The Density of Symbol Systems – A Critique of Nelson Goodman's Notion

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Abstract

Nelson Goodman's theory of symbol systems expounded in his *Languages of Art* has been frequently criticized on many counts (cf. list of secondary literature in the entry "Goodman's Aesthetics" of *Stanford Encyclopedia of Philosophy* and Sect. 3 below). Yet it exerts a strong influence and is treated as one of the major twentieth-century theories on the subject.

While many of Goodman's controversial theses are criticized, the technical notions he used to formulate them seem to have been treated as neutral tools. One such technical notion is that of the density of symbol systems. This serves to distinguish linguistic symbols from pictorial representations (after Goodman entirely rejected resemblance in that role) and is a crucial part of Goodman's explanation of what constitutes aesthetic experience (and so indirectly what is art). Thus its significance for Goodman's theory is fundamental.

The aim of this paper is a detailed, logical analysis of this notion. It turns out that Goodman's definition is highly problematic and cannot be applied to symbol systems in the way Goodman envisaged. To conclude, Goodman's theory is problematic not just because of its controversial theses but also because of logical problems with the technical notions used at its very core. Hence the controversial claims are not simply contestable, but inaccurately expressed.

Keywords Nelson Goodman · Density of symbol systems · Critique · *Languages of Art* · Finite differentiation

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1 Introduction: density in Goodman's aesthetic system

The notion of the density of symbol systems, proposed by Nelson Goodman in his well-known book *Languages of Art. An Approach to a Theory of Symbols* (Goodman, 1968) is undoubtedly one of the central notions in that treatise and in Goodman's aesthetic system. Let us begin with a brief outline of the context in which this notion was introduced and the questions it is designed to elucidate.

In the first chapter, "Reality Remade", Goodman launches an all-out attack on the conviction that the essence of pictorial representation lies in its resemblance to the represented object. Indicating that no degree of resemblance is in itself sufficient for pictorial representation to occur (the fact that two cars of the same model are virtually indistinguishable from one another does not automatically make one of them a representation of the other; neither does a prince represent his portrait, even though the portrait represents the prince and the relationship of similarity is obviously symmetrical), Goodman states that pictorial representation is founded on a symbolic relationship:

> The plain fact is that a picture, to represent an object, must be a symbol for it, stand for it, refer to it; and that no degree of resemblance is sufficient to establish the requisite relationship of reference. […] Denotation is the core of representation and is independent of resemblance. (Goodman, 1968, p. 5)

Not content, however, with the obvious conclusion that the fact of resemblance is insufficient for pictorial representation to occur, he suggests that resemblance actually plays no role whatsoever:

> Is it perhaps the case that if A denotes B, then A represents B just to the extent that A resembles B? I think even this watered-down and innocuous-looking version of our initial formula betrays a grave misconception of the nature of representation. (Goodman, 1968, p. 6)

It might seem that Goodman's robust rejection of the idea that resemblance plays any role in constituting pictorial representation was motivated partly by a wish to clear the way for his own proposition. The resultant void lends credibility to his own explanation, as indeed it would to any other solution that might be put forward as the "sole alternative", given the alleged erroneousness of others. Goodman's solution, presented at the beginning of the last chapter in his book, entitled "Art and the Understanding", reads as follows: representation differs from other instances of denotation in that it is a symbol functioning within a dense symbol system, whereas linguistic notational systems are the exact opposite of density: they are characterized by finite differentiation.

> Nonlinguistic systems differ from languages, depiction from description, the representational from the verbal, paintings from poems, primarily through lack of differentiation – indeed through density […] – in the symbol scheme (of the symbol system). […] A scheme (system) is representational only insofar as it is dense; and a symbol is a representation only if it belongs to a scheme (system) dense throughout. (Goodman, 1968, p. 226)

The key notions of density and lack of differentiation will be explained below. But first let us outline the other considerations that led Goodman to introduce them. They include the perception of a difference between arts which Goodman calls autographic and allographic (this is discussed in the third chapter, "Art and Authenticity"):

> Let us speak of a work of art as *autographic* if and only if the distinction between original and forgery of it is significant; or better, if and only if even the most exact duplication of it does not thereby count as genuine. (Goodman, 1968, p. 113)

Works of art that are not autographic, Goodman calls "allographic":

> […] in music, unlike painting, there is no such thing as a forgery of a known work. […] Copies of the score may vary in accuracy, but all accurate copies […] are equally genuine instances of the score. (Goodman, 1968, p. 112)

Goodman explains the source of this difference by pointing out that allographic arts are always based on a specific notation, which autographic arts are lacking. This in turn gives rise to the question as to what features a notation as a symbol system must have, and extensive research into this problem (in the fourth chapter, "The Theory of Notation") leads the author to formulate the condition of the finite differentiation of notational systems, which he contrasts with the density of the symbol system characteristic of some non-notational systems, in particular painting.

Goodman then uses the notion of density to distinguish between analog and digital systems (and in particular between analog and digital computers). As he wittily observes: "Plainly, a digital system has nothing special to do with digits, or an analog system with analogy". (1968, p. 160). So here too the notion of density comes in handy: "A symbol *scheme* is analog if syntactically dense; a *system* is analog if syntactically and semantically dense" (1968, p. 160); "To be digital a system must be not merely discontinuous but *differentiated* throughout" (1968, p. 161). In fact in (Goodman, 1988, p. 123; cf. also p. 126) he seems to equate those two pairs of terms: "symbols in a dense or 'analog' system and those in a finitely differentiated or 'digital' system". And he continues to use the term "analog" much more often than "dense".

Finally, one last point showing how essential the notion of density is for Goodman: the "symptoms of the aesthetic" that Goodman formulates in his sixth and last chapter, "Art and the Understanding". As he explains it, "A symptom is neither a necessary nor a sufficient condition for, but merely tends in conjunction with other such symptoms to be present in, aesthetic experience." (Goodman, 1968, p. 252) He was prompted to formulate conditions of this sort by the earlier conclusion that it is impossible to find a uniform and invariably binding criterion that unequivocally distinguishes aesthetic experience. Goodman gives four such symptoms: two are the syntactic and semantic density of the symbol system; the other two are "syntactic repleteness" and "exemplificationality". He defines their role as follows:

> Yet if the four symptoms listed are *severally* neither sufficient nor necessary for aesthetic experience, they may be *conjunctively* sufficient and *disjunctively*

necessary; perhaps, that is, an experience is aesthetic if it has all these attributes and only if it has at least one of them. (Goodman, 1968, p. 254)[1]

It is clear, therefore, that since the pair of opposing notions of the "finite differentiation" and the "density" of symbol systems is not just responsible for distinguishing representation from other instances of denotation, pictures from descriptions and autographic from non-autographic (allographic) arts, but is actually a crucial part of Goodman's explanation of what constitutes aesthetic experience (and so indirectly what is art), their significance for Goodman's theory is fundamental. Thus a correct and coherent definition of those notions, corresponding to the intentions linked to them, would seem most crucial to that theory. Unfortunately, and paradoxically, the coherence and clarity with which Goodman defines those notions seems inversely proportional to their enormous weight.

