Reason*, Kant expresses the principle called “axioms of intuition” as follows:

All appearances are, as regards their intuition, **extensive magnitudes**. (Kant, 1998b, A 162, p. 260; 1998a, A 162, p. 286)

And in the second edition:

**All intuitions are extensive magnitudes**. (Kant, 1998b, B 202, p. 260; 1998a, B 202, p. 286)

As to the historical details, I will explain later. In any case, Kant calls the magnitude that goes on increasing by addition as much as you want, “extensive magnitude”

<sup>13</sup> For this, Kant says: “these principles are nothing other than rules of the objective use of the categories.” (Kant, 1998b, A 161/B 200, p. 258; 1998a, A 161/B 200, pp. 284–285). Therefore, if the three categories of “reality,” “negation,” and “limitation” are those that should be applied to the degrees of sensation, then it is a matter of course that the principle of “anticipations of perception” is, as I will discuss later, treated as what is exclusively related to the “intensive magnitude.”

<sup>14</sup> Kant spells the plural form of the German noun “Axiom” as “Axiomen” in every case. For this, see Mellin (1797, pp. 447–452 [Axiomen] and pp. 452–454 [Axiomen der Anschauung]).

(*extensive Größe*). He explains, “I call an extensive magnitude that in which the representation of the parts makes possible the representation of the whole” (Kant, 1998b, A 162/B 203, p. 261; 1998a, A 162/B 203, p. 287). “All appearances are accordingly [...] intuited as aggregates (*Aggregate*)” (Kant, 1998b, A 163/B 204, p. 262; 1998a, A 163/B 204, p. 288).

## 7 The Distinction between “Intuition” and “Sensation”

Here is one thing that we must confirm in advance in order to better understand what Kant calls “extensive magnitude.” It is his distinction between “intuition” and “sensation.”

Kant divides what Locke and Berkeley call “sensation”—that is, what we sense through our five senses—into “intuition” and “sensation.” In the transcendental aesthetic of the first edition, he says:

Besides space, [...] there is no other subjective representation related to something **external** that could be called *a priori* objective. Hence this subjective condition of all outer appearances cannot be compared with any other. The pleasant taste of a wine does not belong to the objective determinations of the wine, thus of an object even considered as an appearance, but rather to the particular constitution of sense in the subject that enjoys it. Colors are not objective qualities of the bodies to the intuition of which they are attached, but are also only modifications of the sense of sight, which is affected by light in a certain way. Space, on the contrary, as a condition of outer objects, necessarily belongs to their appearance or intuition. Taste and colors are by no means necessary conditions under which alone the objects can be objects of the senses for us. They are only combined with the appearance as contingently added effects of the particular organization. Hence they are not *a priori* representations, but are grounded on sensation, and pleasant taste is even grounded on feeling (of pleasure and displeasure) as an effect of the sensation. And no one can have *a priori* the representation either of a color or of any taste: but space concerns only the pure form of intuition, thus it includes no sensation (nothing empirical) in itself, and all kinds and determinations of space can and even must be able to be represented *a priori* if concepts of shapes as well as relations are to arise. Through space alone is it possible for things to be outer objects for us. (Kant, 1998b, A 28–29, p. 104; 1998a, A 28–29, p. 161)

In this passage, Kant’s distinction between intuition and sensation is already shown, but the contrast between the two is not so clear. Kant says that “Space, [...] as a condition of outer objects, necessarily belongs to their appearance or intuition.” On the contrary, he says: “Taste and colors are by no means necessary conditions under which alone the objects can be objects of the senses for us. They are only combined with the appearance as contingently added effects of the particular organization. Hence, they are not *a priori* representations, but are grounded on sensation, and pleasant taste is even grounded on feeling (of pleasure and displeasure) as an effect of the sensation.” Here, although the distinction between space on the one hand and

colors and tastes on the other is clear, it does not yet take on the clear expression that space is “intuition” but colors and tastes are “sensations.”

His description found in the same place of the second edition is different. There, he says:

Besides space, [...] there is no other subjective representation related to something **external** that could be called *a priori* objective. For one cannot derive synthetic *a priori* propositions from any such representation, as one can from intuition in space [...]. Strictly speaking, therefore, ideality does not pertain to them, although they coincide with the representation of space in belonging only to the subjective constitution of the kind of sense, e.g., of sight, hearing, and feeling, through the sensations of colors, sounds, and warmth, which, however, since they are merely sensations and not intuitions, do not in themselves allow any object to be cognized, least of all *a priori*. (Kant, 1998b, B 44, p. 104; 1998a, B 44, pp. 177–178)

Here, he clearly says that colors, sounds, and warmth “are merely sensations and not intuitions.”

In both the first and second editions, by saying that “Besides space, [...] there is no other subjective representation related to something **external** that could be called *a priori* objective” he acknowledges that the representation of space is *a priori*. Moreover, as we have already seen, in the first edition he confirms that whereas the representations of colors and tastes are not *a priori*, the representation of space is *a priori*. In addition, in the second edition he says that the colors, sounds, and warmth “do not in themselves allow any object to be cognized, least of all *a priori*” and that “one cannot derive synthetic *a priori* propositions from any such representation, as one can from intuition in space.” In this manner, he states that from the intuitions in space we can derive synthetic *a priori* propositions.

Actually, in the introductory paragraph of the transcendental aesthetic, he has already mentioned the distinction between the representations of the space and the sensations like colors, tastes, and so on:

I call all representations pure (in the transcendental sense) in which nothing is to be encountered that belongs to sensation. Accordingly the pure form of sensible intuitions in general is to be encountered in the mind *a priori*, wherein all of the manifold of appearances is intuited in certain relations. This pure form of sensibility itself is also called pure intuition. So if I separate from the representation of a body that which the understanding thinks about it, such as substance, force, divisibility, etc., as well as that which belongs to sensation, such as impenetrability, hardness, color, etc., something from this empirical intuition is still left for me, namely extension and form. These belong to the pure intuition, which occurs *a priori*, even without an actual object of the senses or sensation, as a mere form of sensibility on the mind. (Kant, 1998b, A 20–21/B 34–35, p. 94; 1998a, A 20–21/B 34–35, p. 156)

Here, citing some concrete examples, Kant distinguishes what the understanding thinks, what belongs to sensation, and what belongs to pure intuition. According to

him, impenetrability, hardness, and color are sensations; by contrast, extension and form belong to pure intuition. In this manner, Kant shows in advance the distinction between sensation and intuition that we saw above.

As to Kant's distinction between sensation and intuition, as far as we see its concrete examples, while there are differences, it basically follows the basic distinction of revived atomism ("corpuscularian hypothesis" or "Epicureanism") between primary and secondary qualities<sup>15</sup> and Descartes's distinction of the properties of bodies that has a close relation with atomism's distinction.<sup>16</sup> Moreover, the distinction is connected with his view on pure geometry. Kant thinks that space is pure form of the intuition and that pure geometry is established on the basis of such space. That is to say, Kant thinks that space is *a priori* inherent in our sensibilities and that by (productive) imagination's depicting (constructing) the figures of geometrical concepts in this space we get geometrical knowledge qua synthetic *a priori* judgments that are not derived merely from concepts.<sup>17</sup> Since such geometrical axioms derived from the intuition are plural and what Kant tries to show as a "principle" that corresponds to "categories of quantity" is the one concerning such plural axioms based on the intuition, the principle itself is called "axioms of intuition" (*Axiomen der Anschauung*). Owing to such circumstances, here Kant asserts as a "principle" that the spatial "intuition" that is distinguished from "sensation" is nothing but "extensive magnitude" that is treated by geometry. Clearly, Kant regards the principle that the space is an extensive magnitude as the basis of geometry, and specifies it in the "Analytic of Principles." According to him, the principle is the rule for application of the categories of quantity, and he regards a division of judgments as the basis of those categories.

His procedure that derives from the categories of quantity the principle that our spatial intuition is extensive magnitude might, had we glanced that alone, seem not to be so problematic. However, if one sees it in contradistinction to his way of grasping the "intensive magnitude," which is found in the principle called "Anticipations of Perception," one must feel that something is wrong.

<sup>15</sup> As is well known, Kant objects to the corpuscular hypothesis, takes the plenist position, and treats the concept of force as a basic one in his physical theory (Kant, 1968a; Carrier, 2001). However, Kant is well acquainted with Locke's views in the *Essay* and even inherits some part of them. The distinction in question is one such example. For this, see Tomida (2008, p. 267). For Locke's distinction, see Locke (1975, II, viii, 9–26, pp. 134–143) and Tomida (2017c, pp. 90–101).