The aim of the present article is to analyze the notion of density and to present the problems that arise when we attempt to concretize it and refer it in the way Goodman suggests to the fine arts, and to painting in particular. The result of this analysis will be the conclusion that the way in which Goodman defines this notion is untenable: if we wish to express the intuitions that inform it, we have to seek another definition.

2 Symbols, inscriptions, finite differentiation and density

We should start by explaining Goodman's terminological conventions. He uses the term "symbol scheme" in reference to the actual set of characters employed in a system, together with the principles by which they are combined into complex characters ("Any symbol scheme consists of characters, usually with modes of combining them to form others", Goodman, 1968, p. 131), but detached from any meanings or references. In other words, this is the pure syntactic level of a system. A symbol scheme, combined with symbol references, that is, combined with their semantic layer, forms a symbol system. ("A symbol system consists of a symbol scheme correlated with a field of reference". Goodman, 1968, p. 143) Goodman understands characters as classes (consisting of one or more elements) of inscriptions, which are material representatives (tokens) of the character – the only symbols that physically

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[1] To give the full picture, one might mention that in a later work (1977) Goodman added a fifth symptom, "multiple and complex reference, where a symbol performs several integrated and interacting referential functions, some direct and some mediated through other symbols" (Goodman 1978, p. 68), and weakened somewhat the power of those symptoms to designate the aesthetic: "And even for these five symptoms to come somewhere near being disjunctively necessary and conjunctively (as a syndrome) sufficient might well call for some redrawing of the vague and vagrant borderlines of the aesthetic." (1978, p. 68–69). In (Goodman 1984, p. 136) he repeats the whole list of five symptoms and weakens their role even further by saying: "None [of the symptoms] is always present in the aesthetic or always absent from the nonaesthetic; and even presence or absence of all gives no guarantee either way." (1984, p. 137).

However, as the fifth symptom is not related to density, which continues to constitute two symptoms of the aesthetic, all of this is of little relevance to our task, which is the critical examination of the notion of density.

exist, one might say. Goodman expands the meaning of the term "inscription" to encompass more than just written marks: "an inscription is any mark – visual, auditory, etc. – that belongs to a character" (Goodman, 1968, p. 131).

Some real, physical "symbols" (or "inscriptions", according to the adopted terminology) are identified with one another on the strength of the rules in force within a particular symbol scheme, and they are treated as different, but equivalent, specimens of the same symbol. Such an identity relation is of course reflexive (every inscription is identified with itself) and symmetrical (if we identify *a* with *b*, then we identify *b* with *a*). For every inscription, we may define the character as the class of all the inscriptions that are equivalent to it. Goodman notes that in order to speak of notation (and one might add that such a requirement seems natural in relation to any useful symbol system, notational or otherwise), the relation of being a specimen of the same character must be transitive (that is, if we identify *a* with *b* and *b* with *c*, then we also identify *a* with *c*) – a condition that amounts to stating that characters understood as classes of inscriptions must be disjoint. (The equivalence of these two conditions is demonstrated by a simple exercise from set theory.) If that were not the case, then it would be fundamentally impossible to determine what character a given inscription represents, since it might represent several different characters at once.

Thus the disjointness of characters – Goodman's first requirement of a notational scheme[2] – is intended to preclude a situation where one inscription belongs to two different characters, and so represents two characters. In other words, it is meant to ensure that each mark represents no more than one character (exactly one if it is an inscription; none if it is not). In fact, what we really expect of the condition of disjointness may be expressed as follows: we wish to be able to decide unequivocally, when perceiving a mark, what character it represents (or that it represents none). As Goodman observes, however, the condition of disjointness itself does not guarantee this: even if it is met, it may be impossible to determine that a given mark represents a certain symbol and not many others. In other words, the disjointness of symbols is impossible to establish, and a mark cannot be assigned to a particular symbol. To illustrate such a possibility, Goodman gives the following example:

> Suppose, for example, that only straight marks are concerned, and that marks differing in length by even the smallest fraction of an inch are stipulated to belong to different characters. Then no matter how precisely the length of any mark is measured, there will always be two (indeed, infinitely many) characters, corresponding to different rational numbers, such that the measurement will fail to determine that the mark does not belong to them. (Goodman, 1968, p. 135)

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[2] Strictly speaking, Goodman words this condition somewhat convolutedly, speaking of character-indifference among inscriptions of a single symbol. He essentially wishes to state that the relation of being an inscription of the same symbol should be a relation of equivalence. And since the reflexivity and symmetry of such a relation is obvious, we need only demand its transitivity, which, as already mentioned, proves to be equivalent to the condition of the disjointness of symbols.

In other words, in the scheme described above by Goodman – let us call it S – a single character is a class of all straight marks of *precisely* the same length. If we use the designation C_a for the class of all marks of the *precise* length a, the characters of scheme S are classes C_a for all positive real numbers[3] a. If we now measure a particular mark with the precision p and the measurement gives us the length l, that will mean that the actual length of the mark is expressed by a number from the range $(l-p,\ l+p)$, that is, this mark represents one of the infinite number of symbols C_a, where a is a number from that range. In that case, a measurement could only interpret a particular mark as being representative of exactly one character if it were absolutely precise, that is, if the measurement error were equal to 0, and that is impossible. In order to prevent such a situation, Goodman formulates another condition that all notations should meet:

> The second requirement upon a notational scheme, then, is that the characters be *finitely differentiated*, or *articulate*. It runs: *For every two characters K and K' and every mark m that does not actually belong to both,*[4] *determination either that m does not belong to K or that m does not belong to K' is theoretically possible.* "Theoretically possible" may be interpreted in any reasonable way; whatever the choice, all logically and mathematically grounded impossibility […] will of course be excluded. (Goodman, 1968, pp. 135–136)

In the next paragraph, Goodman defines syntactic density and immediately comments on the mutual relationship between those two notions:

> A scheme is syntactically dense if it provides for infinitely many characters so ordered that between each two there is a third. In such a scheme, our second requirement is violated everywhere: no mark can be determined to belong to one rather than to many other characters. (Goodman, 1968, p. 136)

A crucial role in the above definition is held by the notion of the order. Although Goodman presented extensive analysis of quality orders in his earlier book *The Structure of Appearance* (Goodman, 1977, pp. 193–257), first published in 1951,

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[3] If we want all straight marks of any length to be inscriptions in our scheme S, we must, in the above definition, refer to real numbers, which, intuitively speaking, are all the numbers describing the length of all segments of the straight line or – as in our example – of all straight marks of any length. The rational numbers that Goodman speaks about are all fractions formed of two integers. They constitute a proper subset of real numbers: for example, the square root of 2 which represents the length of a diagonal of a square with a side length equal to 1 is an irrational real number.