<sup>16</sup> Descartes thinks that the properties of bodies that we can clearly and distinctly perceive are limited to the quantitative ones (the ones that we can quantify from the mathematical point of view) and that although as to extension, shape and size, situation, motion, quantity, and duration, we can have clear and distinct ideas of them, as to colors, tastes, warmth and coldness, and so on, we cannot have such ideas. For this reason, he omits colors, tastes and the like from the properties of bodies and treats them as what we merely have as sensations. What I mention as "Descartes's distinction of the properties of bodies" is this distinction by Descartes. For this, see also Tomida (2019, p. 201ff.)

<sup>17</sup> For this, see Tomida (2017a, p. 76ff.; 2018, p. 188ff.).



## 8 “Anticipations of Perception”

The principle that Kant calls “Anticipations of Perception,” which corresponds to the “categories of quality” is, in the first edition of the *Critique of Pure Reason*, expressed in the following manner:

In all appearances the sensation, and the **real**, which corresponds to it in the object (*realitas phaenomenon*), has an **intensive magnitude**, i.e., a degree (*Grad*). (Kant, 1998b, A 166, p. 265; 1998a, A 166, p. 290)

In the second edition, he says:

**In all appearances the real, which is an object of the sensation, has intensive magnitude**, i.e., a degree. (Kant, 1998b, B 207, p. 265; 1998a, B 207, p. 290)

His expressions are subtly different from each other, but the subject is, clearly, “sensation.” In brief, he says that our sensations have “intensive magnitudes,” that is, certain “degrees.” Kant sees the sensations from the point of view of three ways of being, namely possessing a certain property completely (reality), possessing it to a certain degree (limitation), and not possessing it at all (negation). The distinction is, for example, being completely white, being white to a certain degree, and not being white at all.

Thus, considering both the “Axioms of Intuition” and the “Anticipations of Perception,” we can understand what Kant is saying. His “categories of quantity” are those of “extensive magnitude,” and in the principle of “Axioms of Intuition” that corresponds to them, among our “sensations” in the ordinary sense of the term (that is, not only the sensations of colors, tastes, and the like, but also those of shapes, sizes and so on), the spatial qualities like shapes and sizes have a property qua “extensive magnitude,” that is, the property of growing bigger through the addition of its part. By contrast, in the principle called “Anticipations of Perception,” Kant says that what he calls “sensations” —colors, tastes, etc.—as distinguished from “intuition,” are “intensive magnitudes” that appear in various degrees.

Originally, there was no reason to restrict the scope of application of the “categories of quality” derived from the “quality of judgments” to the so-called “qualitative” sensation, like that of whiteness. His argument here is too forced and unjust. However, adjoined with the “categories of quantity,” we can see his intention: he wants to emphasize the difference between “extensive” and “intensive” magnitudes, which point has nothing to do with the formal distinction of propositions or judgments in the traditional logic.

## 9 A Comparison with Locke

Before moving on, in order to better understand Kant’s distinction between “extensive magnitude” and “intensive magnitude,” I invoke Locke. In his *Essay* (1690), we find a distinction that corresponds with Kant’s. Locke says:

All the *Ideas*, that are considered as having parts, and are capable of increase by the addition of any equal or less parts, afford us by their repetition the *Idea* of Infinity; because with this endless repetition, there is continued an enlargement, of which there can be no end. But in other *Ideas* it is not so; for to the largest *Idea* of Extension or Duration, that I at present have, the addition of any the least part makes an increase; but to the perfectest *Idea* I have of the whitest Whiteness, if I add another of a less or equal whiteness, (and of a whiter than I have, I cannot add the *Idea*.) it makes no increase, and enlarges not my *Idea* at all; and therefore the different *Ideas* of Whiteness, *etc.* are called Degrees. For those *Ideas*, that consist of Parts, are capable of being augmented by every addition of the least part; but if you take the *Idea* of White, which one parcel of Snow yielded yesterday to your Sight, and another *Idea* of White from another parcel of Snow you see to day, and put them together in your Mind, they embody, as it were, and run into one, and the *Idea* of Whiteness is not at all increased; and if we add a less degree of Whiteness to a greater, we are so far from increasing, that we diminish it. Those *Ideas* that consist not of Parts, cannot be augmented to what proportion Men please, or be stretched beyond what they have received by their Senses [...]. (Locke, 1975, II. xvii. 6, pp. 212–213)