[4] It would seem that the sole purpose of the clause "*that does not actually belong to both*" is to make the definition applicable also to systems that do not fulfil the condition of disjointness of characters understood as classes of inscriptions. In systems that do meet that condition, no mark belongs to (represents) two different characters at the same time, and so that clause is redundant.

At the same time, it should be noted that although in formal terms the disjointness and the differentiation of the symbol scheme may be defined as properties independent of one another, the condition of finite differentiation was conceived as that which enables us to actually recognize disjointness and put it to good use. And so the premise of the disjointness of signs when discussing finite differentiation is perfectly natural.

he does not refer to them here. That would appear to be due to the fact that, in Goodman's own words, that analysis was

> […] designed to apply to *finite* sets of elements. […] it explains why some of the most familiar theorems concerning order in a continuum, such as that there is an element between each two distinct elements, will not hold here. (1977, p. 215)

In that analysis, a crucial role was played by the notion of "besideness", and "two elements are beside each other just in case there is no element between them" (1977, p. 216). Thus this theory concerns finite and non-dense schemes, so it does not apply to dense sets which must needs be infinite, as automatically ensues from the definition of density. Consequently, there is no reason to discuss Goodman's analysis from *The Structure of Appearance* in the present paper, especially since Goodman himself did not do so in *Languages of Art*.

The same applies to the article "Order from Indifference" (Goodman, 1972) which constitutes a more accessible version of the theory of quality orders presented in *The Structure of Appearance*. In Goodman's words, that article makes

> […] the outlines of the matching-calculus more readily available and understandable. In this paper […] I have tried to free the exposition from the peculiar philosophical features and the technical terminology of the more comprehensive system outlined in *SA*. (1972, p. 421)

In this article, the notion of besideness again plays an important role, and Goodman reiterates: "in sensory scaling we are concerned only with finite arrays" (1972, p. 432), which again precludes the possibility of applying its results to the analysis of density.

To end our survey of Goodman's views connected to our subject, we should say that besides the two above-mentioned syntactic conditions that every notational system should meet, Goodman formulates three semantic conditions. Since he speaks of meanings in terms of compliance classes of symbols which are formally analogous to inscription classes on the syntactic level, two semantic conditions – the disjointness of compliance classes and semantic finite differentiation[5] – are defined in entirely analogous terms to those used to define the corresponding syntactic conditions. The same applies to the definition of semantic density. In purely formal terms, therefore, there is no difference between the pair of syntactic notions, namely finite differentiation and density and their mutual correlations, and the pair of semantic notions. In light of this, the whole critique of syntactic notions that will be presented here could be repeated in an identical way in relation to the pair of semantic notions. For the sake of transparency, however, we will confine ourselves to speaking solely of the former. Hereafter, "density" and "finite differentiation" will signify syntactic notions alone.

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[5] The third condition – or actually the first in logical order – is the non-ambiguity of symbols; that is, the possession by each of them of just a single compliance class.

3 Previous critiques of Goodman's notions

Goodman applies the notions of density and finite differentiation to deal with the problems presented in Sect. 1. His proposals have been frequently criticized. Given the topic of the present article, I will mention only those critiques in which the notion of density is invoked. In 1987 Douglas Arrell wrote:

> Nelson Goodman's theory of pictorial representation is the best known and most widely rejected feature of his aesthetics. […] A survey of some forty of the articles and reviews which appeared in the wake of *Languages of Art* reveals that in about three-quarters of them this theory was a major topic of concern, and that overwhelmingly, the concern was to refute it […]. (Arrell, 1987, p. 41)

Those early critiques of Goodman's theory of pictorial representation include Bach (1970), O'Neill (1971) and Harris (1973). Arrell's publication was followed by further critiques, including Peacocke (1987), Kulvicki (2003, 2006), and Blumson (2011). Most often indicated as a counter-example to Goodman's conception were digital pictures, as broadly understood (e.g. Harris speaks of "pictures which have been produced on a typewriter", pp. 326–327), which are limited to a certain finite scheme of symbols and consequently cannot be dense in Goodman's sense, contrary to his thesis that "A scheme (system) is representational only insofar as it is dense; and a symbol is a representation only if it belongs to a scheme (system) dense throughout" (1968, p. 226). (Cf. also Bach, 1970 (p. 126), Peacocke, 1987 (p. 406, note 32), Kulvicki, 2003 (p. 329), Kulvicki, 2006 (p. 26), Blumson, 2011 (pp. 5–6); only Arrell's and O'Neill's critiques invoke different arguments.)

Since pictorial representation is not the focus of this article, we do not have to decide whether such a counter-example (of digital pictures) effectively challenges Goodman's position. It would not have to, as long as Goodman's intention were "some redrawing of the vague and vagrant borderlines" (Goodman, 1978, pp. 68–69[6]) of "pictorial representation", and not a faithful analysis of its meaning as it is typically used. As argued by Cohnitz & Rossberg in the book *Nelson Goodman* (2014), in fact, the goal of Goodman's philosophy is

> […] to overcome the ambiguity and vagueness of natural language […] with the help of modern logic and by way of *explications*. An explication is a type of definition that is a hybrid of nominal definitions[7] and meaning analyses. Since philosophy tries to clarify our way of speaking, it has to depart from the terms and expressions we already use in the language. These terms are assumed to be imprecise and that is why they lead to philosophical problems.

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[6] In this article, this wording of Goodman refers to "the aesthetic".

[7] Among the various uses of this term, the authors have in mind here a stipulative definition, establishing the meaning of a new term.

> If we want to clarify these terms, we have to substitute them with terms that are in the relevant respect clearer than the ones we have started with. (p. 61)
> When Goodman speaks of definitions as the method by which philosophy arrives at *rational reconstructions* of 'induction' or 'pictorial representation' and so on, he has explications in mind. (p. 62)

In this situation, it is clear that not every counter-example can be treated as undermining the proposed explication, since two related, but not necessarily identical meanings of a certain term come into play: the existing meaning present in actual linguistic usage, which at times is vague and ambiguous, and its proposed regulation, intended as clear and less ambiguous. That which is a counter-example when we are thinking about one of the meanings may not be so for the other meaning.[8] Arguing by means of counter-examples is obviously more problematic still – or even downright impossible – in a situation where some notion is elucidated by means of symptoms, which – as we saw earlier – Goodman does in relation to the notion of the aesthetic.

No such difficulties arise, however, in relation to the notion of density, which is the object of analysis in the present article. That is because the definition of this notion is a simple terminological stipulation. In relation to painting – or to symbol systems in general – the term "density" does not have any usage history behind it with which Goodman's definition might enter into conflict. Goodman is simply introducing a new term and a new notion in this area. Alternatively, we might say that Goodman is borrowing this definition from the field of mathematics, specifically from order theory, and attempting to apply it to his analysis of symbol systems. Yet mathematical definitions are also terminological stipulations and are characterized by a lack of any additional, non-explicit content. So any potential difficulties that could arise in the context of the two methods described above (explication in the sense presented above or the enumeration of symptoms) do not threaten analyses of the notion of density and its application to the visual arts and to symbol systems in general. In our analysis, therefore, it will be possible to expect unambiguous answers to the question as to whether the definition of density is met in particular cases and whether it can be applied in keeping with Goodman's intentions.

To return to the overview of earlier critiques of Goodman's conceptions, John Zeimbekis (2012, 2015) questioned the validity of autographic/allographic distinction as envisaged by Goodman. Another topic quite often discussed is the analog–digital distinction. Alternative accounts explicitly disagreeing with Goodman were proposed for example by Lewis (1971) and Haugeland (1981), and more recently by Maley (2011), Frigerio et al. (2013) and Katz (2016).

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[8] So Goodman might say, for example, "digital pictures are only a counter-example to the traditional way of understanding images. However, I believe that such digital pictures are in fact inconsistent with the essence of pictorial representation and therefore I propose redefining 'pictorial representation' using the concept of density." In this particular case, however, Goodman says nothing of the sort. Rather, he seems to accept the validity of the counter-example invoking digital pictures, as in his later work "Representation Re-presented" (Goodman 1988, pp. 126–128) he tries to somehow reformulate his theory in order to accommodate this counter-example.

While Goodman's theses are questioned in various ways in all the articles referred to in this section of the present text, the notion of density itself used to formulate them is almost always understood as easily and naturally applicable in all the contexts that are discussed and thus is treated as a neutral and unproblematic tool. O'Neill (1971) and Haugeland (1981) are the only exceptions, so their reservations toward this notion should be briefly set out and commented on. O'Neill's main objection is that it is not clear

> […] what counts as a separate symbol in a painting or a sculpture? One is inclined to answer, any part of its visible surface which can be identified as representing an identifiable represented object. In that case for any one observer of the work there will only be a finite number of symbols. (1971, pp. 370–371)

This precludes density. And O'Neill excludes the possibility "that we have any use for talking about an infinite number of symbols, say every set of adjoining points on the surface" (p. 371). Naturally, we can – following O'Neill – understand a painting as a complex symbol and ask what are the atomic (or at least more basic) symbols within it. But according to Goodman's definition of density (see above), it is the entire symbol scheme that needs to be infinite. And it is clear that in keeping with Goodman's intention such a scheme includes multiple pictures, as for example the quotation in Sect. 5 below indicates. So "density" can be applied to pictures in other ways besides O'Neill's unsuccessful attempt. Haugeland's objections are even more scant:

> Goodman says a scheme is analog if dense – that is, if between any two types there is a third (see pp. 160 and 136). The main difficulty is that "between" is not well-defined for all cases that seem clearly analog. What, for instance, is "between" a photograph of Carter and one of Reagan? (1981, p. 221)

Of course, one cannot refute Goodman's notion solely on the basis of such a rhetorical question for which there is no apparent clear answer. As becomes clear in Sect. 6, however, this question aptly recognizes one of the problems with the notion of density.

Those two reservations concerning the notion of density had no effect on the widespread trust and confidence with which this notion was used by later authors (and applied to various symbol systems) almost up to the present day (see examples given above). This is also true for the entry "Goodman's Aesthetics" in the *Stanford Encyclopedia of Philosophy*, updated in 2017. Thus a critical appraisal of this notion and its applicability seems desirable.

4 Are all symbol systems dense?

It is clear that the meaning of "density" depends on how we understand ordering in a set of symbols. In traditional schemes, such as numbers or letters of the alphabet, we deal with conventional order relations, which could be used as points of reference. Yet in the case of painting or musical notation, it is by no means clear how the relevant order relation should be understood. And Goodman considers many more abstract – at times artificially

constructed – symbol schemes, in which no pre-imposed, traditional order relations exist. In the first edition of his book, however, he gives no indications as to how the ordering should be understood. We may assume, therefore, that the definition of density refers to any symbol scheme with any order relation established within it. Some remarks (e.g. Goodman, 1968, p. 137, n. 6: "a given set may be dense under one ordering and discontinuous throughout [this is Goodman's term for a set that is not dense itself and does not contain any dense subset] under another") appear to support such an interpretation. If, however – in accordance with Goodman's intentions – density is to entail a lack of finite differentiation, such an interpretation cannot be accepted. This results from two observations.

Firstly, as Goodman himself notes in the passage quoted above, the same set may be dense under one ordering and discontinuous throughout under another. We might even say that in *every* set one may introduce an ordering that is discontinuous throughout, that is, one that is not dense itself and contains no dense subset, and so is the complete opposite of density.[9] With regard to the opposite possibility, of a dense ordering, it obviously does not exist for finite sets (since the very condition of density implies the infinity of the set), but it does occur for all the infinite sets that might be of interest to us.[10] And where we are dealing only with a finite quantity of simple symbols, it is always possible to construct an infinite number of complex symbols, as long as the scheme does not restrict the formation of compounds: by means of just one basic symbol, we can construct symbols composed of two, three, four, etc. repetitions of it. To sum up, practically every symbol scheme can be ordered both in an entirely discontinuous way and in a dense way.