Locke's point is clear. According to him, among the ideas that we perceive through our senses, some are ones, like extension, that extend themselves infinitely by repeating addition, and others are ones, like whiteness, that do not augment by addition. Locke applies the word "Degree" to the latter. Incidentally, in the German edition of the *Essay* translated by Heinrich Engelhard Poley, which Kant might have read, Locke's phrase "therefore the different *Ideas* of Whiteness, *etc.* are called Degrees" is translated as "Daher werden die verschiedenen Begriffe der Weiße u. s. w. Grade genannt" (Locke, 2004, p. 209); that is, "Degrees" is translated as "Grade".<sup>18</sup> Thus, the distinction between "extensive" and "intensive" magnitudes that Kant's principles of the "Axioms of Intuition" and the "Anticipations of Perception" deal with, respectively, corresponds with the distinction that Locke presented in the *Essay* prior to Kant.

In addition, Locke says that every extensive magnitude has parts as follows:

<sup>18</sup> As to how Kant, who did not read English, read Locke's *Essay*, we can surmise from some clues, including his own reminiscence in the *Prolegomena* to the effect that he received a hint (*Wink*) for the distinction between analytic and synthetic judgments from Locke (Kant, 1968b, § 3, p. 270; see also Woolhouse, 1971, p. 11). While Locke was alive, his *Essay* was translated into Latin by Ezekiel Burridge, and the first Latin edition was published in 1701 (Locke, 1701). It was reissued in Leipzig as Locke (1709), and in 1741, its new edition, revised by Gotthelf Heinrich Thiele, was published as Locke (1741). On the other hand, in 1757 a German edition of the *Essay* was published by Heinrich Engelhard Poley (Locke, 2004). Poley's German translation is based on Locke (1727). Therefore, there is a possibility that Kant read Poley's German edition. However, whereas in the *Prolegomena*, Kant expresses Locke's technical term "*Idea*" as "*Vorstellung*," Poley translates it as "*Begriff*." Of course, there is a possibility that although Kant referred to Poley's German translation, he did not follow Poley's wording, but bearing in mind the fact that he was a master of Latin, I suspect that Kant read Locke's *Essay* in a Latin translation.

There is one thing more, wherein *Space and Duration* have a great Conformity, and that is, though they are justly reckoned amongst our *simple Ideas*: Yet none of the distinct *Ideas* we have of either is without all manner of *Composition*, it is the very nature of both of them to consist Parts [...]. (Locke, 1975, II. xv. 9, pp. 201–202)

As seen in Section 6 above, Kant says that “All appearances are [...] intuited as aggregates (*Aggregate*).” Without doubt, Kant’s “Aggregat” corresponds to Locke’s “Composition” in the citation above.

## 10 In the Case of Today’s Natural Science

The distinction in question is still used today in natural science. To express the distinction, one uses the term “extensive property” or “intensive property,” or “extensive variable” or “intensive variable.” In any case, the adjectives of “extensive” and “intensive” have survived.

Here is an example from the *Fundamentals of Engineering Thermodynamics* (2011):

Thermodynamic properties can be placed in two general classes: extensive and intensive. A property is called extensive if its value for an overall system is the sum of its values for the parts into which the system is divided. Mass, volume, energy, and several other properties introduced later are extensive. Extensive properties depend on the size or extent of a system. [...]

Intensive properties are not additive in the sense previously considered. Their values are independent of the size or extent of a system and may vary from place to place within the system at any moment. Thus, intensive properties may be functions of both position and time, whereas extensive properties can vary only with time. Specific volume [...], pressure, and temperature are important intensive properties [...]. (Moran et al., 2011, p. 9)

As an example of “extensive properties,” let us consider mass. If we add the mass of each part together, we get the sum total. In the case of volume, too, the situation is the same. If we add the volume of each part together, we get the sum total. In contrast, with pressure, even if we add the pressure of each part together, we cannot get the pressure of the whole. In the case of the temperature, the situation is the same as the pressure. Even if we add the temperature of each part of a building, we cannot get the temperature of the whole building. If we bear those facts in mind, it may be clear that the distinction that Locke mentions and Kant asserts is important especially in the natural science.

## 11 The Tradition since Ancient Greece

The distinction between “extensive” and “intensive” magnitudes has a long history that traces back to ancient Greece.