Secondly, a change to the ordering of a given symbol scheme does not affect its differentiation, since the definition of finite differentiation contains no reference to an order relation, but only to inscriptions and their classes, that is, symbols. A scheme that is – or is not – finitely differentiated remains the same regardless of how we order its characters.

In particular, every finitely differentiated scheme containing an infinite number of symbols – for instance, the above-mentioned scheme consisting of repetitions of one basic symbol (for example, letters "a" or ones) – can be ordered in a dense way, whereby we obtain an example that contradicts Goodman's thesis that density implies a lack of finite differentiation. Goodman appears not to have realized that even his own examples can be used to undermine that implication. On page 136 (Goodman, 1968), he mentions a symbol scheme comprising ordinary Arabic fractional numerals as an example of a finitely differentiated scheme containing an infinite number of characters. At

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[9] This results directly from Zermelo's famous theorem that every set can be well ordered, that is, linearly ordered in such a way that each of its subsets contains a least (first) element. Since every subset of a well-ordered set is obviously well-ordered itself and every well-ordered set is non-dense, a well-ordered set is discontinuous throughout, in Goodman's sense.

[10] This results, in turn, from the possibility of transplanting a dense ordering from a set of rational or real numbers by means of a one-to-one mapping.

the same time, this scheme is obviously dense in relation to the natural ordering of the numbers. We should not be misled by Goodman's remark on this subject, contained in footnote 5 on the same page: "I am speaking here of symbols only, not of numbers or anything else the symbols may stand for. The Arabic fractional numerals are finitely differentiated even though fractional quantities are not." (Goodman, 1968, p. 136, n. 5). Fractional quantities are only necessary for introducing an order relation in the scheme (that is, among the fractional numerals themselves), after which they may be rejected as their possible meanings. And the thus-established order relation in a symbol scheme (that is, in a set comprising fractional numerals alone) considered *in abstracto*, without any reference to their possible meanings, remains dense, but the scheme, despite this, is finitely differentiated.

Therefore, if we do not impose any further conditions on the ordering of a symbol scheme, we have to state that there is no correlation whatsoever between finite differentiation and density. This conclusion ensues not solely from the above example of fractional numerals, but from the foregoing considerations of the possibility of applying any ordering to a symbol scheme without affecting its differentiation. That would mean, for example, that the syntactically finitely differentiated notational systems of music and language could be ordered in a dense way, and a dense and non-differentiated – according to Goodman – painting scheme in a way that is non-dense and even discontinuous throughout, which would obviously undermine the sense of all Goodman's basic differentiations and essentially his entire theory.

5 The first specification of an order relation

Goodman realized this, and as one of the changes to the second edition of his book, from 1976, he introduced an addendum specifying what ordering we consider to be natural for a symbol scheme:

> […] where density is said to imply lack of differentiation, the ordering in question is understood to be such that any element lying between two others is less discriminable from each of them than they are from each other. (Goodman, 1976, p. 136)

Indeed, imposing such a condition on an order relation seems to be the only sensible (and necessary) way of linking it to the differentiation of characters, if Goodman's assertion that in a dense scheme finite differentiation is violated everywhere is to be true: "no mark can be determined to belong to one rather than to many other characters." (Goodman, 1968, p. 136). The intuitions that inform the formulation of the condition expressed in the above quotation can be presented as follows: the more indiscriminable two inscriptions are, the more difficult it is to distinguish them and assign them to different characters. So if a greater density is to signify a more problematic discrimination of the characters, then the character

b that lies between two others *a* and *c*[11] should be harder to distinguish from each of them than they are from each other; we will designate this relation with the symbol A(*a, b, c*).[12]

Apart from the remark quoted above, which is only given in the second edition, Goodman gives no further indications as to how we should understand the ordering in systems which he considers to be dense, in particular in painting and other fine arts. All the fundamental conclusions of Goodman's theory – the distinction of depiction from description and of autographic from allographic art, and the specification of the symptoms of the aesthetic – are based primarily on his assurances that the symbol scheme of painting[13] is dense. The longest passage that Goodman devotes to this question is this:

> Consider, for example, some pictures in traditional Western system of representation: the first is of a man standing erect at a given distance; the second, to the same scale, is of a shorter man at the same distance. The second image will be shorter than the first. A third image in this series may be of intermediate height; a fourth, intermediate between the third and second; and so on. (Goodman, 1968, p. 226)

As we can see, Goodman merely repeats – this time in a different, less convincing, form – what is essentially his only example of a dense system comprising straight marks of various length. As we will soon see, however, this is by no means sufficient for introducing an order relation in the symbol scheme of painting and actually showing its density.

6 An attempt at defining an order relation in a symbol scheme

So let us try to see what consequences ensue from Goodman's condition linking ordering with character differentiation and how it may be used to define an order relation in schemes with which no order relation is conventionally linked (for example, in the symbol schemes of painting or musical notation), in order to determine

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[11] Hereafter, different letters, *a, b, c, d*, will always denote *different* characters.

[12] Admittedly, such a formulation contains a certain difficulty: as we recall, for Goodman characters are classes of equivalent inscriptions. Therefore, the relation A(*a, b, c*) would have to signify that the inscription class *b* was more difficult to distinguish from classes *a* and *c* than they were from one another. Yet what we actually wish to speak of are the differences between "real, material symbols", that is, inscriptions of characters.

However, since the formulation of such a condition – suggested by Goodman in the passage quoted above – would appear to be necessary in order to uphold assertions of the dependence between density and finite differentiation, we must assume that this difficulty may be somehow overcome, for example by means of a clarification of the following kind: character *b* is more difficult to distinguish from *a* and *c* than they are from one another when such a relation occurs for each set of three inscriptions *a', b', c'* of the characters *a, b, c* respectively.

[13] It is probably clear that this designation does not signify any specific symbol system occurring, for example, in the religious art of a certain culture or in the art of a given era or of a particular artist, but solely paintings or fragments thereof understood as sensory units of perception, which may carry certain meanings and in that sense constitute a symbol scheme. More specifically, every painting as a whole may be treated as a symbol, in most cases as a complex symbol.

– in accordance with Goodman's suggestions – whether they are dense or not. The condition that should be met by a natural ordering of a symbol scheme ("any element lying between two others is less discriminable from each of them than they are from each other" Goodman, 1976, p. 136) can be notated in the following way:

$$a < b < c \Rightarrow A(a,b,c)$$

(The symbol "$a < b$" should not, of course, be read as "b is greater than a", but as "b follows a" or "b is later in order than a".) Since the very definition of the relation A(*a, b, c*) implies the equivalence

$$A(a,b,c) \iff A(c,b,a),$$

combined with the above condition, we obtain also

$$c < b < a \Rightarrow A(a,b,c)$$

and so together:

(1) $$\qquad a < b < c \text{ or } c < b < a \Rightarrow A(a,b,c)$$

It is not clear whether for Goodman the natural order relation should fulfil the reverse implication as well – but that cannot be excluded. In such a situation, we would be dealing with the equivalence

(2) $$\qquad a < b < c \text{ or } c < b < a \iff A(a,b,c)$$

The above conditions (1) and (2) can be used in attempts to define the order relation in any symbol scheme. Since every relation of this kind must meet at least condition (1), it follows that when introducing an ordering we have to observe the following rule:

(I) $a < b < c$ or $c < b < a$ can only be defined when A(*a, b, c*)

If we were ever to define $a < b < c$ in a situation where A(*a, b, c*) did not hold, the order relation would not fulfil condition (1).

If we were to assume, however, that such a relation should fulfil equivalence (2), that would oblige us to proceed according to a stronger rule:

(II)   Always when A(*a, b, c*), and only then, define $a < b < c$ or $c < b < a$

One may doubt whether such rules might suffice to introduce an order relation in a whole – possibly infinite – symbol scheme. After all, we should be able to define the relation between every two characters *a* and *b*, that is, determine whether $a < b$, $b < a$ or neither. And the above rules only give any hints when we are dealing with three characters *a, b, c*. But there is no need to doubt the usefulness of rules (I) and (II) on account of this – possibly surmountable – difficulty, since it can be demonstrated quite easily that they cannot be used in general (that is, in any symbol

scheme) to coherently introduce an order relation that could be used to carry out the distinctions suggested by Goodman.

Since rule (I) has no recommending force, but solely an enabling character (in particular, not introducing on the basis of this rule any relations or specifying $a < b$ solely for two randomly selected a and b will be in keeping with it), we will begin an analysis with the stronger rule (II).

First let us recall that every ordering has at least two natural properties: transitivity (if b follows a and c follows b, then c follows a; in formal notation $a < b$, $b < c$ entails $a < c$) and antisymmetry (if b follows a, then a does not follow b; in formal notation, if $a < b$ then not $b < a$). Let us now consider the four characters $\underline{\text{O}}$, |O, |Δ and $\underline{\Delta}$, and let us call them α, β, γ and δ respectively. For what follows, it is not of essence whether or not the symbol scheme contains other characters as well – we will concentrate solely on attempting to introduce a relation between these four. In order to obtain a certain similarity to the examples put forward by Goodman and at the same time a certain affinity with the field in which we intend to employ the notion of density, that is, with painting, we may also consider the following example: as characters, we take rectangles, each covered with a uniform color (let's say a particular shade of grey) – thus they are objects that might possibly be regarded as one of the forms of twentieth-century painting. We define the symbols α, β, γ and δ as follows: α and β are square pictures measuring 100 cm × 100 cm, γ and δ are rectangles 100 cm × 105 cm; α and δ have exactly the same color, let's say the shade of grey G; β and γ also have an identical color, the slightly lighter shade of grey G'. The two interpretations of the symbols α, β, γ and δ may be represented schematically as follows:
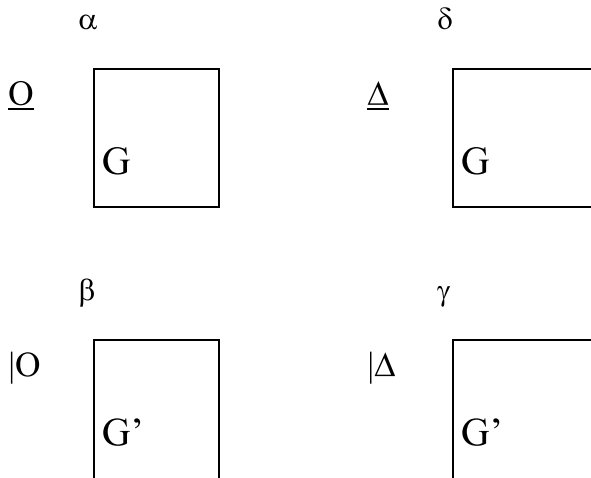
The following argumentation is entirely independent of the choice of one or the other interpretation of the symbols α, β, γ and δ - it looks identical for both.

Since in both interpretations A(α, β, γ), on the basis of rule (II) we have to define either $\alpha < \beta < \gamma$ or $\gamma < \beta < \alpha$. The further reasoning is analogous for both possibilities, so let us take one of them, say

(i) $\alpha < \beta < \gamma$

For γ, δ, α, A(γ, δ, α) holds true, and so we obtain $\gamma < \delta < \alpha$ or $\alpha < \delta < \gamma$. Due to transitivity, the former possibility implies $\gamma < \alpha$, which on account of antisymmetry stands in contradiction to $\alpha < \gamma$ resulting from (i). Thus we are left with

(ii) $\alpha < \delta < \gamma$

For β, γ, δ, in turn, we obtain, on the basis of our rule, either $\beta < \gamma < \delta$ or $\delta < \gamma < \beta$. The former possibility is contradictory with (ii) ($\delta < \gamma$ and $\gamma < \delta$); the latter with (i) ($\beta < \gamma$ and $\gamma < \beta$).

Thus we arrive at the conclusion that even in a remarkably simple four-character scheme the introduction of an order relation on the basis of rule (II) is not possible. Of course, in some schemes – like, for example, the above-described scheme *S*, consisting of straight marks of various length – it may prove successful. But since it is not possible in general – that is, in all possible symbol schemes – rule (II) does not give us a general way of determining whether a given symbol scheme is dense or not. In our modest four-character scheme, this rule proves too strong: it forces us to introduce so many different relations that it inevitably leads to contradiction. Therefore, it must be sufficiently weakened as to not oblige us to introduce contradictory relations. The weakest alternative is rule (I), if we agree that the order relation we introduce must meet condition (1). Yet in order to impart to rule (I), above and beyond that which is merely admissible, a certain recommending force, it may be slightly strengthened (though not to the level of rule (II)):