Plato is said to have coined the abstract noun ποιότης (quality) from the interrogative adjective ποῖος, (–α, –ον), meaning “of what kind?” or “what?” (Plato, 1921, 182a, pp. 148–149). In the same manner, Aristotle coined the abstract noun ποσότης (quantity) from the interrogative adjective πόσος (–η, –ον), or “of what quantity?” (Aristotle, 1933, 1028a19, pp. 310–311).

In the fourth chapter of the *Categories*, Aristotle classified our ways of speaking—categories—into ten sorts. In the list he cites “ποσόν” (quantity or magnitude) and “ποιόν” (kind or quality). As examples of quantity, he mentions “δίπηχυς” (two cubits long) and “τρίπηχυς” (three cubits long), and as examples of quality he mentions “λευκόν” (white) and “γραμματικόν” (grammatical) (Aristotle, 1949a, 1b25–29, p. 5).

In the same ways as Plato and Aristotle, Cicero coined the words to translate “ποσότης” and “ποιότης” into Latin. He coined the abstract noun “quantitas” to mean quantity or magnitude, from the interrogative adjective “quantus (–a, –um),” or “how great?,” “how much?,” or “how many?.” In the same manner he coined the abstract noun “qualitas” to mean “quality,” from the interrogative adjective “qualis (–a, –um),” or “of what sort?,” “of what kind?,” or “of what nature?.”

By adding the adjectives “extensiva” and “intensiva” to the Latin noun “quantitas,” the Latin phrases “quantitas extensiva” and “quantitas intensiva” were formed. In medieval Europe, they were widely used. In Kant’s wording they are “extensive Größe” and “intensive Größe,” respectively. That is to say, the Latin phrases “quantitas extensiva” and “quantitas intensiva” are the roots of Kant’s “extensive Größe” and “intensive Größe.”

As often seen in the European culture, the origin of the way of thinking that divides quantity into two types goes back to Aristotle. In the *Categories*, Aristotle distinguishes quantity and quality as two distinct categories, but at the same time, in its eighth chapter, regarding quality he says:

[Qualities] admit of a more and a less; for one thing is called more pale or less pale than another, and more just than another. Moreover, it itself sustains increase (for what is pale can still become paler) [...]. (Aristotle, 1949a, 10b26–30, p. 31. English translation is from: Aristotle, 1963a, 10b26–30, p. 29.)

That “it itself sustains increase” means that one and the same thing can have a certain quality to a greater degree. Boethius, who translated Aristotle’s *Categories* from Greek to Latin, translated the Greek word “ἐπίδοσις” (increase) by the Latin word “intentio.”<sup>19</sup> It is one of the noun forms of the Latin verb “intendere,” meaning to “stretch,” “extend,” “magnify,” and so on. Here it is used in the sense of “increase” or “augmentation.”

Thus, Aristotle basically distinguishes quantity and quality, but since, regarding quality, he mentions the distinction of its degrees, people begin to think that quality,

<sup>19</sup> See Aristotle (1961, p. 28). Boethius translates the passage from the *Categories* cited above into Latin as follows: “Suscipit autem qualitas magis et minus; album et enim magis et minus alterum dicitur, et iustum alterum magis. Et idem ipsum sumit intentionem (album enim cum sit, contingit illud fieri albius) [...]” (Aristotle, 1961, p. 28)

too, has a sort of “quantity” or “magnitude.” That is to say, they think whereas the so-called “quantity” is “extensive quantity,” the so-called “quality” has an “intensive quantity” and shows it to various degrees. As a result, people begin to think that there are two types of quantity, instead of thinking of the contrast between quantity and quality.

In medieval Europe, this distinction chiefly appears in the form of the distinction between “*quantitas extensiva*” (extensive quantity) and “*quantitas intensiva*” (intensive quantity) or between “*magnitudo extensiva*” (extensive magnitude) and “*magnitudo intensiva*” (intensive magnitude). The Latin word “*intensiva*” is the adjective form of the Latin noun “*intentio*” mentioned above.