(III)   If $a < c$ and A(a, b, c), then define $a < b < c$

On the basis of rule (I) combined with (III) (and of course also on the basis of (I) alone), we can coherently introduce an order relation into the above-described four-character scheme. First, on the basis of (I), we may define $\alpha < \beta < \gamma$. This results in $\alpha < \gamma$. Since A(α, δ, γ), we are obliged, on the basis of (III), to define $\alpha < \delta < \gamma$. In this way, however, the matter is finished. Although A(β, γ, δ) and A(β, α, δ) do hold, since neither $\beta < \delta$ nor $\delta < \beta$, we do not have to introduce any further relations. All that remains, therefore, is $\alpha < \beta < \gamma$ and $\alpha < \delta < \gamma$, as well as, of course, the resultant $\alpha < \gamma$, which does not lead to any contradictions. The relation introduced into our scheme in this way is transitive and antisymmetric; that is, it is an order relation. Despite this, it seems somewhat strange. What it lacks is linearity, that is, the property that for each pair of different elements x and y either $x < y$ or $y < x$ – which is often associated with an order relation. In our case, meanwhile, β and δ do not stand in any relation to one another. The linear order relation has another property besides, resulting from its definition:

every finite set of elements x_1, x_2 ... x_n can be ordered in precisely one way; to wit, $x_{k1} < x_{k2} < ... < x_{kn}$. This explains the name "linearity".

It turns out that linearity, combined with the weaker condition (1) that must be met by the desired order relation, already gives equivalence (2). In order to show this, we must demonstrate that A(a, b, c) entails $a < b < c$ or $c < b < a$. On account of linearity, for any three elements a, b, c precisely one of six possible orderings occurs: $a < b < c$, $c < b < a$, $a < c < b$, $b < c < a$, $b < a < c$ or $c < a < b$. The third and fourth of these imply – on the strength of condition (1) – A(a, c, b), which is contradictory to the assumed premise A(a, b, c), since condition A(a, c, b) means that the difference between characters a and c is smaller than between characters a and b, whereas condition A(a, b, c) implies the reverse – that the second difference is smaller than the first. The fifth and sixth of the possible orderings imply A(b, a, c), which is also contradictory to A(a, b, c). We are left, therefore, with the orderings $a < b < c$ or $c < b < a$, which was to be demonstrated. Therefore, the premise of the linearity of the order relation means that it must fulfil the stronger condition (2), and so when defining such a relation we must proceed in accordance with rule (II), which, as we have shown, leads to contradiction. It ensues from this that in order to introduce an order relation in our four-character symbol scheme without contradiction we must abandon linearity.

So perhaps the secret to the success of our attempts to define a natural order relation in any given symbol scheme lies in relinquishing linearity. And indeed, if we do not demand that each two characters a and b must always stand in the relation $a < b$ or $b < a$ to one another, it is possible to arrive at a sensible definition of the order relation not just in an extremely simple four-character scheme, but also in much richer, infinite schemes – for example, in the above-described scheme in which the inscriptions are all rectangles of different sizes, each uniformly painted some shade of grey, and in which – as in system S – the inscriptions of the same character are only rectangles of precisely the same dimensions and precisely the same shade of grey. We no longer have to restrict ourselves to the four characters of that scheme – we can introduce an ordering in a whole scheme consisting of an infinite number of characters.

To this end, let us describe each of them unequivocally by means of three parameters – length, height and color – and present them as (L, H, C). L and H are positive real numbers, and since the colors that we can place under C are shades of grey, they may be linearly ordered, like numbers, from the darkest, black, to the lightest, white.[14] Using the sign "$\leq$" in the standard way to denote "$<$ or $=$" (for

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[14] Thus such a spectrum of colors from black to white could be identified, for example, with the range of all real numbers from 0 to 1, with zero corresponding to black, one to white and successive real numbers between 0 and 1 to all the intermediate shades of grey. With such an identification, the triples (L, H, C) that describe unambiguously all the elements of our scheme can be treated as triples of numbers, where $L > 0$, $H > 0$, $1 \geq C \geq 0$. The set of all such triples is nothing other than the Cartesian product $R_+ \times R_+ \times [0,1]$. So whilst the model for the dense scheme S is the set R_+, the model for our scheme of rectangular pictures may be the set $R_+ \times R_+ \times [0,1]$. Therefore, an attempt to define the order relation in our scheme of rectangular "pictures" corresponds to introducing an order relation in the set $R_+ \times R_+ \times [0,1]$.

both numbers and colors), we define for two different characters $S_1 = (L_1, H_1, C_1)$ and $S_2 = (L_2, H_2, C_2)$:

$$S_1 < S_2 \iff L_1 \leq L_2 \text{ and } H_1 \leq H_2 \text{ and } C_1 \leq C_2$$

The assumed difference of characters S_1 and S_2 guarantees that not all three relations $L_1 \leq L_2$, $H_1 \leq H_2$, $C_1 \leq C_2$ are equalities, and so the right side of the above equivalence is unequivocally defined. When the condition expressed there is not fulfilled, for example when $L_1 < L_2$ and at the same time $H_1 > H_2$, no relation holds between characters S_1 and S_2. It is not difficult to verify that the relation defined in this way is transitive and antisymmetric, and so it is a (non-linear) order relation. At the same time, it meets condition (1), namely $S_1 < S_2 < S_3$ means that $L_1 \leq L_2 \leq L_3$, $H_1 \leq H_2 \leq H_3$ and $C_1 \leq C_2 \leq C_3$, and so S_2 is less discriminable from S_1 and S_3 than they are from each other (S_2 is a rectangle larger or equal to – in both dimensions – S_1, smaller or equal to S_3, and its color is also an intermediate shade between the shades of symbols S_1 and S_3.) This scheme is also not, in the Goodmanian sense, finitely differentiated, and it appears to be dense (we will discuss that below); even the lack of finite differentiation seems to have something in common with its density, as in scheme S.

Our next example comes closer still to the fine arts. We will consider all rectangular drawings to be inscriptions; that is to say, pictures drawn in pencil, pen, charcoal, and so on, in relation to which we may assume that they are strictly black and white (without any shades of grey) and which are of the same dimensions, for instance 40 cm × 60 cm. (This last restriction is not essential; we could also allow rectangular drawings of different sizes and combine an ordering in terms of size with the ordering presented below relating to drawings of the same size in the same way as we did in the previous example.) So at this point, we consider perfectly normal pictures, that is, products that – in contrast to the previous example – we can ascribe without hesitation to the fine arts: most drawings regarded as works of art fit into this category. However, we must make the idealizing assumption that the whole surface of the picture falls into two complementary and disjoint parts: white and black. The closest to this are probably drawings made in pen or sharp pencil, or some kinds of engravings, since it is in these that one can indicate most unequivocally the places where the drawing implement (or the burin) has touched the white base, and the intensity of the color in those places is relatively even. In such drawings, shades of grey are rendered by various densities of uniformly black lines, such that on closer inspection the brighter and darker areas prove to be thicker or thinner tangles of black lines on a white background. Those lines should not be understood as "not possessing width". On the contrary: if we are to speak at all about a blackened part of a picture's surface, it is obvious that lines must be accorded a certain width, not necessarily the same throughout the whole drawing. Thus it is possible to blacken the whole surface of a drawing by means of a finite number of lines.