The distinction that has been cultivated in this manner is, as a matter of course, used by medieval representative theologians, including Thomas Aquinas and John Duns Scotus.<sup>20</sup>

In the age of Kant, too, not only Kant but other intellectuals mentioned this distinction. For example, in the appendix (*Anhang*) added to the third edition of the *Phädon, or On the Immortality of the Soul* (1769), Moses Mendelssohn, who had a relationship with Kant and competed for a prize with him in an essay contest, wrote as follows:

Namely, there is a magnitude of amount (*Größe der Menge*) (*quantitas extensiva*), which exists in the amount of parts that compose it, and there is a magnitude of strength (*Größe der Kraft*) (*quantitas intensiva*), which is also called **degree** (*Grad*). If several parts are added, then the magnitude of the first type increases, but [for] the degree [to increase, it] requires an internal reinforcement (*eine innerliche Verstärkung*), not a larger extension. If lukewarm water is poured into lukewarm water, then the amount of the water is increased, but not the degree of warmth. Many bodies, which move with equal velocity, when combined, make up a greater mass, but no greater velocity. The degree is as great in each part as in the whole, therefore, the sum of the parts cannot change the degree. If this were to happen, then the forces of the amount must be concentrated in a one, because the internal intensity can be increased according as the extension decreases. (Mendelssohn, 1791, *Anhang: Einige Einwürfe betreffend, die dem Verfasser gemacht worden sind*, pp. 217–218. English translation is mine.)

In this passage Mendelssohn takes up the distinction between extensive and intensive quantities in order to argue that no matter how many unclear concepts of atoms we may pile, we cannot get any distinct concept of an atom. In any case, he expresses the difference between extensive and intensive quantities by the phrases “*Größe der Menge*” and “*Größe der Kraft*,” and adds to them the traditional Latin expressions “*quantitas extensiva*” and “*quantitas intensiva*” in parentheses.

<sup>20</sup> For this, see, e.g. Aquinas (2003, *Quaestio 1, Articulus 2, Responsio*, p. 50) and Scotus (1894, *Distinctio 44, Quaestio 1, Scholium*, p. 179).

## 12 Unfair Use of the Viewpoint of the Traditional Logic

Thus, Kant's view, which regards "extensive magnitude" as a property of spatial "intuition" that geometry should handle, and the "intensive magnitude" as a property of "sensation," has a long prehistory since ancient Greece. Especially since the early modern period, the distinction between "extensive" and "intensive" magnitudes is treated as a key natural-scientific distinction. It is certain that Kant himself, considered it as one of the most important distinctions in natural science. In fact, in the *Metaphysical Foundations of Natural Science* (1786), he mentions it and argues that the velocity is "intensive" and that it must be distinguished from "extensive" quantities (Kant, 1968a, pp. 493–494).

Bearing these situations in mind, we must heed the fact that Kant uses half of his twelve pure concepts of the understanding (categories) to show that intuition has extensive magnitude and that sensation has intensive magnitude. In the case of Kant, the categories concerning extensive and intensive magnitudes account for half of the whole categories, and the reason is that Kant tries to pretend that he derives his twelve categories from the division of the judgments in the traditional logic. According to him, the categories of quantity are derived from the distinction that consists of "singular," "particular," and "universal" judgments, and the categories of quality are derived from the distinction that consists of "affirmative," "negative," and "infinite" judgments. However, whether his treatment of "categories of quantity" is valid or not, the basic problem is that Kant pretends to derive "categories of quality" from the division of "quality of judgments" and that he treats these categories in an unfair manner as the ones that exclusively concern the "sensation" (or intensive magnitude) distinguished from "intuition."

To those who know the history of the traditional logic, it is quite obvious that Kant's progress from "quality of judgments" to "categories of quality," and further to the "anticipations of perception" qua a principle on intensive magnitude, has a problematic abnormality. We can think that Kant wanted to fix, in the form of pure concepts of the understanding, the distinction between "extensive" and "intensive" quantities that is an important division of natural science, and also wanted to assert that the ground of the distinction existed in the traditional division of judgments. But what he really did was to abuse some viewpoints of the traditional logic in an unfair manner.

However, such abuse by Kant rather shows how heavily he emphasized the difference between extensive and intensive magnitudes that had already been important in the natural science of his time. That is to say, in the *Critique of Pure Reason*, Kant tried to prepare a true metaphysics that would ground other sciences including natural science, but actually, his arguments were often led by natural-scientific views that he wanted to "hold fast."<sup>21</sup> In other words, they were seemingly independent of natural science but were, in fact, heavily dependent on it. Thus, what Kant really did is nothing but a circular, "naturalistic" activity with a recourse to natural science to ground it.<sup>22</sup>

<sup>21</sup> For this wording, see Quine (1991, p. 266).

<sup>22</sup> I am obliged to W. V. Quine for his having given moral support to my naturalistic interpretation over a long period of time.

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