Given such premises, we can identify every drawing unequivocally with the black surface SF, which is part of the whole surface of the picture. Since all the pictures are of the same size and rectangular shape, we can superimpose any one

picture onto another in order to compare their blackened surfaces SF_1 and SF_2. They may share a part, one may be contained in the other, or they may be disjoint. Now we can define the order relation as follows:

> For two different drawings D_1 and D_2, we define $D_1 < D_2$ exactly when blackened surface SF_1 is contained in blackened surface SF_2 ($SF_1 \subset SF_2$, i.e. SF_1 is an actual subset of SF_2, that is, it is not equal to SF_2).

It is easy to verify that this relation is transitive and antisymmetric. At the same time, it again seems that the scheme is dense in Goodman's sense and consequently lacking in finite differentiation. Condition (1) also seems to be more or less fulfilled. However, doubts do arise in relation to some examples. For instance, let us consider the following drawings: the first is half black and half white (let's assume that the bottom rectangular half is black and the top half is white); the second is like the first, but with black "teeth" (or any other pattern or ornament no higher than 1 cm) protruding from the black bottom half; the third picture is again like the first, but with the black surface (again rectangular) extending one centimeter higher, with the result that the picture again falls into two rectangles divided by a horizontal line, the bottom one of which, the black rectangle, is slightly larger. This may be presented in diagram form as follows (B = black; W = white):
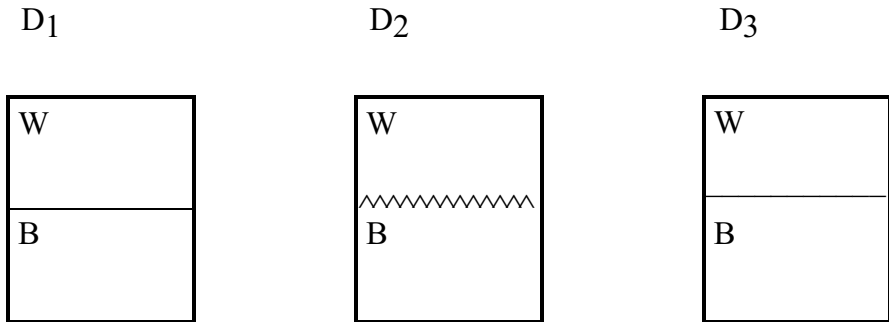

In this case, $SF_1 \subset SF_2 \subset SF_3$, and so, according to our definition, $D_1 < D_2 < D_3$, but it is not certain whether we would be inclined to say that A (D_1, D_2, D_3), that is, that D_2 is less discriminable from D_1 and D_3 than they are from each other. It is not clear, therefore, whether we can consider that such an ordering fulfils Goodman's condition linking the order relation with the differentiation of the characters.

7 Conclusion

Yet regardless of this last difficulty, what standpoint should we adopt towards these seemingly promising examples? Unfortunately, it has to be said that neither of the two order relations presented above – contrary to first impressions – is dense in the

sense of Goodman's definition. According to that definition, a scheme is dense "if it provides for infinitely many characters so ordered that between each two there is a third". (Goodman, 1968, p. 136) Well, in the first example, there is no character between $S_1 = (50,100,C)$ and $S_2 = (100,50,C)$, where C is any fixed color. That is because if for some S_3 it was $S_1 < S_3 < S_2$ or $S_2 < S_3 < S_1$, that would mean, due to transitivity, $S_1 < S_2$ or $S_2 < S_1$, which is contradictory to our definition of the order relation, since the characters S_1 and S_2 do not stand in any relation to one another. Similarly, there can be no character in the second symbol scheme between the pictures

W	
B	

W B
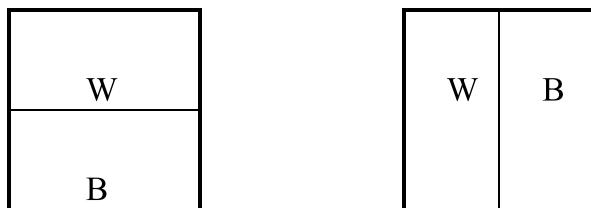
since that would again mean that $S_1 < S_2$ or $S_2 < S_1$, again in contradiction to the fact that, according to the definition of the order, S_1 and S_2 do not stand in any relation to one another. In fact, Goodman's definition of density implies linearity: if between every a and b there is a third, c, that means that $a < c < b$ or $b < c < a$, and so by consequence $a < b$ or $b < a$. Meanwhile, the order relations in the last two schemes are non-linear.[15]

The requirement of linearity that results from density thwarts all hopes of introducing a natural – that is, in accordance with the differentiation of the symbols – ordering in more complex symbol schemes. So if, in accordance with Goodman's definition, every dense symbol scheme is linear, then those symbol schemes which cannot be naturally ordered in a linear way (and painting is certainly one of them) cannot be dense. Goodman would certainly be most disappointed at such a conclusion.

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[15] For the sake of accuracy, one might add that at least in the former scheme, one can easily define a linear order. One example would be the lexicographic order that we use when arranging words alphabetically: the first determinant is the first letter; where that letter is the same for two words, the order is decided by the second letter, and so on. In our case, the order would be decided in the first instance by width. If the width of two pictures was equal, we would compare the height, and if that was also equal, then the shade of grey would be conclusive. The problem is that such an order has nothing to do with the differentiation and discriminability of pictures. By way of example, pictures described by three elements (1, 1, G), (1, 10, G) and (1.01, 1, G), where G denotes a certain constant shade of grey, would be arranged in precisely that order. But the middle picture – of height 10 – would be far more easily discriminable from the other two than they would be from one another. We might even give a completely different color (shade of grey) to that middle picture, making it even more distinctly different from the other two, but according to the definition of the order it would still lie between them.

To conclude, it turns out that Goodman's definition is highly problematic and cannot be applied to symbol systems in the way Goodman envisaged. Consequently, Goodman's theory is problematic not just because of its controversial theses but also because of logical problems with the technical notions used at its very core. His controversial claims are not simply contestable, but inaccurately expressed.

Declarations

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